A Model of Trade with Ricardian Comparative Advantage and Intra-sectoral Firm Heterogeneity*

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Abstract

In this paper, we merge the heterogeneous firm trade model of Melitz (2003) with the Ricardian model of Dornbusch, Fisher and Samuelson (DFS 1977) to explain how the pattern of international specialization and trade is determined by the interaction of comparative advantage, economies of scale, country sizes and trade barriers. The model is able to capture the existence of inter-industry trade and intra-industry trade in a single unified framework. It explains how trade openness affects the pattern of international specialization and trade. The paper examines the generality of Melitz’s firm selection effect in a multi-sectoral setting. Although opening to trade is unambiguously welfare-improving in both countries, trade liberalization can lead to a counter-Melitz effect in the larger country if it is insufficiently competitive in the sectors where it has the strongest comparative disadvantage but still produces. In this case, the productivity cutoff for survival is lowered while the exporting cutoff increases in the face of trade liberalization. This is because the inter-sectoral resource allocation (IRA) effect dominates the Melitz effect in these sectors. Consequently, the larger country can lose from trade liberalization. Some hypotheses related to firms’ exporting behavior across sectors upon opening up to trade and upon trade liberalization are also derived. Analyses of data of Chinese manufacturing sectors confirm these hypotheses.

Keywords: inter-industry trade, intra-industry trade, heterogeneous firms, trade liberalization

JEL Classification codes: F12, F14

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1 Introduction

How do firms’ entry, exit, output and exporting decisions respond to trade integration and trade liberalization? Do they respond differently across sectors? How is trade pattern determined by the interaction between pattern of comparative advantage across sectors and monopolistic competition between firms within the same sector? How are trade pattern and welfare affected by globalization? We try to answer these questions by developing a model of trade with comparative advantage across sectors and intra-sectoral firm heterogeneity.

There are by and large two types of international trade: inter-industry trade and intra-industry trade. It is widely recognized that the former is driven by comparative advantage and the latter by economies of scale. The most widely used models for capturing comparative advantage are of the Ricardian type (e.g. Dornbusch, Fisher and Samuelson (DFS) 1977, Eaton and Kortum 2002) and the Heckscher-Ohlin type. The most notable models used to capture intra-industry trade are probably attributed to Krugman (1979, 1980). More recently, Melitz (2003) extends Krugman’s (1980) model to analyze intra-industry trade when there is firm heterogeneity, thus capturing the selection of firms according to productivity and profit-shifting to firms of higher productivity when a country opens up to trade and trade liberalization. It stimulates much further work in this direction, notably Chaney (2008), Melitz and Ottaviano (2008), and Arkolakis (2010), to name just a few papers.

Most papers analyzing trading economies focus their analysis on the effects attributed to one single trade model. For example, they assume that the world is described by an Armington, Krugman, DFS, Eaton and Kortum, or Melitz model. Thus, they ignore the interaction of the various effects when both comparative advantage and economies of scale are present so that there are both inter-industry and intra-industry trade between countries. This paper proposes a unified framework to capture both inter-industry and intra-industry trade in a single model. By doing so, we have a model that explains how comparative advantage, economies of scale, firm selection and home market effect interact to sort sectors into ones in which only one of the countries produces (where there is inter-industry trade) and ones in which both countries produce (where there is intra-industry trade). Specifically, we modify Dornbusch, Fischer, and Samuelson’s (1977) two-country, multi-sector Ricardian framework by incorporating intra-sectoral firm heterogeneity a la Melitz (2003). A number of testable hypotheses are generated. For example, sectors in which one of the countries has strong comparative advantage would be characterized by inter-industry trade, while sectors in which neither country has strong comparative advantage would be characterized by intra-industry trade. For any given country, the fraction of firms that export is higher for a sector with stronger comparative advantage. These results are supported by Chinese data.

Furthermore, we are able to understand the interaction of the forces attributed to comparative advantage effect, productivity selection effect and home market effect, in the face of trade integration and trade liberalization. We find that we can always decompose the total effect of trade liberalization into those caused by inter-sectoral resource allocation (IRA) effect and firm selection effect according to productivity (which we call Melitz effect).

Although trade integration (switching from autarky to trade) is always welfare-improving, the welfare
effect of trade liberalization (reduction of trade barriers) depends on the relative size of the two countries, the height of trade barriers and the Ricardian technological differences between the two countries. In particular, in the case of trade liberalization, we find that the interaction of the IRA effect, the Melitz effect and home market effect in certain sectors can give rise to an effect that is opposite to what is predicted by Melitz. For lack of a better term, we call this effect counter-Melitz effect. Melitz predicts that trade liberalization leads to an increase in the productivity cutoff for survival but a decrease in the exporting productivity cutoff, and this gives rise to an increase in the average productivity of the firms that serve the domestic market, leading to domestic welfare gains in the sector. The counter-Melitz effect that occurs in our model predicts that, in some sectors, trade liberalization leads to a decrease in the productivity cutoff for survival but an increase in the exporting productivity cutoff, and this gives rise to a decrease in the average productivity of the firms that serve the domestic market, leading to domestic welfare losses in these sectors. This is because the IRA effect dominates the Melitz effect in the sectors where the larger country has the strongest comparative disadvantage and yet still produces. In other words, in the sectors where counter-Melitz effect occurs, the home market effect interacts with the IRA effect to create a force so large that it overwhelms the Melitz effect, leading to losses from trade liberalization. The existence of such sectors in the larger country is attributed to the home market effect. For this reason, they cannot exist in the smaller country.

Therefore, there exist a counter-Melitz effect in the comparative disadvantage sectors and a Melitz effect in the comparative advantage sectors. This implies that, for any given country, in the face of trade liberalization, the fraction of exporters and the share of revenue in total revenue both increase in the comparative advantage sectors but they both decrease in the sectors in which the country has the strongest comparative disadvantage but in which it still produces. We find that this proposition is supported by Chinese data.

In the recent theory literature modeling open economy with heterogeneous firms, our neighbors are Demidova (2008) and Okubo (2009). Demidova (2008) extends Melitz’s (2003) model to a setting with two countries of the same size but are asymmetric in the distribution of the productivity draws of firms. She assumes that there is only one differentiated-good sector, in which both countries produce and trade with each other in equilibrium. In contrast, we assume that the two countries are of different sizes, there is a continuum of sectors, and in equilibrium there are sectors in which only one country produces (with one-way trade) as well as ones in which both countries do (with two-way trade). She assumes a general distribution function for firm productivity whereas we assume the distribution to be Pareto. In both her model and ours, there exists a homogeneous good sector in which both countries produce and trade cost is zero in that sector. Like her, we find that the laggard country may lose from falling trade cost. However, we show that this can only happen in the large country. Okubo (2009) also introduces multiple sectors into the Melitz model, thus making it a hybrid of the multiple-sector Ricardian model and the Melitz model. In the two-sector case he analyzes the general equilibrium effects, allowing the endogenous determination of the relative wage. But the focus of his paper is quite different from ours, though there are some similarities. He mainly focuses on changes in population and the effects on the number of varieties. We mainly focus on how the strength of comparative advantage of a sector affects firm selection in different sectors under trade integration and trade liberalization. We
analyze and obtain closed form solution of the international pattern of specialization and trade as a function of trade barriers, relative country size and Ricardian comparative advantage. We decompose the total effect of trade liberalization into the IRA effect and Melitz effect and explain the condition under which one effect can dominate the other. Most importantly, we identify the conditions under which there is a counter-Melitz effect and a loss from trade liberalization in certain sectors or even the entire economy. We carry out empirical tests of the hypotheses while he does not conduct any empirical tests.

Bernard, Redding and Schott (2007) incorporate firm heterogeneity into a two-sector, two-country Heckscher-Ohlin model, and analyze how falling trade costs lead to the reallocation of resources, both within and across industries. Inter-sectoral resource reallocation changes the ex-ante comparative advantage and provides a new source of welfare gains from trade as well as causes redistribution of income across factors. In contrast to the work of BRS (2007), our paper focuses on how comparative advantage and increasing returns to scale interact to determine inter-sectoral and intra-sectoral resource allocation as well as welfare in the face of trade integration and trade liberalization.

Hsieh and Ossa (2010) build a Ricardo-Krugman-Melitz model with many countries and many sectors, each of which consists of heterogeneous firms engaging in monopolistic competition with each other. They then analyze how real incomes of all countries are affected by productivity growth in one of the countries. The difference with our paper is that they only focus on the case when all countries produce in all sectors. In contrast, we analyze a two-country setting, but we allow for the possibility that countries endogenously specialize in certain sectors and so do not produce in sectors where they have strong comparative disadvantage, and this gives rise to interesting results. We are able to obtain closed form solution to comparative statics with regard to how productivity cutoffs for survival, exporting productivity cutoff, firm number and welfare in each sector are impacted by trade liberalization and other changes.

The paper is organized as follows. Section 2 presents the model with heterogeneous firms in the closed economy and examines the properties of the equilibrium. In section 3, we carry out an analysis of the equilibrium in the open economy. We analyze the pattern of specialization and trade and identify the existence of inter-industry trade as well as intra-industry trade. In section 4, we show the impact of trade integration on the productivity cutoffs, the number of firms and welfare in each sector. An empirical test of the main proposition of the section is carried out. In section 5, we analyze the effects of trade liberalization, and demonstrate the existence of a counter-Melitz effect in certain comparative disadvantage sectors of the large country. Empirical tests of the main propositions of the section are carried out. Section 6 concludes.

## 2 A Closed-economy Model

In this section, we shall describe the features of a closed economy, but where necessary we also touch upon some features of a two-country model when the closed economy opens up to trade. The closed
economy is composed of multiple sectors: a homogenous-good sector, and a continuum of sectors of differentiated goods. There is only one factor input called labor. The homogeneous good is produced using a constant returns to scale technology. It is freely traded with zero trade costs when the country is opened up to trade. Firms are free to choose the sectors into which they enter. We assume that in order to produce a differentiated good, a firm has to pay a sunk cost of entry. After entry, a firm decides whether or not to produce according to whether the expected present discounted value of its economic profit is non-negative after its firm-specific productivity has realized. The economic profit is determined by the following factors. There is a fixed cost of production per period, and a constant marginal cost of production. The fixed cost of production is the same for all firms but the marginal cost of production of a firm is partly determined by a random draw from a distribution. Upon payment of the entry cost \( \phi \), the firm earns the opportunity to make a random draw from a distribution of firm productivity. The draw will determine the firm-specific component of the firm’s productivity (i.e. reciprocal of the unit labor requirement for production). The above characterizations of the model are basically drawn from Melitz (2003). Unlike Melitz, there is another factor that affects the variable cost of production of a firm, which is an exogenously determined sector-specific technological level. In general, this technological level differs across sectors in a country as well as differs across countries within the same sector. The set of sector-specific technological levels across sectors in both countries determine the pattern of comparative advantage across sectors of each country. The above structure is basically drawn from DFS (1977). Our model is therefore a hybrid of Melitz (2003) and DFS (1977).

There are \( L \) consumers, each supplying one unit of labor. Preferences are defined by a nested Cobb-Douglas function:

\[
\ln U = \alpha \ln C_h + \int_0^1 b_k \ln C_k dk
\]

\[
C_k = \left[ \int_0^{\theta_k} c_k(j)^{\rho_k} dj \right]^{\frac{1}{\rho_k}} \text{ with } \int_0^1 b_k dk = 1 - \alpha
\]

where \( \alpha \) denotes the share of expenditure on homogenous goods, \( b_k \) is the share of expenditure on differentiated good \( k \in [0, 1] \); \( \theta_k \) is the endogenously determined mass of varieties in differentiated sector \( k \). The homogeneous good is produced with constant unit labor requirement \( 1/A_h \). The price of the homogeneous good is \( w/A_h \), where \( w \) is the wage, as it is produced and sold under perfect competition. For the differentiated-goods sectors, the exact price index for each sector is denoted by \( P_k \), where

\[
P_k = \left[ \int_0^{\theta_k} p_k(j)^{1-\sigma} dj \right]^{\frac{1}{\sigma}}, \text{ where } \sigma = \frac{1}{1-\rho} > 1
\]

where \( p_k(j) \) denotes the price of variety \( j \) in sector \( k \), and \( \sigma \) denotes the elasticity of substitution between varieties. Cost minimization by firms implies that the gross revenue flow of firm \( j \) in sector \( k \) is given by

\[
r_k(j) = b_k E \left[ \frac{p_k(j)}{P_k} \right]^{1-\sigma}
\]

where \( E = wL \) denotes the total expenditure on all goods.
The labor productivity of a firm in the differentiated-good sector \( k \) is the product of two terms: one is firm-specific, being a random variable following a Pareto distribution \( P(1, \gamma) = 1 - \left( \frac{1}{\varphi} \right)^\gamma \) where \( \varphi \in [1, \infty) \) and \( \gamma \) (\( > \sigma - 1 \)) is the shape parameter of the distribution;\(^1\) the other is \( A_k \), which is exogenous and sector-specific. The labor productivity of a firm is thus equal to \( A_k \varphi_k \). Production labor employed by firm \( j \) in sector \( k \) is a linear function of output \( y_k(j) \):

\[
l_k(j) = f + \frac{y_k(j)}{A_k \varphi_k(j)},
\]

where \( f \) is the fixed cost of production per period, and \( A_k \varphi_k(j) \) is the productivity of firm \( j \) in sector \( k \). Therefore, under monopolistic competition in sector \( k \) the profit-maximizing price is given by

\[
p_k(j) = \frac{w}{\rho A_k \varphi_k(j)} \tag{4}
\]

