Employee Initiative and Managerial Control

Heikki Rantakari*
University of Southern California
Marshall School of Business

December 22, 2010

Abstract

I analyze the interaction between a manager and an employee in a setting where both parties can come up with new ideas for implementation. For low levels of managerial effort, I illustrate the presence of a competition effect, whereby increased effort by the manager increases employee effort. To utilize this competition effect and to maximize employee initiative, the manager should always retain formal authority over which ideas to implement but at the same time limit her involvement in the idea generation stage. Too much involvement will crowd out employee initiative, while too little involvement or control will allow the employee to pursue his pet projects at the detriment of overall organizational goals. Alternatively, if the cost of employee initiative is too large in terms of the compromised quality of managerial ideas, the manager prefers to work alone. The worst possible outcome arises when the manager is involved enough to demotivate the employee but not good enough to generate strong alternatives herself. Additional results illustrate how the manager can rely on competition between employees as a source of ideas and how the optimal level of managerial involvement interacts with the use of performance-based pay.

*contact: rantakar@marshall.usc.edu. I would like to thank Kevin Murphy, John Matsusaka, Tony Marino, Mark Westerfield, Bob Gibbons, Ricardo Alonso and the participants of USC CLEO and MIT Organizational Economics lunch workshops for helpful comments and suggestions. All remaining errors are my own.
1 Introduction

Organizations have become increasingly aware of their employees as a valuable source of ideas and innovation. As a result, there has been increased interest in finding organizational practices that facilitate employee initiative and a pronounced shift in corporations away from traditional command-and-control structures towards more employee participation and empowerment.\footnote{See, for example, Osterman (1994,2000) and Applebaum et al. (2000) and the references therein on the adoption and productivity consequences of High Performance Work Organizations aimed at the increased utilization of the ideas and creativity of the work force.} A similar shift has occurred in the academic literature on the manager-employee interaction. The traditional view of a manager was that of a supervisor and a monitor who makes sure that the subordinates perform their assigned tasks appropriately (Calvo and Wellisz, 1978, for example), with increased managerial monitoring increasing employee effort. There was no role for the allocation of authority because the premise was that the manager knows what tasks the employee should do. More recently, the attention has shifted to examining situations where there is uncertainty over the right task choice, and with a particular focus on the impact of employee discretion on organizational performance.

In the now-classic contribution, Aghion and Tirole (1997) analyze a model where the right action is not known ex ante and instead both the manager and the employee can exert costly effort to learn what the right course of action would be. In other words, the manager can now rely on the employee as a source of information instead of acquiring information herself, introducing a collaborative element to the manager-employee interaction. The main features of this framework are two-fold. First, the efforts of the manager and the employee are substitutes: the harder the manager works on finding out the right course of action, the less valuable it is for the employee to do the same. Second, employee initiative is maximized by formally delegating the right to choose the course of action to the employee, as he is then able to pursue tasks he finds particularly rewarding without the risk of being overruled by the manager. This additional initiative, however, comes at a cost, which is commonly referred to as "loss of control." To the extent that the employee's preferences are not fully aligned with the goals of the firm, the employee can make choices that the manager would like to overrule. It is this tradeoff between increased initiative and loss of control that has become one of the main tradeoffs associated with delegation (and employee discretion in general).

This paper contributes to this literature on motivational consequences of employee discretion by introducing a competitive element to the manager-employee interaction. In the model, a manager and an employee can both work to come up with an "idea," which could be an improvement to an existing product design, a new way of organizing a given production process or the like. Because of resource constraints (or because the ideas are mutually exclusive), only one of the ideas can be implemented and then yields particular payoffs to the manager (the organization as a whole) and the employee. Finally, when choosing their efforts, the parties do not choose only how much effort to exert, but also which types of ideas they are likely to generate. That is, they can focus their attention on ideas that are primarily beneficial to the manager, to the employee, or any combination thereof.

To influence the equilibrium efforts and the resulting outcome, the manager has two control
instruments at her disposal. First, by altering her set of other responsibilities, the manager can influence her own opportunity cost of time and thus how costly it is for her to engage in idea generation itself. I will refer to the equilibrium amount of effort exerted by the manager as the level of managerial involvement. Second, the manager can formally delegate the right to choose which idea to implement to the employee.

The results from the model are two-fold. First, when the manager retains formal authority, I illustrate the presence of a competition effect, whereby increased managerial involvement can increase the expected quality of ideas generated by the employee. When the level of managerial involvement is low, the employee will only pursue his pet projects because he knows that the manager is going to approve them for the lack of better alternatives. Increasing the level of managerial involvement increases the expected quality of managerial ideas, which then initially encourages the employee to work harder: the employee knows that he needs to generate an idea that is more attractive to the manager than her own idea to have his idea chosen for implementation. This competition effect is, however, present only for sufficiently low levels of managerial involvement. The reason is that once the manager becomes too involved, the employee comes to perceive the likelihood that his idea gets selected to be too low to warrant working on it in the first place. The result is then a non-monotone relationship between managerial involvement and employee initiative, with initial involvement motivating but extensive involvement demotivating the employee.

Second, under the assumption that the worker is (weakly) more efficient than the manager in generating ideas, the manager never wants to formally delegate the right to choose to the employee. The simple reason is that it is only by retaining the formal right to choose which idea to implemented that the manager is able to induce the employee to work on generating value to the organization. If the employee had the right to choose, he would focus only on ideas that he finds particularly rewarding. In other words, much like too little managerial involvement, formal delegation would grant the employee too much discretion and actually demotivate the employee.

Given these two results, the optimal organizational arrangement is then straightforward. To maximize employee initiative, the manager should always retain formal authority but limit her level of involvement in idea generation. By retaining formal authority and providing some involvement, the manager is able to motivate the employee through some healthy competition for ideas, while by limiting that involvement she avoids crowding out the initiative of the employee. This maximization of employee initiative comes, however, at a cost, which is the compromised quality of managerial ideas. If these costs are too large relative to the induced initiative, then the manager prefers to work alone and be the sole source of ideas.

The analysis thus identifies two qualitatively different optimal organizational arrangements, where either the manager works alone, or she restricts herself enough so that the employee becomes the main source of ideas. Given these characteristics, one could interpret these two outcomes as "centralization" and "decentralization," respectively. The differences to the existing literature on authority are two-fold. First, in both arrangements, the manager retains formal authority. Instead, we are only influencing her level of involvement in the task itself. Second, the two outcomes arise from a continuous set of alternatives, where the transition from one to the other is discontinuous: the worst possible outcome for the organization occurs in the intermediate case where the man-
ager is involved enough to demotivate the employee but not good enough to come up with strong alternatives herself.

Beyond this pattern of optimal organizational arrangements, two general features of the solution deserve particular attention. The first is the presence of the competition effect, whereby the efforts of the employee and the manager are complements for low levels of managerial involvement. This effect is important because it implies that there are situations where there is unambiguously too little managerial involvement: in the presence of the competition effect, less managerial involvement leads to both loss of control and loss of initiative. Further, while identified here in a particular setting, the competition effect can arise in a variety of situations where it is possible for the employee to "do better" than the manager by working harder, which introduces the competitive element to the manager-employee interaction. For example, beyond generating better ideas, an employee can evaluate more alternatives or find a solution to a given problem faster than the manager by working harder. This is the key element that is absent in Aghion and Tirole (1997) where, if the manager finds her preferred project, there is no way for the employee to do any better.

The second observation relates to the type of ideas generated in equilibrium. While the solution that maximizes employee initiative frequently implements the employee's idea and the employee could thus be viewed as having a significant amount of "real authority" in the terminology of Aghion and Tirole (1997), the employee's idea is implemented only because he is taking into account the manager's preferences when generating ideas. The "authority" that the employee holds in this case is thus partially an illusion, a result that highlights one of the important features of authority relationships. Subordinates often make decisions or take actions that are not overruled not because they truly have authority but because they choose to undertake actions that they know won't get overruled by the superior.

Having established the basic results, I then analyze various extensions to the model and examine how these extensions impact the optimal level of managerial involvement. First, I illustrate how competition between employees functions as a natural substitute for managerial involvement: since each employee can get their idea chosen only by providing higher benefits to the manager than the other idea, the employees end up (partially) internalizing the organizational goals even when they are directly motivated only through their individual payoffs. Second, I consider the optimal scale at which to implement both ideas, thus introducing a "second prize" to the contest. I illustrate how such a second prize can be used as an additional motivational tool that is a complement to managerial involvement.

Third, I consider the role of explicit monetary compensation, both in the forms of a bonus conditional on the acceptance of the employee's idea and pay contingent on the realized value of the implemented idea. I show that a formal bonus functions as a substitute to managerial involvement while pay contingent on the realized value is a complement to managerial involvement, unless it becomes optimal for the manager to stop working altogether. The reason for the distinction is that a bonus based on acceptance biases the manager against accepting the employee's idea and thus functions as a performance threshold analogous to a managerial idea. In contrast, pay based on the realized value of the implemented idea is independent of the choice and simply provides additional motivation to the employee. Finally, I illustrate how allowing the employee to generate an idea first
dominates simultaneous efforts by utilizing the employee’s incentives to pre-empt the manager from generating an idea herself.

The remainder of the paper is organized as follows. Section 2 outlines the related literature and section 3 presents the model. The basic results are derived in sections 4 and 5, and section 6 considers the extensions to multiple employees, the implementation of multiple ideas, use of monetary transfers and sequential efforts. Section 7 concludes.

2 Related Literature

This paper is related to three different strands of literature. The first strand is the literature on delegation and authority already mentioned above. The four contributions closest to the present model are Aghion and Tirole (1997), Newman and Novoselov (2009), Bester and Krähmer (2008) and Armstrong and Vickers (2010).\footnote{The credibility of delegation in the Aghion-Tirole framework is analyzed in Baker, Gibbons and Murphy (1999) in a repeated game setting. In the present setting, the issue of credibility does not arise because formal delegation to the employee is never optimal.} The first two papers both consider the interaction between managerial and employee efforts and the role that the allocation of authority plays in motivating the employee to exert effort in alternative settings.\footnote{The motivational impact of delegation is also analyzed in Zabojnik (2002) but in a setting where the principal exerts no effort and there are thus no effort interactions.} The key difference is that the present paper introduces the competition effect of managerial involvement whereby, conditional on the allocation of authority, increased managerial effort can actually lead the employee to work harder. The competition effect and the suboptimality of formal delegation are, in turn, conceptually related to the latter two papers, even if the consider very different settings.\footnote{Bester and Krähmer analyze the implications of authority on the effort that an employee will exert in implementing a chosen project (and thus deals with ex post instead of ex ante effort) and Armstrong and Vickers analyze the optimal acceptance rule for projects when the project proposals are made by a strategic agent who has access to a random number of alternatives.} These links are discussed in more detail in section 5.1, after the presentation of the results.

