School of Accounting Seminar Series
Semester 2, 2014

The Effect of Information on Uncertainty and the Cost of Capital

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Date: Friday September 26 2014
Time: 3.00pm – 4.00pm
Venue: ASB 220
The Effect of Information on Uncertainty and the Cost of Capital

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Revised Draft: 20 July, 2014

Abstract

It is widely held that better financial reporting makes investors more confident in their predictions of future cash flows and reduces their required risk premia. The logic is that more information leads necessarily to more certainty, and hence lower subjective estimates of firm "beta" or covariance with other firms. This is misleading on both counts. Bayesian logic shows that the best available information can often leave decision makers less certain about future events. And for those cases where information indeed brings great certainty, conventional mean-variance asset pricing models imply that more certain estimates of future cash payoffs can sometimes bring a higher cost of capital. This occurs when new or better information leads to sufficiently reduced expected firm payoffs. To properly understand the effect of signal quality on the cost of capital, it is essential to think of what that information says, rather than considering merely its "precision", or how strongly it says what it says.

Keywords: Information, uncertainty, cost of capital, CAPM, information risk

Acknowledgements: I have been greatly assisted by the editor (Steven Huddart) and two anonymous referees, and also by comments on earlier drafts by Sudipta Basu, Jeremy Bertomeu, Greg Clinch, Tom Dyckman, Bruce Grundy, Marlene Plumlee, Jim Ohlson, Robert Verrecchia, Alfred Wagenhofer, Bob Winkler and participants at the CAR Conference (Kingston 2013).
1 Introduction

This paper concerns the fundamental question of how information about the firm affects its cost of capital. It is widely held that "good" financial reporting resolves uncertainty and brings a lower cost of capital. The economic logic is that (i) more information leads to more certainty, and hence to lower estimates of firm "beta" or covariance with other firms, and (ii) the cost of capital falls when information brings a lower estimate of beta.

The intuition is straightforward. A firm’s cost of capital is the riskless interest rate plus a risk premium. Releasing more information and, in particular, more public information through financial reports and other public disclosures by firms reduces the uncertainty about the size and the timing of future cash flows and, therefore, also the risk premium. (Christensen et al. 2010, p.817)

This common understanding is misleading on both counts. First, the best available information can often leave Bayesian users less certain about future events. Second, where information does bring greater certainty, mean-variance asset pricing models imply that more certain estimates of expected firm payoffs can prompt an increase in the firm's cost of capital. This occurs when unfavorable information implies a sufficiently reduced expected (i.e. mean) future payoff.

The force of the payoff mean on the cost of capital is at first surprising. Conventional wisdom has it that the cost of capital is driven only by the firm’s returns covariance or "beta". However, a simple step of writing the CAPM in terms of cash payoffs rather than returns brings to light the latent underlying effects of both the firm’s expected payoff and its payoff covariance, each of which affects beta.

Given that the firm’s returns beta, and therefore its CAPM cost of capital, react to the estimated mean payoff, accounting information is relevant for its "direction" (favorable or unfavorable) as well as for its precision. Favorable information, which leads to a higher expected payoff, pushes the firm’s cost of capital downwards, all else equal. Conversely, if information leads to greater certainty that the firm’s future payoff will be both low and highly dependent on market-wide factors, then its assessed covariance will increase and its mean will fall, causing a two pronged increase in the cost of capital. In general, information can shift either or both the mean payoff or payoff covariance in either direction, and may therefore push its CAPM cost of capital up or down. Lambert et al. (2007) set out clear proof of this result, but highlight the role of the covariance.

The Bayesian effect of information on beliefs is generally greater for more precise information, however it is wrong to assume that more precise information necessarily reduces the cost of capital merely because it is more precise. Better information can expose grounds for pessimism about firms’ payoffs, or inter-dependencies between the payoffs of different firms, sufficient in either case to increase the market risk premium on average across all firms, or even for all firms individually.
This paper formalizes important general principles regarding information and the cost of capital. I show how restrictive assumptions and overly narrow focus on the payoff covariance rather than mean have led to conclusions that are appealing and consistent with conventional wisdom, but which do not hold in general. The more valid intuition is that, by being informative, accounting information can change users’ probability beliefs and lead to either a higher or a lower cost of capital. Just as new information has an unpredictable effect (up or down) on the share price, it also has a generally unpredictable effect on the CAPM equilibrium cost of capital.

2 The Model

My model is a generalization of Lambert et al. (2007), hereafter LLV, under which the firm’s ex ante (forward looking) cost of capital is determined in equilibrium by a mean-variance CAPM, and incremental information affects the cost of capital by altering investors’ Bayesian assessments of the mean and covariance parameters of the joint distribution of firms’ payoffs.

I treat payoffs as the primitive uncertain quantities, rather than returns. Payoffs are observable and exogenous. Returns are harder to describe and comprehend. They are defined as payoffs relative to equilibrium prices, and are therefore endogenous.

My method (again like LLV) is to price assets on the basis of a joint probability distribution of payoffs, under an assumed utility function, and then work backwards to what those asset prices imply about the cost of capital. The "cost of capital" is understood, therefore, as an ex ante expected return or discount rate. Specifically, if firm j has expected terminal cash payoff \( E[V_j] \), then its ex ante expected return implied by its initial market price \( P_j \) is \( E[r_j] = E[V_j]/P_j - 1 \). My equations show returns as factors \( R_j = (1 + r_j) = V_j/P_j \), rather than as increments \( r_j \).

The technical assumptions of my model are as follows:

(i) The market contains \( n \) risky firms \( (j = 1, 2, \ldots, n) \). Each firm \( j \) has uncertain payoff \( V_j \), realized at time \( t = 1 \).

(ii) The stochastic mechanism by which \( (V_1, \ldots, V_n) \) is generated is stationary (i.e. all firm investment choices are set).

(iii) Asset payoffs have a joint probability distribution \( f(V_1, V_2, \ldots | \cdot) \) as at \( t = 0 \).

(iv) Investors are Bayesian and have the same information, and the same beliefs, \( f(V_1, V_2, \ldots | \cdot) \).\(^1\)

\(^1\)Fama and Miller (1972, p.287) wrote: "It may be helpful to think of the homogeneous expectations assumption as a way of concentrating on the pure effects of ‘objective’ uncertainty on the pricing of investment assets."
(v) Investors are risk averse and have increasing quadratic utility.

(vi) The quality and arrival of information are fixed exogenously (i.e., financial reporting standards are mandatory rather than voluntary).

I assume quadratic utility for two main reasons. First, quadratic utility allows the joint payoff distribution $f(V_1, V_2, \ldots | \cdot)$ to take an arbitrary continuous or discrete form, which is not only realistic but also opens up the broad question of how information can affect beliefs $f(V_1, V_2, \ldots | \cdot)$, without the limitation of a specific parametric family of belief distributions or likelihood functions.

Second, as long as investor wealth remains within the domain for which utility is increasing, quadratic utility is consistent with risk-averse mean-variance preferences. Later, in a numerical example, I show how the effects of information on the cost of capital match up under two mean-variance setups, first under quadratic utility, and then under the LLV pairing of exponential utility and joint normality.

Stock market returns can be skew, fat-tailed, or depart from joint normality in other ways. The payoffs arising from a firm’s activities are likely not joint normal, even if they have normally distributed components. For example, if the payoff from a project or division has a normal distribution with parameters determined randomly, the probability distribution of that payoff is a mixture of normals, and mixture distributions of normals are not normal.

My analysis is *ex post* in the sense that it concerns information that has already been seen by the market. The assumption is that exogenous information arises at time $t = 0$, and is absorbed instantly into the market’s probability distribution $f(V_1, V_2, \ldots | \cdot)$ of time $t = 1$ payoffs. This brings updated expectations and new asset prices at time $t = 0$, which together imply new expected rates of return. These are best described following Gao (2010, p.9) as "conditional" returns, meaning that they are expectations conditioned on the signal that has just been observed.

It is important to distinguish between information already existing and an obligation by which the firm will disclose a certain quality of information at future times. Both my analysis and LLV examine how signals realized at $t = 0$ affect the market’s conditional "forward looking" expected return on capital, as at $t = 0$, conditional on the information observed at $t = 0$. Discussion on how the conditional and unconditional (pre-signal) perspectives relate to one another is added in Section 8.

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2 Quadratic utility is often set aside, mainly because it has a point of satiation where utility starts to decrease in money, and because it is increasingly risk averse. It should be noted however that Levy and Markowitz (1979), Markowitz (1991), Levy (2012) and others have supported portfolio choice based on the approximate validity of risk-averse quadratic utility.

3 Baron (1977, p.1692) and Liu (2004, p.233) discuss how probability mixtures of different assets or payoffs occur in business ventures. See also Winkler (1973, p.399).

4 Beyer et al. (2010, p.308) and Gao (2010, p.21) point out that Lambert et al. (2007) is concerned with conditional (post-signal) expected returns.
3 Quadratic Utility CAPM

In this section, I set out an original derivation of the mean-variance CAPM, as an extension of expected quadratic utility. This derivation requires no assumption about the shape or parametric family of the payoffs distribution of the firm or any other firm in the market. Importantly, it allows the endogenous market risk premium to be calculated (from the utility function) rather than merely assumed. This form of the CAPM is useful for its generality and for numerical illustration of the paper’s main findings. Numerical examples (see Section 7) help to explain how incremental information concerning one asset affects its cost of capital and also affects the endogenous market price-weighted-average cost of capital, $E[R_M]$.