(3) and (4) imply that the flow of profit of the firm is given by

\[
\pi_k(j) = \frac{r_k(j)}{\sigma} - f w = \frac{b_k w L [P_k w^{-1} \rho A_k \varphi_k(j)]^{\sigma-1}}{\sigma} - f w
\]

If a firm draws too low a productivity, it will exit immediately, as the expected present discounted value of its economic profit is negative. Denote the cutoff productivity for a firm to survive in sector \( k \) by \( \overline{\varphi}_k \). We shall call this the productivity cutoff for survival or the operating productivity cutoff. Then, the aggregate (exact) price index in sector \( k \) can be rewritten as

\[
P_k = \left( \theta_k \int_{\overline{\varphi}_k}^\infty [p_k(\varphi)]^{1-\gamma} g(\varphi) \frac{1}{1-G(\varphi)} d\varphi \right) \frac{1}{1-\gamma} = \theta_k^{1-\gamma} p_k(\overline{\varphi}_k),
\]

where \( p_k(\varphi) \equiv \frac{w}{\rho A_k \varphi} \), \( G(\varphi) \) is the c.d.f. of the distribution of the firm-specific component of productivity \( \varphi \) in the sector and \( g(\varphi) \) is its p.d.f. The function \( G(\varphi) \) is the same for all sectors. Moreover,

\[
p_k(\overline{\varphi}_k) = \frac{w}{\rho A_k \overline{\varphi}_k}
\]

where \( \overline{\varphi}_k \) can be interpreted as the “average” productivity in sector \( k \). It can be easily shown that

\[
\overline{\varphi}_k = \left( \int_{\overline{\varphi}_k}^\infty \varphi^{\sigma-1} g(\varphi) \frac{1}{1-G(\overline{\varphi}_k)} d\varphi \right)^{\frac{1}{\sigma-1}} = \left( \frac{\gamma}{\gamma - \sigma + 1} \right)^{\frac{1}{\sigma-1}} \overline{\varphi}_k .
\]

if \( G(\overline{\varphi}_k) \) is the c.d.f. of \( \varphi_k \), which follows a Pareto distribution \( P(1, \gamma) \) (i.e. \( G(\varphi_k) = 1 - \left( \frac{1}{\overline{\varphi}_k} \right)^\gamma \)).

From now on, we shall assume for tractability that \( G(\varphi) \) is a Pareto distribution as described above. The qualitative nature of most of our results will not be affected by this assumption. We assume that there is no discounting of future, and only the stationary equilibrium is considered. We assume that

\(^1\)The assumption \( \gamma > \sigma - 1 \) ensures that, in equilibrium, the size distribution of firms has a finite mean.
in each period, an operating firm faces a constant probability $\delta$ of a bad shock that forces it to exit. Given stationarity, the expected profit of a firm is the same in all periods in which the firm stays in the market. After making a draw from the productivity distribution, a firm will exit immediately if it makes negative expected profits in each period in the future. The zero cutoff profit (ZCP) condition determines the productivity $\bar{\sigma}_k$ of the marginal firm that makes zero expected economic profits in each period:

$$\sigma f w = r_k(\bar{\sigma}_k) = \frac{b_k w L}{\theta_k} \left( \frac{\bar{\sigma}_k}{\bar{\sigma}_k} \right)^{\sigma - 1} = \frac{b_k w L}{\theta_k} \left( \frac{\gamma - \sigma + 1}{\gamma} \right) \quad \text{(ZCP)} \quad (6)$$

As more firms enter, the cutoff productivity increases. This in turn lowers the probability of surviving after entry. So, when the cutoff productivity becomes sufficiently high, there will be no more entry. More precisely, the free entry (FE) condition, which relates the cutoff productivity to the entry cost $f_e$, is given by

$$f_e w = p_{in} \tilde{v} = [1 - G(\bar{\sigma}_k)] \frac{\bar{\sigma}_k}{\delta} \quad (7)$$

where $p_{in} = 1 - G(\bar{\sigma}_k)$ is the ex-ante probability of successful entry; $\tilde{v} \equiv \frac{1}{\delta} \bar{\sigma}_k$ is the present value of the average profit flow of a surviving firm; and $\bar{\sigma}_k \equiv \sigma_k(\bar{\sigma}_k)$ is the average profit flow of a surviving firm, which is equal to $f w \left[ \left( \frac{\bar{\sigma}_k}{\bar{\sigma}_k} \right)^{\sigma - 1} - 1 \right] = f w \left( \frac{\sigma - 1}{\gamma - \sigma + 1} \right)$ according to the ZCP condition (6) and equation (5).\(^2\)

Solving for the above system of 2 equations for 2 unknowns, we can get

$$(\bar{\sigma}_k)^\gamma = \left( \frac{\sigma - 1}{\gamma - \sigma + 1} \right) \frac{f}{\delta f_e} \equiv D_1; \quad \theta_k = \left( \frac{\gamma - \sigma + 1}{\gamma} \right) \frac{b_k L}{\sigma f} \equiv D_2 L$$

**Lemma 1** In the closed economy, the fraction of firms that can successfully enter is the same across sectors. The number of firms in each sector is proportional to the sector’s share in total expenditure.

In fact the result stated in Lemma 1 is independent of the assumption of Pareto distribution. The only unknown in equation (7) is $\bar{\sigma}_k$, as aggregate productivity $\bar{\sigma}_k$ is a function of $\bar{\sigma}_k$ for any distribution, thus $\bar{\sigma}_k$ is endogenously determined and unrelated to $A_k$.\(^3\) Similar logic applies to the result concerning $\theta_k$, for it is endogenously determined by $\bar{\sigma}_k$ following equation (6). Note that the actual cutoff productivity is $A_k \bar{\sigma}_k$, which still differs across sectors. From now on, we assume $\left( \frac{\sigma - 1}{\gamma - \sigma + 1} \right) \frac{f}{\delta f_e} \geq 1$, in order to avoid the corner solution.\(^4\)

The intuition of this proposition is that, an increase in the sector-specific technology will cause a firm’s optimal price to decrease, and following this, the aggregate price for this sector will decrease as

\(^2\)If $\left( \frac{\sigma - 1}{\gamma - \sigma + 1} \right) \frac{f}{\delta f_e} \geq 1$, $\bar{\sigma}_k$ will have corner solution. Even so, Lemma 1 still holds in this case.
well. This price reduction leads to two opposite effects on the profit of a firm: on the one hand, the decline of sectoral aggregate price causes more demand for each firm, which increases the firm’s profit; on the other hand, the decrease of price will reduce the profit. These two effects cancel each other so that the expected profit of each firm does not change. As a result, the fraction of firms that can successfully enter will be the same across sectors, and the number of firms in each sector is proportional to the sector’s share in total expenditure. It is also noteworthy that although the increase in sector-specific technology does not affect the number of firms, it will improve consumers’ welfare due to the increased output of each firm.

### 3 An Open-economy Model

In this section, we consider a global economy with two countries: Home and Foreign. We attach an asterisk to all the variables pertaining to Foreign. We index sectors such that as the index increases Home’s comparative advantage strengthens. In other words, the sector-specific relative productivity $a(k) \equiv a_k \equiv \frac{A_k}{A^*_k}$ increases in $k \in [0, 1]$. Therefore, $a'(k) > 0$.

On the demand side, we assume that consumers in both countries have identical tastes:

$$
\ln U = \alpha \ln C_h + \int_0^1 b_k \ln C_k dk \quad \text{with} \quad \int_0^1 b_k dk = 1 - \alpha
$$

and $C_k = \left( \int_0^{b_k} c(i)^p di + \int_0^{b^*_k} c^*_k(j)^p dj \right)^{\frac{1}{p}}$

On the production side, the labor productivity in the homogeneous good sector are respectively $A_h$ and $A^*_h$ in Home and Foreign. In the rest of the paper, we assume that the homogeneous good sector is sufficiently large so that the homogeneous good is produced in both countries. We also assume that there is no trade cost associated with the homogeneous good. Therefore free trade of homogeneous goods implies that the wage ratio is determined by relative labor productivity in the sector, i.e. $\omega \equiv \frac{w}{w^*} = \frac{A_h}{A^*_h}$, where $w^*$ denotes Foreign’s wage. Without loss of generality, we assume that $\frac{A_h}{A^*_h} = 1$ and normalize by setting $w^* = 1$. Therefore, in equilibrium $w = w^* = 1$. The specification on technology in the differentiated-good sectors is the same as in autarky.

The subscript “$dk$” pertains to a domestic firm serving the domestic market in sector $k$, the subscript “$xk$” pertains to a domestic firm serving the foreign market in sector $k$, and the subscript “$k$” pertains to sector $k$ regardless of which market is being served. For the differentiated-good sectors, each firm’s profit-maximizing price in the domestic market is given, as before, by $p_{dk}(j) = \frac{1}{\rho^{A_k} c_k(j)}$. But Home’s exporting firms will set higher prices in the Foreign market due to the existence of an iceberg trade cost, such that $\tau$ (> 1) units of goods have to be shipped from the source in order for one unit to arrive at the destination. Therefore, the optimal export price of a Home-produced good sold in Foreign is given by $p_{xk}(j) = \frac{\tau}{\rho^{A^*_k} c^*_k(j)}$. Similarly, Foreign’s firms’ pricing rules are given by $p^*_{dk}(j) = \frac{1}{\rho^{A^*_k} c^*_k(j)}$ and $p^*_{xk}(j) = \frac{\tau}{\rho^{A^*_k} c^*_k(j)}$. Here, we assume identical iceberg trade cost $\tau$ for both countries for simplicity.
3.1 Firm entry and exit

According to the firms’ pricing rules, the gross revenue flow and net profit flow of firm \( j \) in differentiated sector \( k \) from domestic sales for Home’s firms are, respectively:

\[
\begin{align*}
    r_{dk}(j) &= b_k L \left[ \frac{p_{dk}(j)}{P_k} \right]^{1-\sigma}, \\
    \pi_{dk}(j) &= \frac{r_{dk}(j)}{\sigma} - f.
\end{align*}
\]

The expressions for the corresponding variables for Foreign’s firms, \( r_{dk}^*(j) \) and \( \pi_{dk}^*(j) \), are defined analogously. The variables \( P_k \) and \( P_k^* \) are the aggregate price index in sector \( k \) of goods sold in Home and Foreign, respectively. Their expressions are given in equation (8) below. Following the same logic, the gross exporting revenue flow and net profit flow of firm \( j \) in sector \( k \) for Home’s firms are, respectively:

\[
\begin{align*}
    r_{xk}(j) &= b_k L^* \left[ \frac{p_{xk}(j)}{P_k^*} \right]^{1-\sigma}, \\
    \pi_{xk}(j) &= \frac{r_{xk}(j)}{\sigma} - f_x.
\end{align*}
\]

The expressions for the corresponding variables for Foreign’s firms, \( r_{xk}^*(j) \) and \( \pi_{xk}^*(j) \), are defined analogously. The variable \( f_x \) is the fixed cost of exporting to be paid at each date, which is the same for all firms. Note that we can interpret \( f_x = \delta f_{ex} \) as the amortized cost of entry into the export market, where \( f_{ex} \) is the one-time fixed cost of entry into the export market. Let \( \varphi_{dk} \) and \( \varphi_{xk} \) denote the cutoffs of the firm-specific productivity for domestic sales and exporting respectively of sector \( k \) for Home firms; \( \varphi_{dk}^* \) and \( \varphi_{xk}^* \) denote the corresponding variables for Foreign. Consequently, the mass of exporting firms from Home is equal to:

\[
\theta_{xk} = \frac{1 - G(\varphi_{xk})}{1 - G(\varphi_{dk})} \theta_{dk} = \left( \frac{\varphi_{dk}}{\varphi_{xk}} \right)^{\gamma} \theta_{dk}
\]

where \( \theta_{dk} \) denotes the mass of operating firms in Home. The corresponding expression relating the variables \( \theta_{xk}^* \) and \( \theta_{dk}^* \) for Foreign are defined analogously. Then, in differentiated-good sector \( k \), the mass of varieties available to consumers in Home is equal to

\[
\theta_k = \theta_{dk} + \theta_{xk}
\]

and \( \theta_k^* \) is defined analogously. The aggregate price indexes are given by:

\[
\begin{align*}
    P_k &= (\theta_k)^{\frac{1}{\sigma}} p_{dk}(\varphi_k), \\
    P_k^* &= (\theta_k^*)^{\frac{1}{\sigma}} p_{dk}(\varphi_k^*)
\end{align*}
\]

