Money and Credit: The Role of the Informal Sector*

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Abstract

We use a search-theoretic model of money to study a seller’s decision to be paid in cash or extend credit. Credit trades are recorded and allows the government to levy income taxes. Accepting cash avoids record-keeping of the transaction and thus taxes. Thus, a seller chooses to be anonymous or not. In the absence of distortionary taxes, only a credit equilibrium exists. Otherwise a monetary equilibrium will exist for sufficiently low inflation rates. For high inflation rates, only credit is used. We then show that heterogeneity of preferences leads to coexistence of money and credit with both being used in some matches.

JEL Codes: E4, E5.

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1 Introduction

Modern monetary theory is founded on several key frictions that make money essential for trade. These frictions are: 1) imperfect recordkeeping over individual trading histories, 2) no public communication of trading histories and 3) limited enforcement. If these frictions are absent then trade credit can be used and this Pareto dominates money as a payment system. The word ‘anonymity’ is commonly used as short-hand to mean that these frictions are present. Since Kiyotaki and Wright (1989,1993), nearly all studies of the medium exchange role of money impose these frictions on the environment in order to give fiat money value. In short, anonymity is part of the environment – it is not a choice. This raises the following question: If agents could choose to be anonymous, would they do so and why?

A common answer to this question is that agents choose to be anonymous in order to evade paying taxes.\(^1\) While recordkeeping, public communication and enforcement allow credit to exist, these features of the trading environment also make it much easier for governments to monitor economic activity and collect taxes. By using money, agents do not create records of transactions and without records of transactions, governments have a difficult time taxing agents’ income. Thus, monetary exchange allows agents to trade anonymously and evade taxes but at the cost of using a less efficient payment system. This is the fundamental tradeoff we study in this paper.

We use the Lagos-Wright (2005) model to study the role of money to evade income taxes. The main difference from Lagos and Wright is we assume there are no information frictions that make money essential for trade. There is a record-keeping technology, communication and enforcement that can be used to facilitate exchange via the use of credit. The question is will agents choose credit to trade with each other? Money is a potential medium of exchange that can also be used. If agents use credit, then the transaction is recorded, reported to the government who can then enforce payment of income taxes by the seller. If money is used, no transaction is recorded and nothing is reported to the government. Hence, cash income is not taxable. We then study how inflation and taxes interact to affect agents’ decisions to use money or credit.

Our key results are as follows. First, in the absence of distortionary taxes money is not essential and credit is used to facilitate trade. Second, with distortionary taxation, monetary equilibria exist as long as the inflation rate is not ‘too high’. If inflation is high enough, agents resort to credit and the value of money goes to zero. Third, if there is no heterogeneity amongst buyers (or sellers) then there may be a monetary equilibrium or a

\(^1\)While our answer is probably the most common explanation, there are others. For example, Kahn and Roberds (2005) assume agents can choose to be anonymous to avoid identify theft.
credit equilibrium but money and credit do not coexist. With heterogeneity, money and credit can coexist as means of payment.

An important finding is that money and credit can coexist. This is not new since there are several modern monetary models which have this result.\footnote{While there are many models where money and credit coexist prominent examples are Cavalcanti and Wallace (1999), Berentsen, Camera and Waller (2007) and Telyukova and Wright (2008). Telyukova and Wright allude to the use of cash to avoid taxes but do not model it.} However, in all of these models, trade occurs with either money or credit. We have this same result plus a new one – some buyers pay with money \textit{and} credit in the same match. These are matches where buyers demand a lot of goods but have low real cash balances. Thus, the terms of trade dictate that the buyer gives all of his cash balances to the seller as partial payment and receives credit for the remaining balance of the transaction.

The structure of the paper is as follows. In Section 2, we describe how our approach and results differ from the existing literature. Section 3 contains the environment and policy actions. In Section 4, the planner allocation is presented. In Section 5, we construct an equilibrium for our economy. In Section 6, we show existence and uniqueness of the equilibrium and characterize it. It also contains some examples and extensions of the basic model. Section 7 concludes. All proofs are in the Appendix.

\section{Brief Literature Review}

The model we study is often referred to as an economy with an ‘informal’, ‘shadow’ or ‘underground’ sector. In general, the informal sector includes illegal activities as well as those aimed at tax evasion.\footnote{Tax evasion is the act of under-reporting taxable income, which is illegal. Tax avoidance is undertaking legal actions to reduce one’s taxable income, such as producing for one’s self (homework).} While tax evasion can be done without the use of cash (barter, falsifying documents, etc.) money seems to play a prominent role in most discussions of the shadow economy. In fact, a common measurement of the shadow economy is the amount of currency in circulation.\footnote{See Tanzi (1983,1999) and Schneider and Enste (2000) for a discussion of this method. For an interesting critique of this method see Thomas (1999).} Although there are models that study the use of money for trading in illegal activities (see Camera (2001) for example), this is not the focus of our analysis; we do not want to get bogged down into a discussion of why some goods or services are illegal. Other models simply assume informal goods are not taxable and/or that producing a good informally requires a less efficient technology (see Koreshkova (2006)). These assumptions again imply that goods are fundamentally different. We take the opposite view – there is nothing fundamental about the goods or the technologies but rather it is the method of payment that affects one’s ability to evade taxes.
There is a substantial literature that looks at the use of money as a way to hide transactions. Nearly all of this literature assumes money is needed in the economy – no explicit microfoundations of money are incorporated into the model. We argue that it is these information frictions that are critical for understanding the role of money and, more importantly, how those frictions affect other aspects of the model, such as tax collection.\(^5\)

Introducing tax evasion as a reason for using money necessarily requires introducing a fiscal authority into the model. In modern monetary models, fiscal policy has recently been introduced by a variety of authors to study the use of the inflation tax as a source of revenues.\(^6\) Among these papers only Aruoba (2010) examines the role of money and tax evasion. However, like all of the models in the Kiyotaki-Wright tradition, anonymity is imposed on the model to rule out the use of credit – it is not a choice of the agents. Again, he defines informal by the goods and location that they are traded as opposed to the method of payment used. Thus, in the absence of distortionary taxation, money is still essential in his model whereas money is not essential in our framework.

3 The Model

The Environment  Time is discrete and each period is divided into two subperiods. There is a generic good that can be produced and consumed in each subperiod. This good is perishable across subperiods. As in Rocheteau and Wright (2005), there is a continuum of agents of measure 1 who are divided into two groups of equal size, called buyers, \(B\), and sellers, \(S\). Buyers wish to consume during the first subperiod but cannot produce and while sellers can produce in the first subperiod but do not wish to consume. In this subperiod, agents meet at random and pairwise in a decentralized market denoted DM. Buyers get utility \(\varepsilon u (q)\) from consuming \(q\) units of the good where \(\varepsilon\) is an idiosyncratic preference shock with distribution \(\mathcal{F}(\varepsilon)\) and compact support \([\underline{\varepsilon}, \overline{\varepsilon}]\). We assume \(u' (q), -u'' (q) > 0\) and \(u(0) = 0\). Sellers incur utility cost \(c (q)\) from producing \(q\) units of the good with the following properties \(c' (q) > 0, c'' (q) \geq 0\) and \(c (0) = 0\).