The role of authority has also been analyzed from various other perspectives. Its impact on the transmission and use of information is analyzed, among others, in Dessein (2002), Alonso, Dessein and Matouschek (2008) and Rantakari (2008), while its role in overall organizational design is analyzed, among others, in Friebel and Raith (2010), Dessein, Garicano and Gertner (2010) and Rantakari (2010). The key difference in the approaches is that in these papers the principal plays no active role beyond that of a potential decision-maker, while here it is the combination of authority and the effort choices that is the key behind the equilibrium outcome.

The second strand is the large literature on tournaments and contests that has followed the contributions of, for example, Tullock (1967), Lazear and Rosen (1981) and Rosen (1986). This link arises because the present model is essentially a contest in ideas: the manager and the employee both exert effort to generate an idea, of which one gets chosen for implementation. The key differences are that in the present setting one of the participants has the right to choose the winner, and that the value of the "prize" is determined endogenously through the efforts of the participants.
which influence the value of the implemented idea. As pointed out by Baye and Hoppe (2003), endogenous-prize innovation tournaments have not received extensive attention in the theoretical literature. Their contribution is a notable exception, but they still consider a very different setting where the payoffs and efforts are one-dimensional, with effort improving both the size of the prize and the likelihood of winning the prize. Finally, the interaction of the effort levels of the participants, which plays an important role in the present analysis, is analyzed, among others, by Dixit (1987) in a standard tournament setting and by Nti (1999) in a Tullock rent-seeking contest.

The third related strand is the broader literature on agency models of innovation, such as Aghion and Tirole (1994), Rotember and Saloner (1994,2000), Murdock (2002), Hellman and Thiele (2009) and Manso (2010). These papers, however, focus on different aspects of the problem and do not consider the role that the manager can play as an alternative source of ideas, which is the key feature of the present model.

3 Model

The basic model consists of a manager (P) and an employee (A). Each party can generate an idea that, if implemented, will yield benefits to both the employee and the organization as a whole. These benefits are captured by a two-dimensional payoff \((v_i, w_i)\), where \(v_i\) is the value of the idea to the manager (the organization) and \(w_i\) is the value of the idea to the employee, with \(i \in \{P, A\}\) indexing the party that generated the idea.

The value of the ideas is ex ante uncertain, but the parties can exert personally costly effort to improve the expected quality of their ideas. In particular, by exerting efforts \((x_{i,v}, x_{i,w})\), the values \((v_i, w_i)\) are drawn from independent exponential distributions with rate parameters \(\lambda_{i,v} = \frac{1}{x_{i,v}}\) and \(\lambda_{i,w} = \frac{1}{x_{i,w}}\). As a result, \(E(v_i|x_{i,v}) = x_{i,v}\) and we can view the parties as choosing the expected quality of their ideas. The cost of influencing the expected quality of the ideas is given by

\[
C_i(x_{i,v}, x_{i,w}) = \frac{c_{i,v}}{2} x_{i,v}^2 + \frac{c_{i,w}}{2} x_{i,w}^2,
\]

where \(c_{i,v}, c_{i,w}\) parameterize how costly it is for the two parties to influence the quality of their ideas in the manager’s and the employee’s dimension, respectively.\(^6\) I assume that \(c_{A,v} \leq c_{P,v}\) and \(c_{A,w} \leq c_{P,w}\), so that the employee is weakly more efficient in generating ideas.\(^7\) The parties choose their efforts simultaneously and non-cooperatively.

---

\(^5\)The endogeneity of the "prize" links the present paper also weakly to models of conflict, such as Skaperdas (1992), which examine the resource allocation problem between productive and appropriative activities. The basic setups are, however, fundamentally different.

\(^6\)The separability of costs is assumed simply for tractability. An alternative formulation with perfectly substitutable efforts is illustrated in a separate Online Supplement.

\(^7\)Note that since we have normalized the effort to measure the expected quality of ideas, the advantage of the agent can come from either the fact that his time is truly more productive because of his proximity to the task, or because his opportunity cost of time is lower.
The manager offers a contract \((\mu, s)\) to the employee. The employee accepts or rejects. The two parties choose simultaneously their levels of effort \((x_i, v_i, x_i, w_i)\). The value of the ideas is observed by both parties. The manager chooses which idea to implement. Payoffs are realized.

Figure 1: Timing of events

After the ideas have been generated, both parties observe the value of the two ideas and the manager chooses her preferred idea for implementation.\(^8\) In this basic model, I assume that both \(v_i\) and \(w_i\) are non-contractible. For example, \(v_i\) could measure the value of a given design innovation in improving a product that is impossible to disentangle from the overall value of the product, or the benefits of the idea are realized only in a sufficiently distant future so that pay-for-performance is infeasible. Similarly, \(w_i\) measures the non-contractible benefits that the employee can get from having his particular idea implemented. It could reflect, for example, an increase in his reputation, an improvement in his general human capital, or simply the enjoyment he can get from continued work on the idea. As a result, when choosing her effort levels and which idea to implement, the manager cares only about benefits \(v_i\), while the employee cares only about the benefits \(w_i\).

In the first stage of the game, the manager offers the employee a contract consisting of two components. First, while the manager cannot commit to a given level of involvement \((x_{P,v}, x_{P,w})\), I assume that her level of involvement can be influenced indirectly by altering her opportunity cost of time. Technically, I capture this by assuming that given the minimum cost of managerial involvement \(C_P(x_{P,v}, x_{P,w})\), the actual cost (as determined by the contract) is \(\bar{C}_P(x_{P,v}, x_{P,w}) = \mu C_P(x_{P,v}, x_{P,w})\), where \(\mu \in [1, \infty)\) determines the opportunity cost of time. In other words, while the manager cannot make herself more efficient in idea generation, she can make herself less efficient.

The second component of the contract is a (fixed) salary \(s\), which is used to satisfy the employee’s participation constraint. The participation constraint is given by \(EU_P + s \geq U_A\), where \(EU_P\) is the net utility the employee expects to earn in the relationship and \(U_A\) is his reservation utility. Thus, the more the employee expects to enjoy the employment relationship, the lower the wage that is needed to ensure participation. In the design stage, the manager thus solves:

\[
\max_{s, \mu} EU_P - s
\]

s.t. \(x_{i,v}, x_{i,w} \in \arg \max_i EU_i\) \quad \forall i \in \{P, A\}

\[
EU_A + s \geq U_A.
\]

This overall game is summarized in Figure 1. First, the manager offers the contract \((\mu, s)\) to the employee. Second, the two parties simultaneously choose their effort levels \((x_{i,v}, x_{i,w})\) to maximize their expected payoffs. Third, the value of the ideas \((v_i, w_i)\) is realized and observed by both parties, and the manager chooses which idea to implement. Finally, the payoffs are realized.

\(^8\)Because formal delegation is never optimal, the analysis focuses on the role of managerial involvement. Formal delegation is discussed in section 5.
3.1 Assumptions and extensions

The model outlined above makes a number of simplifying assumptions to isolate the role of managerial involvement \((x_{P,v}, x_{P,w})\) in motivating the employee. First, I assume that the manager is able to contractually commit to the opportunity cost of time, \(\mu\). In reality, such constructs are not contractible but the opportunity cost of time (in a given task) can still be influenced by altering the other responsibilities of the manager. For example, increasing the span of control increases the opportunity cost of time spent with any given employee. How such considerations can be used to close the model without the assumption of contractible \(\mu\) is illustrated in a separate Online Supplement.

I am also assuming that (i) the manager interacts only with one employee, (ii) only one idea can be implemented in equilibrium, (iii) no contingent monetary transfers are available and (iv) the effort choices are simultaneous. All four assumptions are relaxed in section 6 and their impact on the role of managerial involvement is analyzed.

4 Preliminary Results

To build the intuition behind the equilibrium of the game, it is instructive to establish some preliminary results. First, for the purposes of comparison, I will establish the first-best benchmark to the game. Second, I will examine the interaction between the effort choices of the manager and the employee when those choices are strategic, which will be the key to understanding the equilibrium of the game and the comparative statics analyzed in section 5.

4.1 First-best

If the decision rule and the effort choices of both the manager and the employee were contractible, then the first-best solution would be straightforward. First, we should implement the project with the highest total payoff \(v_i + w_i\). Second, the effort choices of the two parties should solve

\[
\max_{\{x_{i,k}\}} E\left(\max(v_A + w_A, v_P + w_P) \mid \{x_{i,k}\}\right) - C_P(x_{P,v}, x_{P,w}) - C_A(x_{A,v}, x_{A,w}).
\]

The solution to this problem yields (under the assumption that the employee is weakly more efficient than the manager) \(x_{P,v} = x_{P,w} = 0\) and \(x_{A,v} = 1/c_{A,v}, x_{A,w} = 1/c_{A,w}\). In other words, the manager should exert no effort while the employee should choose effort levels to maximize total surplus conditional on zero managerial effort. The intuition is as follows. Given that only one of the ideas can be implemented, the worse idea (and the effort sunk into generating it) is always wasted. As a result, the total optimal efforts of the manager and the employee are substitutes: an increase in the expected quality of one reduces the probability for the other idea being implemented. In the present setting, the substitutability of the efforts is sufficiently strong so that the expected benefit...
from improving the manager’s idea is outweighed by the reduction in the optimal expected quality of the employee’s idea, making it optimal for only one party to be active in equilibrium.\(^9\) Thus, the only reason for the manager to work in the present setting is that the employee has suboptimal incentives to work if the manager does not work.