(8)

where \( \varphi_k \) and \( \varphi_k^* \) denote the aggregate productivity in differentiated sector \( k \) for goods sold in Home and Foreign, respectively. They are given respectively by:

\[
\begin{align*}
    (\varphi_k)^{\sigma-1} &= \frac{1}{\theta_k} \left[ \theta_{dk} (\varphi_{dk})^{\sigma-1} + \theta_{xk}^* \left( \tau^{-1} a_k \varphi_{xk}^* \right)^{\sigma-1} \right], \\
    (\varphi_k^*)^{\sigma-1} &= \frac{1}{\theta_k^*} \left[ \theta_{dk}^* (\varphi_{dk})^{\sigma-1} + \theta_{xk} \left( \tau^{-1} a_k \varphi_{xk} \right)^{\sigma-1} \right]
\end{align*}
\]

(9)

(10)
where \( \hat{\varphi}_{dk} \) (\( \hat{\varphi}_{dk}^* \)) and \( \hat{\varphi}_{zk} \) (\( \hat{\varphi}_{zk}^* \)) denote respectively the aggregate productivity level of all of Home’s (Foreign’s) operating firms and Home’s (Foreign’s) exporting firms. The relationships between \( \hat{\varphi}_{dk} \) and \( \hat{\varphi}_{zk} \), between \( \hat{\varphi}_{dk}^* \) and \( \hat{\varphi}_{dk}^* \), between \( \hat{\varphi}_{zk} \) and \( \hat{\varphi}_{zk}^* \), and between \( \hat{\varphi}_{zk}^* \) and \( \hat{\varphi}_{zk}^* \), are exactly the same as in the closed economy, namely equation (5). That is, \( \hat{\varphi}_{zk} = \left( \frac{\gamma}{\gamma - \sigma + 1} \right)^{\frac{1}{\sigma}} \hat{\varphi}_{sk} \) and \( \hat{\varphi}_{zk}^* = \left( \frac{\gamma}{\gamma - \sigma + 1} \right)^{\frac{1}{\sigma}} \hat{\varphi}_{sk}^* \) for \( s = x, d \). From the above equations, it is obvious that these aggregate productivity measures as well as aggregate price indexes are functions of \( \left( \hat{\varphi}_{dk}, \hat{\varphi}_{dk}^*, \hat{\varphi}_{zk}, \hat{\varphi}_{zk}^*, \theta_{dk}, \theta_{dk}^* \right) \). As will be shown below, as long as \( \frac{\tau f}{\tau} \) is sufficiently large, an entering firm will produce only if it can generate positive present-discounted profit by selling domestically, and export only if it can generate positive present-discounted profit by selling abroad. Then we have the following four zero cutoff profit conditions

\[
\begin{align*}
    r_{dk}(\hat{\varphi}_{dk}) &= b_k L \left( P_k \rho A_k \hat{\varphi}_{dk} \right)^{\sigma - 1} = \sigma f \\
    r_{dk}^*(\hat{\varphi}_{dk}^*) &= b_k^* L^* \left( P_k^* \rho A_k \hat{\varphi}_{dk} \right)^{\sigma - 1} = \sigma f \\
    r_{zk}(\hat{\varphi}_{zk}) &= b_k L \left( \frac{P_k}{\tau} \rho A_k \frac{\hat{\varphi}_{zk}}{\hat{\varphi}_{dk}} \right)^{\sigma - 1} x = \sigma f x \\
    r_{zk}^*(\hat{\varphi}_{zk}^*) &= b_k L^* \left( \frac{P_k}{\tau} \rho A_k \frac{\hat{\varphi}_{zk}^*}{\hat{\varphi}_{dk}} \right)^{\sigma - 1} x = \sigma f x
\end{align*}
\]

Define \( \tilde{\pi}_k \) and \( \tilde{\pi}_k^* \) as the average profit flow of a surviving firm in sector \( k \) in Home and Foreign respectively. It can be easily shown that

\[
\begin{align*}
    \tilde{\pi}_k &= \pi_{dk}(\hat{\varphi}_{dk}) + \left[ 1 - G(\hat{\varphi}_{zk}) \right] \pi_{zk}(\hat{\varphi}_{zk}) = \frac{\sigma - 1}{\gamma - \sigma + 1} \left[ f + \left( \frac{\hat{\varphi}_{dk}}{\hat{\varphi}_{zk}} \right)^{\gamma} f_x \right] \\
    \tilde{\pi}_k^* &= \pi_{dk}^*(\hat{\varphi}_{dk}^*) + \left[ 1 - G(\hat{\varphi}_{zk}^*) \right] \pi_{zk}^*(\hat{\varphi}_{zk}^*) = \frac{\sigma - 1}{\gamma - \sigma + 1} \left[ f + \left( \frac{\hat{\varphi}_{dk}^*}{\hat{\varphi}_{zk}^*} \right)^{\gamma} f_x \right].
\end{align*}
\]

These are analogous to the equation shown in footnote 2 for the closed economy. The potential entrant will enter if her expected post-entry present-discounted profit is above the cost of entry. Hence, the Free Entry (FE) conditions for Home and Foreign are, respectively

\[
\begin{align*}
    f_e &= \left[ 1 - G(\hat{\varphi}_{dk}) \right] \frac{\tilde{\pi}_k}{\delta} = \left( \frac{\sigma - 1}{\gamma - \sigma + 1} \right) f \cdot \left( \frac{\hat{\varphi}_{dk}^{\gamma}}{\delta} + f_x \cdot \hat{\varphi}_{zk}^{\gamma} \right) \\
    f_e^* &= \left[ 1 - G(\hat{\varphi}_{dk}) \right] \frac{\tilde{\pi}_k^*}{\delta} = \left( \frac{\sigma - 1}{\gamma - \sigma + 1} \right) f \cdot \left( \frac{\hat{\varphi}_{dk}^{\gamma}}{\delta} + f_x \cdot \hat{\varphi}_{zk}^{\gamma} \right).
\end{align*}
\]

The derivation of the above two equations are available from the corresponding author’s homepage at http://ihome.ust.hk/~elai/ or upon request.

The condition is \( \frac{\tau f^*}{\tau} > \max \left( \frac{1}{f}, \frac{1}{f_x} \right) \). If this condition is not satisfied, then there may exist some firms that only export. Those firms need to pay a fixed cost of \( f + f_x \) in order to export.

\[
\gamma \tilde{\pi}_{dk} \equiv \pi_{dk}(\hat{\varphi}_{dk}) = \frac{\pi_{dk}(\hat{\varphi}_{dk})}{\delta} - f = \frac{1}{\delta} \left( \frac{\hat{\varphi}_{zk}}{\hat{\varphi}_{dk}} \right)^{\gamma - 1} r_{dk}(\hat{\varphi}_{dk}) - f = f \left( \frac{\hat{\varphi}_{dk}}{\hat{\varphi}_{zk}} \right)^{\gamma - 1} - f = f \cdot \frac{\sigma - 1}{\gamma - \sigma + 1}. \]

The third equality arises from the fact that \( \left( \frac{\hat{\varphi}_{zk}}{\hat{\varphi}_{dk}} \right)^{\gamma - 1} = \frac{\pi_{dk}(\hat{\varphi}_{dk})}{\pi_{zk}(\hat{\varphi}_{zk})} \). The fourth equality comes from the fact that \( \sigma f = r_{dk}(\hat{\varphi}_{dk}) \), which is the ZCP condition above. The fifth equality comes from equation (5). Furthermore, \( \tilde{\pi}_{zk} = f_x \left( \frac{\sigma - 1}{\gamma - \sigma + 1} \right) \) can be derived from similar steps as above by replacing the subscript “d” by “x” and the variable \( f \) by \( f_x \). Finally, \( 1 - G(\varphi) = \varphi^{-\gamma} \).
3.2 General equilibrium

Assuming that both countries produce in sector \( k \), given the wage ratio \( A_h/A_k^* = 1 \), we can solve for \((\varphi_{dk}, \varphi_{dk}', \varphi_{zk}, \varphi_{zk}', \theta_{dk}, \theta_{dk}')\) from the four zero cutoff profit conditions and two free entry conditions (11) to (16) since the aggregate prices are functions of these six variables (for details, please refer to the Appendix). The solutions are given below.

\[
\begin{align*}
(\varphi_{dk})^\gamma &= D_1 \left[ \frac{B - B^{-1}}{B - (a_k)^{\gamma}} \right] \tag{17} \\
(\varphi_{dk}')^\gamma &= D_1 \left[ \frac{B - B^{-1}}{B - (a_k)^{\gamma}} \right] \\
\varphi_{zk} &= \frac{\beta}{a_k} \varphi_{dk} \tag{19} \\
\varphi_{zk}' &= a_k \beta \varphi_{dk}' \tag{20} \\
\theta_{dk} &= D_2 \left[ \frac{BL^* - (a_k)^{\gamma-1}L}{B - B^{-1}} \right] \\
\theta_{dk}' &= D_2 \left[ \frac{BL^* - (a_k)^{\gamma-1}L}{B - B^{-1}} \right] \tag{22}
\end{align*}
\]

where \( B = \tau^\gamma \left( \frac{L}{F} \right)^{\frac{2}{\gamma-1}} \) and \( \beta = \tau \left( \frac{L}{F} \right)^{\frac{1}{\gamma-1}} \). The variable \( B \) and \( \beta \) can be interpreted as summary measures of trade barriers; \( a_k \) can be interpreted as competitiveness of Home in differentiated goods sector \( k \). Recall that \( a_k'(k) > 0 \) is assumed.

The condition \( \tau^{\sigma-1} f_x > f \) is needed to ensure that in both countries there exist some sectors in which some firms produce exclusively for their domestic market. This is exactly the required condition in Melitz (2003) for the same purpose. The rationale of this assumption is explained in the Appendix.\(^8\)

In this paper, a stricter condition \( \frac{L}{F} > \max\left\{ \frac{L}{F}, \frac{L^*}{F^*} \right\} \) is adopted so as to ensure that some firms produce exclusively for their domestic market in all sectors.

According to equations (21) and (22) Home’s firms will exit sector \( k \) when \( \theta_{dk} \leq 0 \), and Foreign’s firms will exit the sector if \( \theta_{dk}' \leq 0 \). This implies that \( B^{-1} \frac{B-(a_k)^{\gamma}}{B (a_k)^{\gamma-1}} < \frac{L}{F^*} < B \frac{B-(a_k)^{\gamma}}{B (a_k)^{\gamma-1}} \) is needed for both countries to produce positive outputs in sector \( k \), otherwise there will be complete dominance by one country in the sector and one-way trade. Rearranging these inequalities, we can sort the sectors into three types according to Home’s strength of comparative advantage. Home will not produce in

\(^8\)In fact, this condition implies \( B > 1 \). [If \( f_x > f \), \( B = \tau^\gamma \left( \frac{L}{F} \right)^{\frac{2}{\gamma-1}} \) is obviously larger than 1. If not, then \( B = \frac{L}{F} \beta^\gamma \geq \beta^\gamma > 1 \). We can argue as follow that \( B > 1 \) is supported by empirical evidence: The firm’s revenue in sector \( k \), \( r_g(j) \), follows a Pareto distribution with parameter \( \frac{1}{\sigma-1} \). According to Axtell (2001), \( \beta^\gamma \) is close to one, which in turn implies that \( \frac{2}{\sigma-1} \) is close to zero. To be more precise, it equals 0.059 according to Axtell’s estimation; therefore \( B \) approaches \( \tau^\gamma \), which must be larger than 1. [For \( \gamma = 5 \) (which must be larger than \( \sigma - 1 \)), and a small \( \tau = 1.1 \), we need \( \frac{L}{F} > 3220 \) in order for \( B \) to be less than 1.]
sector \( k \iff k \leq k_1 \), where \( k_1 \) satisfies

\[
(a_{k_1})^\gamma = \frac{B \left( \frac{L}{L^*} + 1 \right)}{B^2 \frac{L}{L^*} + 1};
\]

and Foreign will not produce in sector \( k \iff k \geq k_2 \), where \( k_2 \) satisfies\(^9\)

\[
(a_{k_2})^\gamma = \frac{B^2 \frac{L}{L^*} + 1}{B \left( \frac{L}{L^*} + 1 \right)}.
\]

Therefore, the solutions to (17) and (22) are valid if and only if \( k \in (k_1, k_2) \). It is clear that \( k \in (k_1, k_2) \) implies that \( (a_k)^\gamma \in (\frac{1}{B}, B) \) for any possible GDP ratio \( L/L^* \), which ensures that the productivity cutoffs will never reach the corner for the sectors in which both countries produce.

