Earnings Management?
Averaging, Sample Selection Bias, and Scaling Lead to Erroneous Inferences

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Abstract

A vast literature following Hayn [1995] and Burgstahler and Dichev [1997] attributes the so-called “discontinuity” in earnings distributions around zero to earnings management. Durtschi and Easton [2005] conclude that the shape of these distributions is not ipso facto evidence of earnings management. Jacob and Jorgensen [2007] question this conclusion based on evidence from an alternate set of analyses, which compares the distribution of fiscal year (t) earnings with a benchmark distribution comprised of a weighted average of six quarters of earnings ending in the third quarter of year t. We show that this comparison cannot provide evidence of earnings management. Rather, the difference between these distributions is a statistical artifact; averaging smooths the distribution of the underlying data. We show that more plausible explanations for the shapes of earnings distributions are sample selection bias and scaling; distributions that are not affected by these research design flaws do not exhibit patterns that suggest earnings are being managed to avoid losses. We reiterate the point in Durtschi and Easton [2005] that before one can draw conclusions regarding the presence/absence of earnings management, evidence beyond the shapes of these distributions must be brought to bear.
1. **Introduction**

A vast literature following Hayn [1995] and Burgstahler and Dichev [1997] attributes the so-called “discontinuity” in earnings distributions around zero to earnings management.\(^1\) Durtschi and Easton [2005] show that the shapes of these distributions are driven by sample selection bias and scaling. Jacob and Jorgensen [2007] introduce an alternate methodology and conclude that their findings suggest that “while scaling and associated selection biases might contribute to the observed discontinuities, they are not primarily responsible for these discontinuities.” We show that this conclusion is not supported by their analyses. Hence we reiterate the point in Durtschi and Easton [2005] that the shapes of frequency distributions of earnings metrics are not *ipso facto* evidence of earnings management; before one can draw conclusions regarding the presence/absence of earnings management, evidence beyond the shapes of these earnings distributions must be brought to bear.

The main point of this paper is that rigorous academic research can not be based on just an appeal to the popularity of the notion that the shapes of the earnings distributions are evidence of earnings management because: (1) evidence that this is so is sparse, perhaps even non-existent; and (2) alternative explanations are often very evident. We illustrate these alternative explanations.

The Jacob and Jorgensen [2007] – hereafter JJ – methodology consists of comparing the distribution of fiscal year (t) earnings with a benchmark distribution,

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which is, essentially, the average of the distributions of three “as-if” years of earnings.\textsuperscript{2} Each of the three “as-if” years span four consecutive quarters: (1) ending with quarter one of fiscal year $t$; (2) ending with quarter two of fiscal year $t$; and (3) ending with quarter three of fiscal year $t$. In other words, the benchmark is a weighted average of quarterly earnings over the six quarters ending in the third quarter of year $t$. JJ argue that earnings measured over these alternate years are less likely to suffer from the effects of managerial income manipulation than earnings of the fiscal year; they claim that differences between the distribution of fiscal year earnings and the distribution of the average of the three “as-if” years of earnings is evidence of earnings management.\textsuperscript{3}

We show that JJ’s conclusions are not supported by their analyses for several reasons. First, the analyses are based on Compustat quarterly data, which are often restated. Second, the distribution of a weighted average of earnings across two years is not a meaningful benchmark because averaging smoothes the distribution of the raw earnings data by combining the idiosyncratic performance in one year with the idiosyncratic performance in the next year. Third, the JJ sample selection criteria lead to the deletion of many more small loss observations than small profit observations. Finally, deflation, which affects the shape of the earnings distribution, has a different effect on the shape of the distribution of deflated fiscal year net income than on the shape of deflated “as-if” year income.

\textsuperscript{2} We provide more detail of the JJ benchmark distribution in section 2.

\textsuperscript{3} Since most of our observations relate to any earnings metric, we use the generic term “earnings distribution” when our description applies to earnings metrics in general. When our observations relate to a specific type of metric such as the distribution of deflated net income or the distribution of earnings per share, we will be specific in our reference.
We present two demonstrations of the effects of averaging on the shape of frequency distributions. First, we compare the distribution of scores from Australian Football League matches with the distribution of a weighted average of scores across matches. Second, we show that discontinuities in earnings distributions, which are evidently not due to earnings management (rather they are the result of sample selection criteria imposed by JJ), are eliminated when net income is averaged over “as-if” years.

Many studies that are based on analyses of distributions of net income around zero focus on the distribution of net income deflated by market capitalization. These studies implicitly assume that deflation will not distort the underlying distribution of net income; Durtschi and Easton [2005] – hereafter DE -- show that it does. Also, since the hypotheses in the extant literature focus on management of earnings to avoid small losses, and since the arguments underlying these hypotheses do not suggest that earnings are managed relative to beginning-of-period market capitalization, DE question whether it is ever appropriate to examine deflated earnings in an earnings management context.

Nevertheless, studies continue to be based on analyses of the distribution of deflated earnings (see, for example, Beaver, McNichols, and Nelson [2007] – hereafter BMN, Cohen, Mashruwala, and Zach (2008), Gunny, Jacob, and Jorgensen [2007], JJ, and Kerstein and Rai [2007]).\(^4\) We extend the analyses in DE and provide additional details regarding the way that deflation by beginning-of-year market capitalization results in the unusual shape of the distribution of deflated net income. In particular we show how the “divot” in the distribution (the piece taken out of the intervals immediately to the left of zero which appears to be deposited immediately to the right of zero causing a

\(^4\) Beaver, McNichols, and Nelson [2007] claim that their “results are not an artifact of deflation”. This conclusion is unsubstantiated by their analyses. We will elaborate on this point.
“mini-peak”) is created. We also elaborate on the observation in DE that the distribution of end-of-year prices differs from the distribution of end-of-quarter prices, providing another reason why the distribution of deflated fiscal-year net income will differ from the distribution of weighted-average deflated net income across years.