In the second subperiod all agents consume and goods are traded in a centralized Walrasian market denoted CM. Agents can also sell labor to competitive firms and are paid \(w\) per unit of labor supplied. Both sets of agents get utility \(U (x)\) from consuming \(x\) units of the good and incur disutility cost \(-h\) from supplying \(h\) units of the good. Time is discounted from the CM to the DM at rate \(\beta_C = \beta < 1\) and from the DM to the CM at rate \(\beta_D = 1\).

\(^5\)In Koreshkova (2006), agents can pay for informal goods with either cash or credit. This misses the fundamental point that credit transactions create records that can be used to levy taxes on agents.

Firms in the CM can produce one unit of output per unit of labor used in production. It then follows that \( w = 1 \). Although we assume that there are different utility and production costs across the two sub-periods, none of our results would change if we assumed \( U(x) = u(q) \) and \( c(q) = q \).

In the CM, agents take prices parametrically with \( \phi \) denoting the goods price of money. In the DM, terms of trade for pairwise meetings are determined by proportional bargaining. This entails distributing a fraction \( \theta \) of the match surplus to the buyer, and fraction \( 1 - \theta \) to the seller. Since buyers cannot produce in the DM some form of payment is needed to entice sellers to trade. We assume that individual trading histories can be costlessly recorded and communicated to other agents. We also assume promises of repayment can be enforced. Consequently, credit is a feasible form of payment in both the DM and the CM. It is important to stress that our assumptions imply that financial markets are fully developed and efficiently operated. Hence, our results are not driven by incomplete financial markets.

We also assume there is a fiat object called money that can be used for payment in either market. The aggregate stock of fiat currency per capita is given by \( M_t \) and grows at the gross rate of \( \gamma = 1 + \pi \) implying \( M_t = \gamma M_{t-1} \). Monetary injections occur in the CM and as payment for goods and services. Finally, for notational purposes we drop the \( t \) subscript and denote time as \(-1\) for \( t - 1 \), \(+1\) for \( t + 1 \) and so on.

**Fiscal Policy** We assume that the government uses distortionary and lump-sum taxes to finance a constant stream of government spending, \( G \), in the CM. The government imposes a linear tax rate \( \tau \) on labor income that can be observed and uses lump-sum taxes \( T \) as needed to balance the budget. All taxes are paid in the CM even if the income was generated in the DM. At this point, we do not need to assume that money is issued by the government. Agents may choose to use another object as a medium of exchange, e.g. a foreign currency that is not controlled by the local government. But as a useful starting point, we will assume that government-issued fiat money is the monetary object in our economy.

Regarding the government’s ability to observe incomes, we assume that all labor income generated in the CM is reported to the government by firms. Thus, regardless of whether wage payments are made in cash or with credit, the income is observed and thus can be taxed. However, income earned by sellers in the DM may or may not be observed by the government depending on the form of payment used. For illustration, suppose in a DM trade, a buyer pays with a combination of cash and credit. The seller extends a loan of size \( \ell \) to the buyer and this is reported to the government, who treats the recorded transaction \( \ell \) as taxable income. However, whatever portion of the transaction that is done with money is not recorded and so there is nothing to report to the government. Furthermore, we assume the
government cannot observe agent’s money holdings in the CM. Consequently, cash income
earned by sellers is unobservable and cannot be taxed.

The government budget constraint is

\[ G = \tau H + \tau L + T + \phi (M - M_{-1}) \]

where \( H \) is aggregate labor income in the CM, \( L \) the reported loan income of all sellers in
the DM and \( T \) is lump-sum tax revenue. The last term is real seigniorage.

4 Planner’s Problem

The planner problem is to maximize the sum of discounted utilities of buyers and sellers:

\[
\max_{x,h,q,} U(x) - h + (\sigma/2) \int q \left[ \varepsilon u(q_x) - c(q_x) \right] dF(\varepsilon)
\]

s.t. \( x = h \)

The first-best allocation yields \( U'(x^*) = 1 \) and \( \varepsilon u'(q^*_x) = c'(q^*_x) \) in each match with buyer preference \( \varepsilon \).

5 Markets and trades

5.1 CM

Buyers During the centralized market buyers also choose how much to consume and work
but also how much money to carry to the next period’s decentralized market. These choices
are made before \( \varepsilon + 1 \) is realized in the next DM. Loan payments must be also be settled.
Hence, at the beginning of the centralized market the problem of a representative buyer
holding \( z \) units of real balances and outstanding real loans \( \ell \) (a liability) is denoted by:

\[
W(z, \ell) = \max_{x,h,z+1} \left\{ U(x) - h + \beta V(z_{+1}) \right\}
\]

s.t. \( x = (1 - \tau) h + z - \ell - \gamma z_{+1} - T \)

where \( \gamma = \phi/\phi_{+1} \) is the inflation rate in the CM from period \( t \) to the \( t + 1 \). This problem
can be rewritten as

\[
W(z, \ell) = (1 - \tau)^{-1} (z - \ell - T) + \max_{x,z_{+1}} \left\{ U(x) - (1 - \tau)^{-1} x - (1 - \tau)^{-1} \gamma z_{+1} + \beta V(z_{+1}) \right\}.
\]
The first-order conditions yield

\[ U'(x) = (1 - \tau)^{-1} \]  
\[ \beta V'(z+1) \leq \gamma (1 - \tau)^{-1} \quad (= 0 \text{ if } z+1 > 0) \]

and the envelope conditions are \( W_z = (1 - \tau)^{-1}, W_\ell = -(1 - \tau)^{-1} \).

**Sellers** During the centralized market sellers choose consumption and how much labor to supply. It is straightforward to show that seller’s will not take money balances into the next DM since they have no need for it. Let the CM value function for a seller be denoted \( W^s(z, -\ell) \) where \( z \equiv \phi m \) and \( \ell \) are his holdings of real balances and loans extended (an asset) measured in units of the CM good. Hence, the value function of a representative formal seller at the beginning of the CM is given by:

\[
W^s(z, \ell) = \max_{x,h} \{ U(x) - h + \beta V^s \}
\]

\[ s.t. \quad x = (1 - \tau) h + z + [1 - \tau (\ell)] \ell - T. \]

where \( V^s \) is the value function entering the next DM for a seller and

\[
\tau (\ell) = \tau \text{ if } \ell \geq 0 \\
\tau (\ell) = 0 \text{ otherwise.}
\]

This function taxes income earned via issuing credit but does not subsidize borrowing by sellers. The idea is to tax income and not financial transactions unrelated to the generation of income. Substituting out for \( h \) using the budget constraint yields

\[
W^s(z, -\ell) = (1 - \tau)^{-1} \{ z + [1 - \tau (\ell)] \ell - T \} + \max_x \{ U(x) - (1 - \tau)^{-1} x + \beta V^s \}
\]

The first-order conditions yield \( U'(x^*) = (1 - \tau)^{-1} \) and the envelope conditions \( W^s_z(z, \ell) = (1 - \tau)^{-1}, W^s_\ell(z, \ell) = (1 - \tau)^{-1} [1 - \tau (\ell)] \). The envelope conditions show that cash has a higher value in the CM if the seller is informal.