When \( k \notin (k_1, k_2) \), one of the countries dominates sector \( k \). In that case, there is no interior solution to some of the equations in the system above, as no firm in the other country has incentive to enter the market, which means that the number of firms in that country solved from the system is negative. Therefore, a different set of equations need to be solved for this case. Without loss of generality, we consider the \textbf{Home-dominated sectors}. As there is no competition from Foreign’s firms when Home’s firms sell in Foreign, the aggregate price indexes become

\[
P_k = (\theta_{dk})^{\frac{1}{\gamma - \sigma}} \frac{1}{\rho A_k \bar{\bar{\varphi}}_{dk}}
\]

\[
P^*_k = (\theta_{xk})^{\frac{1}{\gamma - \sigma}} \frac{\tau}{\rho A_k \bar{\bar{\varphi}}_{xk}}
\]

Accordingly, the two zero cutoff conditions for Home (11) and (13) continue to hold.

As the Free entry condition (15) for Home firms continues to hold, solving the diminished system of three equations (11), (13), (15) for three unknowns, we have

\[
\theta_{dk} = \frac{b_k L}{\sigma f} \left( \frac{\gamma - \sigma + 1}{\gamma} \right) = D_2 L
\]

\[
\theta_{xk} = \frac{b_k L^*}{\sigma f} \left( \frac{\gamma - \sigma + 1}{\gamma} \right) = D_2 \frac{f^*}{f} L^*
\]

\[
(\varphi_{dk})^\gamma = \left( \frac{L + L^*}{L} \right)^{\frac{1}{\gamma - \sigma}} D_1.
\]

Furthermore, we can easily obtain \( (\varphi_{xk})^\gamma = \left( \frac{L + L^*}{L} \right)^{\frac{1}{\gamma - \sigma}} D_1 \) by noting that \( \theta_{xk} = \frac{1 - G(\varphi_{xk})}{1 - G(\varphi_{dk})} \theta_{dk} \). An analogous set of solutions for the Foreign-dominated sectors can be obtained.\(^{10} \quad 11\)

---

\(^9\)Because \( \frac{n^2 \frac{L}{L^*} + 1}{n \left( \frac{L}{L^*} + 1 \right)} > \frac{n \left( \frac{L}{L^*} + 1 \right)}{n \left( \frac{L}{L^*} + 1 \right)} \) holds as long as \( B > 1 \), we always have \( k_1 < k_2 \).

\(^{10}\)They are: \( \theta_{dk}^* = \frac{b_k L^*}{\sigma f} \left( \frac{\gamma - \sigma + 1}{\gamma} \right) = D_2 L^* \); \( \theta_{xk}^* = \frac{b_k L^*}{\sigma f} \left( \frac{\gamma - \sigma + 1}{\gamma} \right) = D_2 \frac{f^*}{f} L^* \); and \( (\varphi_{dk})^\gamma = \frac{L + L^*}{L} D_1 \).

\(^{11}\)The uniqueness of the above equilibrium is proved in an appendix posted on the corresponding author’s homepage at http://ihome.ust.hk/~elai/ or upon request.
Proposition 1. In sectors $k \in [k_2, 1]$, where Home has the strongest comparative advantage, only Home produces, and there is one-way trade. An analogous situation applies to Foreign in sectors $k \in [0, k_1]$. In sectors $k \in (k_1, k_2)$, where neither country has strong comparative advantage, both countries produce, and there is two-way trade.

We show the three zones of international specialization in Figure 1. The upward sloping curve (including the dotted portions) corresponds to equation (21), while the downward sloping curve (including the dotted portions) corresponds to equation (22). The horizontal portion of $\theta_{dk}$ in the diagram corresponds to the equation for $\theta_{dk}$ above when Home dominates sector $k$ completely. The horizontal portion of $\theta^*_{dk}$ corresponds to the analogous equation for Foreign.

Figure 1. Three Zones of International Specialization (assumptions: (i) expenditure shares are equal across sectors; (ii) $L < L^*$).

Pattern of international specialization

In general, the pattern of international specialization depends on the size of the trade barrier, the pattern of comparative advantage of the two countries (i.e. the form of the $a(k)$ function) and the sizes of the two countries.

It is clear that when the trade barrier is very large (but not infinity) both countries produce in all sectors regardless of the form of the $a(k)$ function and the sizes of the two countries: For any given $a(k)$ function, one can always find a sufficiently large $B$ such that the solutions from the above equations for $k_1$ and $k_2$ yield $k_1 < 0$ and $k_2 > 1$. In this case, there is intra-industry trade of all differentiated goods in this case. Provided that $a(k)$ is symmetrical (e.g. $a(k) = 1$ for all $k \in [0, 1]$, or $a(0.5 + j) = 1/a(0.5 - j)$

---

12 The dashed curves represent the equilibrium firm numbers of the original system of equations based on the assumption that both countries produce positive outputs in all sectors.
for all \( j \in [0, 0.5] \), the larger country will be a net exporter of the differentiated goods. This is because of the home market effect attributed to Krugman (1980). At the other extreme, under free trade, Home specializes in the differentiated goods in which it has comparative advantage (i.e. goods for which \( a(k) > 1 \)) while Foreign specializes in differentiated goods in which it has comparative advantage (i.e. goods for which \( a(k) < 1 \)), provided that \( a(0) < 1 \) and \( a(1) > 1 \). In this case, whether or not Home exports the homogeneous good depends on the \( a(k) \) function and the relative size of the two countries. Given that \( a(k) \) is symmetrical, the larger country will be a net importer of the differentiated goods.\(^{13}\) On the other hand, if \( a(1) < 1 \) (i.e. \( a(k) < 1 \) for all \( k \in [0, 1] \)), then Home has comparative disadvantage in all differentiated goods (compared to the homogeneous good). In this case, under free trade, Foreign completely dominates all differentiated-good sectors, while Home specializes in and exports only the homogeneous good. In any case, there is no intra-industry trade when trade barrier falls to zero.

**Lemma 2** When the trade barrier is sufficiently large (but not infinity), all trades in the differentiated goods are intra-industry in nature. If \( a(k) \) is symmetrical, the larger country will be a net exporter of the differentiated goods. When the trade barrier is zero, all trades in the differentiated goods are inter-industry in nature. If \( a(k) \) is symmetrical, the larger country will be a net importer of the differentiated goods.

### 4 Opening up to Trade

In this section, we analyze how opening trade between the two countries impacts the economy of each country, e.g. the productivity cutoffs, the mass of producing and exporting firms, as well as welfare. Before proceeding with the analysis, it is helpful to list the solutions to the relevant variables corresponding to the three types of sectors in Table 1.

\(^{13}\)For example, given that \( a(0.5 + j) = a(0.5 - j) \), if \( L > L^* \) then the value of Home’s imports of differentiated goods exceeds the value of its exports of differentiated goods under free trade, and so Home is a net importer of the differentiated goods and net exporter of the homogeneous good under free trade.
\[
\begin{array}{|c|c|c|c|}
\hline
\text{Sector type} & \text{Foreign-dominated} & \text{Two-way trade} & \text{Home-dominated} \\
\hline
k < k_1 & k_1 < k < k_2 & k > k_2 \\
(\bar{\pi}_{dk})^\gamma & \emptyset & D_1 \frac{L - B - (a_k)}{L - (a_k)} & D_1 + \frac{L - B - (a_k)}{L} \\
(\bar{\pi}_{xk})^\gamma & \emptyset & D_1 \frac{L - B - (a_k)}{L - (a_k)} & D_1 + \frac{L - B - (a_k)}{L} \\
(\bar{\pi}_{dk})^\gamma & D_1 \frac{L + L^*}{L} & D_1 \frac{L - B - (a_k)}{L - (a_k)} & \emptyset \\
(\bar{\pi}_{xk})^\gamma & D_1 \frac{L + L^*}{L} & D_1 \frac{L - B - (a_k)}{L - (a_k)} & \emptyset \\
\theta_{dk} & 0 & D_2 \frac{B - (a_k)}{B - (a_k)} - \frac{1}{L} & D_2 L \\
\theta_{xk} & 0 & D_2 \frac{B - (a_k)}{B - (a_k)} - \frac{1}{L} & D_2 L \\
\theta_{dk}^* & D_2 L^* & D_2 \frac{B - (a_k)}{B - (a_k)} - \frac{1}{L} & 0 \\
\theta_{xk}^* & D_2 \frac{L^*}{L} \theta_{dk} & \left( \frac{\bar{\pi}_{dk}}{\bar{\pi}_{xk}} \right) \theta_{dk} & 0 \\
P_k & (D_2 L)^{\frac{1}{\sigma}} a_k \frac{B}{L} \left( \frac{L}{L + L^*} \right)^{\frac{1}{\gamma}} \frac{1}{\rho_{a_k}, \rho_{B_k}} & (D_2 L)^{\frac{1}{\sigma}} a_k \frac{B}{L} \left( \frac{L}{L + L^*} \right)^{\frac{1}{\gamma}} \frac{1}{\rho_{a_k}, \rho_{B_k}} & (D_2 L)^{\frac{1}{\sigma}} \frac{1}{\rho_{a_k}, \rho_{B_k}} \\
P_k^* & (D_2 L^*)^{\frac{1}{\sigma}} a_k \frac{B}{L} \left( \frac{L}{L + L^*} \right)^{\frac{1}{\gamma}} \frac{1}{\rho_{a_k}, \rho_{B_k}} & (D_2 L^*)^{\frac{1}{\sigma}} a_k \frac{B}{L} \left( \frac{L}{L + L^*} \right)^{\frac{1}{\gamma}} \frac{1}{\rho_{a_k}, \rho_{B_k}} & (D_2 L^*)^{\frac{1}{\sigma}} \frac{1}{\rho_{a_k}, \rho_{B_k}} \\
\hline
\end{array}
\]

Table 1: Solution of the System

\[
D_1 = \left( \frac{\sigma - 1}{\gamma - \sigma + 1} \right) \frac{f}{\delta f_f}; \quad D_2 = \left( \frac{\gamma - \sigma + 1}{\gamma} \right) \frac{b_f}{\sigma_f}; \quad (a_{k_1})^\gamma = \frac{B (L^* + 1)}{B^2 (L^* + 1)}; \quad (a_{k_2})^\gamma = \frac{B^2 (L^* + 1)}{B (L^* + 1)}
\]

4.1 Impacts on productivity cutoffs

In this subsection, we analyze how trade affects the productivity cutoffs from two aspects: within sector and across sectors. First, we look at how trade integration changes the cutoffs within a certain sector. As a result, we find that the impacts of trade integration on productivity cutoffs are the same as in Melitz (2003) in all sectors. Then we compare the cutoffs across sectors upon trade integration. We add a subscript c to all the parameters pertaining to autarky (c=closed economy). It has been shown in Section 2 that the autarky productivity cutoff for survival in Home and Foreign is given by \((\bar{\pi}_{ck})^\gamma = (\bar{\pi}_{ck})^\gamma = \left( \frac{\sigma - 1}{\gamma - \sigma + 1} \right) \frac{f}{\delta_f} = D_1\). If both countries produce, then the equilibrium cutoffs for survival are given by (17) and (18). As \((a_k)^\gamma \in \left( \frac{1}{B}, B \right)\), we have \(\bar{\pi}_{dk} > \bar{\pi}_{ck}\) and \(\bar{\pi}_{dk} > \bar{\pi}_{ck}\).

Recall that if only one country produces, the equilibrium operating cutoffs are given by:

\[
(\bar{\pi}_{dk})^\gamma = \frac{L + L^*}{L} - D_1 > (\bar{\pi}_{ck})^\gamma \quad \text{if only Home produces}
\]

\[
(\bar{\pi}_{dk})^\gamma = \frac{L + L^*}{L^*} - D_1 > (\bar{\pi}_{ck})^\gamma \quad \text{if only Foreign produces}
\]

Hence, the least productive firms in all sectors will exit the market after trade integration. As a result, resources will be reallocated to the most productive firms. Furthermore, \(\bar{\pi}_{dk} > \bar{\pi}_{ck}\), implies that \(\bar{\varphi}_{dk} > \bar{\varphi}_{ck}\), and \(\bar{\pi}_{dk} > \bar{\pi}_{ck}\) implies that \(\bar{\varphi}_{dk} > \bar{\varphi}_{ck}\). Therefore, the average productivity in any sector \(k\) is higher under trade integration than in autarky. Thus we generalize Melitz’s result to a setting where there exist endogenous intra-industry trade and inter-industry trade in a single model.
In the closed economy, the cutoffs for survival are identical across sectors. However, this is not true any more in the open economy. In the sectors where both countries produce, the equilibrium cutoff for survival is an increasing function of the sectoral comparative advantage. More precisely, as \( \alpha_k \) increases, \( \varphi_{dk} \) rises but \( \varphi_{dk}^* \) falls, and, following the free entry conditions (15) and (16), \( \varphi_{xk} \) falls but \( \varphi_{xk}^* \) rises. Thus, we have

**Proposition 2** In sectors where both countries produce, for a given country, a sector with stronger comparative advantage has a higher fraction of domestic firms that export and higher fraction of revenue derived from exporting.