The remainder of the paper proceeds as follows. In section 2, we show that the distribution of an average based on data that spans two fiscal years is not a meaningful benchmark for comparing the distribution of fiscal year data. In section 3, we show how the frequency distributions of net income, earnings per share, and change in earnings per share, which have no unusual patterns at zero, are changed by sample selection criteria in Burgstahler and Dichev [1997] – hereafter BD -- and JJ into distributions that show an irregularity around zero. In section 4, we show that averaging across years removes this irregularity providing another demonstration of the fact that a difference between the distribution of averaged earnings and the distribution of the underlying earnings data is not evidence of earnings management. In section 5, we provide a detailed analysis of the effects of scaling; we show that the divot in distributions of deflated earnings immediately to the left of zero and the mini-peak immediately to the right of zero is evidence of the effects of scaling rather than evidence of earnings management. We present our conclusions in Section 6.

5 Although DE show evidence that supports the conclusion that scaling affects the shape of earnings distributions they do not show evidence that scaling leads to the unusual pattern in the distribution in the immediate vicinity of zero (that is, the divot). This unusual pattern is often the focus of discussion of the BD results. For example, in the textbook, “Financial Reporting and Analysis,” Revsine, Collins, and Johnson [2005] show the discontinuity in the distribution of deflated earnings reported by BD and describe it as follows: “The striking feature of this graph is the discontinuity in the number of firms reporting slightly negative earnings versus slightly positive earnings. Substantially fewer firms fall just below zero compared to those reporting earnings at or just above zero. What appears to be happening is that managers of firms that would otherwise report small losses...are finding ways to prop up earnings in order to move reported profits into the...slightly positive range. One way of doing so, even in troubled times, is to exploit the flexibility in GAAP or to resort to a variety of accounting gimmicks to push earnings into the positive range.” (Emphasis added).
2. **A Weighted-Average is not a Meaningful Benchmark**

The method in JJ relies on a comparison of the distribution of fiscal year t income to a benchmark distribution that is the average of three distributions of net income for three “as-if” years; the first “as-if” year ends with quarter one of year t, the second ends with quarter two of year t, and the third ends with quarter three of year t. The data comprising the average are described in Figure 1. Figure 1 shows how averaging over three “as-if” years effectively assigns a weight of 1/12 to earnings of quarter two of year t-1, 2/12 to earnings of quarter three of year t-1, 3/12 to earnings of quarter four of year t-1, 3/12 to earnings of quarter one of year t, 2/12 to quarter two of year t and 1/12 to quarter 3 of year t.

We argue in this section that this benchmark is not meaningful for two reasons. First, the start-date and end-date of a fiscal year are likely chosen for sound economic reasons rendering a comparison to a benchmark that is a weighted combination of current year and prior year earnings questionable. Second, comparison to a benchmark that is a weighted average of earnings that span two fiscal years will offset idiosyncrasies in the individual years, thus affecting the shape of the earnings distribution.⁶

2.1. **The Fiscal Year is Chosen for Sound Economic Reasons**

The start-date and end-date of a fiscal year is consciously chosen by management for sound economic reasons. Managers, investors, creditors, labor unions, etc., tend to focus on fiscal-period earnings, rather than earnings of “as-if” years that end at dates other than the fiscal-year end. Compensation packages are generally adjusted annually based on fiscal-year performance; investment, marketing, production, and inventory

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⁶ While we focus most of our discussion on the weighted-average, most of the points apply to any one of the “as-if” years, which, implicitly, average idiosyncratic earnings from quarters of the prior year with idiosyncratic earnings of the current year.
decisions and contracts with suppliers are based on a fiscal year. In addition, earnings may contain a seasonal component that is related to the company’s choice of fiscal year; retail companies, for example, often receive a large portion of their revenue in the last fiscal quarter.7

Using the distribution of a weighted-average of earnings across two fiscal years as a benchmark against which to compare the distribution of fiscal-year earnings assumes that the business and economic environment of one fiscal year is essentially the same as that of the next. Specifically, this assumes; that there is no growth, that the competitive and economic environments are the same, that innovations are evenly spread throughout the years, that supply, distribution and sales problems/issues/highlights/surprises are spread evenly over time, and that various contracts for compensation, contracts with suppliers, etc., do not change between fiscal years.

Another problem with combining earnings numbers that span two fiscal years stems from the integral theory of accounting; that is, each quarterly statement is considered an installment in a fiscal year (APB 28). Examples include expenses that can be deferred and allocated over several periods in a fiscal year based on sales volume, production levels, etc. It follows that sets of quarterly earnings numbers that span two fiscal years combine “installments” from two different fiscal years. A second, perhaps more serious problem, is that numbers in interim reports are often restated at the annual audit.

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7 Sixty percent of retail department and variety stores have a fiscal-year end of late-January after the completion of the holiday season when their inventory is depleted. They make investment decisions (particular regarding inventory), re-stock their shelves, and begin a new cycle where the outcome of their investment decisions, advertising strategy, etc. will be realized over the forthcoming fiscal year culminating in the next holiday season when the success or failure of their investment decisions becomes known. By contrast, only 17 percent of retail grocery stores have a January fiscal-year-end, as they do not have the same degree of seasonality in their sales.
2.2. Averaging Changes the Shape of the Distribution

Averaging net income across fiscal years has two effects on the distribution of averaged data; it smoothes the irregularities in the distribution and draws observations toward the mean. Averaging across periods tends to offset idiosyncrasies from one period against idiosyncrasies of another period.\(^8\) It follows that calculating a weighted-average performance measure creates a smoother distribution. We use an analogy of team-scores from the Australian Football League (AFL) to demonstrate this point.

2.3. The AFL Analogy

Similar to a business reporting cycle, each football match has four quarters. If the score for each quarter of a football match is analogous to quarterly earnings of the firm, the score for the match (that is, the sum of the scores for each of the four quarters) is then analogous to the annual net earnings.\(^9\)

We follow the methodology in JJ to create three “as-if” matches which we average together to form a benchmark distribution of match-scores. As in JJ, we add four consecutive quarters of scores (that is, for the four consecutive quarters that end with

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\(^8\) The devil’s advocate may argue that earnings management may be one of these idiosyncrasies. We agree. But we hasten to add: (1) earnings management is just one of many factors that may differ across observations; and (2) researchers should not assume the existence of earnings management if they have not ruled out other, possibly more plausible, explanations.