**5.2 DM**

In the decentralized market buyers observe their idiosyncratic realization of \( \varepsilon \) and with probability \( \sigma \) are randomly matched with a seller. Terms of trade are given by proportional bargaining with threat points given by no trade. The seller has to decide whether or not to
offer credit to the buyer. If it is extended, the buyer decides whether to use it or not. As we show below, the buyer will always use credit if it is offered.

In a match with a buyer of type \( \varepsilon \), the seller produces \( q_\varepsilon \) for the buyer and gives him a loan of size \( \ell_\varepsilon \). The buyer in turn gives the seller \( d_\varepsilon \) units of real balances. The buyer’s surplus is

\[
S^b_\varepsilon \equiv \varepsilon u(q_\varepsilon) + W(z - d_\varepsilon, \ell_\varepsilon) - W(z, 0) \\
= \varepsilon u(q_\varepsilon) - (1 - \tau)^{-1} (d_\varepsilon + \ell_\varepsilon),
\]

while the seller’s surplus equals:

\[
S^s_\varepsilon \equiv -c(q_\varepsilon) + W^s(d_\varepsilon, \ell_\varepsilon) - W^s(0, 0) \\
= -c(q_\varepsilon) + (1 - \tau)^{-1} d_\varepsilon + (1 - \tau)^{-1} [1 - \tau (\ell)] \ell_\varepsilon.
\]

The total surplus in a match of type \( \varepsilon \) is given by:

\[
S_\varepsilon = \varepsilon u(q_\varepsilon) - c(q_\varepsilon) - \tau (\ell) (1 - \tau)^{-1} \ell_\varepsilon.
\]

It is obvious from this expression that using credit, ceteris paribus lowers the match surplus. The reason is the seller has to pay taxes on this income which lowers the net gains from trade. Thus, by lowering the amount of credit extended by one unit, the seller saves \( \tau / (1 - \tau) \) units of labor in the next CM. This creates extra surplus for the buyer and seller to split.

With proportional bargaining, the buyer gets the fraction \( \theta S_\varepsilon \) while the seller gets \( (1 - \theta) S_\varepsilon \). Thus we have

\[
S^b_\varepsilon = \theta [\varepsilon u(q_\varepsilon) - c(q_\varepsilon) - \tau (\ell) (1 - \tau)^{-1} \ell_\varepsilon], \\
S^s_\varepsilon = (1 - \theta) [\varepsilon u(q_\varepsilon) - c(q_\varepsilon) - \tau (\ell) (1 - \tau)^{-1} \ell_\varepsilon].
\]

The buyer’s surplus can be rearranged to obtain

\[
d_\varepsilon = (1 - \tau) [(1 - \theta) \varepsilon u(q_\varepsilon) + \theta c(q_\varepsilon)] - [1 - \theta \tau (\ell)] \ell_\varepsilon.
\]

For \( \ell > 0 \), we have \( |\partial \ell_\varepsilon / \partial d_\varepsilon| = (1 - \theta \tau)^{-1} > 1 \) for \( \theta > 0 \). Bringing in one less unit of real balances increases the loan amount by more than one unit. This is the way in which the seller must be compensated for extending credit.\(^7\) Typically, by using an extra unit of

\(^7\)Alternatively, reducing the loan amount more than 1-for-1 is the way the seller compensates the buyer for bringing in an additional unit of money.
cash rather than credit, a buyer saves principal and interest on a loan, \(1 + i\). The implicit interest here is thus given by

\[ i_{DM} = \frac{\theta \tau}{1 - \theta \tau}. \]  \(5\)

The implicit interest rate is increasing in both the tax rate and the buyer’s bargaining power. The tax rate effect is clear – the higher is \(\tau\) the more costly it is for the seller to extend credit to the buyer. Therefore he charges a higher rate of interest. What is less clear is why the interest rate is increasing in the buyer’s bargaining power. One’s intuition would be that it should go down. The reason is as follows: As \(\theta\) increases, the buyer can extract more \(q\) from the seller. Since the money holdings are given in the match, the seller has to give a bigger loan to the buyer which imposes a tax liability on the seller. In order to compensate the seller, the implicit interest rate that the buyer pays must therefore go up.

The seller faces the following problem:

\[
\max_{q_\varepsilon, d_\varepsilon, \ell_\varepsilon} -c(q_\varepsilon) + (1 - \tau)^{-1} d_\varepsilon + (1 - \tau)^{-1} [1 - \tau (\ell)] \ell_\varepsilon \\
\text{s.t.} \quad 0 \leq d_\varepsilon \leq z, \quad 0 \leq \ell_\varepsilon \\
\theta \left[ \varepsilon u(q_\varepsilon) - c(q_\varepsilon) - \tau (\ell) (1 - \tau)^{-1} \ell_\alpha \right] \leq \varepsilon u(q_\varepsilon) - (1 - \tau)^{-1} (d_\varepsilon + \ell_\varepsilon).
\]

The seller chooses how much output to produce and how the buyer should pay for it subject to the constraint that the buyer receives no less than \(S^b_\varepsilon\). The solution to this problem is as follows. For \(z \geq z^*_\varepsilon \equiv (1 - \tau) [(1 - \theta) \varepsilon u(q^*_\varepsilon) + \theta c(q^*_\varepsilon)]\) we have

\[
d_\varepsilon = z^*_\varepsilon, \quad \tau (\ell) = \ell_\varepsilon = 0 \\
\varepsilon u'(q^*_\varepsilon) = c'(q^*_\varepsilon)
\]

For \(\bar{z}_\varepsilon < z < z^*_\varepsilon\), where \(\bar{z}_\varepsilon\) is defined below, we obtain

\[
d_\varepsilon = z, \quad \tau (\ell) = \ell_\varepsilon = 0 \\
z = (1 - \tau) [(1 - \theta) \varepsilon u(\hat{q}_\varepsilon) + \theta c(\hat{q}_\varepsilon)]
\]

where \(\hat{q}_\varepsilon < q^*_\varepsilon\) and is increasing in \(z\). For \(0 \leq z < \bar{z}_\varepsilon\) we have

\[
d_\varepsilon = z \quad \tau (\ell) = \tau, \\
\ell_\varepsilon = (1 - \theta \tau)^{-1} \left\{ (1 - \tau) [(1 - \theta) \varepsilon u(\hat{q}_\varepsilon) + \theta c(\hat{q}_\varepsilon)] - z \right\} \\
\varepsilon u'(\hat{q}_\varepsilon) = (1 - \tau)^{-1} c'(\hat{q}_\varepsilon)
\]  \(6\)
where $\tilde{q}_\varepsilon < \hat{q}_\varepsilon$ and $\tilde{z}_\varepsilon \equiv (1 - \tau) [(1 - \theta) \varepsilon u(\hat{q}_\varepsilon) + \theta c(\hat{q}_\varepsilon)]$. Note that the critical values, $z^*_\varepsilon$ and $\tilde{z}_\varepsilon$ differ across buyer types. It is straightforward to show that both are monotonically increasing in $\varepsilon$.