The utility function for wealth $W$ is $U(W) = W - \frac{b}{2}W^2$ ($b > 0$). Marginal utility $U'(W) = 1 - bW$ is positive for $W < 1/b$ (this is the domain in which utility is increasing). The investor endowed with initial wealth $W$ maximizes expected utility, $E[W(R_j - R_f)(1 - bW R_{opt})] = 0$, by selecting asset weights $w_j = w_j^*$, subject to asset prices $P_j$, where $R_j = V_j/P_j$ is the return on asset $j$ and $R_f$ is the risk-free return ($\sum_j$ is shorthand for $\sum_{j=1}^n$ throughout). Note that $R_f$ is a known constant and asset returns are all expressed as factors.

Returns on risky assets $j = 1, 2, ..., n$ depend on their equilibrium prices $P_j$. The expected return on asset $j$ is by definition $E[V_j]/P_j$. Importantly, $P_j$ is a constant, set by the market, and the point of the CAPM is to find the theoretical value of $P_j$ (under a given utility function and wealth endowment $W$).

Differentiating expected utility with respect to weight $w_j$ gives the first order condition

$$E[W(\sum_j w_j R_j + (1 - \sum_j w_j) R_f) - \frac{b}{2}(W(\sum_j w_j R_j + (1 - \sum_j w_j) R_f))^2] = 0,$$

where $R_{opt} = \sum_j w_j R_j + (1 - \sum_j w_j) R_f$ is the weighted average return on the investor’s optimal overall portfolio including both risky and risk-free assets.

Using the identity $cov(R_j, R_{opt}) = E[R_j R_{opt}] - E[R_j]E[R_{opt}]$, the first order condition (1) simplifies to pricing equation

$$E[R_j] = R_f + \frac{bW cov(R_j, R_{opt})}{1 - bW E[R_{opt}]}.$$

In total, the investor allocates all her endowment $W$, that is $P_{opt} = W$. The expected return on the optimal portfolio of risky and risk-free assets is thus

$$E[R_{opt}] = \frac{E[V_{opt}]}{P_{opt}} = \frac{\sum_j E[V_j] + (W - \sum_j P_j) R_f}{W}.$$
Substituting (3) into (2), and remembering that \( P_j = E[V_j]/E[R_j] \), the mean-variance CAPM appears in the form of an unusual but insightful equation

\[
P_j = \frac{1}{R_f} \left( E[V_j] - \frac{b \text{cov}(V_j, V_{\text{opt}})}{1 - b E[V_{\text{opt}}]} \right)
\]

(4)

\[
P_j = \frac{1}{R_f} \left( E[V_j] - \frac{b \text{cov}(V_j, \sum_j V_j)}{1 - b \left\{ \sum_j E[V_j] + (W - \sum_j P_j)R_f \right\}} \right).
\]

(5)

Note that \( \text{cov}(V_j, V_{\text{opt}}) = \text{cov}(V_j, \sum_j V_j + (W - \sum_j P_j)R_f) = \text{cov}(V_j, \sum_j V_j) \), since the risk-free component \((W - \sum_j P_j)R_f\) of \( V_{\text{opt}} \) is a constant.\(^5\)

4 Which Parameters Drive Cost of Capital?

A prima facie interpretation of (2) suggests that the single firm parameter affecting its cost of capital is its returns covariance, \( \text{cov}(R_j, R_{\text{opt}}) \), or equivalently its returns "beta", \( \text{cov}(R_j, R_M)/\text{var}(R_M) \), where \( R_M = V_M/P_M = \sum_j V_j/\sum_j P_j \) is the return on the market portfolio. The problem with this rule of thumb is that it does not account for the logical circularity that (ingeniously) constitutes the CAPM equilibrium pricing mechanism. Specifically, the expected return \( E[R_j] \) depends on the returns covariance \( \text{cov}(R_j, R_M) \), but \( R_j = E[V_j]/P_j \) is a function of \( P_j \), and \( P_j \) itself depends on \( E[R_j] \). The full picture of what primitive factors affect the firm’s mean-variance CAPM-implied cost of capital is thus not superficially evident in equation (2).

**Proposition 1** Under a mean-variance CAPM, assuming an increasing risk-averse quadratic utility function, the firm’s cost of capital \( E[R_j] \) is decreasing (increasing) in its expected payoff \( E[V_j] \) given positive (negative) payoff covariance with the market payoff.

By writing \( E[R_j] = E[V_j]/P_j \) and substituting the value of \( P_j \) from (4), the expected return is seen as a function of both the payoff covariance \( \text{cov}(V_j, \sum_j V_j) \) and the payoff mean \( E[V_j] \)

\[
E[R_j] = \frac{E[V_j]R_f}{E[V_j] - b \text{cov}(V_j, \sum_j V_j)/(1 - b E[V_{\text{opt}}])}.
\]

(6)

Equation (6) reveals that the cost of capital for firm \( j \), \( E[R_j] \), decreases (increases) with the expected payoff \( E[V_j] \) whenever \( \text{cov}(V_j, \sum_j V_j) \) is positive (negative). This assumes utility increasing in money, implying positive marginal utility \((1 - b E[V_{\text{opt}}])\), and risk aversion, \( b > 0 \). Note that marginal utility \((1 - b E[V_{\text{opt}}])\) can be taken as constant, provided that \( V_j \) is small relative to \( V_M = \sum_j V_j \).

\(^5\)For equivalent CAPM equations derived under power and log utility functions, see Satchell (2012) and Johnstone (2012).
This finding about the effect of $E[V_j]$ on $E[R_j]$ matches the Proposition 1c proved by LLV (p.392), under an exponential-utility mean-variance CAPM and joint normal payoffs (see also the confirmation of this result in Christensen et al. 2010).

The finding that the mean payoff $E[V_j]$ helps determine the cost of capital is fundamental to accounting standard setting. The value of the firm can be viewed as an expected payoff discounted by a "risk-adjusted" discount factor, that is $P_j = E[V_j]/E[R_j]$. While information about the expected or mean payoff $E[V_j]$ is obviously relevant to users' assessments of the numerator in this formula, the much less obvious point is that the mean also plays a large part in determining the denominator $E[R_j]$ (i.e. the cost of capital or equity discount factor).

LLV not only prove this effect, they give a highly agreeable intuitive explanation for its existence. The explanation is that an increased mean without any increase in covariance amounts to adding a positive constant or risk-free element to the existing expected payoff, thus leading to a new "average" cost of capital for the firm, closer to the risk-free rate. In the limit, as the risk-free component of the firm’s cash flow increases and approaches its mean, the cost of capital converges on the risk-free rate.

5 Direct Effects of Information

The purpose of this analysis is to expand on LLV by examining how, in general, estimates of the firm’s payoff covariance or payoff mean can change with the arrival of information, and how these changes, individually or simultaneously, can alter the cost of capital imposed on the firm by the market. The firm’s activities are assumed fixed, so there are no indirect effects in the LLV (p.404) sense.\(^6\)

5.1 Covariance Effects

The first general rule is that information does not always lead to greater certainty, in either a variance or covariance sense, even when we assume stationarity.

**Proposition 2** The variance of payoff $V_j$ conditional on signal $S$, $\text{var}(V_j|S)$, can be higher than the unconditional or prior variance, $\text{var}(V_j)$.

The precept that information resolves uncertainty is almost lawlike in economics but is not always operational. The completely general (model-free) probability rule concerning information and variance is the law of conditional variance,

$$\text{var}(V_j) = E[\text{var}(V_j|S)] + \text{var}(E[V_j|S]).$$

\(^6\)LLV defined indirect effects as those brought by the firm reacting to its own reported information by altering its business activities (see Beyer et al. 2010, p.308 on the LLV distinction between direct and indirect effects of information). See Gao (2010) for an equilibrium model that links the firm’s reported information to its own investment choices.
This statistical identity implies that the average posterior variance of uncertain $V_j$ under signal $S$ is lower than the prior variance, that is $E[\text{var}(V_j|S)] < \text{var}(V_j)$, but not that the posterior variance is necessarily lower (Gelman et al. 2004, p.37).\footnote{It is well known that within the univariate Bayesian model adopted by LLV that any extra data leads to a lower variance posterior distribution for $V_j$ (e.g. Winkler 2003, p.150). It is apparently less well known that this is not true in general within Bayesian inference. Even in the very similar beta-binomial model, where $V_j \in (0,1)$ has a beta prior and beta posterior distribution, it is possible, albeit atypical, for the posterior to have a larger variance than the prior (Winkler 2003, p.141).}

Newly available information might directly concern $V_j$ and yet add substantially to market uncertainty about $V_j$.\footnote{Rogers et al. (2009, p.90) allow earnings forecasts to add to uncertainty: "...certain types of earnings disclosures, especially those that are unexpected, likely cause investors to revise their assessments of firm or manager type, potentially increasing uncertainty about underlying firm value."} Suppose that $V_j$ is driven by some causal factor or parameter $F$, and that $S$ is Bayesianly indicative of $F$, via the likelihood function $f(S|F)$. The Bayesian predictive distribution of $V_j$ is then

$$\int_F p(V_j|F)p(F|S)dF,$$

assuming that $V_j$ is independent of $S$ when given $F$. Hence, any observation $S$ that shifts the probability distribution of $F$ towards states of $F$ under which $V_j$ has high variance, must generally add to the variance of $V_j$, and conversely.\footnote{The archetypal Bayesian analysis of the expected value of imperfect information (EVII) was laid out by Pratt et al. (1965) and other Bayesian statistical decision theorists in contexts such as oil exploration. These standard calculations, all built around Bayes theorem, show that a geological sample can leave the oil company either more or less certain of oil after seeing the realized test result. See Clemen and Reilly (2001, pp.502-8) and Winkler (2003, pp.296-301) for expositions of EVII demonstrating how sometimes new information leads to a posterior distribution that is "flatter" (less peaked) than the prior distribution.}

Take the hypothetical case of oil exploration. Suppose that credible drill sites are in nature of two types, A and B, where testing is required to identify the type of a given site. In A-type sites the known relative frequency of oil is 0.5 and in B-type sites this frequency is 0.95. The population average frequency of oil is therefore $\rho(0.5) + (1-\rho)0.95$, where $\rho$ is the proportion of A-type sites. If from experience or theory the geologist has $\rho = 0.7$, then his prior probability of oil in a "random" site is 0.635. Now suppose that a technically ideal geological test reveals with certainty the type of site he is looking at. It follows that in 70\% of such tests, the geologist’s probability of oil will fall from a prior of 0.635 to a posterior of 0.5, thus leaving greater (maximum possible) variance, and thus uncertainty, post-test.