\(^9\) The analogy can be expanded by noting other parallels including: both the football coach and corporate management prepare the team (firm) with a strategy prior to each match (year); both the coach and management team have the opportunity to modify this strategy during the match (year) in light of the competition encountered; the firm’s goal is to maximize the year’s profits and the coach’s goal is to maximize the match score; both the match and fiscal year have a natural beginning and end, and that end is a time to tally up and make judgments on the performance of the team or firm, and to think about strategy for the next match (year). In both cases; the competition, the makeup of the team (firm), the strategy, may differ from match (year) to match (year). Just as the fourth quarter in business is the time to “balance out” the firm’s earnings, as influenced by the estimates made in the prior quarter’s installments, similarly the football team’s strategy may change in the fourth quarter depending on the performance in the first three quarters. In combining either income across fiscal years or scores across matches, idiosyncratic superior performance in the previous year (match) may be averaged with idiosyncratic inferior performance in the current year (match). This averaging may (likely will) result in a change in the shape of the distribution of scores (earnings).
quarter one of the current match, with quarter two of the current match, and with quarter three of the current match). We then average the three “as-if” match-scores in the same way that JJ average their three “as-if” years of earnings and we use that average as the benchmark expected scores for the current match.

Whether one believes this analogy is sufficiently sound is, in some ways, irrelevant as the point of our analyses is to show the inevitable effects of averaging on the shape of frequency distributions; the distribution is smoothed and observations are drawn toward the mean.

Our data comes from 22 rounds of matches among the 16 teams that have comprised the AFL over the years 1997 to 2006. In other words we analyze data for 220 matches for each of 16 teams (a total of 3,420 matches). The same 16 teams have comprised the AFL for these 10 years. Points in the AFL are scored in two different ways: goals (when the ball is kicked between the two goal posts) are awarded six points and behinds (when the ball is kicked between the goal post and a behind post) are awarded one point. The winner is the team that has the highest total score by the end of the match. If the scores are even at the end of play, the match is a draw.

Figure 2 demonstrates the effect of averaging scores across football matches. The dark line, which represents the average score of three “as-if” matches, is noticeably smoother than the light line which shows the actual distribution of match scores. This smoothing effect of averaging is almost inevitable, as match-specific events/scores in one match are averaged with match-specific events/scores in another.

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101997 represents a natural year to start the analysis because the teams comprising the AFL have been the same from that year until today.
11 One point is also awarded if the ball hits a goal post, or if an opposition player sends the ball between the goal posts by touching it with any part of the body.
2.4. **Differences between the Benchmark Distribution (Averaged Data) and the Distribution of Raw Data are not Evidence of Earnings Management**

In the next two sections of the paper we show that, similar to our football analogy, differences between the distribution of averaged net income and the distribution of fiscal-year net income may not be used as evidence of earnings management. We provide details of the results from our analysis of the distribution of net income and we summarize the results of similar sets of analyses of change in net income, earnings per share, and change in earnings per share.

We show, in section 3, that the distribution of all observations of annual fiscal period net income available on Compustat in our sample period does *not*, in seeming contradiction to BD and JJ, exhibit an irregularity around zero net income. Next we apply the JJ (and/or the BD) sample selection criteria and show that the resulting distribution then exhibits a distinct irregularity. In other words, there is a discontinuity in this distribution that is due to sample selection bias; it is not due to earnings management. We then show, in section 4, that the weighted-average of these earnings does not exhibit this discontinuity. Again this demonstrates that the weighted-average smoothes the earnings distribution and therefore is not an appropriate benchmark for evaluating the distribution of fiscal-year earnings. More importantly, we show that for a sample where the null hypothesis of no earnings management is descriptive of the distribution, averaging leads to a distribution, which if used as the benchmark would lead to rejection of the null; of course this is nonsensical.
3. **Sample Selection Bias Leads to an Irregularity in Earnings Distributions**

For each set of analyses, we obtain the required data from the Compustat files spanning 1977 to 2006. This sample period includes the periods used by BD (1976 to 1994), JJ (1981 to 2001), and DE (1983 to 2003) as well as the more recently available years. We replicate the relevant findings in BD, JJ and DE and we find that their results hold in our sample. To be consistent with all three papers, we eliminate regulated firms.

3.1. **The Effect of Sample Selection on the Distribution of Net Income**

The line above the white (top) section in Figure 3 shows the frequency distribution of all observations of reported annual net income (annual Compustat data item 172) for 1977 to 2006. We partition firms into $100,000 intervals of net income as in DE, JJ, and Dechow, Richardson, and Tuna [2003]. As in DE there is no discontinuity in this distribution around zero.

The JJ sample selection criteria, which severely censor the data, affect the shape of the distribution of annual net income. The line above the gray area in Figure 3 shows the observations that survive the first JJ sample selection criterion; these observations have four consecutive quarters of observations of net income on the Compustat quarterly file. This criterion leads to deletion of sixteen percent of the observations. More importantly, a very large proportion of small net income observations, particularly small negative net income, are deleted. For example, 51 percent of the smallest negative net income observations (those in the interval $0 to -$100,000) are eliminated while only 42 percent of the smallest positive net income numbers are eliminated. Requiring four
consecutive quarters of data has introduced a discontinuity into the distribution which has nothing to do with earnings management.\textsuperscript{12}

Requiring four quarters of consecutive net income data is not the only selection criterion imposed by JJ. Like BD, JJ deflate net income by beginning-of-year market capitalization to obtain a distribution that is similar to BD, Figure 1. When the additional data requirement of the availability of beginning-of-year market capitalization is included, even more observations are lost; the black (lowest) area in Figure 3 graphs the observations that survive this deletion criterion. Now, 73 percent of the observations in the lowest negative net income interval are deleted while 65 percent of the observations in the lowest positive net income interval are deleted.