The nature of this solution is that if the buyer has sufficiently high real balances, he acquires the first-best quantity and pays with cash. No credit is used and the transaction is not recorded. If the buyer’s real balances are somewhat lower (below $z^*_\varepsilon$) the seller chooses not to extend credit and accepts only cash. However, rather than the first-best quantity he produces something less. Again, the transaction is not recorded. Finally if real balances are low enough, then the seller takes all of the cash and gives the buyer enough credit to acquire $\tilde{q}_\varepsilon$. The portion of the transaction involving credit is recorded and the seller pays taxes on that earned income in the CM.

There are several key observations from this solution. First, for $z < \tilde{z}_\varepsilon$ the quantity traded, $\tilde{q}_\varepsilon$, is independent of how much money is exchanged. This means that even if the buyer has no cash, trade still occurs in the DM via the use of credit. Second, the critical values for money balances, $z^*_\varepsilon$ and $\tilde{z}_\varepsilon$, are functions of the buyer’s preference parameter. For high $\varepsilon$ buyers, the first-best quantity is much larger so $z^*_\varepsilon$ is larger as well. The reverse is true for low $\varepsilon$ buyers. Hence, for a given amount of real balances, a buyer may get the first-best quantity using only cash if he has a low preference shock whereas he gets $\tilde{q}_\varepsilon < q^*_\varepsilon$ and pays with cash and credit if he has a high preference shock. If the cash constraint is binding for a measure of the $\varepsilon$ buyers and $\ell_\varepsilon = 0$ for these buyers, then we have

$$\frac{\partial \tilde{q}_\varepsilon}{\partial \varepsilon} = -\frac{(1 - \theta) u(\hat{q}_\varepsilon)}{(1 - \theta) \varepsilon u'(\hat{q}_\varepsilon) + \theta c'(\hat{q}_\varepsilon)} < 0.$$  

Why is this the case? An increase in $\varepsilon$ expands the trade surplus and some of this surplus must be transferred to the seller for $\theta < 1$. Since the buyer has no additional money to give to the seller and no credit is being extended, the seller simply reduces the quantity of goods produced. One way to think of this is that the seller charges a higher nominal ‘price’, $m/\hat{q}_\varepsilon$, per unit of goods received. Finally, the buyer will always accept credit if offered. Why? Since the seller is essentially choosing $q_\varepsilon, d_\varepsilon, \ell_\varepsilon$ to maximize the match surplus then this also maximizes the payoff to the buyer. Refusing to use credit would simply lower his payoff.

### 5.3 Optimal money buyer’s money holdings

Buyers must choose the optimal amount of real balances to carry from the CM to next period’s DM. The key tradeoffs of this intertemporal choice are the cost of carrying money (given by the inflation rate) vis-a-vis the expected benefit of using money for trades in the
DM. Specifically, the buyer’s intertemporal optimization is:

\[
\max_z (\beta - \gamma)z + \beta\sigma\theta \int_{\xi(z)}^{\varepsilon(z)} \left[ \varepsilon u(q^*_z(z)) - c(q^*_z(z)) \right] d\mathcal{F}(\varepsilon) + \\
+ \beta\sigma\theta \int_{\varepsilon(z)}^{\xi(z)} \left[ \varepsilon u(\hat{q}_z(z)) - c(\hat{q}_z(z)) \right] d\mathcal{F}(\varepsilon) + \\
+ \beta\sigma\theta \int_{\hat{\varepsilon}(z)}^{\xi(z)} \left[ \varepsilon u(\tilde{q}_z(z)) - c(\tilde{q}_z(z)) - \tau \ell (1 - \tau)^{-1} \ell(z) \right] d\mathcal{F}(\varepsilon).
\]

Here, the function \(\varepsilon^*(z)\) is a value such that all decentralized trades with preference shock lower than \(\varepsilon^*(z)\) are not constrained. In turn, \(\tilde{\varepsilon}(z)\) captures the lowest value of the preference shock such that the DM bargaining problem requires a positive loan. The properties of the proportional bargaining solution derived before imply functions \(\varepsilon^*(z)\) and \(\tilde{\varepsilon}(z)\) are well defined, increasing, and satisfy \(\tilde{\varepsilon}(z) \geq \varepsilon^*(z)\) for all \(z\).\(^8\)

The tradeoffs faced by the buyer can be easily seen by computing how a marginal increase in real holdings affects the buyer’s intertemporal objective. The derivative of the buyer’s objective (7) with respect to real balances yields

\[
(\beta - \gamma) + \beta\sigma\theta \int_{\varepsilon(z)}^{\tilde{\varepsilon}(z)} \frac{[\varepsilon u'(\hat{q}_z) - c'(\hat{q}_z)]}{[(1 - \theta)\varepsilon u'(\hat{q}_z) + \theta c'(\hat{q}_z)]} d\mathcal{F}(\varepsilon) \\
+ \beta\sigma\theta \tau (1 - \tau)^{-1} (1 - \theta\tau)^{-1} \int_{\tilde{\varepsilon}(z)}^{\xi(z)} d\mathcal{F}(\varepsilon)
\]

This expression is fairly intuitive. The first term is the cost of bringing money into the DM. The derivative of the second term of the buyer’s objective is zero because the expected payoff of unconstrained trades does not change by bringing more money. The second term in (8) is the expected increase in the buyer’s surplus that results from bringing more money to constrained trades that do not use credit. The last term in (8) reflects the fact that bringing more money lowers the size of loans in transactions where credit is used, and thus the tax extracted from the match. This tax savings is partially passed to the buyer, as dictated by the bargaining solution.

6 Equilibrium

For the reminder of this paper we will focus our attention on symmetric stationary equilibria. Symmetry requires all similar agents to undertake the same actions. We say that a stationary

\(^8\)Our appendix provides a formal proof for these results.
equilibrium is monetary when buyers carry a strictly positive amount of real balances from the centralized market to next period’s decentralized market.

Stationary equilibria is an income tax rate, $\tau$, and a collection of sequences of lump-sum transfers, prices, money holdings, and time-invariant allocations of consumption and hours worked at the centralized market, and terms of trade functions for the DM,

$$\{T_t, \phi_t, x_t^b, h_t^b, x_t^s, h_t^s, m_t\}, q_e, \ell_e, d_e,$$

such that: (a) The money holdings, consumption and hours worked allocations for the buyers are optimal taking as given the tax rate, lump-sum transfers and terms of trade functions; (b) the consumption and hours worked allocations solve the seller’s problem $x_t^s, h_t^s$; (c) the money demanded by buyers equals the money supply; (d) equilibrium prices $\phi_t$ grow at the same rate as the money supply; (e) the government’s budget constraint is balanced.

The main theoretical results of this paper are summarized by the following proposition:

**Proposition 1** A unique stationary equilibrium exists. Further, there are three classes of equilibria: (i) Buyers hold enough money so that all trades are unconstrained only at the Friedman rule; (ii) There is a high enough inflation rate, $\bar{\gamma} \equiv \beta [1 + \sigma \theta \tau (1 - \theta \tau)^{-1}]$, such that buyers hold no money and thus all DM transactions are based on credit; (iii) There are intermediate inflation rates such that buyers carry a positive amount of money. If credit is used in a match, it is used simultaneously with money.

