The Effect of Ambiguity Aversion on Risk Reduction and Insurance Demand*

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Abstract

In this paper, we show that ambiguity aversion always raises the demand for self-insurance but may well decrease the demand for self-protection. We also show that ambiguity aversion raises the optimal insurance coverage. Finally, we characterize the optimal insurance design under ambiguity aversion. We exhibit a case in which the straight deductible contract is optimal.

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1 Introduction

How does the ambiguity about the probability of the possible consequences of our acts drive our decisions? Almost all models used in insurance economics up to now have implicitly assumed that ambiguity does not matter for decisions by using the Savagian subjective expected utility. This approach is problematic because of the empirical evidence showing that individuals do not behave in that way (see, e.g., Camerer and Weber, 1992). The most famous observation illustrating the violation of the subjective expected utility theory is the Ellsberg (1961)’s paradox. It is by now fairly well recognized that people exhibit ambiguity aversion, which can be thought as an aversion to any mean-preserving spread in the space of probabilities. For example, ambiguity-averse agents prefer the lottery that yields a gain of 100 with a probability 1/2 to another lottery in which the probability of earning 100 is uncertain, but with a subjective mean of 1/2 (for example if the agent believes that the probability of earning 100 is either 1/4 or 3/4 with equal probability).1 This psychological trait differs from risk aversion, which is an aversion to any mean-preserving spread in the payoffs of the lottery. Since the seminal work by Gilboa and Schmeidler (1989), decision theorists have proposed various decision models that exhibit a form of ambiguity aversion.

In this paper, we examine the effect of ambiguity aversion on optimal risk exposure decisions and on the valuation of risk reduction. In particular, we derive interpretable sufficient conditions to sign the effect of ambiguity aversion on self-protection, self-insurance and insurance coverage decisions. We also generalize the optimality of a deductible insurance in a model with ambiguity-averse preferences. We consider the theory of ambiguity as axiomatized by Klibanoff, Marinacci and Mukerji (2005). This theory captures well the idea that mean-preserving spreads in probabilities reduce the welfare of ambiguity averse agents. Also, this theory permits to separate the effect of ambiguity aversion from that of risk aversion. Therefore our results permit to examine whether the effect of ambiguity aversion on insurance demand differs from that of risk aversion.

There has been several papers studying the effect of risk aversion on insurance demand in subjective expected utility models. A well-known result is that risk aversion increases the demand for insurance, i.e. it raises the coverage rate, and it reduces the straight deductible. Indeed, in the case

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1See, e.g., Halevy (2007) for recent experimental evidence.
of coinsurance, this result is a direct consequence of the well-known Pratt’s result that risk aversion decreases the demand for the risky asset. Similarly, it can be shown that risk aversion always increases the demand for self-insurance (Ehrlich and Becker, 1971) and increases the demand for a deductible (Schlesinger, 1981). In contrast, the effect of risk aversion on self-protection is not clear (Dionne and Eeckhoudt, 1985). An activity of self-protection consists in investing to reduce the probability of an accident. In fact, no general result can be obtained to sign the effect of risk aversion on the intensity of self-protection (Sweeney and Beard, 1992): to sign the comparative statics analysis of risk aversion on self-protection, it is necessary to specify the value of the probability of loss and to make further assumptions on the utility function (Dachraoui et al., 2004; Eeckhoudt and Gollier, 2005).

Intuitively, ambiguity aversion reinforces risk aversion. Under this intuitive view, ambiguity aversion should raise the demand for insurance, and it should have an ambiguous effect on self-protection. Following Gollier (2009), we show that ambiguity aversion is not equivalent to an increase in risk aversion. For example, Gollier shows that ambiguity aversion may raise the demand for an ambiguous risky asset in a simple one-risky-one risk-free-asset portfolio model. In general, the effect of ambiguity aversion on the optimal decision is complex, but strongly connected to the comparative statics analysis of the effect of risk. Gollier (1995) summarizes the main findings on this comparative statics analysis in the case of a portfolio choice problem, which is further examined in Abel (2002). Strong restrictions on the utility function are required to guarantee that ambiguity aversion reduces the optimal risk exposure.

In this paper, we are mostly interested in uncertain contexts with only two states of nature, loss or no-loss. This simplified structure of uncertainty allow us to get much simpler results to sign the effect of ambiguity aversion. Our results are driven by the observation that the behavioral effect of ambiguity aversion is as if an expected utility maximizer would use a pessimistic distribution of his beliefs when he acts. That is, ambiguity aversion raises the perceived probability of loss. It happens that the comparative statics of a perceived increase in the probability of loss is simpler than the one of more risk aversion within a subjective expected utility model. Namely, an increase in the perceived probability of loss raises the willingness-to-pay for self-insurance, and it reduces the willingness-to-pay for self-protection. It also raises the optimal insurance coverage rate.