It is evident that the sample selection criteria in JJ severely censor the data and this censoring affects the shape of the earnings distribution in ways that likely have nothing to do with earnings management. Other studies also invoke severe sample selection criteria and erroneously infer that the resulting shape of the earnings distribution is due to earnings management. For example, BMN select firms with positive sales, positive assets, and beginning of year market capitalization; they observe that this leads to the elimination of more observations in the intervals immediately to the left of zero than in the intervals immediately to the right. We do not disagree that it is extremely difficult, perhaps impossible, to create a sample where variables necessary for subsequent tests do not exclude more observations from either the left or the right of zero. Given that

\textsuperscript{12} Commentators on this paper have made the observation that it is possible that Compustat has four quarters of consecutive data only for firms that have managed earnings (that is, non-managers do not have four quarters of consecutive data). This seems unlikely. If this is the assertion, more evidence must be provided before we can conclude that the shape of the distribution is due to earnings management rather than sample selection bias.
difficulty, serious consideration must be given to the effect of sample selection before drawing conclusions based on evidence of differences across the zero threshold.

3.2. The Effect of Sample Selection on the Distribution of Earnings per Share

JJ also examine the distribution of earnings per share in a manner that is very similar to their analysis of net income and deflated net income. However, rather than using reported earnings per share, they use Compustat quarterly data item 27. According to Compustat, in interim quarters (the basis of the JJ benchmark): (1) quarterly data item 27 will equal (within four cents deviation) the sum of the prior four quarters of basic earnings per share, excluding extraordinary items; and, (2) one to three quarters of a four quarter total may include restated data whereas the others may not. In addition, from the data available in the Compustat files, it appears that the JJ sample selection criteria remove a larger proportion of firms reporting zero profits and small losses than firms reporting small profits.

It is difficult to measure the complete effect of sample selection bias for quarterly Compustat data item 27 because of the way each observation of data item 27 is created. Figure 4 describes the creation of data item 27. Notice in Figure 4 that each observation of quarterly data item 27 is created by "adding four quarters of available for common and dividing by the 12-months moving shares figure."13 This means that, to meet the JJ sample selection criteria, where an observation only appears if there are complete observations for quarterly data item 27 for all quarters in a fiscal year, there must be seven consecutive quarters of income data available for any particular company. So, for example, if an observation of net income data for quarter three in the prior year is missing, this will preclude creation of data item 27 for quarter two of the current year. If

13 See Compustat manual.
quarter two of the current year is missing, the entire current year would be deleted from the JJ analysis.

Quarterly Compustat data item 27 for quarter four is fiscal year earnings per share. We can objectively determine which of these fiscal year earnings are deleted by JJ because there is a comparable data item on the Annual Compustat file (data item 58). The dark line in Figure 5 shows the frequency distribution of annual earnings per share (data item 58); the light line shows those observations that survive to be included in the JJ sample. Thirty percent of the smallest negative earnings per share observations (-$0.01) are missing and 29 percent of the smallest positive earnings per share observations ($0.01) are missing. This hints at what may be happening in the benchmark data. There is, however, no way to determine what effect the sample selection criteria used by JJ have on the distributions used in the benchmark weighted average of three observations of quarterly data item 27 because there is no comparable reported “as-if” annual earnings per share datum.

In sum, we question whether meaningful conclusions can be drawn from the JJ analysis of earnings per share because Compustat states that, in interim quarters (the JJ benchmark), quarterly data item 27 will equal (within four cents deviation) the sum of four quarters of basic earnings per share, excluding extraordinary items and that one to three quarters of a four quarter total may include restated data whereas the others may not. In addition, from the data we can examine, it appears the JJ sample selection criteria remove a larger proportion of firms reporting zero and small losses than firms reporting small profits.
4. **Averaging Smoothes Irregularities in Earnings Distributions**

JJ plot the distribution of “as-if” year net income for each of the three years ending in quarters one, two and three using the sample-selection criteria we described in section 3.1; the sample for each of the three “as-if” years and the sample for the fiscal year are formed separately. We first replicate the JJ analyses using these four samples of observations and then we repeat these analyses for the sub-set of observations that have data for each of the “as-if” years and for the fiscal year to ensure that the JJ results are indeed due to averaging rather than due to differences among the samples.

These JJ distributions of “as-if” year net income for each of the three years ending quarters one, two, and three are replicated in Figure 6, Panels A, B, and C. JJ average these three distributions and compare that average to the actual distribution of fiscal year net income.14 The distributions of averaged net income (light line) and the distribution of fiscal year net income (dark line) are shown in Panel D. JJ argue that, the difference between the distribution of averaged net income and the distribution of fiscal year net income (which exhibits a selection-induced discontinuity) is evidence that their results “do not support the … DE… assertions that the [BD] results on the discontinuity at zero in earnings is attributable to scaling”.

The observation that a discontinuity in the earnings, which is due to sample selection, is removed by averaging provides no evidence either for or against the effects of scaling on the earnings distribution. All that JJ have shown is that averaging earnings from quarters of one year with earnings from quarters of the next years to form “as-if” annual earnings smoothes the earnings distribution.

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14 As in JJ, we present the percentage of observations in each cell; this then facilitates comparison across distributions drawn from samples of different size.
There is a possibility (albeit remote) that the differences between the distributions in Figure 6 are due to differences in the samples. To ensure that this is not so, we repeat the analyses for the sub-sample of observations for which JJ observe income for each of the “as-if” years and for the subsequent fiscal year. The results are presented in Figure 7. Again the averaging effect of combining quarterly net income observations across consecutive fiscal years is evident. In short, we have demonstrated that a distribution created under the null hypothesis of no earnings management has been changed by averaging. In other words, the JJ methodology cannot be used to provide evidence of earnings management.

4.1. Averaging of Earnings per Share

As shown in Figure 8 averaging has the same effect on the distribution of quarterly data item 27 as it does on the distribution of net income: averaging smooths the distribution. The dark line is the distribution of the average of data item 27 for “as-if” years ending in quarters one, two, and three; the light line is distribution of data item 27 for quarter four. Once again, the distribution of the average of the observations is smoother than the distribution of the raw data. The difference between the dark and light line is due to averaging; it is not evidence of earnings management.