### 4.2 Expected payoffs and effort choices

The next step is to solve for the expected payoffs and the equilibrium effort choices of the two parties when their behavior is strategic. First, since the manager cares only about her payoff \(v\) and can pick which idea gets implemented, we can write her expected payoff (derived in Appendix A.1) as

\[
EU_P = E \left( \max(v_A, v_P) \right) x_{A,v} x_{P,v} - \mu C_P(x_{P,v}, x_{P,w}) = \frac{x_{A,v}^2 + x_{P,v}^2}{x_{A,v} + x_{P,v}} - \mu \left( \frac{c_{P,v}}{2} x_{P,v}^2 + \frac{c_{P,w}}{2} x_{P,w}^2 \right),
\]

and so the first-order conditions are

\[
x_{P,w} : -\mu c_{P,w} x_{P,w} = 0 \\
x_{P,v} : \frac{(2x_{A,v} + x_{P,v})}{(x_{A,v} + x_{P,v})} x_{P,v} - \mu c_{P,v} x_{P,v} = 0.
\]

The manager will naturally never exert any effort to generate benefits to the employee, since she gets no benefits from that effort. The level of effort the manager exerts in her dimension, on the other hand, depends on both \(x_{A,v}\) and \(\mu\). Intuitively, the harder the employee works, the less likely it is that the manager ends up implementing her idea. Thus, to the manager, the two efforts are substitutes and \(\frac{\partial x_{P,v}}{\partial x_{A,v}} < 0\). Similarly, the higher the opportunity cost of her time, the lower her effort level and so \(\frac{\partial x_{P,v}}{\partial \mu} < 0\). Since \(x_{P,w} = 0\) in equilibrium, the remainder of the analysis will focus on the role of \(x_{P,v}\), which I will refer to as the level of managerial involvement.

To the employee, the situation is analogous to a contest. If \(v_A \leq v_P\), the manager chooses her idea and the employee receives the benefits \(E(w_P|x_{P,w}) = 0\). If \(v_A > v_P\), the manager will choose the employee’s idea and he receives \(E(w_A|x_{A,w}) = x_{A,w}\). The efforts in the employee’s dimension thus determine the size of the expected rewards to the employee while the efforts in the manager’s dimension determine the likelihood for each idea to be chosen for implementation. We can then write the employee’s payoff as

\[
EU_A = \Pr(v_A > v_P|x_{A,v}, x_{P,v}) E(w_A|x_{A,w}) - C_A(x_{A,v}, x_{A,w}) = \frac{x_{A,v}}{(x_{A,v} + x_{P,v})} x_{A,w} - \frac{c_{A,v} x_{A,v}^2}{2} - \frac{c_{A,w} x_{A,w}^2}{2},
\]

and so the first-order conditions are

\[
x_{A,w} : \frac{x_{A,v}}{(x_{A,v} + x_{P,v})} - c_{A,w} x_{A,w} = 0 \\
x_{A,v} : \frac{x_{P,v}}{(x_{A,v} + x_{P,v})} x_{A,w} - c_{A,v} x_{A,v} = 0.
\]

\(^9\)Of course, this conclusion can be reversed if the cost functions are sufficiently convex and so the crowding-out effect is smaller.
The employee’s incentives to work in his dimension are thus driven by the probability that his idea is actually implemented. The more likely it is that the employee gets his idea through, the more motivated he is to work on increasing the value of that idea to himself. The employee’s incentives to exert effort to benefit the manager come, in turn, from influencing the probability of getting his idea implemented. The two efforts are thus complements. The higher the probability of acceptance, the more the employee is willing to invest on generating benefits to himself, and the larger the benefits, the harder the employee is willing to work to get his idea implemented.

The next question is how these choices are influenced by the level of managerial involvement.

4.3 The motivational consequences of managerial involvement

The final step before analyzing the equilibrium outcome of the game is to examine how the employee’s effort choices depend on the level of managerial involvement, \( x_{P,v} \).11 The reason for performing this additional step is that while the effort choices are simultaneous and thus determined by the Nash equilibrium of the game, the manager can indirectly influence her (and thus the employee’s) choices in the design stage by altering her opportunity cost of time. Analysis of the employee’s best-response functions gives us the following proposition:

**Proposition 1 Impact of managerial involvement:**

(i) An increase in the manager’s effort \( x_{P,v} \) unambiguously reduces the employee’s effort in his dimension but will increase his effort in the manager’s dimension for sufficiently low levels of involvement:

\[
\frac{\partial x_{A,w}}{\partial x_{P,v}} < 0 \text{ while } \frac{\partial x_{A,v}}{\partial x_{P,v}} > 0 \text{ if and only if } \frac{x_{A,v}}{x_{P,v} + x_{A,v}} \geq \frac{2}{3}.
\]

(ii) An increase in the manager’s effort \( x_{P,v} \) will increase the employee’s total effort for sufficiently low levels of involvement:

\[
\frac{\partial (x_{A,w} + x_{A,v})}{\partial x_{P,v}} > 0 \text{ if and only if } \frac{x_{A,v}}{x_{P,v} + x_{A,v}} \geq \Pr(v_A > v_P) = \frac{2}{3} \left(1 + \frac{c_{A,v}}{3c_{A,w}} \left(1 + \frac{c_{A,v}}{c_{A,w}} - 1\right)\right).
\]

**Proof.** See Appendix A.1

The first part of the proposition illustrates the impact of managerial involvement on the two dimensions of employee effort. First, an increase in the level of involvement unambiguously reduces the employee’s incentives to work in his dimension. Intuitively, an increase in \( x_{P,v} \) reduces the equilibrium probability that the employee’s idea is implemented, leading to a reduction in \( x_{A,w} \).
Second, and which is the key part of the proposition, the relationship between managerial involvement and employee effort in the manager’s dimension is non-monotone in the level of involvement, with the employee initially working harder but later becoming demotivated as we increase the level of involvement.

To build the intuition behind this second result, note that while the employee’s incentives to work in his own dimension are determined by the probability of winning, his incentives to work in the manager’s dimension are determined by the marginal contribution of that effort to the probability of winning. If the level of managerial effort is very low, the expected value of the manager’s idea, \( v_P \), is low. As a result, the employee has no incentives to exert any significant amounts of effort because \( \Pr(v_A > v_P) \) will be large even for low levels of \( x_{A,v} \) and any additional effort will have only a small effect in increasing that probability further. Similarly, if the level of managerial involvement is very high, the employee again has no incentives to exert any significant amounts of effort but now for the opposite reason: given the high level of managerial involvement, \( \Pr(v_A > v_P) \) will remain low despite the effort and so the marginal contribution of additional effort is again low. It is thus only intermediate levels of managerial involvement that succeed in motivating the employee to work harder. Such levels set a performance target for the employee that is sufficiently challenging so that he needs to exert effort to meet it but not so challenging as to prevent him from trying in the first place. As a result, we obtain the non-monotone relationship between \( x_{A,v} \) and \( x_{P,v} \), where managerial involvement first increases employee effort as the employee attempts to meet the increasingly stringent threshold in the hope of realizing his benefits \( x_{A,w} \) but later becomes demotivating as the employee comes to view the threshold to be too difficult to meet. This simple intuition is developed further in the Online Supplement.

The second part of the proposition highlights that not only can the level of managerial involvement increase how hard the employee works on the manager’s dimension, but this effect is sufficiently strong to lead to an improvement in the overall quality of the employee’s ideas (as measured by the total expected benefit \( x_{A,w} + x_{A,v} \)). In other words, for low enough \( x_{P,v} \), managerial involvement generates a positive competition effect, whereby the employee is encouraged to work harder. Finally, because \( x_{A,w} \) is decreasing in \( x_{P,v} \), the overall quality of the employee’s idea will start decreasing in the level of managerial involvement sooner than the quality of that idea in the manager’s dimension.

### 5 Equilibrium

Having analyzed how the employee’s effort choices are influenced by the level of managerial involvement, we can now consider how they are jointly determined by the manager’s opportunity cost of time and, in particular, the implications of that equilibrium to the total surplus generated, as illustrated in Figure 2.

When \( \mu \rightarrow \infty \), the level of managerial involvement goes to zero. As a result, the employee knows that he will get his idea implement even with minimal attention to the manager’s benefits because the manager will have no better alternatives at her disposal. Therefore, \( x_{A,v} \rightarrow 0 \) while the fact that
Pr($v_A > v_P$) → 1 means that he will work very hard on generating benefits to himself. If we lower $\mu$, the manager increases her level of involvement, $x_{P,v}$. As discussed above, the initial response of the employee is to start working harder in the manager’s dimension to protect the benefits he gets from having his idea implemented. While the employee simultaneously lowers $x_{A,w}$, the overall quality of the employee’s idea increases, leading to an increase in total surplus. This increase in surplus highlights the basic benefit of some managerial involvement: it provides competitive pressure on the employee by limiting what ideas get implemented in equilibrium and thus encourages him to work harder.

As $\mu$ decreases (and $x_{P,v}$ increases) further, the total surplus starts to decrease. This decrease begins for two reasons. First, as the probability of having his idea implemented decreases, the employee starts to become demotivated and the expected quality of his ideas starts to decrease. Second, since the manager becomes more involved, she begins to choose her idea with increasing frequency. This result has an additional social cost because the manager chooses her idea whenever $v_P > v_A$, even if the employee’s idea would yield a higher total surplus. This decrease in surplus thus illustrates the basic cost of intermediate levels of managerial involvement: by limiting the frequency at which the employee gets his idea implemented, it crowds out employee initiative without providing strong alternatives herself.

Finally, as $\mu$ becomes sufficiently small ($x_{P,v}$ becomes sufficiently large), the total surplus starts to increase again. The reason is that at this point, the employee is already putting in very little effort in both dimensions and so has his idea implemented only with a very low probability. Thus, any further reduction in the quality of the employee’s idea is outweighed by the improvement in the manager’s idea that results from further reductions in her opportunity cost of time. Further, the effective bias in the project choice is decreased because the employee’s idea is now unlikely to have a large private component $w_A$ which the manager ignores in the project selection stage.

In summary, the expected total surplus is a non-monotone function of the manager’s opportunity
cost of time, leading to two local maxima. The first is a "limited involvement" equilibrium, where \(0 < x_{P,v}^* (\mu) < \frac{1}{2} x_{A,v}^*\) and the manager intentionally limits her involvement to motivate the employee to work harder. The second is a "do-it-alone" equilibrium, where the manager simply works alone. The worst possible outcome is the situation where the manager is involved enough to demotivate the employee but not good enough to come up with strong alternatives herself.