In the last section of the paper, we also examine the problem of the opti-
nal insurance design with more than two states of nature. In particular, we examine the robustness of the celebrated Arrow (1971) result to the introduction of ambiguity aversion. We focus our analysis to the case in which the ambiguity is limited to the probability of occurrence of an accident, but the distribution of loss given an accident is unambiguous. We show that the straight deductible contract, which is optimal under subjective expected utility, remains optimal under ambiguity aversion. The only effect of ambiguity aversion is to reduce the level of the optimal deductible.

2 Risk Reduction

2.1 Preliminary results and intuitions

We first focus on the willingness to pay (WTP) for self-protection and self-insurance. We consider two extreme cases, a risk elimination and an infinitesimal risk reduction. These cases allow us to understand how ambiguity aversion affects the basic trade-offs induced by a costly risk reduction. Interestingly the effects of ambiguity aversion on self-protection and on self-insurance have opposite signs. In the next subsection, we will then study the effect of ambiguity aversion in a more general model combining both self-insurance and self-protection efforts.

We assume that the decision-maker may loose \( L > 0 \) with probability \( p(\theta) \). Self-protection corresponds to a reduction in the probability \( p(\theta) \), and self-insurance corresponds to a reduction in the loss \( L \). The probability \( p(\theta) \) is ambiguous in the sense that it depends upon an unknown parameter \( \theta \). The ambiguity takes the form of a probability distribution for \( \theta \). For the sake of simplicity, we consider a discrete support \( \{1, \ldots, n\} \) for the random variable \( \theta \). Let \( q(\theta) \) denote the subjective probability that the true value of the parameter be \( \theta \), with \( \sum_{\theta=1}^{n} q(\theta) = 1 \).

Suppose that the true value of \( \theta \) is known, and let \( U(\theta) \) denote the expected utility reached for that specific \( \theta \). It is defined as follows:

\[
U(\theta) = (1 - p(\theta))u(w) + p(\theta)u(w - L)
\]

in which \( w \) is initial wealth, and \( u \) is the vNM utility function. We say that the agent is ambiguity-neutral if he evaluates his welfare ex ante by the expected value \( E_\theta U(\theta) \), i.e., if he uses expected utility with the mean probability of loss \( E_\theta p(\theta) \). This agent is indifferent to any mean-preserving
spread in the probability of loss. In accordance to the resolution of the Ellsberg paradox, let us assume alternatively that the agent dislikes mean-preserving spread in the probability of loss, i.e., that he is ambiguity-averse. Following the work by Klibanoff, Marinacci and Mukerji (2005), let us assume that the agent evaluates his welfare ex ante by the certainty equivalent of the random variable $U(\tilde{\theta})$ by using an increasing and concave valuation function $\Phi$. The concavity of $\Phi$ expresses ambiguity aversion, i.e., an aversion to mean-preserving spreads in the random probability of loss $p(\tilde{\theta})$. The ex ante welfare is equal to

$$\Phi^{-1}(E_{\tilde{\theta}}\Phi(U(\tilde{\theta}))) = \Phi^{-1}\left(\sum_{\theta=1}^{n} q(\theta)\Phi(U(\theta))\right).$$

We first study the WTP for risk elimination under ambiguity aversion. Risk elimination can be viewed as an ultimate form of either self-protection (where the final probability of loss equals zero) or self-insurance (where the final loss equals zero). The WTP $P$ under ambiguity aversion for risk elimination is then defined by:

$$u(w - P) = \Phi^{-1}(E_{\tilde{\theta}}\Phi(U(\tilde{\theta}))).$$

The special case of a linear $\Phi$ function corresponds to ambiguity-neutrality. In that case, the WTP for the elimination of risk is denoted $P_0$, which is defined by $u(w - P_0) = E_{\tilde{\theta}}U(\tilde{\theta})$. Observe that under $\Phi$ concave, namely under ambiguity aversion, we have

$$\Phi(u(w - P)) = E_{\tilde{\theta}}\Phi(U(\tilde{\theta})) \\
\leq \Phi(E_{\tilde{\theta}}U(\tilde{\theta})) = \Phi(u(w - P_0))$$

Therefore $P_0$ is always less than $P$ under $\Phi$ concave. In other words, ambiguity aversion always raises the WTP for risk elimination. The intuition is that eliminating the risk also eliminates all the ambiguity associated with the risk. Therefore this is no surprise that ambiguity averse agents are willing to pay an extra premium for risk elimination.