4.2. Analysis of Changes in Earnings per Share

JJ do not tabulate results for their analysis of change in earnings per share but they note that they perform a “similar analysis” on changes in primary earnings per share. They conclude that there are significantly fewer than expected decreases of one cent in fiscal year earnings per share from the previous year.\footnote{The JJ notion of the “expected” distribution is the weighted-average distribution.} Considering the problems inherent in quarterly data item 27 (that is, each observation requires four quarters of data...
and it is, according to Compustat, imprecise) we question whether any conclusions can be drawn regarding earnings management based on an analysis of changes in this data item.

In our analysis (un-tabulated) we find no discontinuity in the distribution of changes in quarterly data item 27 even using the JJ sample selection criteria. Therefore, we posit that the JJ results are due to the effects of averaging the benchmark.

To demonstrate this point, we compare change in fiscal year earnings per share (from t-1 to t) to the average of that change and the change from fiscal years t-2 to t-1. The results of these analyses are shown in Figure 9. The averaging effects that were observed in the analyses of earnings levels are more pronounced in these analyses of earnings changes. The difference to which JJ refer when stating that the chances of observing the sequence of expected frequencies is 1 in 131,072 is, in effect, the difference between the two lines in this figure (the dark line representing the distribution of the average change in annual earnings per share and the light line representing the change in annual earnings per share). This result suggests that the JJ comparison to the changes in “as-if” year earnings per share reflects no more than the effects of averaging; it cannot be interpreted as evidence of earnings management.

5. The Effects of Scaling

Many studies that are based on analyses of distributions of net income around zero focus on the distribution of net income deflated by market capitalization (see, for example, BMN, BD, Gunny, Jacob, and Jorgensen [2007], JJ, and Kerstein and Rai

\[\text{16}\textrm{ To create their “difference” in earnings distribution, JJ require seven quarters of quarterly data item 27 (that is, 11 consecutive quarters of earnings per share data). We add just one more quarter in our analyses.}\]
These studies implicitly assume that deflation will not distort the underlying distribution of net income; DE show that it does.

In this section, we extend the analyses in DE and provide additional details regarding the way that deflation by beginning-of-year market capitalization results in the unusual shape of the distribution of deflated net income. In particular, we show how the “divot” in the distribution immediately to the left of zero and the “mini-peak” immediately to the right of zero are created. For this analysis, we use the annual Compustat data, as in BD.

We also elaborate on the observation in DE, related to the JJ analysis, that the distribution of end-of-year prices differs from the distribution of end-of-quarter prices, providing another reason why the distribution of (deflated) fiscal-year net income will differ from the distribution of weighted-average (deflated) net income across years. For this second set of analyses, we use annual net income as calculated by JJ.

5.1. Difference in Market Capitalization of Companies Reporting Profits and Companies Reporting Losses

DE report that stock prices are lower for each dollar amount of loss than for the same dollar amount of profit. In a similar vein, Figure 10 shows, for each $100,000 net income interval, the 25th-percentile, median, and 75th-percentile of the distribution of beginning-of-year market capitalization (that is, price – Annual Compustat data item 199 – times number of shares outstanding – Annual Compustat data item 25).¹⁷

Figure 10 demonstrates that, like prices, the market capitalization of companies reporting a loss tends to be smaller than for companies reporting the same dollar amount of profit. For example, companies/observations with annual net income in the smallest

¹⁷ Figure 10 is similar to Figure 6 in DE, which shows the distribution of prices for each cent of earnings per share.
negative interval (between $0 and -$100,000) have a median market capitalization of $2.33 million while observations in the smallest positive net income interval (between $0 and $100,000) have a median market capitalization of $4.45 million. The difference in these medians may be used to demonstrate the point made by DE that scaling will cause observations with positive net income to be drawn away from zero to a lesser extent than observations with negative net income (-$0.1m/$2.33m = -0.0420 while $0.1m/$4.45m = 0.0223).  

The larger deflator to the right of zero will cause an upward step in the distribution of deflated net income at zero; but this step is not evident. Rather the distribution of deflated net income has a divot immediately to the left of zero and a mini-peak immediately to the right of zero. In the next section we divide observations into two sub-samples; those in the net income interval on either side of zero (-$100,000 to $100,000) and all other observations in order to: (1) show that the upward step in the distribution is evident for a subset of observations; and (2) show how the divot and mini-peak are formed.

5.2. Detailed Analysis of the Effects of Scaling

Figure 11, Panel A plots the distribution of net income deflated by beginning-of-year market capitalization (net income – Annual Compustat data item 172 divided by (price – Annual Compustat data item 199 – times number of shares outstanding – Annual Compustat data item 25)) for companies reporting net income between -$100,000 and $100,000.

BMN claim that the distribution of market capitalization is symmetric around zero. This claim is based on visual inspection of Figure 5, Panel B in their paper, which is the distribution of the mean of market capitalization analogous to the distribution of the quartiles of market capitalization in our Figure 10. The claim is not supported by the results in their figure. For example, the mean market capitalization for the observations in the $100,000 interval to the left of zero is $8.34m, while the mean market capitalization for the observations in the cell immediately to the right of zero is $13.79m (when the BMN sample selection criteria are applied, these means increase to $8.54m and $13.97m).
$100,000.\textsuperscript{19} Figure 11, Panel B shows the percentage of observations in each deflated net income interval. There are 2,383 observations with net income between $0 and $100,000 that meet the BD sample selection criteria, and 1,559 observations between $0.00 and -$100,000.

The step in the distribution of deflated-net-income at zero caused by a larger deflator to the right of zero is evident in Figure 11, Panels A and B. After deflation, 39 percent (929/2383) of the observations with positive net income are in the deflated-net-income interval (0.005) immediately to the right of zero. For these observations beginning-of-year price per share is at least 200 times earnings per share. On the other hand, only 23.6 percent (368/1559) of the observations with negative net income are in the deflated-net-income interval (-0.005) immediately to the left of zero; not surprisingly a smaller percentage of loss observations have a price per share that is greater than 200 times the amount of the per share loss than observations that have a price per share that is greater than 200 times the amount of per share profit.