It is reassuring nonetheless that, by corollary of the law of conditional variance, information $S$ produces "on "average" a reduction in variance. This is consistent with the theoretical property of Bayes that enough data concerning $V_j$ will "in the long run" make $V_j$ certain (assuming a stationary process).\footnote{Barry and Winkler (1976) hold that data loses its relevance over time in nonstationary worlds and that uncertainty cannot be eliminated, but might wax and wane as the causal process shifts and observation runs to catch up.}
Proposition 3 The covariance of asset payoff \( V_j \) and asset payoff \( V_k \) conditional on signal \( S \), \( \text{cov}(V_j, V_k|S) \) can be higher than the unconditional or prior covariance, \( \text{cov}(V_j, V_k) \).

The probability identity that connects information and the covariance is the law of conditional covariance, sometimes known as the law of total covariance,

\[
\text{cov}(V_j, V_k) = E\left[\text{cov}(V_j, V_k|S)\right] + \text{cov}\left(E[V_j|S], E[V_k|S]\right).
\]

Since the covariance of the conditional expectations \( \text{cov}(E[V_j|S], E[V_k|S]) \) can be negative while the prior covariance \( \text{cov}(V_j, V_k) \) is positive, it follows immediately from this identity that the expected conditional covariance over the possible signal events \( S, E[\text{cov}(V_j, V_k|S)] \), need not be lower than the prior covariance.

Hence, as a general rule, assuming nothing but probability theory, the CAPM measure of risk or uncertainty, namely \( \text{cov}(V_j, V_k) \), need not shift towards zero upon receipt of imperfect information \( S \). Moreover, prior to observing \( S \), the investor’s expectation may be that the covariance will increase upon revelation of signal outcome \( S \). This occurs where the sample space (set of all feasible values) of signal \( S \) is known before \( S \) is realized, and are such that \( \text{cov}(E[V_j|S], E[V_k|S]) \) is negative.

Intuition and the law of conditional covariance are clearly in agreement. Imagine that data exhibits a much stronger covariance between two variables than the prior covariance. Data like this must logically have potential (under some formal Bayesian models at least) to bring an increase in the observer’s subjective assessment of the covariance. Similarly, new information can always reveal that two variables have a stronger common driver than was previously understood, and hence rational models of statistical inference must sometimes produce a higher subjective posterior covariance between two random variables.

The same of course goes for the variance, where data or fundamental analysis must have the facility under Bayesian probability to indicate that the quantity in question is more variable or volatile than was previously appreciated. This will not surprise portfolio managers whose subjective estimates of firm beta or returns covariance often increase with new information, rather than approaching zero.

5.2 Mean Effects

LLV (p.392) prove that according to the CAPM applied to typical stocks (with positive covariance with the market), information prompting a lower assessment of the expected payoff \( E[V_j|\cdot] \) has an upward effect on the required return (i.e. there is an inverse mathematical relationship between expected future payoff and CAPM required return). Thus, if new information increases the estimated mean \( E[V_j|\cdot] \) but leaves the covariance \( \text{cov}(V_j, V_M|\cdot) \) unchanged, the revised (conditional) CAPM price \( P_j|\cdot = E[V_j|\cdot]/E[R_j|\cdot] \) of the firm is higher for two reasons: (i) \( E[V_j|\cdot] \) is higher, and (ii) the firm’s cost of capital \( E[R_j|\cdot] \) is lower.
5.3 Relative Effects of $E[V_j | r]$ and $\text{cov}(V_j, V_M | r)$

In finance it is generally established that portfolio optimization is more sensitive to the estimated mean return than the estimated returns variance or covariance (e.g. Best and Grauer 1991). For this reason, Chopra and Ziemba (1993) argue that the bulk of effort should be spent on estimating mean returns rather than covariances.\footnote{Verrecchia (2001, p.174) makes general comments regarding the usual dominance of first-moment over second-moment effects.}

The relative sensitivity of optimal portfolio weights to the mean return rather than the covariance suggests that perhaps the cost of capital is similarly sensitive to the mean payoff. Partial derivatives of the quadratic utility CAPM cost of capital (6) support this suspicion. To see this, first define $\sqrt{\text{cov}(V_j, V_M)}$ as the square root of the payoff covariance, thereby constructing a variable with dimension dollars rather than dollars-squared. Now examine the respective derivatives

$$\frac{\delta E[R_j]}{\delta E[V_j]} = \frac{-b \text{cov}(V_j, V_M)(1 - b E[V_{opt}]) R_f}{(E[V_j] - b \text{cov}(V_j, V_M) + E[V_j] E[V_{opt}])^2},$$

and

$$\frac{\delta E[R_j]}{\delta \sqrt{\text{cov}(V_j, V_M)}} = \frac{2b \sqrt{\text{cov}(V_j, V_M)} E[V_j](1 - b E[V_{opt}]) R_f}{(E[V_j] - b \text{cov}(V_j, V_M) + E[V_j] E[V_{opt}])^2}.$$

It can be seen from these derivatives that the ratio of the change in cost of capital brought by a (small) $\delta$ increase in $E[V_j]$ versus the change brought by a $\delta$ increase in $\sqrt{\text{cov}(V_j, V_M)}$ is equal to

$$-\frac{\text{cov}(V_j, V_M)}{2E[V_j]},$$

and is independent of the risk aversion parameter $b$.

It appears that for firms with material positive payoff correlation with the market, this ratio will commonly be very large (i.e. large negative), indicating that the mean payoff is a significantly stronger force than the payoff covariance. Given sufficient positive correlation between the stock and market payoffs, and a sufficiently large market payoff variance $E[(V_M - \overline{V}_M)^2]$, the square root of the payoff covariance, $\sqrt{E[((V_j - \overline{V}_j)(V_M - \overline{V}_M)]}$, will tend to be a very large quantity in absolute terms relative to the expected payoff $E[V_j]$ of a single stock.\footnote{Thinking empirically, with positive correlation the average value of the period-by-period $(V_j - \overline{V}_j)(V_M - \overline{V}_M)$ will be generally be very large when $(V_M - \overline{V}_M)$ is commonly large.}

This conclusion rests on how strongly positively correlated stocks are in general and how large is the market payoff variance and the stock's payoff variance. Ultimately, therefore, the relative sensitivity of the cost of capital to the mean is an empirical property of any given market, since it depends on whether, for typical stocks, the $\text{cov}(V_j, V_M)$ is large relative to the mean $E[V_j]$. Note for instance that from the first of the partial derivatives above, the cost of capital is highly insensitive
to the mean payoff when the payoff covariance itself is low (i.e. the stock and market have near zero correlation). That is sensible, since from (6) the cost of capital is a constant (the risk-free rate) when \( \text{cov}(V_j, V_M) = 0 \), regardless of \( E[V_j] \).

6 Lambert et al. (2007)

The results proved by LLV have been widely misinterpreted. In this section I make note of how the literature has understood LLV and compare LLV with my more general results. The main point is that the Bayesian conclusions found by LLV, if taken as universals rather than as artifacts of just one subclass of Bayesian probability models based on narrow assumptions, are unjustified.

6.1 Interpretations of Lambert et al. (2007)

LLV has been extremely influential, and is taken as theoretical grounds for believing that more information leads economically to a lower cost of capital. The following passages show succinctly how LLV is widely interpreted in accounting research:

...in Lambert et al.’s (2007) model, accounting information is not an independent risk factor, but does affect the cost of capital by influencing investors’ assessments of the covariance of firm cash flows with those of the market, and thereby affecting beta. (Core et al. 2008, p.21)

Lambert et al. (2007) examine whether and how public accounting reports and disclosures affect a firm’s cost of equity capital in the presence of diversification. Using a model that is consistent with the CAPM and allows for multiple securities whose cash flows are correlated, they demonstrate that the quality of accounting information ...has a direct effect on the assessed covariance of a firm’s cash flows with other firms’ cash flows, suggesting that earnings quality can affect the cost of capital via a firm’s beta. (Beyer et al. 2010, p.308)

Lambert et al. (2007, 2009) suggest that firms with more precise information about future cash flows have lower conditional covariances with the market, and, as a consequence, lower conditional betas and low expected returns. (Ogneva 2012, p.1418)

Similarly, see Leuz and Wysocki (2008, p.9) and Leuz and Schrand (2011, p.7) and many similar passages in the empirical research literature.