In the particular example of Figure 2 (drawn for fully symmetric parties), it is clear that the optimal choice is the limited involvement equilibrium. In general, however, the solution will depend on the relative costs of the two parties. To analyze this choice, it is instructive to define two parameters. First, let \(\frac{c_{P,v}}{c_{A,v}}\) be the relative efficiency of the employee, as it measures how much cheaper it is for the employee to improve the manager’s payoff relative to the manager doing it herself. Second, let \(\frac{c_{P,v}}{c_{A,w}}\) be the relative importance of the employee’s private benefits, as it measures how much costlier it is to the manager to improve her payoff than it is for the employee to improve his payoff. Then, the equilibrium choice of managerial involvement is summarized in the following proposition:

**Proposition 2** Optimal choice of managerial involvement:

(i) For all cost parameters, there exists a local "limited involvement" equilibrium where both parties exert positive effort and the effort levels induced by \(\mu^*\) satisfy \(x_{P,v} < \frac{1}{2} x_{A,v}^*\). Further, the level of managerial involvement is increasing in both \(\frac{c_{P,v}}{c_{A,v}}\) and \(\frac{c_{P,v}}{c_{A,w}}\) until \(\mu^* = 1\).

(ii) For \(\frac{c_{P,v}}{c_{A,v}}\) and \(\frac{c_{P,w}}{c_{A,w}}\) sufficiently small, there also exists a local "do-it-alone" equilibrium with \(\mu^* = 1\) and only the manager works in equilibrium.

(iii) The local do-it-alone equilibrium becomes the global maximum whenever \(\frac{c_{P,v}}{c_{A,v}}\) and \(\frac{c_{P,w}}{c_{A,w}}\) are sufficiently small. Otherwise, the limited involvement equilibrium is preferred.

**Proof.** See Appendix A.2

This solution is illustrated in Figure 3. Consider first the limited involvement equilibrium and the determination of \(\mu^*\). First, since we know that some managerial involvement is better than none, we know that this equilibrium always exist. Second, to understand why the level of managerial involvement is increasing in both the employee’s relative efficiency and the relative importance of his private benefits, consider the following. If we increase the employee’s relative productivity \(\frac{c_{P,v}}{c_{A,v}}\), the employee achieves the same quality of ideas at a lower cost. Therefore, other things constant, the employee will increase \(x_{A,v}^*\), which in turn allows the manager to also generate better ideas without the fear of demotivating the employee. Similarly, if we increase the relative importance of the employee’s benefits \(\frac{c_{P,w}}{c_{A,w}}\), the harder the employee is willing to compete to get his idea implemented. Thus, we can again lower \(\mu\) to increase the competitive pressure on the employee.\(^{12}\)

---

\(^{12}\)In this case, the total effect on \(\mu\) is, however, smaller because we are at the same time making the employee’s dimension relatively more important in determining the total surplus generated.
While the limited involvement equilibrium always exists (even if at the corner of $\mu^* = 1$), the do-it-alone equilibrium may not exist for the simple reason that when the employee is sufficiently efficient and motivated, no equilibrium effort level by the manager demotivates the employee enough to get us to the second increasing portion of the total surplus function. However, as we decrease $\frac{c_{P,v}}{c_{A,v}}$ or $\frac{c_{P,w}}{c_{A,w}}$, it becomes harder and harder to motivate the employee to work, leading the manager to choose higher and higher $\mu^*$. Eventually, the benefits of this limited involvement become so small relative to its cost in terms of the compromised quality of managerial ideas that instead of restricting herself further, the manager simply chooses $\mu^* = 1$ and the employee stops working altogether.

**Role of formal authority:** A natural question to ask is whether the manager could do better by formally delegating the right to choose which project to implement to the employee. In the present setting, the answer is no. The simple reason is that if the manager delegated the decision-authority to the employee, the employee would never exert any effort in the manager’s dimension. It is only the combination of formal authority and limited involvement by the manager that is successful in motivating the employee. The only case where formal delegation can arise as the optimal solution is when the manager is more efficient than the employee in generating benefits to the employee ($c_{P,w} > c_{A,w}$), in which case we may want to leverage the manager’s ability to benefit the employee instead of the employee’s ability to benefit the manager. Otherwise, formal delegation cannot be optimal.\(^\text{13}\)

\(^{13}\)Alternatively, if the manager is unable to control her opportunity cost of time, formal delegation, if feasible, may
An application: A particular feature of the solution is that while the level of managerial involvement can be continuously controlled, the optimal solution takes one of two qualitatively different forms: either the manager works alone, or she explicitly restricts her ability to generate ideas to induce the employee to become the main source of ideas. Thus, while there is no role for formal delegation of authority in the model, the optimal outcomes resemble a discrete choice between a "centralized" and "decentralized" organizational arrangements. It is just that the outcomes are not achieved by formally delegating authority to an employee but through the manager indirectly committing herself not to be too involved in the task at hand.

The discreteness in the optimal arrangement points out a potential challenge in organizational growth. Consider the owner-entrepreneur of a startup. Presumably, at the initial stages of the firm, it is the entrepreneur that is the main source of ideas. As the firm grows, the importance of the administrative tasks grows and this increases the opportunity cost for the entrepreneur to continue to work on new ideas. As $c_{P,v}$ grows, it can become optimal for the entrepreneur to start relying on the ideas of the employees for further improvement. But to achieve these benefits, the entrepreneur needs to achieve a discrete reduction in her level of involvement.

A potential solution to this problem is the hiring of a professional manager. The reason is that a professional manager is less likely to be intimately knowledgeable of the underlying production technology of the firm and thus less able to generate new innovations, achieving a level of commitment to lower levels of managerial involvement not available to the entrepreneur herself. In other words, while the transition from owner-managers to professional management is often justified on the basis of the comparative advantage of professional managers in managing the increasingly important administrative tasks, the present analysis suggests an additional advantage, which is their comparative disadvantage in the productive tasks themselves, which achieves the necessary reduction in the level of managerial involvement to realize the benefits of employee initiative when transitioning to a more decentralized structure.

5.1 Formal and Real Authority Revisited

In a now-classic contribution, Aghion and Tirole (1997) (henceforth AT) consider a closely related model where both the manager and the employee can exert effort to find the payoffs to a set of available projects, with formal authority determining who has the right to choose which project to implement while real authority is defined as whose preferred project is actually implemented. They find that the efforts of the manager and the employee are substitutes, and to increase employee motivation, the manager can delegate formal authority to the employee. In contrast, I find that the manager never wants to delegate authority to the employee and that the two efforts can be complements. It is therefore instructive to understand the sources of these differences better and how these differences relate to other recent contributions to the literature.

then be the only way for the manager to get out of the region of intermediate involvement which can be the worst possible outcome for total surplus.
First, the complementarity of efforts arises in the present model because the employee can always generate a better idea (in expectation) than the manager by working harder. This effect is absent in AT, where if the manager finds out the payoffs to the underlying projects, there is no way for the employee to do any better. This difference is relevant for two reasons. First, when this competition effect is present, there are situations where less managerial involvement is unambiguously bad because it leads to both loss of control and loss of initiative. Second, such ability to "do better" seems to be present in many real-world situations. Beyond the particular example of improving an idea, consider, for example, a model of project evaluation but where effort influences the number of projects evaluated. Now, if the manager decides to evaluate more projects, the employee can do better by evaluating even more projects. Or consider a dynamic setting where the manager and the employee are trying to find a solution to a given problem. If the manager starts working harder to find a solution faster, the employee can work even more intensely in the hope of finding a solution before the manager.

It is, however, worth noting that the ability of the employee to "do better" than the manager is not alone enough for the competition effect to arise. The second requirement is that there is enough conflict between the manager and the employee over the preferred alternatives. The reason is that the more benefits the employee can derive from the manager’s effort, the less he comes to care about having his preferred alternative implemented. In essence, as we increase the degree of alignment, we change the underlying game from a contest with complementary efforts to a teams problem with substitutable efforts and the associated free-riding problem. Indeed, some alignment can be worse than no alignment.\textsuperscript{14}

Second, the result that formal delegation is never optimal is related to the complementarity of efforts but it also highlights the influence that the allocation of formal authority has on the type of activities that get performed.\textsuperscript{15} In particular, it is only by retaining formal authority that the manager is ever able to induce the employee to work on satisfying organizational goals that are outside his personal interests. This result is related to Bester and Krähmer (2008), who also find that formal delegation to an employee is rarely optimal. While they analyze a very different setting where the alternatives are readily available but the decision-making stage is followed by an implementation stage, the underlying reason for the result is the same. In their setting, when choosing which task to request from the employee, the manager needs to take into account the employee’s preferences because that influences the employee’s effort choice on the task. Here, the manager retaining authority induces the employee to take into account the manager’s preferences when choosing what type of ideas to work on. In contrast, if the decision authority was delegated to the employee in either setting, the employee would not take into account the manager’s preferences when making his choices. In other words, the right allocation of formal authority can achieve some effective alignment of interests in the behavior of the parties even in the absence of explicit monetary incentives.

This last observation also highlights a key interaction between formal and real authority as defined in AT. While the employee has his project selected for implementation with a very high

\textsuperscript{14}Role of incentive alignment is discussed further in section 6.2.2.

\textsuperscript{15}In AT, efforts are one-dimensional and only influence the probability of discovery.
probability in the limited involvement equilibrium and could thus be considered as having significant real authority, this high frequency occurs only because the employee is devoting enough effort to satisfying the manager’s preferences. If we constructed an equilibrium where the employee had the same amount of real authority in terms of frequency of implementation but also granted him formal authority, the expected type of projects implemented would be significantly different and more favorable to the employee. In other words, the manager finds it beneficial to grant the employee a significant amount of "real" authority only because she knows that the fact that she retains formal authority significantly restricts what the employee will do in equilibrium.\footnote{An extreme example of this logic is provided in section B.6 of the Online Supplement, which considers a sequential variant of the current model. In a setting where only the expected quality of the first idea can be observed before generating the second idea, the optimal solution is to have the employee generate the first idea, and where the employee works hard enough to just prevent the manager from ever generating her own idea, thus giving the employee full "real" authority, even if the equilibrium arises only because the expected benefit realized by the manager is now so large that she is happy not to work at all.}

The motivational benefit of delegation is also qualified by the contributions of Newman and Novoselov (2009) and Che and Kartik (2009), who both find that delegating authority to the employee reduces his incentives to work. Both papers consider settings analogous to AT, but with the key difference that while in AT, information increases the preference conflict between the two parties, in these papers information decreases conflict.\footnote{In AT, both parties prefer to do nothing when they are uninformed while differ in their preferred project when informed, while in the other two papers, the parties disagree in the absence of information while they would prefer the same decision if fully informed.} Therefore, when the principal retains control, the motivation of the agent is increased because of the additional persuasion motive of information acquisition, where the agent is willing to work harder to bring the principal’s decision closer to his preferred decision. This persuasion motive is clearly conceptually related to the competition effect identified in the present paper. The key difference is that the persuasion motive operates only through the allocation of formal authority (given the allocation, the two efforts remain substitutes), whereas the competition effect operates under a given allocation of formal authority.\footnote{In other words, in both Newman and Novoselov and Che and Kartik, agent effort would be maximized by the principal retaining formal authority and not exerting any effort, while in the present setting agent effort is maximized by the principal retaining formal authority and exerting a limited amount of effort.}