Stocks fall in the second interval to the left and to the right of zero (that is, deflated net income between -0.010 and -0.005 and between 0.005 and 0.010) if their price per share is between 100 and 200 times earnings per share. Again, the percentage of loss observations with prices in this range (12.8 percent) is less than the percentage of profit observations (15.5 percent) with prices in this range. Prices for observations in the third interval to the left and to the right of zero range from 66.67 to 100 times earnings and the percentage of loss observation in this interval (7.6 percent) is slightly less than the percentage of profit observations in this interval (8.3 percent).

\textsuperscript{19} The interval width shown in Figure 10 is 0.005 as in JJ and BD.
Since the percentage of the negative net income observations in the three deflated-net-income intervals next to zero (43.99 percent) is much less than the percentage of positive net income observations in the three deflated net income intervals next to zero (62.87 percent), we see a greater percentage of negative net income observations in the deflated net income intervals where prices are less than 66.67 times the amount of the loss than in the deflated positive net income intervals where prices are greater than 66.67 times the amount of the profit.

Figure 11, Panel C plots the distribution of net income deflated by beginning-of-year market capitalization for companies reporting net income less than -$100,000 and greater than $100,000. The peak in this distribution to the right of zero is in the interval 0.050 to 0.055 (that is, the price-to-forward earnings multiple is in the range 18.18 to 20). The lower frequency of observations in the intervals to the right and to the left of this peak reflects the notion that next-period realized earnings are less and less likely to be valuation sufficient as they become more and more extreme. In other words, high price-earnings multiples reflect transitorily low earnings and low price-earnings multiples reflect transitorily high earnings; price captures the entire stream of future expected earnings, not next-period realized earnings.

The transitory nature of earnings is implicit in the entire distribution of negative earnings; a non-zero price reflects the market’s expectation that positive earnings are expected in the future. The peak in the deflated net income distribution to the left of zero in the interval -0.015 to -0.020 is a manifestation of expectations of future earnings that lead to prices that draw transitorily negative net income observations toward zero but
these prices are such that more observations tend to be drawn to the interval that is almost at zero but they are not drawn all the way to the interval immediately to the left of zero.

Figure 11, Panel D combines Panels A and C to recreate BD Figure 1. The light (lowest) section is the distribution of deflated net income for observations of net income between -$100,000 and $100,000. The dark (uppermost) section is the distribution of deflated net income for all other observations of net income. This figure shows that: (1) the larger (smaller) proportion of small positive (negative) net income observations (less (greater) than $100,000) that are drawn into the interval 0 to 0.005 (0 to -0.005) by deflation leads to the mini-peak immediately to the right of zero; and (2) the smaller proportion of observations with net income less than -$100,000 that are drawn into the deflated net income interval -0.005 to 0 compared to the proportion of observations that are drawn to the next three negative net income intervals (-.005 to -.020) leads to the divot immediately to the left of zero.

5.3. Scalers Differ Between Fiscal Year Loss Observations and “as-if” Year Loss Observations

Figure 12 captures the spirit of DE footnote 32 where it is noted that prices may be different at the beginning of the fiscal year than they are at the beginning of the three JJ “as-if” years. This figure shows the distribution of beginning-of-year market capitalization (price – Quarterly Compustat data item 14 -- times shares outstanding – Quarterly Compustat data item 61) for the “as-if” years ending in quarters one through three and the distribution of beginning-of-fiscal-year market capitalization for each net income interval.

Figure 12 shows that, while there are differences in the distributions of market capitalization on both sides of zero, there is a more noticeable difference in the
distributions to the left of zero. Specifically, firms reporting negative net income tend to have higher market capitalization at the beginning of the “as-if” years than at the beginning of the fiscal year. The effect is that when JJ compare deflated fiscal year income to the average of the three deflated “as-if” year’s income, deflation will draw fiscal year negative net income observations toward zero to a lesser extent than it will draw the “as-if” year observations toward zero. By contrast, there are no such distinct differences to the right of zero in the deflators of positive net income. This difference in deflators for negative net income will contribute to the JJ conclusion that there are too few observations of small negative net income in the distribution of fiscal year income; but this difference in the number of observations is not due to earnings management.

5.4. Dealing with Scale Differences between Profit and Loss Observations

The arguments for analyzing the distribution of deflated net income rather than the distribution of net income are: (1) deflation is an attempt to homogenize firms (DeGeorge, Patel and Zeckhauser [1999]), and (2) firms are drawn from a broad range of firm sizes (BD). But these studies do not provide reasons why heterogeneity or differences in firm size may affect the analyses. More importantly, the implicit assumption is that deflation will not distort the underlying distribution of net income. The analyses in DE and in Sections 5.1, 5.2, and 5.3, above, suggest that it does.

Also, firms do not manage earnings deflated by beginning-of-period price; they may manage net income or earnings per share. It follows that the distributions that may be used to show evidence of earnings management are the distributions of net income and
of earnings per share. DE show that these distributions do not show evidence of an irregularity at zero.\textsuperscript{20}

Studies that recognize the fact that deflation affects the shape of the net income distribution attempt to control for scale in their research design. For example, Kerstein and Rai [2007] – hereafter KR -- examine movements among the four BD deflated net income intervals around zero. In their analysis movements constitute their coded dependent variable in a logistic regression. They attempt to control for the effect of scaling by adding log of market capitalization as another independent variable. While this may help, the effects of scale are likely to remain because the relation between the magnitude of net income and magnitude of market capitalization, which affects the classification of the data in KR, likely will not be captured by simply adding log of market capitalization as another explanatory variable. An obvious way of avoiding spurious effects of scaling is to do the analyses in KR using movements among net income and earnings per share intervals rather than movements among deflated earnings intervals.