To provide more insight into the equilibrium behavior of the model, note that if a bond existed in the CM that was traded and held in the CM, then its interest rate would be $1 + i_{CM} = (1 + \pi)(1 + r) = \gamma / \beta$. Thus a monetary equilibrium exists when $\gamma < \bar{\gamma}$ which using (5) and the definition of $\bar{\gamma}$ yields

$$i_{CM} \leq \sigma \theta \tau (1 - \theta \tau)^{-1} = E(i_{DM}).$$

From a buyer’s perspective, acquiring a unit of money in the CM rather than a CM bond will cost the buyer $i_{CM}$ units of goods in the next CM. This is the opportunity cost of acquiring money in the CM. This additional unit of money saves $i_{DM}$ for the buyer since he does not have to borrow as much in the event he meets a seller. So a buyer will always take in real balances as long as the opportunity cost is lower than the marginal benefit. When that breaks down, the buyer prefers to simply use credit. Alternatively, if the costs of carrying money are relatively low compared to the income tax distortion, then buyers bring money to buy goods. If the cost of acquiring money is too large, agents do not value money even
though it can be used to hide labor income – the tax saving does not compensate for the cost of acquiring money.

Note that the larger is $\tau$ the larger is $\hat{\gamma}$. This also is true for an increase in $\theta$. Both parameters make money more valuable in trade which increases the real demand for money and thus the quantity of goods exchanged. Consequently, $\hat{q}_e$ is traded over a wider range of real balances. This has the effect of crowding out credit trades.

It should be stressed that the buyer does not pay the income taxes associated with the credit transaction. Thus, he would always prefer to use credit. It is the seller who benefits from the cash transaction. So the seller must induce the buyer to bring cash into the DM by sharing the tax saving with him. Note that if we have seller-take-all, $\theta = 0$, then $i_{DM} = 0$. If the seller does not share any of the tax savings associated with cash, the buyer will not bring any in and uses credit as a means of payment.

The derivative of the buyer’s objective (8) and Proposition 1 imply the following relationships.

**Corollary 2** For a given parameterization: (i), a higher inflation rate lowers the money holdings of buyers thus increasing the measure of trades where credit is used; (ii) higher income taxes increase the return of money and thus lower the measure of trades where credit is used.

Hence, according to our model, increasing inflation results in a smaller informal sector, while increasing taxes increases the size of the informal sector.

### 6.1 Examples

**Homogenous buyers** In order to understand how heterogeneity affects the model, suppose $F(z)$ is degenerate. We have

$$(1 - \tau) V'(z) = \begin{cases} 
1 & \text{if } z \geq z^*_\epsilon \\
1 + \frac{\sigma(\hat{\eta}u'(\hat{q}_e) - \hat{c}'(\hat{q}_e))}{(1 - \hat{\theta})\hat{u}'(\hat{q}_e) + \hat{\theta}c'(\hat{q}_e)} & \text{if } \bar{z}_\epsilon \leq z \leq z^*_\epsilon \\
1 + \sigma \theta \tau (1 - \theta \tau)^{-1} & \text{if } z \leq \bar{z}_\epsilon 
\end{cases}$$

The first-order condition for $z$ in the CM yields the following solutions for $q_\epsilon$

$$q_\epsilon = q^*_\epsilon \quad \text{for } \gamma = \beta$$

$$\gamma - \beta \leq \frac{\sigma(\hat{\eta}u'(\hat{q}_e) - \hat{c}'(\hat{q}_e))}{(1 - \hat{\theta})\hat{u}'(\hat{q}_e) + \hat{\theta}c'(\hat{q}_e)} \quad \text{for } \beta < \gamma \leq \bar{\gamma}$$

$$q_\epsilon = \bar{q}_\epsilon < q^*_\epsilon \quad \text{for } \bar{\gamma} \leq \gamma$$
The goods price of money is then
\[
\phi = M^{-1} (1 - \tau) \left[ (1 - \theta) \varepsilon u(\hat{q}_\varepsilon) + \theta c(\hat{q}_\varepsilon) \right] \quad \text{for } \beta < \gamma \leq \tilde{\gamma}
\]
\[
\phi = z = 0 \quad \text{for } \gamma > \tilde{\gamma}.
\]

For \( \gamma \leq \tilde{\gamma} \) agents only use money to trade while for \( \gamma > \tilde{\gamma} \) all buyers resort to credit to pay the sellers. In short, no monetary equilibrium exists. This also means that there is no equilibrium where money and credit coexist. Thus, in order to have a monetary equilibrium where money and credit coexist, we cannot have a degenerate distribution over \( \varepsilon \).

**Two state example** Consider the follow 2-point distribution \( \varepsilon \in \{\varepsilon, \bar{\varepsilon}\} \) where \( \bar{\varepsilon} \) occurs with probability \( \lambda \). Conjecture that the spread between these two values is small enough so that \( z^*_\varepsilon > \bar{z}_\varepsilon \). We have the unique solutions for \( q_\varepsilon \)

\[
q_\varepsilon = q^*_\varepsilon, \quad q_\bar{\varepsilon} = q^*_\bar{\varepsilon} \quad \text{for } \gamma = \beta
\]

\[
q_\varepsilon = q^*_\varepsilon, \quad \frac{\gamma - \beta}{\beta} = \frac{\sigma \lambda \theta [\varepsilon u'(\hat{q}_\varepsilon) - c'(\hat{q}_\varepsilon)]}{(1 - \theta) \varepsilon u'(\hat{q}_\varepsilon) + \theta c'(\hat{q}_\varepsilon)} \quad \text{for } \beta < \gamma \leq \gamma_1
\]

\[
0 = (1 - \theta) \left[ \varepsilon u(\hat{q}_\varepsilon) - \bar{\varepsilon} u(\hat{q}_\varepsilon) \right] - \theta \left[ c(\hat{q}_\varepsilon) - c(\hat{q}_\bar{\varepsilon}) \right] \quad \text{for } \gamma_1 < \gamma < \gamma_2
\]

\[
q_\varepsilon = \bar{q}_\bar{\varepsilon}, \quad q_\bar{\varepsilon} = \bar{q}_\bar{\varepsilon} \quad \text{for } \gamma \leq \gamma
\]

where \( \gamma_1 \) is derived from the following two equations

\[
0 = (1 - \theta) \left[ \varepsilon u(\bar{q}_\varepsilon) - \bar{\varepsilon} u(q^*_\varepsilon) \right] - \theta \left[ c(q^*_\varepsilon) - c(\bar{q}_\varepsilon) \right]
\]

\[
\gamma_1 = \beta + \frac{\sigma \lambda \theta [\varepsilon u'(\bar{q}_\varepsilon) - c'(\bar{q}_\varepsilon)]}{(1 - \theta) \varepsilon u'(\bar{q}_\varepsilon) + \theta c'(\bar{q}_\varepsilon)}
\]