Contrary to the common understanding of LLV shown in these quotes, Gao (2010) observes correctly that although it is taken in the literature that more or better disclosure reduces the firm’s cost of capital, by reducing its payoff covariance, the fact is that LLV show that more or better disclosure can increase the cost of capital.

11
A common theme in the previous literature is that disclosure reduces the cost of capital by reducing the conditional variance (or covariance) of the firm’s future cash flow. One exception is Lambert et al. (2007)...

They point out that cost of capital may increase with disclosure quality if disclosure changes both the mean and variance of the firm’s cash flow. However, they do not link this result directly to disclosure quality. (Gao 2010, p.20)

It is apparent from the clash between Gao’s reading of LLV and the more universal view, expressed in those several quotes above, that the implications of LLV are subject to some confusion. In what follows I summarize LLV and explain how this confusion has arisen.

6.2 Summary of Lambert et al. (2007)

The asset pricing model derived by LLV assumes a Bayesian joint normal posterior distribution \( f(V_1, V_2, \ldots, V_n|\Phi) \) of firm payoffs \( V_j \), where \( \Phi \) is the available information, along with an exponential utility function. The resulting asset pricing model (LLV pp.413-4) is of the form

\[
P_j = \frac{1}{R_f} \left( E[V_j|\Phi] - \frac{1}{\Upsilon} \text{cov}(V_j, \sum_j V_j|\Phi) \right),
\]

where \( \Upsilon > 0 \) specifies the market’s risk tolerance under the assumed exponential utility function \( U(W) = \Upsilon (1 - \exp[-\frac{1}{\Upsilon} W]) \), \( \frac{1}{\Upsilon} = \frac{\sum_j E(V_j|\Phi) - R_f \sum_j P_j}{\text{var}(\sum_j V_j|\Phi)} \) is the market price of risk in equilibrium, and \( R_f \) is the risk-free return factor.\(^{13}\) Using their CAPM equation (7), the LLV findings are that the cost of capital \( E[R_j|\Phi] \) is (i) increasing in the covariance between asset \( j \) and "the market" (the sum of all asset values), \( \text{cov}(V_j, \sum_j V_j|\Phi) \), and (ii) decreasing (increasing) in the expected asset value \( E[V_j|\Phi] \) when \( \text{cov}(V_j, \sum_j V_j|\Phi) \) is positive (negative). Of these, finding (i) is extremely well known, and (ii) is very interesting and rarely if ever raised (see Johnstone 2014). The same findings hold under my quadratic utility CAPM, regardless of the asset payoff distribution, as shown above and conjectured by LLV (p.393).

At this point LLV (p.393) note correctly that simultaneous changes in \( E[V_j|\Phi] \), and \( \text{cov}(V_j, \sum_j V_j|\Phi) \), driven by some change in information \( \Phi \), are not likely to be "proportionate" in the sense that their separate effects cancel out, such as to leave the cost of capital unchanged. The implication, therefore, is that changes in

\[^{13}\text{Note that } \Upsilon = N\tau \text{ where in LLV’s proof there are } N \text{ traders in the market each with CARA utility } U(W) = \tau (1 - \exp[-\frac{1}{\tau} W]) \text{ and individual risk tolerance } \tau > 0.\]
extant information $\Phi$ can lead to re-assessments of $E[V_j|\Phi]$ and $\text{cov}(V_j, \sum_j V_j|\Phi)$ that jointly drive the CAPM cost of capital either up or down. LLV do not develop this broader argument. Instead they target the effect of information on the covariance, $\text{cov}(V_j, \sum_j V_j|\Phi)$, and conclude that more information leads necessarily to a lower (i.e. nearer zero) assessment of the firm’s covariance with the market, and therefore a lower cost of capital.

...we show that higher quality accounting information and financial disclosures affect the assessed covariances with other firms, and this effect unambiguously moves a firm’s cost of capital closer to the risk-free rate. (Lambert et al. 2007, p.387)

### 6.2.1 Information and the Variance

The LLV model of information is an elementary Bayesian standard (e.g. Winkler 2003, pp.149-150). Uncertain quantity $V_j$, representing firm payoff, has an assumed normal prior distribution $V_j \sim N(\mu_0, \sigma_0)$. A random sample of $m$ iid observations $(x_1, x_2, ..., x_m)$ is drawn from a normal distribution $N(V_j, \sigma_x)$, where $\sigma_x$ is taken as a known constant. Each draw $x_j$ is thus an unbiased estimate of $V_j$ with known precision $1/\sigma_x^2$. The posterior belief distribution $f(V_j|S)$ of payoff $V_j$ is $N(\mu_1, \sigma_1)$, where

$$\mu_1 = \frac{(1/\sigma_0^2) \mu_0 + (m/\sigma_x^2) \bar{x}}{(1/\sigma_0^2) + (m/\sigma_x^2)}$$

and

$$\frac{1}{\sigma_1^2} = \frac{1}{\sigma_0^2} + m \frac{\sigma_x^2}{1/\sigma_0^2 + (m/\sigma_x^2)}.$$

It follows that no matter what the sample evidence, or how small the sample, the posterior variance of uncertain future payoff $V_j$ is always less than its prior variance. That is, $\sigma_1^2 < \sigma_0^2$ for any $m > 0$. LLV (p.395) hold that this one Bayesian model "formalizes the notion that accounting information and disclosure reduce the assessed variance of the firm’s end-of-period cash flow". By that way of thinking, certainty can be accumulated monotonically - merely by generating more data, contrary to the general Bayesian law of conditional variance.

### 6.2.2 Information and the Covariance

LLV consider the effect of information concerning $V_j$ on the investor’s assessment of the covariance, $\text{cov}(V_j, \sum_j V_j|\Phi)$, separately from its effect on the variance $\text{var}(V_j|\Phi)$. Ultimately, their aim is to show that like the assessed variance, the assessed covariance approaches zero whenever new information is disclosed.

One way to test this proposition would have been to extend their univariate Bayesian model of inference about $V_j$ to the matching case of multivariate normal $f(V_1, V_2, ..., V_n|\phi)$. Under this model, the unknown payoff vector $V = (V_1, V_2, ..., V_n)$ has a conjugate multivariate normal prior distribution $V \sim N(\mu_0, \Sigma_0)$, where $\mu_0$ is the vector of prior means and $\Sigma_0$ is the prior covariance matrix. The data or information about $V$ might then be conceived as a random sample of $m$ iid observations
\[\{(x_1, x_2, \ldots, x_n)_1, \ldots, (x_1, x_2, \ldots, x_n)_m\}\], where each observation is drawn from a multivariate normal distribution \(N(\mathbf{V}, \Sigma_x)\) with some known covariance matrix \(\Sigma_x\). As in the univariate case, each sample observation \((x_1, x_2, \ldots, x_n)_i\) is an unbiased estimate of the parameter vector \((V_1, V_2, \ldots, V_n)_i\), with fixed and known precision \(\Sigma_x\).

By a standard result, the Bayesian posterior distribution of \(\mathbf{V}\) is then multivariate normal \(\mathbf{V} \sim N(\mu_1, \Sigma_1)\) with

\[
\mu_1 = \frac{(1/\Sigma_0) \mu_0 + (m/\Sigma_x) \bar{x}}{(1/\Sigma_0) + (m/\Sigma_x)} \quad \text{and} \quad \frac{1}{\Sigma_1} = \frac{1}{\Sigma_0} + \frac{m}{\Sigma_x},
\]

where \(\bar{x} = (\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n)\) is the vector of observed sample means. Note that in this model the posterior covariance, like the posterior variance above, is influenced only by the precision of the sample observation, not by its actual realized value, \(\bar{x}\).

The main implications of this model from the LLV standpoint are (i) the posterior variance \(\text{var}(V_j|\bar{x})\) is always less than the corresponding prior variance \(\text{var}(V_j)\), as in the univariate model, and (ii) given that the prior covariance \(\text{cov}(V_j, V_k)\) and (known) sampling covariance \(\text{cov}(x_j, x_k)\) are of the same sign, the posterior covariance \(\text{cov}(V_j, V_k|\cdot)\) is closer to zero than the corresponding prior covariance, for all \(k \neq j\). Note that in regard to (ii), if the prior and sampling covariances \(\text{cov}(V_j, V_k)\) and \(\text{cov}(x_j, x_k)\) are of opposite signs, then the posterior covariance \(\text{cov}(V_j, V_k|\cdot)\) can be further from zero than the prior covariance \(\text{cov}(V_j, V_k)\).