\textsuperscript{20} BMN state that DE “argue that share deflation considerably mitigates the discontinuity at zero in the distribution of earnings” and “that share deflation is superior to deflation by other financial variables because the number of shares does not differ systematically between loss and profit observations and thus will not induce a spurious discontinuity.” DE do not make either of these arguments. Rather, they argue that firms do not manage earnings deflated by financial variables; and, though it is possible that firms manage earnings per share, the distribution of earnings per share does not show evidence of earnings management. DE observe that the number of shares outstanding does not differ significantly between loss and profit observations (while the distribution of financial variables does differ significantly between loss and profit observations). Further DE argue that even if the number of shares outstanding did differ between loss and profit observations, such a difference would be moot for a number of reasons: (1) earnings per share itself (rather than net income deflated by beginning of period market capitalization) is the focus of attention by investment analysts and investors, thus is the most likely object of management, (2) management of shares outstanding is a highly visible activity that will be ineffective if the intent is to obscure what is actually happening in the firm, and (3) management of earnings per share from negative to positive is unlikely to be done by managing shares outstanding.
6. Conclusions

In this paper we have examined conclusions regarding earnings management that have been drawn from comparisons of distributions of earnings and earnings changes. We show that a weighted average than spans consecutive years is a faulty benchmark, which: (1) makes little economic sense; and, (2) is unreliable because of the characteristics of quarterly data on which the analyses are based. We note that quarterly data are often, and inconsistently, restated; thus using such data to find evidence of a nuance such as earnings management to avoid a loss is unreliable, at best. In addition, and perhaps most importantly, we have shown how averaging smooths the distribution of the raw data.

Next we show that sample selection criteria eliminate more small loss observations than observations of small profits creating irregularities in earnings distributions around zero. We reiterate the point made in DE that, if a sample selection criterion leads to the deletion of more observations of small losses than observations of small profits, the shape of earnings distributions in the vicinity of zero cannot be used as evidence of earnings management.

Finally, we demonstrate how the choice of deflator contributes to the final shape of the (deflated) earnings distributions. We show that market capitalization for observations of small losses tends to be less than for observations of small profits and show how this difference in market capitalization tends to draw observations of small losses toward zero to a lesser extent than observations of small profits. We also show that market capitalization (the deflator) for loss firms differs between the JJ benchmark
years and the fiscal year, further contributing to the results JJ report regarding the differences between their deflated benchmark earnings and deflated fiscal year earnings.

We conclude, as do DE, that the observed shapes of earnings distributions around zero is not *ipso facto* evidence of earnings management; rather, additional evidence beyond the shape of the distribution must be brought to bear. In this paper we have provided more plausible explanations for the shape of these distributions; sample selection bias, scaling, and averaging. The distributions that are not affected by these research-design flaws do not exhibit patterns that suggest earnings are being managed to avoid losses.
REFERENCES


Figure 1 shows how JJ create the numerator used in their analysis. Fiscal year net income is the sum of quarterly net income for quarters one through four of a fiscal year (Quarterly Compustat data item 69), divided by beginning of year market capitalization: price (Quarterly Compustat data item 14) multiplied by number of shares outstanding (Quarterly Compustat data item 61). This fiscal year is compared to the average of net income calculated for three “as-if” years. These three “as-if” years of net income are the sum of four consecutive quarters of Quarterly Compustat data item 69 ending in quarter 1, quarter 2, and quarter 3 of year t. The income of each “as-if” year is deflated by beginning-of-year market capitalization for that year. In other words, JJ assign a weight of 1/12 to earnings of quarter two of year t-1, 2/12 to earnings of quarter three of year t-1, 3/12 to earnings of quarter four of year t-1, 3/12 to earnings of quarter one of year t, 2/12 to quarter two of year t and 1/12 to quarter 3 of year t. The sample selection process requires seven quarters of continuous quarterly net income data.
Figure 2: The Effect of Averaging on Scores from Australian Football League Games

Figure 2 shows the frequency distribution of the scores for 22 matches between the 16 teams in the Australian Football League from 1997 to 2006: a total of 3,420 matches are in the sample. The dark line is the average of three “as-if” matches where the average is calculated in the same way as in JJ. Each “as-if” match consists of four consecutive quarters ending: (1) in quarter one of the match in week t; (2) in the second quarter of the match in week t; and (3) in the third quarter of the match in week t. The light line is the frequency distribution of actual match scores.
Figure 3: The Effect of Sample Selection Criteria on the Frequency Distribution of Net Income

Figure 3 shows the effect of the JJ sample selection requirements on the frequency distribution of annual net income. Intervals are $100,000 wide as in DE and Dechow et al (2003). The line above the white (uppermost) section shows the frequency distribution of all available observations of annual net income (Annual Compustat data item 172) for the 108,299 observations between -$5,000,000 and $7,000,000. 29,998 observations are < -$5,000,000 and 29,558 observations are > $7,000,000. The line above the gray (middle) section plots the frequency distribution of fiscal year net income computed as in JJ as the sum of four quarters of data (Quarterly Compustat data item 69) after removing the observations that do not meet the first JJ sample selection criteria of having four consecutive quarters of data for that fiscal year; 134,495 of the 161,700 observations that meet this sample selection criteria are shown. The black (bottom) section plots fiscal year net income computed as in JJ as the sum of four quarters of data (Compustat data item 69) after removing those observations that do not meet both JJ sample selection criteria: four consecutive quarters of data and beginning-of-year market capitalization (Quarterly Compustat data item 14 times Quarterly Compustat data item 61).
Figure 4 describes the Compustat computation of quarterly data item 27. Compustat adds “four quarters of available for common and divides by the 12-months moving shares figure.” Data 27 for quarter four “will be the same as the annual earnings figure reported to shareholders, in interim quarters this figure will equal (within four cents deviation) the sum of four quarters of Earnings per Share (basic) – Excluding extraordinary items.”
The dark line in Figure 5 is the frequency distribution of annual earnings per share without extraordinary items (Compustat annual data item 58). Observations are shown between -$0.50 and $0.50, and 117,405 of the 185,401 observations available from the Annual Compustat file between 1977 and 2006 are displayed in the figure. Intervals have one-cent width. The light line is the frequency distribution of earnings per share without extraordinary items (Compustat data item 58) for firm-years that meet the JJ sample selection criteria (that is, four consecutive quarters of data item 27 are available on the Quarterly Compustat file).
Figure 6: Frequency distribution for undeflated net income for three “as-if” years