The first equation yields a value \( \bar{q}_\varepsilon \) associated with the low \( \varepsilon \) money balances just binding while the second comes from the FOC. Similarly, we obtain \( \gamma_2 \) from

\[
0 = (1 - \theta) \left[ \varepsilon u(\bar{q}_\varepsilon) - \bar{\varepsilon} u(q^*_\varepsilon) \right] - \theta \left[ c(q^*_\varepsilon) - c(\bar{q}_\varepsilon) \right]
\]

\[
\gamma_2 = \beta + \frac{\sigma \theta (1 - \lambda) [\varepsilon u'(\bar{q}_\varepsilon) - c'(\bar{q}_\varepsilon)]}{(1 - \theta) \varepsilon u'(\bar{q}_\varepsilon) + \theta c'(\bar{q}_\varepsilon)} + \lambda \sigma \theta \tau \left( 1 - \theta \tau \right)^{-1}
\]
As before, for $z \leq \tilde{z}_\varepsilon$ for $\gamma > \tilde{\gamma}$ no monetary equilibrium exists; only a credit equilibrium exists. We now have a range of inflation rates such that money and credit trades coexist; those for $\tilde{z}_\varepsilon \leq z \leq \tilde{z}_\varepsilon$. For $z$ in this range, the high $\varepsilon$ buyers do not have enough cash so they acquire $\tilde{q}_\varepsilon$ with a combination of cash and credit.

Figure 1 shows the different possible equilibria. For $\gamma = \beta$ we get the first best. For $\gamma_1 \leq \gamma \leq \gamma_2$ we have a money only equilibrium where $\tilde{q}_\varepsilon < q^*_\varepsilon$ and $\tilde{q}_\varepsilon < q^*_\varepsilon$. For $\gamma_2 \leq \gamma \leq \tilde{\gamma}$ we have coexistence of money and credit. The high $\varepsilon$ buyer is using both cash and credit to acquire $\tilde{q}_\varepsilon$ while the low $\varepsilon$ buyer continues to use only cash. Finally at $\tilde{\gamma}$ the credit equilibrium exists for both buyers and after that, the credit only equilibrium exists.

**Productivity Differentials** Rather than differences in utility across buyers we could assume that sellers differ by their productivities (either permanent or temporary). Assume that sellers’ utility cost of producing is given by $c(q, \alpha)$ where $\alpha$ is a productivity parameter with $c_\alpha(q, \alpha), c_{\alpha\alpha}(q, \alpha) < 0$. Consider a 2-point distribution $\alpha \in \{\underline{\alpha}, \overline{\alpha}\}$. In this case, we can redo the bargaining solutions and derive the surpluses as before. We can show existence of equilibrium as before: for $\gamma = \beta$ we get the first-best allocation and for $\gamma > \tilde{\gamma}$ we have the credit-only equilibrium. In between, we can show that a monetary equilibrium exists but uniqueness is difficult to prove without further restrictions (such as imposing buyer-take-all,
\( \theta = 1 \). For a 2 point distribution, linear utility and a CES cost function, we can derive a unique equilibrium shown graphically in Figure 2.

For \( \gamma = \beta \) we get the first best. For \( \gamma_1 \leq \gamma \leq \gamma_2 \) we have a money only equilibrium where \( \hat{q}_\alpha < q_\alpha^* \) and \( \hat{q}_\alpha < q_\alpha^* \). For \( \gamma_2 \leq \gamma \leq \tilde{\gamma} \) we have coexistence of money and credit. The high \( \alpha \) sellers accept cash and extend credit to let the buyer acquire \( \tilde{q}_\alpha \) while the low \( \alpha \) seller continues to accept only cash. Finally at \( \tilde{\gamma} \) both sellers extend credit to the buyer and above that only the credit equilibrium exists. The only difference between this version of the model and the \( \varepsilon \) version (which holds for all utility and cost specifications) is that \( \hat{q}_\pi > \hat{q}_\omega \) when all buyers are cash constrained, as occurs at \( z_0 \). In the \( \varepsilon \) difference set-up the reverse was true, \( \hat{q}_\omega > \hat{q}_\pi \), as can be seen in Figure 1 for \( z_0 \). The difference arises because an increase in \( \alpha \) lowers the seller’s cost of production and thus the match surplus. The seller must transfer some of this surplus to the buyer but his cash holdings are fixed. Thus, the seller produces more output for the same amount of real balances. In short, high \( \alpha \) sellers charge a lower price.

In the \( \alpha \)-model, for \( \gamma_2 \leq \gamma \leq \tilde{\gamma} \) large producers (the high productivity sellers) use credit while low productivity sellers are paid in cash. This is consistent with empirical evidence that large firms tend to operate in the formal economy while small firms are the ones most likely to produce solely for cash in the informal sector.
7 Conclusion

We constructed a micro-founded model of money in order to study the use of cash to evade taxes in the informal sector. We exploit the key frictions that give money an essential role in allocation resources across agents – record-keeping of individual trading histories, public communication of trading histories and enforcement – to show how they also enable governments to collect tax revenues from agents. A key finding of the model is that with heterogeneity of types across either buyers or sellers, we will have co-existence of money and credit trades for a wide variety of inflation rates. However, as inflation increases, monetary trade declines and agents are induced to move into the formal sector and use credit as a means of payment. We also show that when there are productivity differences across agents, high productivity firms will tend to be involved in credit transactions while low productivity firms resort to cash as a means of payment. Our results highlight a key point – using money to evade taxes is not the result of incomplete or under-developed financial markets. Its about avoiding record-keeping.

There are several interesting extensions of our work that could be done. First, we do not explore optimal fiscal policy analysis. This seems like an obvious next step. Second, we do not take our model to the data or calibrate it to see how well it matches existing evidence on the size of the informal sector or the relationship between inflation and the size of the informal sector. We leave those interesting exercises to future work.
References


Appendix

Bargaining solution.

\[
\max_{d_\varepsilon, q_\varepsilon, \ell_\varepsilon} -c(q_\varepsilon) + (1 - \tau)^{-1} \phi d_\varepsilon + (1 - \tau)^{-1} [1 - \tau (\ell)] \phi \ell_\varepsilon
\]

\[s.t. \quad 0 \leq d_\varepsilon \leq m, \quad 0 \leq \ell_\varepsilon
\]

\[\varepsilon u(q_\varepsilon) - (1 - \tau)^{-1} \phi (d_\varepsilon + \ell_\varepsilon) \geq \theta [\varepsilon u(q_\varepsilon) - c(q_\varepsilon) - \tau (\ell) (1 - \tau)^{-1} \phi \ell_\varepsilon].
\]

where

\[\phi \ell_\varepsilon = [1 - \theta \tau (\ell)]^{-1} \{(1 - \tau) [(1 - \theta) \varepsilon u(q_\varepsilon) + \theta c(q_\varepsilon)] - \phi d_\varepsilon\}.
\]

Let \(\phi \lambda_m\) denote the Lagrangian multiplier on the upper bound on \(d_\varepsilon\), \(\phi \lambda_\ell\) be the multiplier on non-negative lending and \(\lambda_s\) be the multiplier on the buyer’s surplus constraint. We can ignore the lower bound on \(d_\varepsilon\) for now. The FOC are

\[d_\varepsilon : 0 = (1 - \tau)^{-1} - \lambda_m - \lambda_s (1 - \tau)^{-1}
\]

\[q_\varepsilon : 0 = -c' (q_\varepsilon) + \lambda_s [(1 - \theta) \varepsilon u'(q_\varepsilon) + \theta c'(q_\varepsilon)]
\]

\[\ell_\varepsilon : 0 = (1 - \tau)^{-1} [1 - \tau (\ell)] + \lambda_\ell - (1 - \tau)^{-1} \lambda_s [1 - \theta \tau (\ell)]
\]

For \(q_\varepsilon > 0, \lambda_s > 0\).