Moreover, \( \varphi_{xk} > \varphi_{dk} > \varphi_{dk} > \varphi_{dk} \) iff Home is more competitive in sector \( k \) \( (\alpha_k > 1) \), while \( \varphi_{xk} > \varphi_{xk} > \varphi_{dk} \) \( \varphi_{dk} \) iff \( \alpha_k < 1 \). This result and Proposition 2 are summarized by Figure 2 below.

**Figure 2.** How productivity cutoffs vary across sectors in which both countries produce (Home’s comparative advantage increases with \( \alpha_k \))

### 4.2 Empirical Tests of Proposition 2

Our model predicts that a sector with stronger comparative advantage will have a higher fraction of firms that export. Furthermore, the share of revenue derived from exporting increases with the comparative advantage of a sector. The National Bureau of Statistics of China (NBSC) obtains annual reports from all state enterprises and large- and medium-sized non-state enterprises (with sales above 5 million RMB) in the manufacturing sector, and sort all firms into 4-digit-level industries according to the Chinese Industry Classification (CIC) system. We obtain this dataset and calculate the fraction of
firms that export and share of revenue derived from exporting in each 2-digit CIC industry. In order to test Proposition 2, we need to establish a proper measure of China’s comparative advantage in each industry. A common measure of comparative advantage without regard to factor-intensity is “revealed comparative advantage” (RCA), which was first introduced by Balassa (1965b). The formula is given by

\[ RCA_{C,k} = \frac{X_{C,k}}{X_{W,k}} \times \frac{X_C}{X_W} \]

where the subscript \( C \) refers to China, \( X_{C,k} \) denotes exports from China in industry \( k \), \( X_C \) denotes total exports from China. The subscript \( W \) refers to the world. Therefore \( X_{W,k} \) and \( X_W \) are the corresponding variables for the world. As each CIC 2-digit industry usually contains one or several SITC 2-digit industries, we match each 2-digit industry based on CIC with a group of 2-digit industries based on SITC.\(^{14}\) We calculate the revealed comparative advantage of each of the CIC 2-digit sectors in China in the year 2000, based on the trade flow data calculated by Feenstra. The relationship between revealed comparative advantage and export propensity (i.e. fraction of firms that export) as well as revenue share from exporting are shown in Figure 3.

![Figure 3.](image)

Proposition 2 is by and large confirmed by the data, as shown Figure 3. It is clear that the points can be fitted with an upward sloping curve in each diagram, meaning that both export propensity and revenue share from exporting increase with the strength of revealed comparative advantage of a sector. For robustness check, we also tried the labor to non-labor cost ratio and labor to capital ratio as proxies for a sector’s comparative advantage, and get positive and statistically significant relationships in the regressions.

\(^{14}\)Usually, a CIC 2-digit industry contains one or several SITC 2-digit industries. We merge several SITC 2-digit sectors together to form a group of sectors, which exactly matches one CIC 2-digit sector (for some sectors, we merge two CIC sectors together to get a more accurate match). Each CIC 2-digit sector represents one sector \( k \) in the formula above. The matching at the 4-digit level becomes very tedious. So we did not go further to test the theory at the 4-digit level.
4.3 Impacts on the masses of firms

In this subsection, we focus on how trade will affect the mass of firms in each sector. As in the previous subsection, a subscript \( c \) denotes all the variables referring to the closed economy.

Recall that if both countries produce, then
\[
\theta_{dk} = D_2 \left[ BL - \frac{B - (a_k)^\gamma}{B - B^{-1}} L^* \right] < D_2 L = \theta_{ck}; \quad \theta_{*dk} = D_2 \left[ BL^* - \frac{B(a_k)^{\gamma - 1}}{B - B^{-1}} L^* \right] < D_2 L^* = \theta_{*ck}.
\]

If only one country produces, then
\[
\theta_{dk} = \frac{b_k L}{\sigma f} \left( \frac{\gamma - \sigma + 1}{\gamma} \right) = D_2 L = \theta_{ck} \text{ if only Home produces} \quad \theta_{*dk} = \frac{b_k L^*}{\sigma f} \left( \frac{\gamma - \sigma + 1}{\gamma} \right) = D_2 L^* = \theta_{*ck} \text{ if only Foreign produces}
\]

We summarize the above findings in the following proposition:

**Lemma 3** In sectors where only Home produces under trade, the number of domestic firms in Home is the same as in autarky. In sectors where both countries produce under trade, the number of domestic firms in each sector decreases in Home after opening up to trade.

Obviously, this proposition applies equally to Foreign.

When both countries produce after trade, we have
\[
\theta_{dk} = \left( \frac{\gamma - \sigma + 1}{\gamma} \right) \left[ BL - \frac{B - (a_k)^\gamma}{B - B^{-1}} L^* \right].
\]
It decreases with \( k \), as \( \frac{B - (a_k)^\gamma}{B - B^{-1}} \) decreases with \( a_k \). Therefore, we have the following lemma:

**Lemma 4** For any given country, in sectors where both countries produce under trade, the domestic firm number to expenditure ratio increases with the strength of comparative advantage of the sector.

4.4 Impacts on welfare

For sectors in which **both countries produce**, i.e., when \( (a_k)^\gamma \in \left( \frac{B(\frac{1}{2} + 1)}{B^2 f + 1}, \frac{B^2 L^* + 1}{B(\frac{1}{2} + 1)} \right) \), we can write Home’s aggregate price index in the sector as:
\[
P_k = (\theta_{dk} + \theta_{*zk}) \frac{1}{\sigma} p_{dk}(\tilde{\varphi}_k) = \left( \theta_{dk} + \theta_{*zk} \frac{f_x}{f} \right) \frac{1}{\sigma} p_{dk}(\tilde{\varphi}_{dk})
\]
Substituting the equilibrium values of \( \theta_{dk}, \theta_{*zk}, \theta_{*zk}, \theta_{zk} \) into the above equation, we find that \( \theta_{dk} + \theta_{*zk} \frac{f_x}{f} = \theta_{ck} \). Therefore, we can simplify the price index as:
\[
P_k = (\theta_{ck}) \frac{1}{\rho A_k \tilde{\varphi}_{dk}}
\]
Then, Home’s real wage in terms of goods in this sector is given by:

$$\frac{1}{P_k} = (\theta_{ck})^{z-1} \rho A_k \tilde{\varphi}_{dk} = \left( \frac{B - B^{-1}}{B} \right)^{\frac{1}{\gamma}} \frac{1}{P_{ck}} > \frac{1}{P_{ck}}$$  \hspace{1cm} (23)

In a sector where Foreign completely dominates, i.e. when \((a_k)^{\gamma} \in \left(0, \frac{B(\frac{1}{B}+1)}{B^2} \right)\), Home’s real wage in terms of goods in this sector is given by:

$$\frac{1}{P_k} = (\theta_{xk})^{z-1} \rho A_k \tilde{\varphi}_{xk} = a_k^{-\frac{1}{\gamma}} B^{-\frac{1}{\gamma}} \left( \frac{L + L^*}{L} \right)^{\frac{1}{\gamma}} \frac{1}{P_{ck}} > \frac{1}{P_{ck}}$$  \hspace{1cm} (24)

In a sector where Home completely dominates, i.e. when \((a_k)^{\gamma} \in \left(\frac{B^2 L^* + 1}{B^2 L^* / L} \right) \infty\), Home’s real wage in terms of goods in this sector is given by:

$$\frac{1}{P_k} = (\theta_{dk})^{z-1} \rho A_k \tilde{\varphi}_{dk} = \left( \frac{L + L^*}{L} \right)^{\frac{1}{\gamma}} \frac{1}{P_{ck}} > \frac{1}{P_{ck}}$$  \hspace{1cm} (25)

Therefore, the welfare increases after trade integration. The following proposition and Figure 4 summarize the analysis above.

**Proposition 3** Welfare increase in both countries after they open up to trade with each other.

![Figure 4. Welfare Impact of Trade Integration (w = w* = 1 by assumption and normalization).](image)

In the next section, we perform comparative statics concerning the effects of trade liberalization. Unlike Dornbusch et al (1977), the relative wage is directly determined by the relative productivity in the homogeneous-good sector in our model.\(^{15}\)

\(^{15}\)We have also tried the version without the homogeneous sector, and relative wage is determined by balance of trade, as in Dornbusch et al. (1977). In that case, the effect on welfare is highly ambiguous, but still the results are very different from Dornbusch et al. (1977), unless we made the fully symmetric assumption like Okubo (2009).
5 Trade liberalization

Trade liberalization is interpreted as a reduction of the iceberg trade cost $\tau$, which lowers $B = \tau^\gamma \left( \frac{1}{B} \right)^{\gamma-1+1}$. As (23) shows, welfare in each country in the sectors where both countries produce just depends on the production cutoff $\varphi_{dk}$ and $\varphi_{xk}^*$ respectively, as they directly determine aggregate price index $P_k$ and $P_k^*$. Differentiating them with respect to $B$, we have

\[
\frac{d}{dB} (\varphi_{dk})^\gamma = \frac{2B^{-1} - (1 + B^{-2}) (a_k)^\gamma}{[B - (a_k)^\gamma]^2} \\
\frac{d}{dB} (\varphi_{xk})^\gamma = \frac{2B^{-1} - (1 + B^{-2}) (a_k)^{-\gamma}}{[B - (a_k)^{-\gamma}]^2}
\]

which shows that $\varphi_{dk}$ increases with $B$ (and so does $\varphi_{xk}^*$, according to equation (20)) if and only if $(a_k)^\gamma < \frac{2B}{1+B^2}$. Moreover, $\varphi_{dk}$ increases with $B$ (and so does $\varphi_{xk}^*$, according to equation (19)) if and only if $(a_k)^\gamma > \frac{1+B^2}{2B}$. \footnote{An increase in $B$ as a result of an increase in $\tau$ or an increase in $f_x/f$ would both lead to an increase in $\beta$. Therefore, according to equation (20), an increase in $B$ as a result of an increase in $\tau$ or $f_x/f$ leads to increases in $\varphi_{x,k}$ and $\varphi_{d,k}$. Similarly, according to (19), an increase in $B$ as a result of an increase in $\tau$ or $f_x/f$ leads to increases in $\varphi_{x,k}$ and $\varphi_{d,k}$, following the same logic.} Comparing $(a_{k_1})^\gamma$ and $(a_{k_2})^\gamma$ with these two thresholds, we see that the pattern of specialization (which is determined by the values of $k_1$ and $k_2$) also depends on the relative size of the two countries.

To analyze the impacts of trade liberalization, refer to equations (23) to (25) and to Appendix C. Figure 5 summarizes the effects. Figure 5 shows the pattern of international specialization for any given $L/L^*$. Without loss of generality, we only focus on the case when $L/L^* > 1$. The zone to the left of the $k_1$ curve corresponds to sectors completely dominated by Foreign. The zone to the right of the $k_2$ curve corresponds to the sectors completely dominated by Home. The figure also shows, for any given value of $L/L^*$, the signs of the welfare effects of trade liberalization on Home and Foreign in different sectors. The upper sign inside a rectangle indicates the sign of Home’s welfare change due to an infinitesimal decrease in $\tau$, and the lower sign indicates the sign of Foreign’s welfare change. For example, for $L/L^* < B^2$, when there is an infinitesimal reduction of $\tau$, Home gains in the Foreign-dominated sectors (the zone to the left of the $k_1$ curve), loses in the two-way trade sectors to the right of the $k_1$ curve but to the left of the vertical line $(a_k)^\gamma = \frac{2B}{1+B^2}$ (this corresponds to the slanted-hatched zone), gains in the two-way trade sectors to the right of the vertical line $(a_k)^\gamma = \frac{2B}{1+B^2}$ but to the left of the $k_2$ curve (this corresponds to the vertically-hatched zone), and does not gain or lose in the Home-dominated sectors (the zone to the right of the $k_2$ curve).

Note that Figure 5 indicates that there is a counter-Melitz effect for Home (the larger country) in the slanted-hatched zone, in the sense that $\varphi_{dk}$ decreases and $\varphi_{xk}^*$ increases in response to trade liberalization. However, the Melitz effect dominates in Home in the vertically-hatched zone, in the
sense that $\overline{\varphi}_{dk}$ increases and $\overline{\varphi}_{sk}$ decreases in response to trade liberalization. These results reflect the algebraic derivation above. Note that there is no counter-Melitz effect for Foreign, the smaller country.

Figure 5. Welfare Effects of Trade Liberalization (infinitesimal reduction of $\tau$). In each region, the upper sign inside the rectangle indicates the welfare change of Home and the lower sign indicates the welfare change of Foreign.

The short arrows indicate the movement of lines as $\tau$ falls.