Rather than continuing via a multivariate Bayesian model, LLV set out a model that is not explicitly Bayesian but which leads to a parallel result. The LLV model begins by presuming a joint normal probability distribution \(f(V_j, V_k, Z_j, Z_k)\), where \(V_j\) and \(V_k\) are the uncertain payoffs of assets \(j\) and \(k\) respectively, and \(Z_j = V_j + \epsilon_j\) and \(Z_k = V_k + \epsilon_k\) are unbiased noisy measurements of \(V_j\) and \(V_k\). These estimates are characterized by normally distributed errors \(\epsilon_j\) and \(\epsilon_k\) (independent of \(V_j\) and \(V_k\)) with fixed variances and covariance, \(\text{var}(\epsilon_j), \text{var}(\epsilon_k), \text{and} \text{cov}(\epsilon_j, \epsilon_k)\). Taking advantage of the covariance properties of the joint normal distribution, LLV show that

\[
\text{cov}(V_j, V_k|Z_j) = \text{cov}(V_j, V_k) \frac{\text{var}(\epsilon_j)}{\text{var}(V_j) + \text{var}(\epsilon_k)}.
\]

On this basis, LLV conclude that the covariance of asset \(j\) with asset \(k\), conditional upon observation of signal \(Z_j\), is always closer to zero than the unconditioned (prior) covariance of assets \(j\) and \(k\). This effect, which is stronger the smaller the observation error variance \(\text{var}(\epsilon_j)\), holds for asset \(j\) with respect to all other single assets \(k\), and hence the covariance of asset \(j\) with the market sum of all assets,

\[
\sum_j \text{cov}(V_j, V_k|Z_j) + \text{var}(V_j|Z_j),
\]

shifts toward zero, not only because the conditional variance \(\text{var}(V_j|Z_j)\) goes toward zero, but also because each of the covariance terms is impacted the same way.

\[\footnote{This is a recognized convenience of such models (Christensen and Feltham 2003, p.78; Verrecchia 2001, p.174, Zhang 2013). See caveats in Christensen et al. (2010, footnotes 3 and 16).} \]
At this point in the LLV analysis, any discrete observation $Z_j$ leads to a lower (nearer zero) covariance for asset $j$, that is $|\text{cov}(V_j, V_k|Z_j)| < |\text{cov}(V_j, V_k)|$ for all $k$. But this conclusion does not remain in the more general case of a multivariate observation, such as $(Z_j, Z_k)$. LLV recognize, and show by a longer proof (pp.416-7), that within their model the joint signal $(Z_j, Z_k)$ leads necessarily to a reduced posterior covariance between $V_j$ and $V_k$ only when the covariances of cash flows and measurement errors, $\text{cov}(V_j, V_k)$ and $\text{cov}(\epsilon_j, \epsilon_k)$, are of the same sign. When this is not the case, LLV (p.402) find that it is difficult to "sign" the effect of information $(Z_j, Z_k)$ on the covariance between $V_j$ and $V_k$ (i.e. the covariance can increase or decrease). This latter concession is an internal weak spot in the LLV argument, particularly given that signals about different individual firms rarely exist in isolation.

The essence of LLV's argument is captured by their second and final proposition (Proposition 2, p.399) which holds that $\text{cov}(V_j, V_k|Z_j)$ is closer to zero than $\text{cov}(V_j, V_k)$ for any new information $Z_j$ (and all $k$). This proposition is taken as grounds to conclude that all new information leads to a covariance closer to zero, and thus ceteris paribus to a lower cost of capital for firms with positive covariance with the market. As in the case of the variance, this conclusion is not consistent with the more general law of conditional covariance. It has the implication that any accounting information that leaves the estimated payoff mean higher or unchanged must lead to a lower firm beta.

7 Numerical Illustration

The following example serves several purposes. First it exhibits the consistency between my quadratic utility CAPM and the common textbook mean-variance CAPM. Second, it involves multivariate normal payoffs, so that mean-variance asset prices and returns can also be calculated under the LLV exponential-utility CAPM. Third, and most importantly, it gives some intuitive insight into how information about firm fundamentals alters both firm and market costs of capital.

In the LLV set up, information is modelled as if the firm’s unknown future business payoff $V_j$ can be randomly sampled, like draws from an urn. This is intended as an abstract depiction of information rather than as an attempt to model the causation of firm payoff $V_j$. An alternative approach, designed to resemble an economic model, is set out below. The point of this analysis is to demonstrate how under a simple causal model, incremental information about fundamentals can drive the firm’s cost of capital either up or down, and can have the same effect on the market risk premium as a whole.

For simplicity, imagine a market offering just two risky assets, with uncertain payoffs, $V_1$ and $V_2$ respectively. From (5) the general equations for the prices under

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15 The fact that this relationship no longer holds necessarily within the LLV model once there are two or more concurrent pieces of information $(Z_j, Z_k, ...)$, is acknowledged in discussion (pp.402-3).
quadratic utility of the two assets are

\[ P_1 = \frac{1}{R_f} \left( E[V_1] - \frac{bcov(V_1, V_1 + V_2)}{1 - b(E[V_1 + V_2] + (W - P_1 - P_2)R_f)} \right) \]  (8)

and

\[ P_2 = \frac{1}{R_f} \left( E[V_2] - \frac{bcov(V_2, V_1 + V_2)}{1 - b(E[V_1 + V_2] + (W - P_1 - P_2)R_f)} \right) \]  (9)

Solving (8) and (9) simultaneously and then eliminating either \( P_1 \) or \( P_2 \) leads to a quadratic equation for \( P_j \) \((j = 1, 2)\), with two possible solutions

\[ P_j = \frac{1}{R_f} \left[ E[V_j] - \frac{\sigma_{j,M} \left\{ (1 - bR_f W) \pm \sqrt{(1 - bR_f W)^2 - 4b^2(\sigma_{1M} + \sigma_{2M})} \right\}}{2b(\sigma_{1M} + \sigma_{2M})} \right], \]

where for notational convenience \( \sigma_{j,M} \) denotes \( \text{cov}(V_j, V_1 + V_2) \). Unfortunately, it is impossible to generalize about which of these solutions is economically sensible, although commonly only

\[ P_j = \frac{1}{R_f} \left[ E[V_j] - \frac{\sigma_{j,M} \left\{ (1 - bR_f W) - \sqrt{(1 - bR_f W)^2 - 4b^2(\sigma_{1M} + \sigma_{2M})} \right\}}{2b(\sigma_{1M} + \sigma_{2M})} \right], \]

lies in the sensible range between 0 and \( E[V_j] \).

Suppose that underlying \( V_1 \) and \( V_2 \) there are two causal factors, \( X \sim N(1, 1) \) and \( Y \sim N(1, 1) \). These are independent normal random variables with unit means and variances. Now imagine that the risky asset payoffs are known to be \( V_1 = cX + Y \), and \( V_2 = X + dY \) respectively, where \( c \) and \( d \) are real constants. The quantities \( V_1 \) and \( V_2 \) are therefore bivariate normal, and their parameters are \( E[V_1] = c + 1 \), \( E[V_2] = d + 1 \), \( \text{var}(V_1) = c^2 + 1 \), \( \text{var}(V_2) = d^2 + 1 \), and \( \text{cov}(V_1, V_2) = c + d \). It follows then that \( \text{cov}(V_1, V_M) = \text{var}(V_1) + \text{cov}(V_1, V_2) = c^2 + 1 + c + d \), and similarly \( \text{cov}(V_2, V_M) = d^2 + 1 + c + d \).

We now plug these values into equations (8) and (9). For example, letting \( c = 1 \), \( d = 3 \), \( W = 10 \), \( R_f = 1.10 \) and \( b = 0.025 \), the resulting prices are \( P_1 = 1.6254 \) and \( P_2 = 3.1865 \), implying expected return factors \( E[R_1] = E[V_1]/P_1 = 2/1.6254 = 1.2305 \) and \( E[R_2] = E[V_2]/P_2 = 4/3.1865 = 1.2553 \). The resulting endogenous return on the "market portfolio" of risky assets is the price-weighted average of these two individual expectations, or equivalently \( E[V_1 + V_2]/(P_1 + P_2) = 1.2469 \) (i.e. 24.69%).

It is reassuring to test these numerical results for consistency with the usual form of the CAPM as seen in most textbooks. Importantly, note that it is not possible to calculate \( R_M \) from the usual textbook CAPM. However, having obtained \( R_M \) under an assumed quadratic utility function, it is possible to check all results for consistency with the usual CAPM equation. Relevant calculations are as follows. First, the
The variance of the market return is $\sigma_M^2 = \text{var}(V_M) = \frac{\text{var}(V_1, V_M) + \text{var}(V_2, V_M)}{P_M} = c^2 + d^2 + 2(c + d + 1)$. The "returns beta" of asset 1 is $\beta_1 = \text{cov}(V_1, V_M) / \sigma_M^2 = \text{cov}(V_1, V_M) / (P_1 P_M \sigma_M^2) = 0.8638$. Similarly, $\beta_2 = \text{cov}(V_1, V_M) / (P_2 P_M \sigma_M^2) = 0.8638$. Hence, the price-weighted average returns beta equals one, and, by the usual form of CAPM, $E[R_1] = R_f + \beta_1 (E[R_M] - R_f)$, we have $E[R_1] = 1.10 + 0.8688(1.2469 - 1.10) = 1.2305$, and similarly $E[R_2] = 1.10 + 1.05706(1.2469 - 1.10) = 1.2553$. These return factors match those calculated above.

To observe the possible effects of information regarding the parameters $c$ and $d$, consider the results plotted in Figure 1. This figure shows the required rates of return on each of the two assets, and on the market sum of these assets, all as functions of parameter $c$ (holding $d = 3$ constant).

![Figure 1](image)

Plot of Expected Return Factors of Individual Assets and the Market as Functions of $c$ (with $0 \leq c \leq 5$)

Under Quadratic Utility $U(W) = W - \frac{b}{2} W^2$

(with $W = 10$, $b = 0.025$, $d = 3$ and $R_f = 1.10$)

These results are merely for example, but are sufficient to show how new information about the unknown $c$ can shift the cost of capital of an individual asset, or of the market as a whole, up or down. Underlying these changes in the cost of capital, information about $c$ influences estimates of both asset mean $E[V_1]$ and covariance $\text{cov}(V_1, V_M)$.