Finally, the competition effect is also conceptually related to Armstrong and Vickers (2010). In their setting, an agent who cares only about his payoffs has access to an uncertain number of projects and chooses which one to propose for implementation. Their question of interest is the design of the optimal acceptance rule in terms of payoffs to the agent and the principal, given the strategic behavior of the agent. They show that the optimal acceptance rule will be biased against the agent relative to the rule that would maximize the principal’s payoff in the absence of strategic behavior by the agent. The key tradeoff when designing the acceptance rule is that if it is too slack, the agent can recommend projects that primarily benefit him even if he has alternatives with higher overall payoffs available, while too strict of a decision rule will lead to too few projects being implemented because they fail to meet the standard. In my framework, the principal lacks commitment power to a given acceptance rule, but the performance threshold that the employee needs to meet is generated endogenously through the realized quality of the manager’s idea, with a similar fundamental tradeoff: if the expected threshold is too slack, the employee will pursue his pet projects, while if the expected threshold is too strict, the employee becomes demotivated from trying in the first place.
6 Extensions

To isolate the role of managerial involvement as a tool for motivating the employee and its implications for organizational performance, the analysis of the previous two sections made a number of simplifying assumptions. These simplifying assumptions are relaxed in this section and their impact on the role of managerial involvement is discussed. Section 6.1 considers competition between employees and establishes the substitutability between managerial involvement and employee competition. Section 6.2 considers the possibility of implementing both ideas on a continuous scale. Allowing for (limited) implementation of the second idea reduces the demotivating impact of managerial involvement and can thus be used as a complementary motivational instrument.

Section 6.3 considers the use of monetary transfers, both in the form of bonuses paid conditional on adopting the employee’s idea and profit-sharing arrangements based on the realized value of the implemented idea. Here, I find that bonuses paid to the employee upon the implementation of his idea is a substitute for managerial involvement and, in the present setting, does strictly better. Pay contingent on the value generated by the implemented idea, on the other hand, can be either a substitute or a complement to managerial involvement. It is a complement as long as managerial involvement is useful in equilibrium, but once a sufficient degree of incentive alignment is achieved, it becomes optimal for the manager to stop working altogether to avoid free-riding and the cost of duplicated efforts.

Finally, section 6.4 considers the possibility of sequencing the moves. The main result is that allowing the employee to move first increases the payoff to both parties relative to the simultaneous-move case. The reason is that allowing the employee to move first utilizes his incentives to pre-empt the manager from generating an idea of her own. Given that a detailed analysis of the extensions is relatively lengthy, this section summarizes only the key results from each of the extensions, with more complete analysis available in a separate Online Supplement.

6.1 Competition between employees

The first extension to consider is what happens if the manager has access to multiple employees that can compete in the provision of ideas for implementation. To analyze this question, consider a variant of the model where, in addition to the manager, there are now two employees that both can generate ideas for implementation. The payoffs to the ideas are now three-dimensional, with \((v, w^i, w^j)\) indicating the payoffs to the manager, employee \(i\) and employee \(j\), respectively. I continue to assume that the dimensions are independent both within and across the ideas, conditional on the effort levels chosen by the participants.\(^{19}\) As a result, the manager continues to work only on her dimension if at all, while the employees will exert effort both on the manager’s dimension, to influence the likelihood that their idea is chosen or implementation, and on their own dimension, to influence the size of the reward if their idea is chosen for implementation.

\(^{19}\)Externalities across employees are discussed in the Online Appendix.
Two main results follow from the analysis. First, the presence of the second employee significantly reduces the role for managerial involvement. The simple reason is that employee motivation depends on the amount of competition in improving \( v \) and the resulting probability of having their idea implemented, not on who is providing that competitive pressure. As a result, while neither employee directly cares about the manager’s payoff \( v \), they both end up exerting positive levels of effort in that dimension even in the absence of any managerial involvement. This direct competitive pressure between the employees, in turn, reduces the need for managerial involvement. Indeed, as long as the second employee is chosen as to maximize total surplus, the optimal level of managerial involvement is generally either zero or so small that choosing that level is no longer incentive-compatible to the manager.\(^{20}\)

The second result is that as long as the manager chooses optimally the second employee \((c_{j,v}, c_{j,w_j})\), subject to the constraint that he is (weakly) less efficient than the first employee, then competition between employees always generates more surplus than the manager competing against a single employee. The intuition is simple. Recall that in the limited involvement equilibrium, it was always the case that \( x_{P,v} \leq \frac{1}{2} x_{A,v} \). As a result, by hiring a sufficiently inefficient employee \( j \), the manager is able to induce \( x_{j,v} = x_{P,v} \) and thus generate the same expected surplus to both employee \( i \) and herself. However, employee \( j \) now generates additional benefits \( x_{j,w} \) to himself whenever his idea is chosen, thus giving a strict improvement in the expected total surplus.\(^{21}\) Of course, the downside of having two employees is that while it generates more surplus than one employee, it also involves an additional employee. As illustrated in the Online Supplement, when the employees are costly, the manager-employee interaction continues to dominate competition between employees for a wide range of parameters.

\[ 6.2 \quad \text{Implementing multiple ideas} \]

The second extension considers relaxing the assumption that only one of the ideas can be implemented. The reason for relaxing this assumption is that from the perspective of ex post efficiency, implementing only one idea is inefficient because both ideas have positive value. Such restrictions can, however, be ex ante efficient because they influence the effort choices of the two parties. In particular, if both ideas were always implemented, then the employee would never have any incentives to work on generating value to the manager. The question is then if we allowed partial (or probabilistic) implementation of the second idea, how are the employee incentives (and the total surplus generated) impacted?\(^{22}\)

To examine this question, suppose that the organization can (costlessly) commit to the amount of resources it has available for implementing the two projects. Each project can be implemented at

\[^{20}\text{The reason why low levels of involvement are no longer incentive compatible to the manager is that she realizes value to her effort only when she selects her idea, which then needs to be better than both of the employees’ ideas. As a result, the managerial payoff becomes strongly convex in } \( x_{P,v} \) with negative returns within } [0, x_{P,v}] \text{ due to the fact that } x_{i,v} \text{ and } x_{j,v} \text{ will now be positive even when } x_{P,v} = 0.\]

\[^{21}\text{On the other hand, a symmetric tournament can be dominated by the manager-employee relationship even from the perspective of total surplus.}\]

\[^{22}\text{Of course, this discussion will then not apply to situations where the ideas are mutually exclusive.}\]
scale \( \alpha \in [0, 1] \), and implementing an idea at scale \( \alpha \) generates a total payoff of \( \alpha (v_i + w_i) \). Since the manager will always implement her preferred idea at full scale, let \( \alpha \) denote the scale at which the second idea is implemented, which then essentially amounts to a second prize in the contest for ideas. In particular, from the perspective of the employee, he will now receive \( \alpha x_{A,w} \) even when his idea is not chosen by the manager for full-scale implementation while he gets an additional \((1 - \alpha) x_{A,w}\) if he wins the contest. In other words, the employee’s expected payoff becomes

\[
EU_A = \frac{x_{A,v}}{x_{A,v} + x_{P,v}} (1 - \alpha)x_{A,w} + \alpha x_{A,w} - \frac{c_{A,w}}{2} x_{A,v}^2 - \frac{c_{A,w}}{2} x_{A,w}^2.
\]

The first question of interest is how does \( \alpha \) impact both the level of employee effort and the role of managerial involvement. The results are two-fold. First, contrary to the standard logic of second prizes, a sufficiently small second prize can actually motivate the employee, so that \( \frac{\partial x_{A,w}}{\partial \alpha} > 0 \). The intuition for this result is as follows. First, increasing \( \alpha \) directly decreases the value of winning the contest, which is now \((1 - \alpha)x_{A,w}\). This is the standard reason why second prizes demotivate participants. Second, the employee will now receive at least \( \alpha x_{A,w} \) from working on his idea. This guarantee of at least some returns, in turn, motivates the employee to work harder in his dimension, increasing \( x_{A,w} \) and thus indirectly increasing also the first prize. This indirect increase in \( x_{A,w} \) can then outweigh the direct decrease in \((1 - \alpha)\), leading to increased motivation for the employee. In other words, even from a purely motivational perspective, it is sometimes better to allow the employee to (partially) implement his idea, independent of how valuable that idea is to the organization. The reason is that by knowing that they can receive at least some of the benefits related to their ideas, the employees can be encouraged to work harder not only on generating benefits for themselves but also to the organization as a whole.

The second result is that introducing a second prize makes the employee effort less sensitive to managerial involvement: \( \frac{\partial}{\partial \alpha} \left| \frac{\partial x_{A,v}}{\partial x_{P,v}} \right| < 0 \). The reason is that increasing \( \alpha \) makes \( x_{A,w} \) less sensitive to the probability of winning (and thus the level of managerial involvement). The downside of this effect is that, as a result, the maximal effort that can be induced from the employee in the manager’s dimension is decreasing in \( \alpha \). The benefit is that higher levels of managerial involvement become much less damaging to employee initiative and, as a result, it becomes possible to reduce \( \mu \) and thus increase the quality of managerial ideas while limiting the resulting demotivating impact on the employee by simultaneously increasing \( \alpha \). Therefore, the size of the second prize and the level of managerial involvement are complements. Indeed, when considering the optimal choice of \((\mu, \alpha)\), we find that \( \mu^* = 1 \) so that the manager is no longer constrained in her level of involvement while \( \alpha^* < 1 \) as long as \( c_{A,v} < c_{P,v} \) to continue to induce effort from the employee also in the manager’s dimension.

### 6.3 Incentive contracts

The third question of interest is how the optimal level of managerial involvement is changed if the organization has access to contingent monetary transfers. This subsection analyzes the impact of
two such forms of transfers: bonuses paid on project choice and contingent compensation based on the value of implemented projects.