As $\tau$ decreases, the curves for $k_1$ and $k_2$, as well as the vertical lines corresponding to $(a_k)^{\gamma} = \frac{2B}{1+B^2}$ and $(a_k)^{\gamma} = \frac{B^2+1}{2B}$, will all shift, with the directions of shifts shown by the small arrows in Figure 5. For any given $L/L^*$, as $\tau$ (and therefore $B$) decreases from a large number, $k_1$ increases, $k_2$ first decreases
then increases, \((a_k)^\gamma = \frac{2B}{1+B^2}\) increases while \((a_k)^\gamma = \frac{B^2+1}{2B}\) decreases. Depending on the range of \([a_0, a_1]\) and the value of \(L/L^*\), it is possible that the \(k_1\) or \(k_2\) curve (or both) may situate outside the range \(k \in [0, 1]\) for some or all values of \(L/L^*\). For example, if \(a_0 < a_{k_1}\) for a given value of \(L/L^*\), then no Foreign-dominated sector exist for that value of \(L/L^*\). This is because as \(L/L^*\) gets sufficiently large, the home-market effect in Home gets so strong that Home can compete even in the sector in which it has the strongest comparative disadvantage, namely sector \(k = 0\).

Figure 5 can be summarized by the following lemma and proposition:

**Lemma 5** When the two country are sufficiently similar technologically in the sense that \(\frac{2B}{1+B^2} < (a_k)^\gamma < \frac{1+B^2}{2B}\) for all \(k\), trade liberalization weakly improves the welfare in both countries.

**Proposition 4** Suppose Home is larger than Foreign. In the sectors where Home has the strongest comparative disadvantage but still produces, there is a counter-Melitz effect in the sense that \(\varphi_{dk}\) decreases while \(\varphi_{rk}\) increases in the face of trade liberalization, leading to welfare losses in these sectors. Foreign, the smaller country, will never lose from trade liberalization as it will never experience any counter-Melitz effect.

Proposition 4 deserves more discussion, as it highlights one of the most important results of this paper. If Home is the larger country, the sectors in which it will lose from trade are defined by \(\left\{k \mid (a_{k_1})^\gamma < (a_k)^\gamma < (a_{k_2})^\gamma \text{ and } (a_k)^\gamma < \frac{2B}{1+B^2}\right\}\). The first condition indicates that the sector is a two-way trade sector. The second condition indicates that Home’s welfare decreases with a reduction in \(B\) (and therefore reduction in \(\tau\)) provided that the first condition holds. In other words, these are sectors where the larger country has strong comparative disadvantage yet still produces. In fact, there is a counter-Melitz effect in such circumstance, i.e. \(\varphi_{dk}\) decreases while \(\varphi_{rk}\) increases in the face of trade liberalization, leading to a decrease in the average productivity of firms serving the Home market, thus lowering welfare. We can explain this phenomenon by decomposing the total effect of trade liberalization into two effects below. We shall analyze from the perspective of Home and Home’s firms.

1. The inter-sectoral resource allocation (IRA) effect as \(B\) decreases — trade liberalization leads to resources in Home (as well in Foreign) being re-allocated away from the differentiated-good sectors in which it has comparative disadvantage to the sectors in which it has comparative advantage (these include other differentiated-good sectors and the homogeneous good sector). Note that the IRA effect does not take place in the one-way trade differentiated-good sectors. The IRA effect tends to reduce the welfare of the comparative disadvantage differentiated-good sectors and raise the welfare of the comparative advantage differentiated-good sectors. Define \(n_k\) and \(n_{ck}\) as the masses of potential entrants in the open economy and autarky respectively. Then \(\theta_{dk} = n_k \left[1 - G\left(\varphi_{dk}\right)\right]\) and \(\theta_{ck} = n_{ck} \left[1 - G\left(\varphi_{ck}\right)\right]\). Re-allocation of resource (labor) across sectors explains why, in the differentiated-good sectors in which Home has the strongest comparative disadvantage, the mass of potential Home entrants \((n_k)\) decreases, while, in the same sectors, the mass of potential Foreign entrants \((n_k^*)\) increases. In Figure 5, we show a dotted downward sloping curve. To the left (right) of it, \(n_k\) decreases (increases) as \(\tau\) decreases. It
follows that the counter-Melitz effect will occur only to the left of the curve.\footnote{Note that if $a_k$ and $b_k$ are both constant for all $k$, then $n_k$ is the same for all $k$, even as trade cost decreases. As $a_k$ deviates from being a constant, the IRA effect kicks in. In this case, under trade liberalization, $n_k$ increases (decreases) for the sectors in which Home has comparative advantage (disadvantage).} Let us analyze from the perspective of Home’s firms in the comparative disadvantage sectors. As $n_k^h$ increases, Foreign’s market becomes more competitive (as there are more firms in Foreign) and so $r_{dk}(\varphi)$ decreases for all $\varphi$. \textbf{This creates pressure for an increase in $\overline{\varphi}_{dk}$ (i.e. only the more productive Home firms can profitably export now).} As $n_k$ decreases, $\theta_{dk}$ also decreases. This leads to the expansion of the sizes of the surviving Home firms. Thus, $r_{dk}(\varphi)$ increases for all $\varphi$. \textbf{This creates pressure for a decrease in $\overline{\varphi}_{dk}$ as some less productive firms which were expected to be unprofitable before can be expected to be profitable now.} In other words, the exporting firms in Home, which are most productive, have to shrink, and so they release resources to the less productive firms. The least productive surviving firms in Home would expand, and the marginal firm that were not profitable before now becomes profitable.\footnote{To see the effect more starkly, consider the case when $L/L^*$ is very large. In this case, in the sectors where counter-Melitz effect occurs, Home has more firms than Foreign does. The market share of Foreign’s firms in these sectors cannot be too high as Foreign’s resource ($L^*$) is too small compared with Home’s resource ($L$). Therefore, a decrease in $n_k$ (as well as $\theta_{dk}$) leads to an increase in the size and revenue of each Home firm that remains. Therefore $r_{dk}(\varphi)$ increases for all $\varphi$.}

2. The Melitz effect (intra-sectoral resource allocation effect as $B$ decreases). It is welfare-improving for Home for all sectors except the Home-dominated sectors and the homogeneous-good sector. Here, we ignore the IRA effect, i.e. suppose that the masses of potential entrants $n_k$ and $n_k^h$ were to remain fixed. In other words, the expected toughness of competition for an exporting firm from both countries is unchanged. As a result, the export revenue of a typical exporting firm will increase as trade cost falls. \textbf{This creates pressure for both $\overline{\varphi}_{sk}$ and $\overline{\varphi}_{dk}$ to decrease.} Meanwhile, this will force the least productive firms in each country to exit (as there are more firms exporting to the domestic market). \textbf{This creates pressure for both $\overline{\varphi}_{dk}$ and $\overline{\varphi}_{dh}$ to increase.} The decrease in prices of imports and the increase in average productivity of Home’s firms raises Home’s welfare. This is the Melitz effect. However, in the sectors in which Home has the strongest comparative disadvantage, the IRA effect counteracts the Melitz effect. This is because the Melitz effect causes trade liberalization to increase Home firms’ advantage in selling to Foreign, whereas the IRA effect causes trade liberalization to increase the disadvantage of Home firms in selling to Foreign (as $n_k$ decreases and $n_k^h$ increases). If the IRA effect dominates, we will have a counter-Melitz effect. This will be the case in the sectors where Home has very strong comparative disadvantage yet still produces. For Foreign, the IRA effect is always positive and so it reinforces the Melitz effect. Therefore, there cannot be counter-Melitz effect for the small country.

When the IRA effect in a sector is negative (i.e. if it is a comparative disadvantage sector) and if it is large enough to dominate the Melitz effect, the sectoral welfare falls upon trade liberalization. Figure 5 shows that this can occur only in the larger country. The fact that there exist sectors in which the larger country (Home) has very strong comparative disadvantage yet still produces is because Home has a large demand for the differentiated goods and so it attracts Home’s firms to produce there to serve the large market while saving the transportation cost. This is the home market effect, which can be
attributed to Krugman (1980).\textsuperscript{19} In such sectors, the IRA effect dominates the Melitz effect, leading to welfare loss in Home in these sectors.\textsuperscript{20}

As the IRA effects in the comparative advantage sectors are positive, there are welfare gains to these sectors upon trade liberalization. Can these gains offset the losses in the comparative disadvantage sectors mentioned above? The answer is, it depends. If Home’s relative size is large, and the Foreign-dominated sector is small, then the gains cannot offset the losses. For example, when $B = 2$, $L/L^* = 6$, $\gamma = 1.05$ (and therefore $a_{k_1} = 0.5$ and $a_{k_2} = 0.7258$), and suppose $a_0 = 0.52$ (and therefore $k_1 < 0$, which means that there does not exist any sector in which Foreign completely dominates). Then, Home will unambiguously lose from trade liberalization, as it loses in the sectors where $k \in [0, k_2]$, and does not gain or lose in the sectors where $k \in [k_2, 1]$ and in the homogeneous good sector.

Therefore, we end this section with the following two testable propositions:

**Proposition 5** Consider the sectors in which both countries produce. For any given country, in the face of trade liberalization, the fraction of exporters increases in the comparative advantage sectors but decreases in the sectors in which the country has the strongest comparative disadvantage, if the country is large compared with the rest of the world.

**Proposition 6** Consider the sectors in which both countries produce. For any given country, in the face of trade liberalization, the fraction of revenue derived from exporting increases in the comparative advantage sectors but decreases in the sectors in which the country has the strongest comparative disadvantage, if the country is large compared with the rest of the world.

### 5.1 Empirical Tests of Propositions 5 and 6

Propositions 5 and 6 predict the existence of a counter-Melitz effect: For a large country, like China, in the sectors where it has the strongest comparative disadvantage but still produces, the fraction of firms that export and the share of revenue derived from exporting will both decrease upon trade liberalization. Can we find any evidence to support the existence of the counter-Melitz effect? This section shows that we indeed find evidence of such an effect.

We test the theory at 4-digit CIC level, using both the NBSC enterprise survey and the Chinese customs data. To get a panel of variables, we need to first tackle the problem caused by a major revision to the CIC classification in the year 2002. In order to have a consistent definition of sectors, we follow Brandt, Van Biesebroeck & Zhang (2011) by adjusting the CIC 4-digit industry type of each firm in the

\textsuperscript{19}The home market effect is clearly demonstrated by the fact that the set of differentiated goods produced by Home increases (i.e. $k_1$ decreases) while the set of differentiated goods produced by Foreign decreases (i.e. $k_2$ decreases) as $L/L^*$ increases. This is shown in Figure 5 by the fact that the $k_1$ curve and $k_2$ curves are both upward sloping in $L/L^*$.

\textsuperscript{20}This home market effect explains why there exist such sectors in the larger country only. The fact that the threshold $a_k$ that demarcates a switching of the dominance of IRA effect versus the Melitz effect is independent of $L/L^*$, together with the fact that the set of goods produced by Home expands beyond this threshold $a_k$ as $L/L^*$ increases, explains why there exist sectors in which the IRA effect dominates the Melitz effect in the large country.
NBSC dataset, to have the same industry type code representing the same industry both before and after year 2002. After adjusting the CIC code for each sector, we aggregate the variables at the 4-digit sector level and obtain a panel of aggregate variables (e.g. mass of firms, mass of firms that export, total revenue, total exporting revenue, etc.) from the years 2000 to 2006, and calculate the variables we need.

In order to test the effect of trade liberalization, we also need to establish a proper measure of trade cost. As transportation cost is hard to measure and should not vary much in a few years’ time, we take the tariff rate, which decreases a lot after China joined the WTO, as the measure of trade cost. It is also noteworthy that the tariff rates for different sectors are different, which is not consistent with the assumption of our model. Fortunately, it turns out that the results and equations of the model will not be qualitatively affected by the heterogeneity of trade costs across sectors. Therefore, we take into consideration the heterogeneity of tariff rates across sectors in calculating which sectors is predicted to exhibit the counter-Melitz effect according to the theory.

We calculate the import volume-weighted tariff rate of all the goods imported by the firms of a sector, and take it as the tariff rate of the sector. To avoid the aggregation bias and potential mismatch of the CIC and HS codes, we merge the NBSC data with the detailed firm level import data for the years 2000-2006 from the Chinese customs authority. We use only the data from the firms which can be matched. For each sector, we take the values of all the goods imported by the firms in this sector and calculate the aggregate import value by summing up the values of imports of all firms in this sector. The tariff is then calculated as

$$\tau_t = \frac{\sum_{g=1}^{G_{it}} v_{gt} \tau_{gt}}{\sum_{g=1}^{G_{it}} v_{gt}},$$

where \(\tau_{gt}\) is the 8-digit HS level tariff of an imported good \(g\) at year \(t\) (the Most-Favored-Nation tariff rate is adopted, as is commonly used in the literature), \(v_{gt}\) is the aggregate import volume of good \(g\) for all firms in sector \(i\) in that year, and \(G_{it}\) is the number of imported inputs by sector \(i\) at year \(t\). Later, we shall use these tariff data to test our theory.