Figure 2 is a repeat of Figure 1 except that it is based on the LLV exponential utility function (see above) rather than my assumed quadratic utility function. The two markets exhibit different expected returns for the same value of $c$, due to their different utility functions. The asset prices and implicit expected returns found using LLV's pricing equation (7), derived under exponential utility and joint normal payoffs, were found to be consistent with the usual textbook CAPM. As above, this cross-check involves taking the endogenous market return calculated using the LLV model,
plugging this into the usual CAPM, and showing that the resulting CAPM individual asset returns are the same as those produced endogenously by the LLV model.

**Figure 2**
Plot of Expected Return Factors of Individual Assets and the Market as Functions of $c$ (with $0 \leq c \leq 5$)
Under Exponential Utility $U(W) = \Upsilon (1 - \exp \left[-\frac{1}{\Upsilon} \right] W)$
(with $\Upsilon = 20$, $d = 3$ and $R_f = 1.10$)

**Figure 3**
Plot of Expected Return Factor for Asset 1 as a Function of its Payoff Mean and Payoff Covariance
Under Exponential Utility $U(W) = \Upsilon (1 - \exp \left[-\frac{1}{\Upsilon} \right] W)$
(with $\Upsilon = 20$, $d = 3$ and $R_f = 1.10$)

Figure 3 is another plot of Asset 1 expected returns. These are the same returns as shown in Figure 2, but are plotted as a function of the asset’s mean payoff $E[V_1]$ and payoff covariance $\text{cov}(V_1, V_M)$, rather than of the underlying constant $c$. Just as in Figure 2, the plotted points are found by holding $d = 3$ constant and letting $c$ take a range of possible values (here between 0 and 4). Note that the expected return on Asset 1 increases monotonically with its payoff covariance and decreases monotonically with its expected payoff, as per LLV.
8  *Ex Ante* Disclosure Obligations

Christensen et al. (2010) and Gao (2010) emphasize that, for the purpose of evaluating accounting standards, it is essential to consider how prices and costs of capital might react to both the promise of better information (the *ex ante* perspective) and to its realization (the *ex post* perspective). In principle, choice of one information system over another is made on the basis of its *ex ante* expected utility, which is a weighted average of the conditional expected utilities of the user’s best actions (investment portfolios) under each of the possible signal realizations (weighted by the prior probabilities of each possible observation).  

*Ex ante* evaluation of an information system depends therefore on first understanding how beliefs and investment choices will be altered upon each possible signal outcome. The *ex ante* (pre-signal) and *ex post* (post-signal) perspectives are thus inseparable.

Consider specifically the probability relationship between future signal $S$ and current assessments of the CAPM parameters of the firm. By the law of iterated expectations, $E[E[V_j|S]] = E[V_j]$. This means that although it is understood that the conditional expectation $E[V_j|S]$ can differ greatly from the current expectation $E[V_j]$, and that the calculation of $E[V_j|S]$ hinges intrinsically on the error properties of $S$, the current (*ex ante*) expected value of $E[V_j|S]$ is $E[V_j]$, and is unaffected by the error properties (likelihood function) of future signal $S$. The less obvious but equivalent point is that the current unconditional (*ex ante*) covariance assessment, $\text{cov}(V_j, V_M)$, is also unaffected by the attributes of the future signal, notwithstanding that the expected conditional (on $S$) covariance does not equal the unconditional covariance and is affected by properties of $S$. Demonstration of this point is provided in the Appendix.

8.1 Christensen et al. (2010)

Christensen et al. (2010) consider both *ex ante* and *ex post* effects of exogenous information by assuming a two period model in which the uncertain payoff $d$ from a single-asset market is paid at $t = 2$ and an unbiased signal $y$ concerning $d$ occurs at midpoint $t = 1$. As in LLV, the signal and the uncertain asset payoff are assumed joint normal, hence (contrary to my more general model) all new information reduces investor uncertainty about payoff $d$. Findings, in the case of homogeneous beliefs and public signal $y$ can be reconstructed and summarized as follows.

First, the precision of the $t = 1$ signal makes no difference to beliefs formed at $t = 0$, consistent with Bayesian logic, as explained above. More specifically, introducing subscripts to denote time $t$ and taking the risk-free interest rate as zero, the price $P_0$

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16 In Bayesian theory, this is sometimes called preposterior analysis (O’Hagen 1994, p.84; Winkler 2003, p.267), and is the framework for choice between competing experiments.

17 Put succinctly, today’s expectation of tomorrow’s expectation equals today’s expectation.
of the asset at \( t = 0 \) is
\[
P_0 = E_0[d] - c \text{var}_0(d),
\]
where \( c \) is a positive constant capturing the level of market risk aversion. Neither the expected payoff \( E_0[d] \) nor the variance \( \text{var}_0(d) \) is affected by the (known) precision of the forthcoming \( t = 1 \) signal, as explained above.

Second, the expected return at \( t = 0 \) for the period \( t = 0 \) to \( t = 2 \), written as a factor, is
\[
\frac{E_0[d]}{P_0} = \frac{E_0[d]}{E_0[d] - c \text{var}_0(d)},
\]
reconﬁrming, as in LLV and Gao (2010, p.20), that the forward-looking cost of capital is decreasing in mean payoff \( E_0[d] \).

Third, although the return expected at \( t = 0 \) over the two-period life of the asset is not altered by anticipation of a signal of given quality at \( t = 1 \), the time intervals over which that return is expected to accumulate are affected. A good way to show this is to consider \( P_0 \) in terms of the \( t = 1 \) payoff, \( P_1 \)
\[
P_0 = E_0[P_1] - c \text{var}_0(P_1),
\]
where \( P_1 = E_1[d] - c \text{var}_1(d) \). From there
\[
\text{var}_0(P_1) = \text{var}_0(E_1[d] - c \text{var}_1(d)) = \text{var}_0(E_1[d]),
\]
since in this model \( \text{var}_1(d) \) is a known constant at \( t = 0 \). Hence, the return expected at \( t = 0 \) over period \( t = 0 \) to \( t = 1 \) becomes
\[
\frac{E_0[P_1]}{P_0} = \frac{E_0[E_1[d] - c \text{var}_1(d)]}{E_0[P_1] - c \text{var}_0(P_1)} = \frac{E_0[d] - c \text{var}_1(d)}{E_0[E_1[d] - c \text{var}_1(d)] - c \text{var}_0(E_1[d] - c \text{var}_1(d))} = \frac{E_0[d] - c \text{var}_1(d)}{E_0[d] - c \text{var}_1(d) - c \text{var}_0(E_1[d])}.
\]

In Bayesian analysis, the term \( \text{var}_0(E_1[d]) \) is known as the "preposterior variance", and describes the variance (before observing signal \( y \)) of the mean \( E_1[d] \equiv E[d|y] \) of the posterior distribution of \( d \) conditioned on \( y \). Using results provided by Raiffa and Schlaifer (1961, p.297), and mirrored by Christensen et al. (2010, pp.826-7),
\[
\text{var}_0(E_1[d]) = \text{var}_0(d) - \text{var}_1(d).
\]

Hence, from (12) the return expected at \( t = 0 \) over period \( t = 0 \) to \( t = 1 \) is
\[\text{It follows from (13) that the two expressions (10) and (11) for asset price } P_0 \text{ are equivalent.}\]
\[
\frac{E_0[d] - c \var_1(d)}{E_0[d] - c \var_0(d)}.
\] (14)

It follows therefore that if signal \( y \) is completely uninformative (\( \var_1(d) = \var_0(d) \)), the return factor (14) expected over period \( t = 0 \) to \( t = 1 \) equals one, which is the risk-free rate, implying that all of the return expected at \( t = 0 \), over the asset’s two-period life, is expected to accrue in the second period. At the other extreme, where \( y \) reveals \( d \) with certainty (\( \var_1(d) = 0 \)), all of the lifetime expected return \( E_0[d]/P_0 \) is expected in the first period, and the return expected in the second period is just the risk-free rate (merely as reward for waiting). Between these two extremes, the general rule is that a signal \( y \) with higher (known) precision brings a higher expected return over the first period, and a commensurately lower expected return thereafter.

This model suggests that if better accounting (e.g. higher "earnings quality") adds to the rate at which uncertainty is resolved, higher returns are expected to arise earlier. It follows on this basis that firms which commit to better accounting can be expected to show higher average realized returns, at least in the short term.

It is important to remember that the Christensen et al. (2010) model differs from my model in two ways, due to its assumption of joint normality between payoff and signal. This assumption, in keeping with most models in the noisy rational expectations literature, implies that (i) it is known at \( t = 0 \) that all new information will resolve some (known) amount of uncertainty (i.e. will reduce the perceived variance of \( d \), by a known amount), and (ii) the conditional variance, upon receiving the signal at \( t = 1 \) is influenced only by signal quality (precision), and not by signal realization \( y \) (see my footnote 14). A more general Bayesian model should allow the signal arising at \( t = 1 \) to sometimes heighten uncertainty about the payoff \( d \) coming at \( t = 2 \), in a variance or covariance sense. In economic reality, uncertainty about any future payoff \( d \) can go up or down, depending on what news is received, in which case expected future returns rise and fall, and are reallocated in time, as each new signal appears.