Case 1: \(\lambda_m = \lambda_\ell = 0\). In this case we have

\[d_\varepsilon : \lambda_s = 1
\]

\[q_\varepsilon : 0 = \varepsilon u'(q_\varepsilon) - c' (q_\varepsilon)
\]

\[\ell_\varepsilon : \lambda_s = \frac{1 - \tau (\ell)}{1 - \theta \tau (\ell)}
\]

The first and last expressions require \(\tau (\ell) = 0\) and thus \(\ell = 0\). Thus the solution has \(\ell = \tau (\ell) = 0\) and \(q_\varepsilon = q^*_\varepsilon\) with \(d^*_\varepsilon = m^*_\varepsilon \equiv \phi^{-1} (1 - \tau) [(1 - \theta) \varepsilon u(q^*_\varepsilon) + \theta c(q^*_\varepsilon)] \leq m.\)

Case 2: \(\lambda_m = 0, \lambda_\ell > 0\). In this case we have \(\ell = \tau (\ell) = 0\) and

\[d_\varepsilon : \lambda_s = 1
\]

\[q_\varepsilon : 0 = \varepsilon u'(q_\varepsilon) - c' (q_\varepsilon)
\]

which yields the same solution as before. So if \(\lambda_m = 0\), then \(\ell = \tau (\ell) = 0\). In short, a buyer would never take out a loan and keep some cash.

Case 3: \(\lambda_m > 0, \lambda_\ell > 0\). In this case we have \(\ell = \tau (\ell) = 0, d_\varepsilon = m\) and \(q_\varepsilon\) solves \(\phi m = \)
(1 − τ) [(1 − θ) εu(q_e) + θc(q_e)].

Case 4: \(\lambda_m > 0, \lambda_e = 0\). In this case we have \(d_e = m\) and

\[
\begin{align*}
q_e & : 0 = -c' (q_e) + \lambda_s [(1 - \theta) \varepsilon u'(q_e) + \theta c'(q_e)] \\
\ell_e & : 0 = (1 - \tau)^{-1} [1 - \tau (\ell)] - (1 - \tau)^{-1} \lambda_s [1 - \theta \tau (\ell)]
\end{align*}
\]

and \(\ell_e\) is given by

\[
\phi \ell_e = [1 - \theta \tau (\ell)]^{-1} \{(1 - \tau) [(1 - \theta) \varepsilon u(q_e) + \theta c(q_e)] - \phi m\}.
\]

If \(\ell_e = 0\) then \(\tau (\ell) = 0\) and we have

\[
\begin{align*}
\varepsilon u'(q_e^*) &= c'(q_e^*) \\
\phi m &= (1 - \tau) [(1 - \theta) \varepsilon u(q_e^*) + \theta c(q_e^*)]
\end{align*}
\]

but this can only be satisfied if

\[
\phi m = m_e^* \equiv \phi^{-1} (1 - \tau) [(1 - \theta) \varepsilon u(q_e^*) + \theta c(q_e^*)].
\]

If this does not hold then we must have \(\ell_e > 0\) and \(\tau (\ell) = \tau\). As a result we have

\[
\varepsilon u'(\tilde{q}_e) = (1 - \tau)^{-1} c'(\tilde{q}_e)
\]

and

\[
\phi \ell_e = (1 - \theta \tau)^{-1} \{(1 - \tau) [(1 - \theta) \varepsilon u(\tilde{q}_e) + \theta c(\tilde{q}_e)] - \phi m\}
\]

In this case we see that \(\tilde{q}_e\) is independent of \(m\) so \(\ell_e > 0\) requires \(m < \phi^{-1} (1 - \tau) [(1 - \theta) \varepsilon u(\tilde{q}_e) + \theta c(\tilde{q}_e)] \equiv \tilde{m}_e\).

**Demand for Money.** Here, we derive the properties of the cut-off functions \(\varepsilon^*(z)\) and \(\bar{\varepsilon}(z)\). We start by showing the money holdings required to obtained the unconstrained allocation are increasing in the preference shock \(\varepsilon\). For it, consider the solution \(q_e^*\) to:

\[
\varepsilon u'(q_e^*) = c'(q_e^*)
\]
\[ \frac{\partial q^*_\varepsilon}{\partial \varepsilon} = -\frac{u'}{\varepsilon u'' - c''} \]

Hence, if costs are convex, the previous derivative is always positive. For the buyer to capture a proportion \( \theta \) of the total surplus, we must have:

\[ z^*_\varepsilon \equiv (1 - \tau) \left[ (1 - \theta) \varepsilon u(q^*_\varepsilon) + \theta c(q^*_\varepsilon) \right]. \]

Finally,

\[ \frac{\partial z^*_\varepsilon}{\partial \varepsilon} = (1 - \tau)[(1 - \theta)(u(q^*_\varepsilon) + \varepsilon u'(q^*_\varepsilon)) + \theta c'(q^*_\varepsilon)] \frac{\partial q^*_\varepsilon}{\partial \varepsilon} > 0. \]

Observe also that \( \varepsilon^*(z) \) can be defined implicitly, at each \( z \), as the solution to

\[ z = (1 - \tau) \left[ (1 - \theta) \varepsilon^* u(q^*_\varepsilon) + \theta c(q^*_\varepsilon) \right]. \]

Since \( z^*_\varepsilon \) is increasing in \( \varepsilon \) it follows that \( \varepsilon^*(z) \) in increasing in \( z \). □

To understand why \( \tilde{\varepsilon}(z) \geq \varepsilon^*(z) \), consider a shock \( \varepsilon_0 \) larger than, but close enough to \( \varepsilon^*(z) \). If credit is going to be used then the surplus obtained has to be larger than what is attainable with money only. This is true because, other things equal, loans reduce the total surplus. What is attainable with money only (in a constrained trade)? In these types of trades output is given by

\[ z = (1 - \tau) \left[ (1 - \theta) \varepsilon_0 u(q^*_\varepsilon) + \theta c(q^*_\varepsilon) \right]. \]