6.3.1  **Bonuses based on project choice**

To consider the role of bonuses, suppose now that, in addition to the private benefits $w_A$ that the employee receives from having his idea implemented, the identity of which project is implemented is verifiable and so the employee’s compensation contract can also include a bonus $b$ that is paid when his idea is selected.

The first effect of such a bonus is to change the acceptance rule of the manager. Since the manager is responsible for paying the bonus, the presence of a positive bonus will bias the manager against accepting the employee’s idea, with the new acceptance rule given by $v_A \geq v_P + b$. This impact of a positive bonus on the manager’s decision rule immediately highlights the result that bonuses and managerial involvement are substitute tools for motivating the employee: both provide a threshold level that the employee needs to meet to have his idea implemented. The relative advantage of using a monetary bonus $b$ is that not only does it provide a threshold that the employee needs to meet to have his idea implemented, but it also provides an additional reward for meeting that target. Indeed, increasing both $b$ and $\mu$ in a fashion that leaves $\Pr(v_A \geq v_P + b)$ constant unambiguously increases employee effort. The relative disadvantage of using a monetary bonus is that now, instead of realizing the benefits $v_P$ when the employee’s idea is not implemented, the manager does not have access to an alternative project. As a result, the motivational advantage of $b$ needs to be large enough to outweigh the loss in surplus when the employee’s idea is not implemented. In the present setting, the incentive effect dominates, implying that the optimal combination of $(\mu, b)$ involves $\mu \to \infty$, while the generality of this conclusion is unknown and left for future research.

6.3.2  **Incentive alignment through profit-sharing**

An alternative means of motivating the employee is through profit-sharing, where the two parties are compensated as a function of the overall value of the project. The advantages of such a plan are two-fold. First, since the manager must make the payment to the employee independent of the project choice, the project selection problem is not further distorted as in the case of bonuses. Indeed, to the extent that the manager starts placing weight on the employee’s payoff, the project selection is improved. Second, since the manager has excessive incentives to work in the no-alignment equilibrium (as illustrated in section 5), some incentive alignment can be achieved for free by simply transferring some of the manager’s payoff to the employee (instead of constraining the manager from working too hard). Indeed, given the first-best solution outlined in section 4.1, achieving first-best would be trivial in the present setting: since only the employee should work in equilibrium, we can simply make the employee a full residual claimant on both $v$ and $w$.

In many cases, however, the extent of profit-sharing that is feasible is limited. Therefore, it is still of some interest to examine whether some profit-sharing is better than no profit-sharing and what
are the consequences of limited profit-sharing for managerial involvement. To examine this question, suppose that the payoffs to the manager and the employee are now given by \((1 - s_v)v + s_ww\) and \(s_vv + (1 - s_w)w\) respectively, with \(s_v\) and \(s_w\) measuring the extent of profit-sharing. While we can solve the equilibrium only numerically, it reveals a number of interesting observations. First, the amount of profit-sharing and the level of managerial involvement are generally complements. The reason is that increasing \(s_v\) makes the employee inherently more motivated to work in the manager’s dimension and thus reduces the demotivating impact of managerial involvement, while increasing \(s_w\) leads the manager to generate better ideas and make better choices, thus reducing the bias in the equilibrium idea choice caused by managerial involvement. The exception is that once the employee is sufficiently aligned with the manager, it becomes optimal for the manager to stop working altogether to eliminate the incentives for free-riding that arise once the projects preferred by the manager and the employee become sufficiently similar.

Second, the observation that increasing the level of profit-sharing reduces the competition effect of managerial involvement by making the projects generated (and preferred) by the two parties more similar sheds light on the impact of profit-sharing on total surplus. First, increasing the employee’s share of the manager’s payoff always improves total surplus: such a change directly motivates the employee to work harder, which allows the manager to work harder as well. Increasing the manager’s share of the employee’s payoff, on the other hand, generally decreases total surplus. The reason is that as the manager comes to place more weight on the employee’s payoff, two things occur. First, the manager will now generate projects that are more attractive to the employee when implemented. This reduces the employee’s incentives to compete for have his idea implemented. Second, the manager’s idea choice itself becomes more favorable to the employee. As a result, the employee can shift his attention from generating benefits to the manager to generating benefits to himself and still maintain the same probability of winning. Both effects reduce the employee’s incentives to exert effort on the manager’s dimension and are detrimental to total surplus.

### 6.4 Sequencing of efforts

The final extension to consider is what happens if we allow either the manager or the employee to move first, present their idea and then the second-mover can decide whether to try to generate an even better idea. The obvious advantage of such sequencing is that if the first idea generated is sufficiently good, then the effort that would be sunk into generating the second idea in the simultaneous-move game is no longer wasted. But despite this advantage, the relative impact of sequencing on total surplus depends on the order of moves.

First, allowing the manager to move first generally decreases the maximal total surplus attainable. The reason for this result is that, as discussed in section 5, the limited involvement equilibrium occurs in the region of complementary efforts. As a result, the manager perceives that by working harder, he is able to increase the employee effort \(x_{A,v}\) \(\left(\frac{\partial x_{A,v}}{\partial \mu_{p,v}} > 0\right)\). From the perspective of total surplus, the manager thus has even more excessive incentives to work than in the simultaneous-move case and we need to increase the manager’s opportunity cost of time even further, leading to additional waste.
In contrast, allowing the employee to move first unambiguously increases the maximal total surplus attainable. The reason is that, in addition to saving on managerial effort when the employee's idea is perceived to be good enough by the manager, the employee now has additional incentives to preempt the manager from exerting effort in the first place \( \frac{\partial x_{P,u}}{\partial x_{A,u}} < 0 \). In other words, it is simply the threat of managerial involvement that can motivate the employee to work hard enough so that the actual frequency of such involvement is limited.

7 Conclusion

In this paper, I analyzed how managerial involvement in idea generation impacts employee incentives to come up with new ideas. I illustrated the presence of a competition effect, whereby increased managerial effort in coming up with new ideas increased the expected quality of the ideas generated by the employee for sufficiently low levels of involvement. To utilize this competition effect, the manager should always retain formal authority over which ideas to implement but restrict her level of involvement in the actual generation of ideas. Formally delegating the right to choose which ideas to implement or otherwise being too little involved would allow the employee to pursue his pet ideas at the expense of overall organizational goals. Similarly, being too involved would crowd out employee initiative as the employee then perceives that the likelihood of having his idea implemented is too low to warrant working on it in the first place. In short, to encourage employee initiative, the manager should always be somewhat involved to provide some healthy competition for the employee's ideas but not too involved to avoid killing his initiative.

Beyond this main result of the paper, three general features of the solution are worth noting. First, while the strategy of limited involvement maximized employee initiative, it came at the cost of compromised quality of managerial ideas. As a result, the optimal organizational arrangement could take two qualitatively different forms. Either the manager explicitly restricted her ability to develop ideas to encourage employee participation, or the manager forewent employee participation altogether and simply worked alone. The worst possible outcome arose for the intermediate case where the manager was involved enough to demotivate the employee but not good enough to generate strong alternatives herself. This aspect of the solution thus highlighted the non-monotone impact that the level of managerial involvement can have on organizational performance, where some restrictions on the manager could reduce but enough restrictions would improve organizational performance.

The second aspect was the complementarity between managerial and employee efforts for low levels of managerial involvement. This aspect was worth noting for two reasons. First, this effect is relevant because when the managerial and employee efforts are complements, there can be unambiguously too little managerial involvement, with further reductions leading to both loss of control and loss of initiative, instead of the much-discussed tradeoff between the two. Second, while identified only in a particular setting, this complementarity can arguably arise whenever the employee has both the ability and the desire to "do better" than the principal. These conditions which seem to be met in many real-world situations, where the employee can generate a better idea, search through more alternatives or find a solution to given problem faster simply by working harder.
The third aspect was the role of formal authority in influencing the type of actions chosen by the employee. The reason for the manager to retain formal authority in the present model is that it is only by retaining formal authority that the manager is able to induce the employee to work on satisfying organizational goals. If the employee was delegated the right to choose which ideas to implement, he would naturally focus only on ideas that are individually beneficial to him. In contrast, when the manager retains formal authority, the employee knows that the idea that he will present to the manager needs to be sufficiently attractive to her to have the possibility of being implemented. In other words, while in the case of limited managerial involvement, the employee frequently gets his idea chosen for implementation and could thus be considered having a significant amount of "real" authority, his idea is chosen in equilibrium only because he is anticipating the response of the manager when choosing which types of ideas to work on. Thus, the allocation of formal authority affects not only the level of effort, which has been the focus so far, but also the allocation of effort in meaningful ways.

Finally, I considered the rich set of interactions that the level of managerial involvement can have with other motivational instruments. First, I illustrated how managerial involvement is a substitute to both competition between employees and the use of bonuses based on the implementation of the employee’s idea. The reason was that from the perspective of the employee, all three instruments provided a performance threshold that he needed to meet to have his idea implemented. Second, I illustrated how managerial involvement is a complement to both the introduction of a "second prize" in terms of partial implementation of the less-preferred idea and the use of pay contingent on the realized value of the implemented idea. The reason was that both instruments (when used right) directly increased the employee’s motivation and reduced the demotivating impact of higher levels of managerial involvement.
References


A  Proofs and derivations

A.1 Proof of proposition 1

The first step is to derive the expected payoffs, for which we need $E (w_A|x_{A,w})$, $Pr(v_A > v_P|x_{A,v}, x_{P,v})$ and $E (\max(v_A, v_P))$. Now, by assumption $E (w_A|x_{A,w}) = x_{A,w}$. To establish $Pr(v_A > v_P|x_{A,v}, x_{P,v})$, note that for a given $v_A$, the probability that the agent wins is $F_P(v_A|x_{P,v}) = 1 - e^{-\lambda_P v_A}$ while the probability of any given realization of $v_A$ is given by $f_A(v_A|x_{A,v}) = \lambda_A e^{-\lambda_A v_A}$ (where $\lambda_i = \frac{1}{x_{i,v}}$), so the expected probability of winning is simply

$$\Pr(v_A > v_P|x_{A,v}, x_{P,v}) = \int_0^\infty (1 - e^{-\lambda_P v_A}) \lambda_A e^{-\lambda_A v_A} dv_A = \frac{\lambda_P}{\lambda_P + \lambda_A} = \frac{x_{A,v}}{x_{A,v} + x_{P,v}}.$$  