Paradoxically, it is possible to define signal characteristics such that it is known before observing the \( t = 1 \) signal that whatever the realization of that signal (i.e. no matter what is actually observed at \( t = 1 \)), the conditional (at \( t = 1 \)) expected return for the ensuing period between \( t = 1 \) to \( t = 2 \) can only be higher than the unconditional (at \( t = 1 \)) expected return for that same period. This can be understood intuitively once we allow for the instantaneous effect of the \( t = 1 \) signal on the asset price at \( t = 1 \). Specifically, it can be known in advance that the signal due at \( t = 1 \) has feasible values that will lead to either a higher price and a lower expected return, or a lower price and a lower expected return, where the new expected return is relative to the new \( t = 1 \) price.\(^{21}\)

\(^{19}\)The importance of point (ii) is stressed by Gao (2010, pp.10, 12).
\(^{20}\)Christensen et al. (2010) consider only variance, because theirs is a single asset market.
\(^{21}\)A proof is available from the author.
9 Empirical Considerations

A given firm has share price \( P_t \) at time \( t \) and produces mandatory accounting reports at the end of each fixed time period, \( t \). Let the report or signal arising at time \( t \) be \( S_t \), so \( P_t \) is conditioned on \( S_t \). The question that arises in empirical accounting research is whether the firm’s cost of capital over period \( t \), represented by \( R_t = P_t/P_{t-1} \), is affected by properties of \( S_{t-1}, S_t, S_{t+1}, S_{t+2}, \ldots \). Assume that \( P_{t-1} \) is ex-dividend and \( P_t \) is cum-dividend, as usual in empirical returns.

Under rational expectations, any theoretical proposition that empirical returns can be expected to be decreasing in accounting signal quality requires that \textit{ex ante} expected returns are decreasing in signal quality. In the following analysis, I show that the \textit{ex ante} expected return over any period \( t \) is not necessarily decreasing in the quality of information already received, \( S_{t-1}, S_t, S_{t+1}, \ldots \), nor in the quality of information yet to be received, \( S_t, S_{t+1}, S_{t+2}, \ldots \). For notational simplicity, I treat information \( S_{t-1} \) as the conjunction of all signals received to date \( t-1 \), and similarly \( S_t \) as the conjunction of all future signals.

(i) At \( t-1 \), the end-of-period payoff is the stock price \( P_t \). Expected returns \( E_{t-1}[R_t] = E_{t-1}[P_t]/P_{t-1} \) will be lower when \( S_{t-1} \) leads to a higher estimated expected payoff \( E_{t-1}[P_t] \) or a lower estimated covariance between \( P_t \) and the market, \( \text{cov}_{t-1}(P_t, P_{M;t}) \).

There is no necessary positive or negative association between the quality of \( S_{t-1} \) and the direction or magnitude of its effect on \( E_{t-1}[P_t] \). Instead, the signal \( S_{t-1} \) can bring good or bad news with respect to the expected future price \( E_{t-1}[P_t] \), regardless of its quality. Similarly, \( \text{cov}_{t-1}(P_t, P_{M;t}) \) can also be higher or lower, no matter what the quality of signal \( S_{t-1} \). For instance, accounting information \( S_{t-1} \), whatever its attributes, might alter beliefs about whether earnings are predominately market-dependent or more firm-specific and idiosyncratic, thus changing the assessed covariance, \( \text{cov}_{t-1}(P_t, P_{M;t}) \), upwards or downwards. It follows therefore that expected returns \( E_{t-1}[R_t] \) are not linked in any one direction to the quality of signal \( S_{t-1} \).

(ii) Anticipation of a more informative future signal \( S_t \) can increase or decrease \( \text{cov}_{t-1}(P_t, P_{M;t}) \). The main point here is that \( P_t \) is conditioned on \( S_t \) and can be strongly influenced by \( S_t \), so its anticipated statistical behavior, specifically its assessed covariance with \( P_{M;t} \), must be affected by properties of \( S_t \). As a simple example, if it were known with certainty at \( t-1 \) that \( P_t \) is determined only by \( S_t \), and also that \( S_t \) is not correlated with the market \( P_{M;t} \), then \( \text{cov}_{t-1}(P_t, P_{M;t}) = 0 \).

To see that the covariance \( \text{cov}_{t-1}(P_t, P_{M;t}) \) can increase with signal quality, start with the more straightforward proposition that \( \text{var}_{t-1}(P_t) \) is often higher when the anticipated \( S_t \) is more informative. This says merely that a more informative signal is disposed to have a bigger effect on stock price. Now imagine that this variation in the future stock price \( P_t \) is believed at \( t-1 \) to be highly correlated with the market price.
Bayesianly, that could occur in various ways, one of which is that the stock is positively correlated with the market and $S_t$ has very good error probabilities under any market state. It would then be the case, for example, that a high $P_{M,t}$ and a "positive" signal $S_t$ gives high probability of high $P_t$.

To see that $\text{cov}_{t-1}(P_t, P_{M,t})$ can decrease with signal quality, there are again many possibilities. One obvious case is where $S_t$ will provide a strong signal, positive or negative, about the firm’s idiosyncratic lines of business. It is then foreseen at $t - 1$ that the firm’s stock price $P_t$ will be more influenced by $S_t$ than by the market $P_{M,t}$, thus reducing the assessed $\text{cov}_{t-1}(P_t, P_{M,t})$.

It follows from the argument in (i) and (ii) that, in general, expected returns $E_{t-1}[R_t]$ are not necessarily increasing or decreasing in signal quality, either in regard to signals already received or signals yet to be realized.\(^{22}\) This may help explain theoretically why empirical work linking accounting quality to stock returns has been said repeatedly to offer "mixed" results.\(^{23}\)

### 9.1 Signalling by Signal Quality

There is a plausible Bayesian rationale for an empirical connection between signal quality and the cost of capital that has not been mentioned in the literature, and which is consistent with LLV and my general model. If accounting information quality is viewed as partly voluntary (endogenous) rather than entirely mandatory (exogenous) then adoption of more stringent or transparent accounting policies might convey confidence from within the firm regarding its future cash flows. If that positive "signal" is accepted by the market as grounds for material upward revision in the firm’s expected cash flows, then the relative force of the mean payoff as a determinant of the firm’s equity discount rate (under CAPM) may bring about an observable reduction in its cost of capital.

Note that this argument departs radically from the conventional view that better information drives the cost of capital solely via its effect on risk or covariance. It says that the voluntary provision of better information can be grounds for a lower cost of capital by way of its Bayesian effect on the estimated mean (expected cash flow).

There is a well known proposition that the endogenous quality of earnings reports (and other financial disclosure) is influenced by the "true" state of underlying earnings and operations. It is suggested for example that there is a generally positive correlation between the direction of accounting signals and their quality (i.e. successful growing firms tend to produce higher quality earnings measures; e.g. Miller

\(^{22}\)That might be why Barth et al. (2013, pp.207, 209, 213) suggest that only information-asymmetry arguments can be relied upon to justify the widely hypothesized link between better accounting and lower discount rates.

\(^{23}\)Also getting in the way of such work is the practical issue that accounting information may not be sufficiently informative, within the supply of all other information, to have a marginal "first order" effect on returns (Zimmerman 2013).
2002). The implicit CAPM effect of the estimated mean payoff on the equilibrium cost of capital adds further substance to this argument. Any incentive for successful or profitable firms to volunteer "quality" accounting (e.g. to be conservative in earnings measurement) is reinforced when the favorable signals carried by that level of disclosure lead to both higher expected future payoffs and a lower discount rate. Similarly, the converse suggests that firms for which things have turned negative will be reluctant to make this any more evident through "good" accounting, since any reduction in market assessments of their expected future cash flows can lead to both a smaller numerator and a higher discount rate, implying a twofold reduction in share price.

10 Conclusion

The widely held view that better financial reporting leads unambiguously to a lower cost of capital is misleading on two grounds. First, information of great precision or Bayesian quality can bring increased uncertainty, and hence higher required returns. Second, where incremental information does indeed lead to greater certainty about the firm's future cash flows, that heightened certainty can be relied upon to reduce the cost of capital only when the signal on which it rests is "positive" (i.e. good news). For sufficiently "negative" signals, the opposite generalization is true. The resulting (conditional) cost of capital depends therefore not only on the quality of the signal (a better signal will generally have a more pronounced effect on the investor's beliefs) but also on the direction of that signal, or, in other words, on what it "says".

Regulators should not assume that better financial reporting standards lead mechanically to lower costs of capital. The premise that accounting can of itself reduce the cost of capital has led to a fixation on that one factor rather than on the more overriding task of valuing the firm. Bertomeu (2013, p.476) describes this orientation as a "current trend in the literature toward reductio cost of capital in which complex policy evaluations have been transformed into whether cost of capital decreases".

For valuation purposes, financial statements should assist users to assess firms' expected payoffs and payoff covariances. The value of the firm, and its cost of capital, both depend on both parameters. It will be very rare if not impossible to identify types or instances of accounting information that are relevant to one of these parameters and not the other. That would require for example a change in the joint distribution of payoffs $f(V_1, V_2, ..., V_n)$ that changes the mean $E[V_j|\cdot]$ but does not alter the covariance, $\text{cov}(V_j, V_M|\cdot)$.$^{24}$ Remarkably, even this unlikely occurrence will affect both "numerator" and "denominator" in asset valuation, since the discount rate (denominator), or CAPM cost of capital, is influenced by both payoff mean and covariance (as shown by LLV).

$^{24}$Note that the payoff covariance can be written as $\text{cov}(V_j, V_M) = E[V_j V_M] - E[V_j] E[V_M]$, which shows how a change in mean payoff $E[V_j]$ might tend to have an effect on $\text{cov}(V_j, V_M)$. 