Furthermore, \( \tilde{q}_{\varepsilon_0} \) must converge to \( q^*_\varepsilon \) as \( z \rightarrow z^*_\varepsilon \). Hence, the surplus attainable with money only when \( \varepsilon_0 \) approaches \( \varepsilon^*(z) \) from above, is also close to the optimal one. What is attainable with loans? Observe that output under a credit trade is given by

\[ \tilde{\varepsilon} u'(\tilde{q}_\varepsilon) = (1 - \tau)^{-1} c'(\tilde{q}_\varepsilon), \quad (9) \]

because of the tax wedge, \( \tilde{q}_\varepsilon < \hat{q}_{\varepsilon_0} \). Thus, a trade with loans attains a strictly lower output, involves a strictly positive loan and results then in a lower surplus. It follows that trades with loans can only occur for shocks strictly higher than \( \varepsilon^*(z) \). □

**Buyer's objective.** Consider now the derivative of the objective function evaluated at strictly positive money holdings:

\[ (\beta - \gamma) + \beta \sigma \theta \tau (1 - \tau)^{-1} (1 - \theta \tau)^{-1} \int_{\tilde{\varepsilon}(z)}^{\varepsilon} dF(\varepsilon) + \]
the derivative of the objective is
\[ + \beta \sigma \theta \int_{\hat{\varepsilon}(\xi)}^{\tilde{\varepsilon}(\xi)} \frac{[\varepsilon u'(\hat{q}_e) - c'(\hat{q}_e)]}{[(1 - \theta) \varepsilon u'(\hat{q}_e) + \theta c'(\hat{q}_e)]} d\mathcal{F}(\varepsilon) + \]
\[ + \beta \sigma \theta f(\tilde{\varepsilon}) \frac{\partial \tilde{\varepsilon}(\xi)}{\partial z} [\varepsilon^* u(q^*_e(z)) - c(q^*_e(z)) - (\varepsilon^* u(\tilde{q}_e(z)) - c(\tilde{q}_e(z)))] + \]
\[ + \beta \sigma \theta f(\hat{\varepsilon}) \frac{\partial \hat{\varepsilon}(\xi)}{\partial z} [\hat{\varepsilon} u(\hat{q}_e(z)) - c(\hat{q}_e(z)) - (\hat{\varepsilon} u(\tilde{q}_e(z)) - c(\tilde{q}_e(z)) - \tau (1 - \tau)^{-1} \ell(z))]. \]

The last two terms of this derivative take into account that bringing more money changes the types of trades the buyer may face. Specifically, higher money holdings allow more unconstrained trades to occur (the previous to last term), which obviously reduces the number of constrained trades. Higher real holdings also increases the return of constrained trades, which simultaneously reduces the set of trades where credit is employed. These last two terms of the derivative of the buyer are, nevertheless, equal to zero. The previous to last term is zero because it evaluates the surplus exactly at the shock where the constraint starts binding. At that point the two surpluses are thus equal to each other. The last term is zero because it evaluates surpluses exactly at the point where buying with credit is equivalent to buying under a constrained trade.

**Uniqueness of equilibrium.** For ease of presentation, we start with the case where inflation is not too high, namely, \((\beta - \gamma) + \beta \sigma \theta \tau (1 - \tau)^{-1} (1 - \theta \tau)^{-1} > 0\).

(a) For all money holdings \(0 < z < z_0\) where \(z_0\) is the largest holding such that \(\tilde{\varepsilon}(z_0) = \hat{\varepsilon}\). Because of the above assumption, the derivative of the buyer’s objective function is positive. Optimal money holdings must be higher than \(z_0\).

(b) for money holdings \(z_0 < z < z_1\), where \(z_1\) is the highest value of money holdings such that \(\varepsilon^*(z_1) = \tilde{\varepsilon}\) the derivative of the objective is
\[ (\beta - \gamma) + \beta \sigma \theta \tau (1 - \tau)^{-1} (1 - \theta \tau)^{-1} \int_{\tilde{\varepsilon}(z_1)}^{\tilde{\varepsilon}(\xi)} d\mathcal{F}(\varepsilon) + \]
\[ \beta \sigma \theta \int_{\tilde{\varepsilon}}^{\hat{\varepsilon}(\xi)} \frac{[\varepsilon u'(\hat{q}_e) - c'(\hat{q}_e)]}{[(1 - \theta) \varepsilon u'(\hat{q}_e) + \theta c'(\hat{q}_e)]} d\mathcal{F}(\varepsilon). \]

We know \(\varepsilon u'(\hat{q}_e) - c'(\hat{q}_e) > 0\). But since \(\hat{\varepsilon}(z)\) is increasing in \(z\) the second term in the sum is less than one. The sign of this derivative depends on the specific functional forms and parameterizations chosen. It is easy to impose additional regularity conditions such that \(\frac{[\varepsilon u'(\hat{q}_e) - c'(\hat{q}_e)]}{[(1 - \theta) \varepsilon u'(\hat{q}_e) + \theta c'(\hat{q}_e)]}\) is decreasing in real money holdings. Under these conditions it suffices to check the value of this derivative at \(z_1\). If it is negative, then there is a unique zero for the derivative of the buyer’s objective in the \(z_0 < z < z_1\) range.

(c) Finally, for money holdings \(z > z_1\) we have \(\tilde{\varepsilon}(z) > \varepsilon^*(z) > \hat{\varepsilon}\). The derivative of the
objective is
\[(\beta - \gamma) + \beta \sigma \theta \tau (1 - \tau)^{-1} (1 - \theta \tau)^{-1} \int_{\hat{\xi}(z)}^{\bar{\xi}} d\mathcal{F}(\xi) + \]
\[
\beta \sigma \theta \int_{\hat{\xi}(z)}^{\bar{\xi}(z)} \frac{[\varepsilon u'(\hat{q}_\varepsilon) - c'(\hat{q}_\varepsilon)]}{[(1 - \theta) \varepsilon u'(\hat{q}_\varepsilon) + \theta c'(\hat{q}_\varepsilon)]} d\mathcal{F}(\xi).
\]
We now show that as \(z\) increases to \(z^*_z\) then the last two terms of the function vanish. This proves the objective is decreasing near \(z^*_z\). Hence, the derivative of the objective, if positive at \(z_1\) must have a unique zero in the range \(z_1 < z < z^*_z\) whenever \(\frac{[\varepsilon u'(\hat{q}_\varepsilon) - c'(\hat{q}_\varepsilon)]}{[(1 - \theta) \varepsilon u'(\hat{q}_\varepsilon) + \theta c'(\hat{q}_\varepsilon)]}\) is decreasing in \(z\).

Uniqueness of equilibrium holds generally because the results in Wright (2010) can be applied to our model even when \(\frac{[\varepsilon u'(\hat{q}_\varepsilon) - c'(\hat{q}_\varepsilon)]}{[(1 - \theta) \varepsilon u'(\hat{q}_\varepsilon) + \theta c'(\hat{q}_\varepsilon)]}\) is not decreasing in real money holdings. Equilibrium for high inflation rates, that is, when \((\beta - \gamma) + \beta \sigma \theta \tau (1 - \tau)^{-1} (1 - \theta \tau)^{-1} < 0\) imply the objective in decreasing in money holdings up to \(z = z_1\). Computing the optimum requires thus a direct comparison of the surplus obtained with money holdings higher than \(z_1\) and the surplus obtained when buyers carry zero money holdings and all transactions are based on credit. ■