To account for the heterogeneity of tariff rates across sectors, we modify some of our equations. Allowing for different tariff rates in different sectors does not qualitatively affect the results listed in Table 1. The only change needed in Table 1 is to change \(B\) to \(B_k = \tau_k^\gamma \left(\frac{f}{\tau}\right)^{\frac{1}{\gamma}-1}\) and \(\beta\) to \(\beta_k = \tau_k \left(\frac{f}{\tau}\right)^{\frac{1}{\gamma}}\). It follows that Propositions 5 and 6 continue to hold. However, different sectors have different threshold for \(\alpha_k\): \((a_k)^\gamma < \frac{2B_k}{1+B_k^2}\), where \(B_k = \tau_k^\gamma \left(\frac{f}{\tau}\right)^{\frac{1}{\gamma}-1}\). The theory predicts that the counter-Melitz effect will occur in the two-way trade sectors in which the relative productivity \(a_k\) is less than \(\frac{2B_k}{1+B_k^2}\). In section 4.2, it has been shown that the fraction of firms that export in each sector increases with the strength of comparative advantage of the sector. This fraction can be written as

$$\frac{\theta_{xk}}{\theta_{dk}} = \left(\frac{\tau_{dk}}{\tau_{xk}}\right)^\gamma = \frac{1}{B_k} \cdot \frac{B_k (a_k)^\gamma - 1}{B_k - (a_k)^\gamma} \cdot \frac{f}{f_x},$$

21 The detail of the matching can be find at http://www.econ.kuleuven.be/public/N07057/CHINA/appendix/

22 The choice of the years is constrained by the availability of the customs data, which we will use and merge with the NBSC data.
which is less than \( B_k^{-2} f_k \), if and only if \((a_k)^2 < \frac{2B_k^2}{f_k} \). As Axtell (2001) shows that \( \frac{1}{\alpha} \) is close to 1, we have \( B_k \approx \tau_k^\gamma \) and so we use \( \tau_k^\gamma \) to approximate for \( B_k \). We use the average tariff rate in each 2-digit CIC industry to proxy for \( \tau_k \), and take \( \gamma \) to be 7.\(^{23}\) Then for each sector, we calculate the ratio of the fraction of exporting firms and \( B_k^{-2} \), and rank the twenty-nine 2-digit CIC industries according to this ratio, which we call RATIO. Table 2 shows the ranking of these 2-digit CIC sectors and the corresponding sectoral information that determine the RATIO. Note that a lower RATIO is assigned a lower ranking. A higher RATIO implies that China has stronger comparative advantage in that sector. We choose the ten sectors with the lowest RATIOs to test for the counter-Melitz effect and the ten sectors with the highest RATIOs to test for the Melitz effect. We run (i) the fraction of exporting firms and (ii) the share of exporting revenue, on the tariff rate for each sector, while controlling for the year and industry (2-digit CIC level) fixed effects and other relevant variables (including employment, capital-labor ratio, average wage in each 4-digit sector). Amongst these variables, employment stands for the size of the sector, and is used to control for the economies of scale; \( K/L \) = capital-labor ratio is used to control for production technique; average wage is used to control for the variable costs and worker skill. The right hand side variables we choose are similar to those of Bernard, Jensen and Schott (2006, Table 4). The results are shown in Tables 3A and 3B. (CA stands for comparative advantage.)

< Table 2, 3A and 3B about here >

From Tables 3A and 3B, we see that all the coefficients for the variable “Tariff” are significant in both regressions. Most importantly, the signs are distinctly positive for the ten comparative-disadvantage sectors and distinctly negative for the ten comparative-advantage sectors. In other words, both the fraction of exporting firms and the share of exporting revenue decrease (increase) with trade liberalization for the sectors in which China has comparative disadvantage (comparative advantage). Therefore, we conclude that, consistent with our theory, there exists counter-Melitz effect in the sectors where China has the comparative disadvantage but still produces; while the Melitz effect continues to hold in the sectors in which China has comparative advantage. The coefficients of the other right hand side variables make sense too. For example, higher \( K/L \) signifies stronger comparative disadvantage, which lowers exporting ratio and share of export revenue according to our theory. Higher wage signifies higher labor quality, which induces higher propensity to export.\(^{24}\)

\(^{23}\)Axtell (2001) shows that firm revenue distribution in US manufacturing firms can be approximated by a Pareto distribution with shape parameter larger than but close to one. In the context of our model, the shape parameter of the Pareto distribution of firm revenue is equal to \( \gamma/(\sigma - 1) \), where \( \sigma \) is the elasticity of substitution in our model. To be consistent with Axtell, therefore, we assume that \( \gamma \) is approximately equal to \( \sigma - 1 \). In the literature, the estimated value of \( \sigma \) is usually between 5 and 10. In Eaton & Kortum (2002), the estimates for \( \sigma \) are 3.6, 8.28 and 12.86. The values of \( \sigma \) used by Anderson and van Wincoop (2004) are 5, 8 and 10. To be consistent with their findings, we take \( \sigma \) to be the around the median of both sets of results, which is approximately 8 and therefore \( \gamma \) is taken to be about 7 (= \( \sigma - 1 \)). In fact, the results do not change much if we assume \( \gamma \) to be 5, 7, 8 or 10.

\(^{24}\)It is interesting to note that higher employment is correlated with lower export propensity in the comparative disadvantage sectors but higher export propensity in the comparative advantage sectors. The latter result is normal, and is consistent with the literature, such as the work of Bernard, Jensen and Schott (2006). The former result can perhaps be explained by the fact that a higher employment in the comparative disadvantage sector in the face of trade liberalization signifies that some structural inefficiencies were at play (e.g. existence of some non-tariff protectionist measures). This would mean lower competitiveness in the export markets.
Robustness of the Result

The choice of the ten sectors with the lowest RATIOs to stand for comparative disadvantage sectors and the ten sectors with the highest RATIOs to stand for comparative advantage sectors may sound a bit arbitrary. Therefore, we check the robustness of the results by varying the set of sectors we choose. We run six regressions for each set of sectors we choose: we run (i) the fraction of exporting firms and (ii) the share of exporting revenue, on the tariff rate for each set of sectors, while controlling for the year and industry (2-digit CIC level) fixed effects and other relevant variables (including employment, capital-labor ratio, average wage in each 4-digit sector). Each regression is the same as the corresponding one shown in Tables 3A and 3B, though the set of sectors used is different. The result is shown in Figure 6.

<table>
<thead>
<tr>
<th>Regressor</th>
<th>ten sectors with weakest CA</th>
<th>ten sectors with strongest CA</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Tariff</strong></td>
<td>0.667*** (0.070) 0.443*** (0.083) 0.321*** (0.080) -0.460*** (0.052) -0.154*** (0.043) -0.101*** (0.040)</td>
<td><strong>Tariff</strong> 0.443*** (0.083) 0.321*** (0.080) -0.460*** (0.052) -0.154*** (0.043) -0.101*** (0.040)</td>
</tr>
<tr>
<td>log(employment)</td>
<td>-0.011*** (0.003)</td>
<td>0.022*** (0.003)</td>
</tr>
<tr>
<td>log(K/L)</td>
<td>-0.087*** (0.010)</td>
<td>-0.114*** (0.014)</td>
</tr>
<tr>
<td>log(wage)</td>
<td>0.165*** (0.018)</td>
<td>0.177*** (0.025)</td>
</tr>
<tr>
<td>Industry fixed effect</td>
<td>NA Yes Yes NA Yes Yes</td>
<td>Industry fixed effect NA Yes Yes NA Yes Yes</td>
</tr>
<tr>
<td>Year fixed effect</td>
<td>NA Yes Yes NA Yes Yes</td>
<td>Year fixed effect NA Yes Yes NA Yes Yes</td>
</tr>
<tr>
<td>Observations</td>
<td>938 938 938 742 742 742</td>
<td>Observations 938 938 938 742 742 742</td>
</tr>
</tbody>
</table>

Note: ***Significant at the 1% level; **Significant at the 5% level; *Significant at the 10% level.

Table 3A

<table>
<thead>
<tr>
<th>Regressor</th>
<th>ten sectors with weakest CA</th>
<th>ten sectors with strongest CA</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Tariff</strong></td>
<td>0.231*** (0.062) 0.337*** (0.075) 0.232*** (0.073) -0.504*** (0.055) -0.201*** (0.044) -0.153*** (0.042)</td>
<td><strong>Tariff</strong> 0.337*** (0.075) 0.232*** (0.073) -0.504*** (0.055) -0.201*** (0.044) -0.153*** (0.042)</td>
</tr>
<tr>
<td>log(employment)</td>
<td>-0.007** (0.003)</td>
<td>0.013*** (0.004)</td>
</tr>
<tr>
<td>log(K/L)</td>
<td>-0.089*** (0.009)</td>
<td>-0.139*** (0.014)</td>
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<tr>
<td>log(wage)</td>
<td>0.099*** (0.017)</td>
<td>0.072*** (0.026)</td>
</tr>
<tr>
<td>Industry fixed effect</td>
<td>NA Yes Yes NA Yes Yes</td>
<td>Industry fixed effect NA Yes Yes NA Yes Yes</td>
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<tr>
<td>Year fixed effect</td>
<td>NA Yes Yes NA Yes Yes</td>
<td>Year fixed effect NA Yes Yes NA Yes Yes</td>
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<td>Observations</td>
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<td>Observations 938 938 938 742 742 742</td>
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</tbody>
</table>

Table 3B
The horizontal axis in Figure 6 indicates what sectors are included when running the regressions. Note that there is a set of bars on the left and another set of bars on the right. On the left, a number \( x \) on the horizontal axis indicates that sectors from the sector with ranking number 1 to the sector with ranking number \( x \) are included in the regressions. Therefore, as \( x \) increases, the number of comparative disadvantage sectors included in the regressions increase. On the right, a number \( x \) indicates that sectors from the sector with ranking number \( x \) to the sector with ranking number 29 are included in the regression. Therefore, as \( x \) decreases, the number of comparative advantage sectors included in the regressions increase. The vertical axis indicates the number of regressions for which the coefficient for the variable “Tariff” is statistically significant when the corresponding set of sectors indicated on the horizontal axis is included in the regressions. In the figure, the darkest bars represent the number of coefficients with the right sign (positive for the left group of sectors and negative for the right group of sectors) and significant at 1% level. The second darkest bars represent the number of coefficients with the right sign and significant at 5% level (but not significant at 1% level). The lightest bars represent the number of coefficients with the right sign, but not significant at 5% level. From the figure, it is clear that the counter-Melitz effect becomes significant when we include sufficiently large number of sectors with the smallest RATIOs (the figure shows that 5 is a sufficiently large number of sectors), and the effect remains significant till we include the 13 sectors with the smallest RATIOs. Likewise, the

\[ \text{Figure 6.} \]

\[ \text{Robustness of Empirical Results} \]

\[ \text{No. of significant coefficients for “Tariff”} \]

\[ \text{Set of sectors included in regressions} \]

\[ 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29 \]

\[ \text{Positive but not significant} \]

\[ \text{Positive and Significant at 5%} \]

\[ \text{Positive and Significant at 1%} \]

\[ \text{Negative but not significant} \]

\[ \text{Negative and Significant at 5%} \]

\[ \text{Negative and Significant at 1%} \]

\[ 25 \text{For example, a number 9 on the horizontal axis indicates that sectors with rankings 1 through 9 are included in the regressions. Corresponding to that, all six regressions similar to the ones shown in Table 3A and 3B yield positive and significant coefficients (at 1% level) for variable “Tariff”. A number 21 on the horizontal axis indicates that sectors with rankings 21 through 29 are included in the regressions. Corresponding to that, five of the regressions similar to the ones shown in Table 3A and 3B yield negative and significant coefficients (at 1% level) for “Tariff”, while one regression yields negative and significant coefficient at 5% level for “Tariff”.} \]
Melitz effect becomes significant when we include sufficiently large number of sectors with the largest RATIOs (the figure shows that two is a sufficiently large number of sectors), and the effect remains significant till we include the 11 sectors with the highest RATIOs. The coefficients for the sectors at both ends of the ranking are mostly not very significant, probably due to the limited sample size (too few observations).\textsuperscript{26} The coefficients are mostly not very significant when the sectors in the middle of the ranking are included. This is consistent with our theory, as they are sectors at the margin, and neither the Melitz effect nor the counter-Melitz effect dominate. Thus, the total effect is ambiguous.

Table 4 shows the results of the OLS regressions when the data of all sectors are pooled together.

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<tr>
<th>Year 2000-2006</th>
<th>Fraction of exporting firms</th>
<th>Share of exporting revenue in total revenue</th>
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<td>Tariff</td>
<td>-0.038 0.011</td>
<td>-0.097*** -0.046 -0.046</td>
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<td></td>
<td>(0.034) (0.034) (0.032)</td>
<td>(0.037) (0.037) (0.035)</td>
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<tr>
<td>log(employment)</td>
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<td>(0.002) (0.002)</td>
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<td></td>
<td>(0.007)</td>
</tr>
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<td>log(K/L)</td>
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<td>(0.007)</td>
</tr>
<tr>
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<td>(0.007)</td>
<td>(0.013)</td>
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<td>log(wage)</td>
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<td>(0.013)</td>
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<td>(0.013)</td>
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<tr>
<td>Observations</td>
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</tbody>
</table>

Table 4

Note: ***Significant at the 1% level; **Significant at the 5% level; *Significant at the 10% level.