24
Better financial reporting is not as a matter of principle about reducing the cost of capital. From an investment-under-uncertainty perspective, the role of information is to facilitate more accurate probability forecasts of future cash flows. On average, more information leads to more accurate probability forecasts, and more accurate probability forecasts will sometimes bring a rationally higher cost of capital. A good way to understand this is to consider how the price per dollar of insurance can sometimes increase when the insurer gains better information about the probability or amount of a possible claim.

Conversely, a lower cost of capital can be the product of bad information or less accurate probabilities, leading to over-pricing and, by the principles of economic Darwinism, inevitable economic decline. Accounting has an obvious role in helping markets to avoid mispricing of capital. In simple terms, accounting information should help investors to assess the relevant statistical parameters of firms’ cash flows more accurately, and hence to cause some firms a higher cost of capital, and other firms a lower cost of capital. Think again of the insurance analogy, and how further information can lead the insurance provider to either increase or decrease a given individual’s life insurance premium, or indeed all premia, or the average of all premia, across the market.

Appendix

Proof that the covariance does not depend on the quality of future information. A way to prove this point is to write the prior (ex ante) covariance in terms of the expected posterior (conditional) covariance. For notational convenience, the value $V_j$ of the given asset $j$ is written without the subscript.

$$\text{cov}(V, V_M) = E[\text{cov}(V, V_M|S)] + \text{cov}(E[V|S], E[V_M|S]).$$

(15)

Each of the two terms on the right of this equation is influenced by signal quality, but this influence is cancelled out when the two terms are added together.

More specifically, these two terms are

$$E[\text{cov}(V, V_M|S)] = E[E(VV_M|S) - E(V|S)E(V_M|S)]$$


and

$$\text{cov}(E[V|S], E[V_M|S]) = E[E(V|S)E(V_M|S)]$$

$$- E[E(V|S)]E[E(V_M|S)]$$


Hence, from (15) the covariance is

$$\text{cov}(V, V_M) = E[VV_M] - E[V]E[V_M],$$

and is unaffected by the error characteristics of $S$. 


References


The Effect of Information on Uncertainty and the Cost of Capital

David Johnstone
University of Sydney
Australia
How Does New or Better Information Affect the Cost of Capital?

More information always equates to less uncertainty, and people pay more for certainty. ...the end result is that better disclosure results in a lower cost of capital. (Foster FASB 2003)

Contrary argument says:

Point (i):
Information often brings greater uncertainty
\(i.e.\) posterior probabilities further from 0 or 1

Point (ii):
By the CAPM, any change in beliefs (e.g. more certainty) can increase the cost of capital
Definition of Cost of Capital

Cost of capital = CAPM equity discount factor

\[ E[R_j] = \frac{E[V_j]}{P_j} \]

Conditional Cost of Capital

Conditional cost of capital given signal \( S \)

\[ E[R_j|S] = \frac{E[V_j|S]}{P_j|S} \]
The Model - Generalization of LLV (2007)
Lambert, Leuz and Verrecchia (2007)

(i) One-period quadratic-utility (mean-variance) CAPM

(ii) Homogeneous joint beliefs $f(V_1, V_2, ... V_n | \cdot)$
over uncertain $t = 1$ payoffs $V_j$

(iii) Arbitrary payoff distribution $f(V_1, V_2, ... V_n | \cdot)$

(iv) Bayesian inference (a là Markowitz)

"Of course, none of us know probability distributions of security returns. But, I was convinced by Leonard J. Savage, one of my great teachers at the University of Chicago, that a rational agent acting under uncertainty would act according to "probability beliefs" where no objective probabilities are known; and these probability beliefs or "subjective probabilities" combine exactly as do objective probabilities." (Markowitz, 1991)
Bayes Treatment of Information Risk

Information risk is captured by the likelihood function
\[ f(S|V_1, V_2, ... V_n) \]

By Bayes theorem, information risk is assimilated within
\[ f(V_1, V_2, ... V_n | S) \]

\[ f(V_1, V_2, ... V_n | S) \propto f(V_1, V_2, ... V_n)f(S|V_1, V_2, ... V_n) \]

\[ \propto \text{prior} \times \text{likelihood} \]
Point (i)
Info can increase uncertainty

\[ p(H) = E[p(H|e)] \]

So \( p(H|e) \) can be higher or lower than \( p(H) \)

\[ p(H|e) = 0.5 \] implies maximum uncertainty

*The most precise accounting information (e.g. cash sales) can add to uncertainty*
Model-Free Relationship Between Information and Uncertainty

(a) Where uncertainty is variance

The Law of Conditional Variance

$$\text{var}(V_j) = E \left[ \text{var} \left( V_j | S \right) \right] + \text{var} \left( E \left[ V_j | S \right] \right)$$

$$\rightarrow E \left[ \text{var} \left( V_j | S \right) \right] < \text{var}(V_j)$$

Information reduces uncertainty only "on average"
(b) Where uncertainty is covariance

The Law of Conditional Covariance

$$\text{cov} \left( V_j, V_k \right) = E \left[ \text{cov} \left( V_j, V_k | S \right) \right] + \text{cov} \left( E \left[ V_j | S \right], E \left[ V_k | S \right] \right)$$

$$\rightarrow E \left[ \text{cov} \left( V_j, V_k | S \right) \right] \text{ need not be less than } \text{cov} \left( V_j, V_k \right).$$

New data can indicate higher covariance between variables than previously believed
Point (ii)
More certain beliefs can increase the cost of capital $E[R_j]$

$$E[R_j] = E[V_j]/P_j = \frac{E[V_j]R_f}{E[V_j] - \kappa \text{cov}(V_j, V_M)}$$

lower $E[V_j]$ \quad \text{higher \ \text{cov}(V_j, V_M)} \quad \rightarrow \text{higher cost of capital } E[R_j]$

More certainty that $V_j$ will be low brings

(i) lower mean payoff $E[V_j]$

(ii) higher discount factor $E[R_j]$

$\rightarrow$ Twofold empirical effect on $P_j$
Ideal Accounting Information

"Better accounting" should expose "poor" stocks and cause them a higher cost of capital

... and cause "good" stocks a lower cost of capital

Better resource allocation via better discrimination
Parameter or Estimation Risk

More information can produce more certain parameter estimates (i.e. tighter posterior distributions)

But the estimated parameter still shifts up or down

$\rightarrow$ conditional cost of capital up or down

Think about what the information says, ...not just its precision
Conclusions

We have subjective probability distribution \( f(V_1, V_2, \ldots V_n | \cdot) \)

Accounting information aids probability assessment
i.e more accurate \( f(V_1, V_2, \ldots V_n | \cdot) \)

To do its job, accounting information must sometimes bring greater uncertainty

New information can move subjective \( E[R_j | \cdot] \) up or down, just like prices go up or down

Better accounting information can bring a higher cost of capital for a single firm or even the whole market
One Specific Bayesian Model

$V_j$ is the uncertain $t = 1$ payoff

Bayesian prior

$V_j \sim N(\mu_0, \sigma_0)$

Information $x$ is unbiased draw from a normal distribution with mean $V_j$ with fixed precision $\sigma_x$

$x \sim N(V_j, \sigma_x)$

Bayesian posterior distribution of $V_j$ after $m$ draws is

$V_j \sim N(\mu_1, \sigma_1)$

where

$$
\mu_1 = \frac{(1/\sigma_0^2) \mu_0 + \left( m/\sigma_x^2 \right) \bar{x}}{\left( 1/\sigma_0^2 \right) + \left( m/\sigma_x^2 \right)}
\quad \text{and} \quad
\frac{1}{\sigma_1^2} = \frac{1}{\sigma_0^2} + \frac{m}{\sigma_x^2}.
$$
Uncertainty in Markets

Prediction Market Price of a $1 Bet on Cubs to Beat Florida Marlins

Expired: 15 Oct 4:03AM
Numerical Example

Two underlying independent causal factors, \( X \sim N(1, 1) \) and \( Y \sim N(1, 1) \). Now imagine \( V_1 = cX + Y \), and \( V_2 = X + dY \), where \( c \) and \( d \) are real constants.

The quantities \( V_1 \) and \( V_2 \) are therefore bivariate normal, and their parameters are \( E[V_1] = c + 1 \), \( E[V_2] = d + 1 \), \( \text{var}(V_1) = c^2 + 1 \), \( \text{var}(V_2) = d^2 + 1 \), and \( \text{cov}(V_1, V_2) = c + d \).

Hence, \( \text{cov}(V_1, V_M) = \text{var}(V_1) + \text{cov}(V_1, V_2) = c^2 + 1 + c + d \), and similarly \( \text{cov}(V_2, V_M) = d^2 + 1 + c + d \).

Plug these values into the equations for \( P_1 \) and \( P_2 \)

The resulting endogenous return on the "market portfolio" of risky assets is then \( E[V_1 + V_2]/(P_1 + P_2) \)
Plot of Expected Return Factors of Both Assets and the Market as Functions of $c$ (with $0 \leq c \leq 5$)

$(W = 10, b = 0.025, d = 3 \text{ and } R_f=1.10)$
Quadratic Utility (Mean-Variance) CAPM

\[ P_j = \frac{1}{R_f} \left\{ E[V_j] - \kappa \text{ cov}(V_j, V_M) \right\} \]

\( \kappa \) is a positive constant under increasing risk averse quadratic utility