To establish $E (\max(v_A, v_P))$, we know that the CDF of the maximum is $F_P(v|x_{P,v}) F_A(v|x_{A,v})$ and so the pdf is

$$f_P(v|x_{P,v}) F_A(v|x_{A,v}) + F_P(v|x_{P,v}) f_A(v|x_{A,v}) = \lambda_P e^{-\lambda_P v} (1 - e^{-\lambda_A v}) + \lambda_A e^{-\lambda_A v} (1 - e^{-\lambda_P v}),$$

and so the expectation becomes

$$\int_0^\infty \lambda_P e^{-\lambda_P v} (1 - e^{-\lambda_A v}) dv + \int_0^\infty \lambda_A e^{-\lambda_A v} (1 - e^{-\lambda_P v}) dv = \frac{1}{\lambda_A} + \frac{1}{\lambda_P} - \frac{1}{(\lambda_A + \lambda_P)}$$

$$= x_{A,v} + x_{P,v} - \frac{1}{x_{A,v} + x_{P,v}} = x_{A,v} + x_{P,v} - \frac{x_{P,v} x_{A,v}}{(x_{A,v} + x_{P,v})} = \frac{x_{A,v}^2 + x_{P,v} x_{A,v} + x_{P,v}^2}{x_{A,v} + x_{P,v}},$$

which then establishes the expected payoffs of

$$EU_P = E (\max(v_A, v_P)|x_{A,v}, x_{P,v}) - \mu C_P(x_{P,v}, x_{P,w})$$

$$= \frac{x_{A,v}^2 + x_{P,v} x_{A,v} + x_{P,v}^2}{x_{A,v} + x_{P,v}} - \mu \left( \frac{c_P}{2} x_{P,v}^2 + \frac{c_p}{2} x_{P,w}^2 \right),$$

$$EU_A = Pr(v_A > v_P|x_{A,v}, x_{P,v}) (E (w_A|x_{A,w}) - E (w_P|x_{P,w})) + E (w_P|x_{P,w}) - C_A(x_{A,v}, x_{A,w})$$

$$= \frac{x_{A,v}}{(x_{A,v} + x_{P,v})} (x_{A,w} - x_{P,w}) + x_{P,w} - \frac{c_A}{2} x_{A,v}^2 - \frac{c_A}{2} x_{A,w}^2.$$  

From the expected payoffs, we then obtain the first-order conditions

$$x_{P,v} : \quad x_{P,v} \left( \frac{x_{P,v} + 2 x_{A,v}}{(x_{P,v} + x_{A,v})^2} \right) = \tilde{c}_{P,v} x_{P,v} \quad x_{P,w} : \quad 0 = \tilde{c}_{P,w} x_{P,w}$$

$$x_{A,v} : \quad \left( \frac{x_{P,v}}{(x_{P,v} + x_{A,v})^2} \right) (x_{A,w}) = c_{A,v} x_{A,v} \quad x_{A,w} : \quad \frac{x_{A,v}}{(x_{P,v} + x_{A,v})^2} = c_{A,w} x_{A,w},$$

where $\tilde{c}_{P,i} = \mu c_{P,i}$. Thus, it is immediate that $x_{P,w} = 0$ since the manager receives no benefits from generating value to the employee. For $x_{P,v}$, we can rearrange the FOC to give

$$(x_{P,v} + 2 x_{A,v}) = \tilde{c}_{P,v} (x_{A,v} + x_{P,v})^2 \rightarrow x_{P,v} = \frac{(1 - 2 \tilde{c}_{P,v} x_{A,v}) \pm \sqrt{1 + 4 \tilde{c}_{P,v} x_{A,v}}}{2 \tilde{c}_{P,v}}.$$
Then, using the principal’s solution and the agent’s choice of $x_{A,w}$, we can write the agent’s choice of $x_{A,v}$ as

$$
((1 - 2c_{P,v} x_{A,v}) \pm \sqrt{1 + 4c_{P,v} x_{A,v}}) c_{A,v} c_{A,v} = \frac{c_{A,w}}{4c_{P,v}} (1 \pm \sqrt{1 + 4c_{P,v} x_{A,v}})^3.
$$

It is easy to confirm that the smaller root is never an equilibrium (it would imply negative effort by the manager). For the larger root, we can write

$$(1 + \sqrt{1 + 4y})^3 = 4 (1 + \sqrt{1 + 4y} + 3y + y\sqrt{1 + 4y}),$$

where $y = c_{P,v} x_{A,v}$, so that we can rearrange the employee’s problem to

$$4y (2\phi (1 - \phi) - y^2 + y\phi (5 + \phi)) = 0,$$

where $\phi = \frac{c_{A,v}}{c_{A,w}}$ is the relative (in)efficiency of the manager. So the roots are

$$y = \frac{\phi (5 + \phi) \pm \sqrt{\phi^2 (5 + \phi)^2 + 8\phi (1 - \phi)}}{2}.$$

Now, we can go back to the assumed principal’s choice, which gives only the negative root as the equilibrium (the larger root would again lead to negative effort by the principal). Thus, the solution is defined recursively through

$$x_{A,v} = \frac{\phi (5 + \phi) - \sqrt{\phi^2 (5 + \phi)^2 + 8\phi (1 - \phi)}}{2c_{P,v}},$$

$$x_{P,v} = \frac{(1 - 2c_{P,v} x_{A,v}) + \sqrt{1 + 4c_{P,v} x_{A,v}}}{2c_{P,v}},$$

$$x_{A,w} = \frac{x_{A,v}}{(x_{P,v} + x_{A,v}) c_{A,w}} = \frac{2c_{P,v} x_{A,v}}{(1 + \sqrt{1 + 4c_{P,v} x_{A,v}}) c_{A,w}},$$

which is the unique equilibrium of the game with both parties exerting positive effort. For the employee, the zero effort equilibrium always exists while the manager always puts in positive effort.

Finally, note that the agent will stop working even in this equilibrium whenever $\phi \leq 1$. In other words, if the manager is sufficiently efficient, the employee will never put in any effort.

From here, we can establish the obvious result that $\frac{d x_{P,v}}{d \mu} \leq 0$, so that even after accounting for the equilibrium adjustments in the employee’s effort levels, the effort level of the principal is decreasing in $\mu$, so that it makes sense to consider the impact of $x_{P,v}$ on the effort levels of the employee. To perform this examination, we can return to the employee’s FOCs, where substituting in $\frac{x_{A,v}}{(x_{P,v} + x_{A,v}) c_{A,w}} = x_{A,w}$ allows us to write the choice of $x_{A,v}$ as

$$\left(\frac{x_{P,v}}{(x_{P,v} + x_{A,v})^2}\right) \frac{1}{c_{A,w} c_{A,v}} - 1 = 0,$$

which rearranges further to
\[
\left( \frac{x_{P,v}}{c_{A,v}c_{A,v}} \right)^{1/3} - x_{P,v} = x_{A,v}.
\]
As a result, \( \frac{\partial x_{A,v}}{\partial x_{P,v}} > 0 \iff x_{A,v} \geq 2x_{P,v} \), which establishes the second part of proposition 1(i).

For part (ii), note first that
\[
x_{A,v} + x_{A,w} = x_{A,v} + \frac{x_{A,v}}{\left(x_{P,v} + x_{A,v}\right)c_{A,w}} = x_{A,v} \left( 1 + \frac{1}{\left(x_{P,v} + x_{A,v}\right)c_{A,w}} \right).
\]
Then, substituting in the expression for \( x_{A,v} \) from above gives us
\[
x_{A,v} + x_{A,w} = \left( \frac{1}{x_{P,v}} \right)^{1/3} - x_{P,v} + \frac{1}{c_{A,w}} - x_{P,v} c_{A,w}^{1/3},
\]
where \( t = c_{A,w} c_{A,v} \). Thus, \( \frac{\partial (x_{A,v} + x_{A,w})}{\partial x_{P,v}} > 0 \) if and only if \( \frac{1}{x_{P,v}} \left( \frac{2}{3} x_{P,v} - 1 \right) \frac{1}{c_{A,w}} > 0 \), which we can rearrange to
\[
0 > 3t^{1/3} c_{A,w} x_{P,v}^{2/3} + 2t^{2/3} x_{P,v} - c_{A,w}.
\]
Let \( x_{P,v} = y \); we have \( 0 > 3 \left( c_{A,w} c_{A,v} \right)^{1/3} c_{A,w} y^{2} + 2 \left( c_{A,w} c_{A,v} \right)^{2/3} y - c_{A,w}, \) or
\[
y \leq \overline{y} = \frac{-(c_{A,w} c_{A,v})^{2/3} + \sqrt{(c_{A,w} c_{A,v})^{4/3} + 3c_{A,w}^{2/3} c_{A,v}^{1/3}}}{3c_{A,w} (c_{A,w} c_{A,v})^{1/3}} = \frac{c_{A,v}}{3c_{A,w}} \left( \sqrt{1 + 3c_{A,w} c_{A,v} - 1} \right),
\]
so that
\[
x_{P,v} \leq \frac{c_{A,v}}{2c_{A,w}} \left( \sqrt{1 + 3c_{A,w} c_{A,v} - 1} \right) = \left( \frac{4c_{A,w} + 3c_{A,v}}{2c_{A,w}} \right) \left( \sqrt{1 + 3c_{A,w} c_{A,v} - 1} \right) - \left( \frac{4c_{A,v} + 9c_{A,w}}{2c_{A,v}} \right).
\]
Finally, recalling that \( \left( \frac{x_{P,v}}{c_{A,w} c_{A,v}} \right)^{1/3} - x_{P,v} = x_{A,v} \), we can write the condition as
\[
\frac{x_{A,v}}{x_{A,v} + x_{P,v}} \geq \Pr(v_{A} \geq v_{P}) = \frac{\left( \frac{1}{x_{P,v}} \right)^{1/3} - x_{P,v}}{\left( \frac{1}{x_{P,v}} \right)^{1/3}} = \frac{\frac{c_{A,v}}{3c_{A,w}} \left( \sqrt{1 + 3c_{A,w} c_{A,v} - 1} \right) - c_{A,w} \left( \frac{1}{x_{P,v}} \right)^{1/3} \left( \sqrt{1 + 3c_{A,w} c_{A,v} - 1} \right)}{1 + \left( \frac{1}{x_{P,v}} \right)^{1/3} \left( \sqrt{1 + 3c_{A,w} c_{A,v} - 1} \right)} = \frac{\overline{y}^{1/3} - c_{A,w}}{\overline{y}^{1/3} c_{A,w}} \left( \sqrt{1 + 3c_{A,w} c_{A,v} - 1} \right) = \frac{2}{3} \left( 1 + \frac{c_{A,v}}{c_{A,w}} \right) \left( \sqrt{1 + 3c_{A,w} c_{A,v} - 1} \right).
\]
Finally, for the first part of the proposition, write
\[
x_{A,w} = \frac{x_{A,v}}{\left(x_{P,v} + x_{A,v}\right)c_{A,w}} = \frac{\left( \frac{x_{P,v}}{c_{A,w} c_{A,v}} \right)^{1/3} - x_{P,v}}{\left( \frac{x_{P,v}}{c_{A,w} c_{A,v}} \right)^{1/3} - x_{P,v}} c_{A,w} = \frac{1}{c_{A,w}} \left[ 1 - \left( c_{A,w} c_{A,v} \right)^{1/3} x_{P,v}^{2/3} \right]
\]
to establish \( \frac{\partial x_{A,w}}{\partial x_{P,v}} < 0 \).
A.2 Proof of proposition 2