In Table 4, most of the coefficients are not statistically significant and even the signs of the coefficients are ambiguous. This is consistent with our theory, as some sectors exhibit counter-Melitz effect while others exhibit Melitz effect, and therefore the total effect maybe ambiguous and statistically insignificant. This result contrast with that obtained by Bernard, Jensen & Schott (2006). They use plant level data of the U.S. and run a similar regression of the probability of exporting on change in trade cost. They get negative sign (Melitz effect) at 10% level of significance for that regression. The main reason for the difference might be that, unlike China, the U.S. does not have many sectors in which it has strong comparative disadvantage but in which it still produces. As a result, fewer plants in the U.S. would show counter-Melitz effect. Therefore, the overall outcome is dominated by the Melitz effect. Moreover, the coefficient is also not very significant statistically as the Melitz effect is countered by the IRA effect in some sectors.

\textsuperscript{26}Some 2-digit industries contain fewer than ten 4-digit sectors. Thus the degree of freedom of the regression may be limited if we choose too few 2-digit industries for testing our propositions.
6 Conclusion

In this paper, we merge the heterogenous firm model of Melitz (2003) with the Ricardian model of Dornbusch et al (1977) to form a hybrid model to explain how the pattern of international specialization and trade is determined by the interaction of comparative advantage, economies of scale, country sizes and trade barriers. The model is able to capture the existence of inter-industry trade and intra-industry trade in a single unified framework. It explains how trade openness and trade liberalization affects the pattern of international specialization and trade. The paper examines the generality of Melitz’s firm selection effect in response to trade liberalization in a multi-sectoral setting.

For any given country, the model predicts that the fraction of exporting firms as well as the share of export revenue in total revenue increases with the strength of comparative advantage of a sector. Empirical tests based on Chinese manufacturing data for the years 2000-2006 confirm this proposition.

Although opening to trade is welfare-improving in both countries, trade liberalization can lead to a counter-Melitz effect in the larger country if it is insufficiently competitive in the sectors in which it has the strongest comparative disadvantage but in which it still produces. In this case, the productivity cutoff for survival is lowered while the exporting cutoff increases in the face of trade liberalization. This is because the inter-sectoral resource allocation (IRA) effect dominates the Melitz effect in these sectors. In these sectors, trade liberalization leads to decreases in the fractions of exporting firms, contrary to the prediction of Melitz (2003). Empirical evidence in the years 2000-2006 confirms that the fraction of exporting firms as well as the share of export revenue in total revenue both decreased in some Chinese comparative-disadvantage sectors in the face of trade liberalization, consistent with our hypothesis.
Appendixes

A Solving for the System

In this appendix, we will show how to solve the model for the sectors where both countries produce. In other words, we solve for \((\bar{\tau}_{dk}, \bar{\tau}_{dk}^\ast, \bar{\varphi}_{xk}, \bar{\varphi}_{xk}^\ast, \theta_{dk}, \theta_{dk}^\ast)\) from the system constituted of the four zero cutoff profit conditions and two free entry conditions. Combining the two zero cutoff conditions for firms serving the Home market, (11) and (14), we have

\[
\frac{\bar{\tau}_{xk}}{\bar{\varphi}_{dk}} = \alpha_k \beta
\]  

(26)

Similarly, combining those for firms serving Foreign’s market, (12) and (13), we can get

\[
\frac{\bar{\tau}_{xk}}{\bar{\varphi}_{dk}} = \frac{\beta}{\alpha_k}
\]  

(27)

where we recall that \(\beta = \tau \left( \frac{L}{f} \right)^{-1} \).

Equations (26), (27), and the FE conditions (15), and (16) now form a system of four equations and four unknowns, \(\bar{\tau}_{dk}, \bar{\varphi}_{xk}, \bar{\varphi}_{dk}^\ast\) and \(\bar{\varphi}_{xk}^\ast\). Solving, we obtain (17), (18), (19) and (20).

Then recall that the aggregate price indexes are given by

\[
P = \varphi_1 \varphi^{-1} (\phi_1 \phi^{-1})_{\bar{\varphi}_{dk}}
\]  

and

\[
P^\ast = \varphi_1^\ast \varphi^{-1} (\phi_1^\ast \phi^{-1})_{\bar{\varphi}_{dk}^\ast}
\]  

Substituting these price indexes into Zero Cutoff Conditions (11) and (12), and with the help of equation (9) and (10), we have

\[
\sigma_f = b_k L \left( \frac{\bar{\tau}_{dk}}{\bar{\varphi}_{k}} \right)^{\sigma-1} = \left( \frac{\gamma - \sigma + 1}{\gamma} \right) \cdot \frac{b_k L}{\theta_{dk} + \theta_{dk}^\ast \frac{L}{f}}
\]  

(28)

\[
\sigma_f^\ast = b_k L^\ast \left( \frac{\bar{\tau}_{dk}^\ast}{\bar{\varphi}_{k}} \right)^{\sigma-1} = \left( \frac{\gamma - \sigma + 1}{\gamma} \right) \cdot \frac{b_k L^\ast}{\theta_{dk}^\ast + \theta_{dk}^\ast \frac{L}{f}}
\]  

(29)

From the equilibrium productivity cutoffs (17) and (18) in both countries, we get

\[
\left( \frac{\bar{\tau}_{dk}}{\bar{\varphi}_{dk}} \right)^\gamma = \frac{B - (a_k)\gamma}{B - (a_k)}
\]  

(30)

Therefore, the number of exporting firms in Home and Foreign are respectively:

\[
\theta_{xk} = \left( \frac{\bar{\tau}_{dk}}{\bar{\varphi}_{xk}} \right)^\gamma \theta_{dk} = \left( \frac{a_k}{\beta} \frac{\bar{\tau}_{dk}}{\bar{\varphi}_{dk}} \right)^\gamma \theta_{dk}
\]  

(31)

\[
\theta_{xk}^\ast = \left( \frac{\bar{\tau}_{dk}^\ast}{\bar{\varphi}_{xk}} \right)^\gamma \theta_{dk}^\ast = \left( \frac{1}{\alpha_k \beta} \frac{\bar{\tau}_{dk}^\ast}{\bar{\varphi}_{dk}^\ast} \right)^\gamma \theta_{dk}^\ast
\]  

(32)

Equations (28), (29), (30), (31), (32) then imply (21) and (22).

\(\theta_{xk}\) and \(\theta_{xk}^\ast\) can be obtained by substituting (30), (21), (22) into (31) and (32) respectively.
B The Rationale for $\tau^{\sigma-1}f_{x} > f$

In this appendix, we explain why we need the assumption $\tau^{\sigma-1}f_{x} > f$. If $B > 1$, we restrict $(a_{k})^\gamma$ to be within $(B^{-1}, B)$, in order to avoid corner solution. To make sure that there exist some sector $k$ in which only the firms with the higher productivity will export (i.e. $\varphi_{dk} < \varphi_{zk}$ for some $k$ and $\varphi_{dk} < \varphi_{zk}$ for some $k$), we need $(a_{k})^\gamma$ to lie in the interval $\left(\frac{(\delta_{f}+1)B}{L^{2}}B^{2+1}, \frac{(\delta_{f}+1)B}{L^{2}}\right)$ for some $k$. For this to be true, we need $\tau^{\sigma-1}f_{x} > f$. For the case $B < 1$, in order to make $\varphi_{dk} < \varphi_{zk}$ for some $k$ and $\varphi_{dk} < \varphi_{zk}$ for some $k$, we also need $\frac{(\delta_{f}+1)B}{L^{2}} > \frac{L^{2}B^{2+1}}{(\delta_{f}+1)B}$, which also implies that $\tau^{\sigma-1}f_{x} > f$ following similar argument.

Hence, we assume that $\tau^{\sigma-1}f_{x} > f$ in order to guarantee that in both countries there exist some sectors in which some firms produce exclusively for their domestic market in both countries.

C Welfare Impact of Trade Liberalization

In this appendix, we will prove how the real wage change after trade liberalization in three cases. Without loss of generality, we assume that $L > L^\ast$.

1. **Foreign-dominated sectors:** $k \in (0, k_1)$. The real wage in terms of aggregate goods of sector $k$ in this zone in Home and Foreign are, respectively:

$$\frac{1}{P_k} = (\theta^\ast_{xk})^\frac{1}{\gamma-1} \rho A^\ast_{k} \varphi^\ast_{zk} \frac{1}{r} = \rho A^\ast_{k} B^{-\gamma} \left(\frac{L + L^\ast}{L} D_1\right)^\frac{1}{\gamma} \left(\frac{\gamma}{\sigma + 1} D_2 L^\ast\right)^\frac{1}{\gamma-1}$$

$$\frac{1}{P_k} = (\theta^\ast_{dk})^\frac{1}{\gamma-1} \rho A^\ast_{k} \varphi^\ast_{zk} = \rho A^\ast_{k} \left(\frac{L + L^\ast}{L} D_1\right)^\frac{1}{\gamma} \left(\frac{\gamma}{\sigma + 1} D_2 L^\ast\right)^\frac{1}{\gamma-1}$$

Since trade liberalization will increase $\frac{1}{P_k}$ as $B$ falls, the real wage in Home will be improved. However, the real wage in Foreign, $\frac{1}{P_k}$, is not related to the trade barriers. That’s, trade liberalization does not affect the real wage in Foreign.

2. **Both countries produce:** $k \in (k_1, k_2)$. The real wage in Home and Foreign are equal to:

$$\frac{1}{P_k} = (\theta^\ast_{ck})^\frac{1}{\gamma-1} \rho A^\ast_k \varphi^\ast_{dk} = \rho A^\ast_k \left(D_1 \frac{B - B^{-1}}{B - (a_k)^\gamma} \right)^\frac{1}{\gamma} \left(\frac{\gamma}{\sigma + 1} D_2 L^\ast\right)^\frac{1}{\gamma-1}$$

$$\frac{1}{P_k} = (\theta^\ast_{ck})^\frac{1}{\gamma-1} \rho A^\ast_k \varphi^\ast_{dk} = \rho A^\ast_k \left(D_1 \frac{B - B^{-1}}{B - (a_k)^\gamma} \right)^\frac{1}{\gamma} \left(\frac{\gamma}{\sigma + 1} D_2 L^\ast\right)^\frac{1}{\gamma-1}$$

This zone is divided into two cases:

(a) Scenario A: $(a_k)^\gamma < \frac{2B}{1+B^2}$.

Note that $\frac{B - B^{-1}}{B - (a_k)^\gamma}$ decreases but $\frac{B - B^{-1}}{B - (a_k)^\gamma}$ increases as trade barrier $B$ falls, as $(a_k)^\gamma < \frac{2B}{1+B^2}$. Therefore, the real wage in Home will decline, but the real wage in Foreign rises.
(b) Scenario B: \((a_k)^\gamma \in \left( \frac{2B}{1+B^2}, \frac{1+B^2}{2B} \right)\).

Since both \(\frac{B-B^{-1}}{B-(a_k)^\gamma}\) and \(\frac{B-B^{-1}}{B-(a_k)^\gamma}\) increase as trade barrier \(B\) falls when \((a_k)^\gamma \in \left( \frac{2B}{1+B^2}, \frac{1+B^2}{2B} \right)\), the real wages in both countries increase in this zone.

3. Home-dominated sectors: \(k \in (k_2, 1)\). Real wages are given by

\[
\frac{1}{P_k} = (\theta_{dk})^{\frac{1}{\gamma-1}} \rho A_k \hat{\varphi}_{dk} = \rho A_k \left( \frac{L+L^*}{L} \right)^{\frac{1}{\gamma}} \left( \frac{\gamma}{\gamma-1} D_2 L \right)^{\frac{1}{\sigma-1}}
\]

\[
\frac{1}{P^*_k} = (\theta_{zk})^{\frac{1}{\gamma-1}} \rho A_k \hat{\varphi}_{zk} \frac{1}{\tau} = \rho A_k B^{-\frac{1}{\gamma}} \left( \frac{L+L^*}{L^*} \right)^{\frac{1}{\gamma}} \left( \frac{\gamma}{\gamma-1} D_2 L^* \right)^{\frac{1}{\sigma-1}}
\]

It is clear that real wage in Home is unchanged but that in Foreign increases as \(B\) falls.
References


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<th>2-digit CIC Sector</th>
<th>Ranking</th>
<th>RATIO</th>
<th>Exporting Ratio</th>
<th>$\tau_k-1$</th>
<th>Revenue$^\dagger$</th>
<th>Exporting Revenue$^\dagger$</th>
<th>Number of firms</th>
<th>Employment</th>
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</tr>
<tr>
<td>Food Processing</td>
<td>29</td>
<td>29.627</td>
<td>0.1327888</td>
<td>0.4714715</td>
<td>3.75E+08</td>
<td>4.09E+07</td>
<td>10897</td>
<td>1744098</td>
</tr>
</tbody>
</table>

$^\dagger$ In thousand yuan

Table 2. Ranking of Chinese sectors according to RATIO (strength of comparative advantage) for the year 2000