Panel A: Frequency distribution of net income for “as-if” year ending quarter one of the fiscal year

Panel B: Frequency distribution of net income for “as-if year ending quarter two of the fiscal year
Panel C: Frequency distribution net income for “as-if” year ending quarter three of the fiscal year

Panel D: Frequency distribution fiscal year net income and the distribution of the average of the three “as-if” years shown in Panels A, B and C
Figure 6 compares the distribution of fiscal year net income for “as-if” years and the fiscal year. As in JJ, we present the percentage of observations in each cell; this then facilitates comparison across distributions drawn from samples of different size. Panel A shows the frequency distribution of un-deflated net income for the “as-if” year ending quarter one of the fiscal year; 40,506 of the 122,774 observations with the requisite data are shown. Panel B shows the frequency distribution of un-deflated net income for the “as-if” year ending quarter two of the fiscal year; 40,667 of the 123,357 observations with the requisite data are shown. Panel C shows the frequency distribution of un-deflated net income for the “as-if” year ending quarter three of the fiscal year; 40,847 of the 123,599 observations with the requisite data are shown. Intervals are $100,000 wide. Net income is the sum of Quarterly Compustat data item 69 over the four quarters of the fiscal year. Each observation has four consecutive quarters of data. The sample selection criteria are those used by JJ. Panel D shows the frequency distribution of un-deflated net income for the fiscal year (dark line) as compared to the average of the three “as-if” years. The fiscal year line shows 43,536 of the 128,477 observations.
Figure 7: The Averaging Effect of Combining Quarterly Net Income Observations across Consecutive Fiscal Years

Figure 7 compares the un-deflated fiscal year net income to the three “as-if” years of net income for all observations for which there are 7 consecutive quarters of net income (Quarterly Compustat data item 69). The fiscal year ends in quarter four year t. The three “as-if” years end in quarter one year t, quarter two year t and quarter three year t. There are 146,180 observations that have seven consecutive quarters of data item 69. The figure shows the observations that fall into the net income intervals between -$1,000,000 and $1,000,000. The fiscal year shows 28,226 of the available observations while “as-if” years ending in quarter one, quarter two and quarter three show 30,161, 29,840 and 29,147 observations, respectively.
Figure 8: The Effect of Averaging on the Shape of the Distribution of Earnings per Share

Figure 8 shows the frequency distribution of earnings per share without extraordinary items – 12 month moving (Quarterly Compustat data item 27) for observations that meet the JJ sample selection criteria (that is, four consecutive quarters of data item 27 in a fiscal year). The dark line shows the distribution of the average of three data 27 items: quarter one, quarter two, and quarter three of a fiscal year and represents 61,501 of the 144,642 observations that meet the sample selection criteria. The light line shows the frequency distribution of the fiscal year (that is, quarter four) data item 27 and represents 60,970 of the 144,642 observations that meet the JJ sample selection criteria.
Figure 9 shows the effect of averaging on the distribution of changes in earnings per share excluding extraordinary items – 12 month moving average (Compustat quarterly data item 27). The figure compares the change in data item 27 from quarter four (t-1) to quarter four (t) to the average of two changes: (1) quarter four (t-2) to quarter four (t-1), and (2) quarter four (t-1) to quarter four (t).
Figure 10: The Distribution of the Deflator (beginning-of-period market capitalization) used by BD

Figure 10 shows the distribution (25th percentile, median and 75th percentile) of market capitalization (that is, price – Compustat data item 199 times number of shares outstanding – Compustat data item 25) for all net income observations between -$5,000,000 and $7,000,000 that meet the BD sample selection criteria. Observations are placed in $100,000 net income intervals.
Figure 11: Effect of Deflation on the Distribution of (Deflated) Net Income

Panel A: Distribution of Deflated Net Income for Observations of Net Income between -$100,000 and $100,000; number of Observations

Panel B: Distribution of Deflated Net Income for Observations of Net Income between -$100,000 and $100,000; Percentage of Observations
Panel C: Distribution of Deflated Net Income for Observations of Net Income greater than $100,000 and less than $100,000

Panel D: Distribution of Deflated Net Income for all Observations of Net Income
Figure 11 Panel A shows the frequency distribution of net income between -$100,000 and $100,000 when those observations are deflated by their beginning of period market capitalization. The numerator of each observation is Compustat data item 172. The denominator is beginning-of-year market capitalization (that is, price – Annual Compustat data item 199 times number of shares outstanding – Annual Compustat data item 25). Panel B shows the distribution of the percentage of net income between -$100,000 and $100,000 that falls into each bin after deflation. Panel C shows the frequency distribution of net income less than -$100,000 and greater than $100,000 when those observations are deflated by their beginning of period market capitalization. The numerator of each observation is Annual Compustat data item 172. The denominator is beginning-of-year market capitalization (that is, price – Annual Compustat data item 199 times number of shares outstanding – Annual Compustat data item 25). Panel D combines Panel A and Panel C to recreate the distribution made noteworthy by BD 1997.
Figure 12: The Distribution of the Deflator (beginning-of-period market capitalization) by whether the deflator is for an “as-if” year or fiscal year.

Figure 12 focuses on observations of net income between -$2,000,000 and $2,000,000 and plots both beginning-of-fiscal-year market capitalization and the average of the beginning-of-year market capitalization for the three “as-if” years, which are used as the JJ benchmark. The light lines represent the average of beginning-of-year market capitalization for the three “as-if” years ending in quarters one, two, and three of year t (price -- Quarterly Compustat data item 14 times shares outstanding -- Quarterly Compustat data item 61). The dark lines represent the beginning-of-fiscal-year market capitalization when using quarterly data (price -- Quarterly Compustat data item 14 times shares outstanding -- Quarterly Compustat data item 61).