When choosing $\mu$, we are maximizing $EU_P + EU_A$. Since the equilibrium effort choices maximize individual payoffs, we can write $\frac{d}{d\mu} (EU_P + EU_A) = 0$ as

$$-\frac{\partial}{\partial \mu} C_P(x_{P,v}) + \left( \frac{\partial EU_P}{\partial x_{A,v}} \frac{\partial x_{A,v}}{\partial x_{P,v}} + \frac{\partial EU_A}{\partial x_{P,v}} \right) \frac{\partial x_{P,v}}{\partial \mu} = 0,$$

since all other effects will have a second-order impact on the expected surplus. Let the total cost be $\frac{\mu x_{P,v}}{2}$, so that $\frac{\partial}{\partial \mu} C_P(x_{P,v}) = \frac{\mu x_{P,v}}{2}$. Further, we have that

$$\frac{\partial EU_P}{\partial x_{A,v}} = -\frac{\mu x_{A,v}}{c_{A,w}(x_{A,v} + x_{P,v})^2}, \quad \frac{\partial EU_A}{\partial x_{P,v}} = -\frac{\mu x_{P,v}}{c_{A,w}(x_{A,v} + x_{P,v})^2}.$$

so that we can write the above as

$$\frac{x_{A,v}}{(x_{A,v} + x_{P,v})^2} \left( (x_{A,v} + 2x_{P,v}) \frac{\partial x_{A,v}}{\partial x_{P,v}} - \frac{x_{A,v}}{c_{A,w}(x_{A,v} + x_{P,v})} \right) \frac{\partial x_{P,v}}{\partial \mu} - \frac{\mu x_{P,v}}{2} x_{P,v} = 0,$$

the solution to which gives us $\mu^*$. Now, from the proof of proposition 1 we know that $x_{A,v} = \left( \frac{x_{P,v}}{t} \right)^{1/3} - x_{P,v}$ and so

$$\frac{\partial x_{A,v}}{\partial x_{P,v}} = \left( \frac{(x_{P,v})^{1/3}}{3x_{P,v}} - 1 \right),$$

where $t = c_{A,w}c_{A,v}$. Similarly, from the manager’s first-order condition we get

$$\left( \frac{x_{P,v} + 2x_{A,v}}{(x_{A,v} + x_{P,v})^2} \right) - \mu c_{P,v} = 0 \Leftrightarrow \left( 2\left( \frac{x_{P,v}}{t} \right)^{1/3} - x_{P,v} \right) - \mu c_{P,v} = 0,$$

so that using the implicit function theorem, we can solve for $\frac{\partial x_{P,v}}{\partial \mu}$:

$$\frac{\partial x_{P,v}}{\partial \mu} = -\frac{3tc_{P,v} \left( \frac{x_{P,v}}{t} \right)^{5/3}}{2\left( \frac{x_{P,v}}{t} \right)^{1/3} + x_{P,v}}.$$

Thus, we can write the marginal impact as

$$\frac{\partial x_{P,v}}{\partial \mu} = -\frac{3tc_{P,v} \left( \frac{x_{P,v}}{t} \right)^{5/3}}{2\left( \frac{x_{P,v}}{t} \right)^{1/3} + x_{P,v}} \left( \left( \frac{x_{P,v}}{t} \right)^{1/3} + x_{P,v} \right) \left( \left( \frac{x_{P,v}}{t} \right)^{1/3} - x_{P,v} \right) - \frac{c_{P,v} x_{P,v}}{2},$$

which, after some simplification can be written as

$$\frac{c_{P,v} x_{P,v}}{2c_{A,w} \left( \frac{x_{P,v}}{t} \right)^{1/3} + 2\left( \frac{x_{P,v}}{t} \right)^{1/3}} \left( \frac{x_{A,w}}{x_{P,v}} + x_{P,v} \right) \left( \frac{x_{P,v}}{t} \right)^{1/3} - \frac{2c_{A,w} x_{P,v}^2}{c_{A,w} x_{P,v}^2 - 7c_{A,w} x_{P,v}^2},$$

Thus, the sign of the first-order condition is determined by
From this, it is then straightforward to establish the basic properties of the solution. In particular, it permits only two solutions in $x_{P,v}$, and thus $\mu$. Suppose first that $c_{P,v}$ is sufficiently low that we can induce any $x_{P,v}$ desired. In practice, there is an upper bound on $x_{P,v}$, given by $\mu = 1$ but let us first ignore this constraint. Now, to begin the analysis, let us establish a benchmark of what is the sign when $x_{A,v} \to 0$ ($x_{P,v} \to \frac{7}{9}$). At this point, the above simplifies to $-3\frac{c_{A,v}}{t}$, which means that restricting the manager would decrease total surplus. Intuitively, at this point the employee works so little that any extra incentives provided to the employee by restricting the manager are outweighed by the damage in terms of compromised managerial ideas. Thus, we have that for $x_{P,v} \geq \frac{1}{\sqrt{9}}$, $\frac{d}{dp} (EU_P + EU_A) < 0$.

For $x_{P,v} \in [0, \frac{1}{\sqrt{9}}]$, let $x_{P,v} = \frac{4}{9}$, with $\alpha \in [0, 1]$, which allows us to write the above condition as

$$6\left(\frac{x_{A,v}}{x_{A,v} + x_{A,v}}\right)^{1/3} + 6x_{P,v} \left(\frac{1}{x_{A,v}} + x_{P,v}\right) + \left(\frac{c_{A,v} + c_{A,v}}{x_{P,v}}\right)^{1/3} - 2\alpha^2 - 12x_{P,v} - 7c_{A,v}x_{P,v}^2.$$

We can now consider how the sign of this expression depends on $x_{P,v}$ (and thus $\mu$). Suppose first that $c_{P,v}$ is sufficiently low that we can induce any $x_{P,v}$ desired. In practice, there is an upper bound on $x_{P,v}$, given by $\mu = 1$ but let us first ignore this constraint. Now, to begin the analysis, let us establish a benchmark of what is the sign when $x_{A,v} \to 0$ ($x_{P,v} \to \frac{7}{9}$). At this point, the above simplifies to $-3\frac{c_{A,v}}{t}$, which means that restricting the manager would decrease total surplus. Intuitively, at this point the employee works so little that any extra incentives provided to the employee by restricting the manager are outweighed by the damage in terms of compromised managerial ideas. Thus, we have that for $x_{P,v} \geq \frac{1}{\sqrt{9}}$, $\frac{d}{dp} (EU_P + EU_A) < 0$.

For $x_{P,v} \in [0, \frac{1}{\sqrt{9}}]$, let $x_{P,v} = \frac{4}{9}$ with $\alpha \in [0, 1]$, which allows us to write the above condition as

$$6\left(\frac{x_{A,v}}{x_{A,v} + x_{A,v}}\right)^{1/3} + 6x_{P,v} \left(\frac{1}{x_{A,v}} + x_{P,v}\right) + \left(\frac{c_{A,v} + c_{A,v}}{x_{P,v}}\right)^{1/3} - 2\alpha^2 - 12x_{P,v} - 7c_{A,v}x_{P,v}^2.$$

Now, if we let $\alpha^2 = y$, the condition for equilibrium becomes

$$6\left(\frac{x_{A,v}}{x_{A,v} + x_{A,v}}\right)^{1/3} \sqrt{y} (1 - y^2) + (6y - 2 - 7y^2) = 0.$$

From this, it is then straightforward to establish the basic properties of the solution. In particular, it permits only two solutions in $\alpha \in [0, 1]$, for which the first is the local maximum and the second a local minimum. To establish their existence, note that for $y = 0$ ($\mu \to \infty$), the above is negative, as it is when $y = 1$ (above). In other words, whenever the level of managerial involvement is either very low or very high, then restricting that involvement is damaging to expected surplus. To establish the location of the local maximum, take the point of $\alpha\frac{\partial x_{A,v}}{\partial x_{P,v}} = 0$ or $\left(\frac{7}{9} = x_{P,v}\right)$. At this point, the expression is positive, which implies that the solution lies in the region of complementarity (and correspondingly, the local minimum exists in the region of substitutability). In other words, we have that $\frac{d}{dp} (EU_P + EU_A) < 0$ for $x_{P,v} < x_{P,v}^1$, $\frac{d}{dp} (EU_P + EU_A) > 0$ for $x_{P,v} \in (x_{P,v}^1, x_{P,v}^2)$ and $\frac{d}{dp} (EU_P + EU_A) < 0$ for $x_{P,v} > x_{P,v}^2$, where $x_{P,v}^1 < \frac{1}{3\sqrt{7}}$ and $x_{P,v}^2 < \frac{1}{\sqrt{7}}$.

Finally, note that I have only analyzed the solution in terms of $x_{P,v}$ and not $\mu$. But from above we know that $x_{P,v}$ is monotone decreasing in $\mu$, with maximum $x_{P,v}$ obtained at $\mu = 1$. Thus, for the existence of the optima, we can conclude that if $x_{P,v} < x_{P,v}^2$, then there exists only a single optimum, which is interior and occurs at $x_{P,v}^1$. If $x_{P,v} > x_{P,v}^2$, then we know that there also exists a boundary solution at $\mu = 1$. As to the impact of $c_{A,v}$ and $c_{A,w}$ on the optimal choice of $\mu$, the result unfortunately can be established only numerically because it holds only around the equilibrium. The intuition, however, is straightforward and provided in the main text.