Next we consider the effect of an infinitesimal risk reduction on self-protection and self-insurance. We start by examining the case of self-protection. Let $P(\varepsilon)$ denote the WTP for a reduction $\varepsilon$ in the ambiguous probability of loss:

$$E_{\tilde{\theta}}\Phi\left([1 - p(\tilde{\theta}) + \varepsilon]u(w - P(\varepsilon)) + (p(\tilde{\theta}) - \varepsilon)u(w - L - P(\varepsilon))\right) = E_{\tilde{\theta}}\Phi(U(\tilde{\theta}))$$

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We selected a model in which the preventive effort reduces the probability of accident by the same absolute amount in all states $\theta$. This means that the preventive effort affects the risk but has no impact on the degree of ambiguity. Straightforward computations lead to

$$P'(0) = \frac{(u(w) - u(w - L))E_\theta\Phi'[U(\tilde{\theta})]}{E_\theta\left(((1 - p(\theta))u'(w) + p(\theta)u'(w - L))\Phi'[U(\tilde{\theta})]\right)}$$  \hspace{1cm} (2)$$

This can be rewritten as follows:

$$P'(0) = \frac{(u(w) - u(w - L))}{(1 - \tilde{p})u'(w) + \tilde{p}u'(w - L)}$$ \hspace{1cm} (3)$$
in which $\tilde{p}$ is the distorted probability of accident that is defined as:

$$\tilde{p} = \frac{E_\theta p(\tilde{\theta})\Phi'(U(\tilde{\theta}))}{E_\theta\Phi'(U(\tilde{\theta}))}.$$  \hspace{1cm} (4)$$

We see from equation (3) that the WTP is the ratio of the utility difference in the two states and of the ambiguity-distorted expected marginal utility. The effect of ambiguity aversion is to transform the probability of accident $p = E_\theta p(\tilde{\theta})$ into a distorted probability $\tilde{p}$. We hereafter demonstrate that this ambiguity-distorted probability of accident is larger than the ambiguity-neutral one: $\tilde{p} \geq p$. Suppose without loss of generality that $p(\theta)$ is increasing in $\theta$. It implies from (1) that $U(\theta)$ is decreasing in $\theta$. Under ambiguity aversion, $\Phi'$ is decreasing, which implies that $\Phi'(U(\tilde{\theta}))$ is increasing in $\theta$. By the covariance rule, it implies that the numerator of in equation (4) is larger than $E_\theta p(\tilde{\theta}) E_\theta\Phi'(U(\tilde{\theta}))$. It implies that

$$\tilde{p} \geq E_\theta p(\tilde{\theta}) = p.$$  \hspace{1cm} (5)$$

The effect of this increased perceived probability of accident raises the ambiguity-distorted expected marginal utility, since $u'(w)$ is smaller than $u'(w - L)$ in the denominator of (3). The increased expected marginal utility reduces the willingness to invest in preventive actions. Thus, it reduces the WTP for self-protection. It is noteworthy that this effect disappears under risk neutrality. Note also that one can sign the effect of ambiguity aversion on self-protection, which is in contrast to the impossibility result concerning the effect of risk aversion on self-protection.
We now study the effect of ambiguity aversion on the willingness to pay for an infinitesimal loss reduction, that is we examine the case of self-insurance. Let $I(\varepsilon)$ the WTP for a reduction $\varepsilon$ in the loss:

$$E_\theta \Phi((1 - p(\theta))u(w - I(\varepsilon)) + p(\theta)u(w - L + \varepsilon - I(\varepsilon))) = E_\theta \Phi(U(\tilde{\theta}))$$

Straightforward computations lead to

$$I'(0) = \frac{u'(w - L)E_\theta p(\theta)\Phi'(U(\tilde{\theta}))}{E_\theta((1 - p(\theta))u'(w) + p(\theta)u'(w - L))\Phi'(U(\tilde{\theta}))}$$

$$= \frac{\hat{\theta}u'(w - L)}{(1 - \hat{\theta})u'(w) + \hat{\theta}u'(w - L)}$$

Ambiguity aversion raises the WTP compared to ambiguity neutrality if and only if

$$I'(0) \geq \frac{u'(w - L)\overline{\theta}}{(1 - \overline{\theta})u'(w) + \overline{\theta}u'(w - L)}.$$ 

This is the case if and only if $\hat{\theta} \geq \overline{\theta}$. We have shown above that this inequality always holds true since $\Phi'$ and $U$ are both decreasing in $\theta$. Therefore, ambiguity aversion always raises the WTP for infinitesimal self-insurance. The intuition is that ambiguity aversion raises the perceived probability of accident, which in turn raises the expected marginal cost of spending money under risk aversion (as for self-protection), but also raises the marginal benefit of self-insurance. The first effect is negative, and the second effect is positive. The result tells us that the second effect always dominates.

We can summarize our finding in the following proposition.

**Proposition 1** Suppose that the decision-maker is risk-averse. Then, ambiguity aversion has the following effects:

- It reduces the willingness to pay for a marginal investment in self-protection;
- It raises the willingness to pay for a marginal investment in self-insurance.

These effects are reversed under ambiguity-loving ($\Phi$ convex). Observe ambiguity aversion has contradictory effects for self-protection and self-insurance. Nevertheless, remember that the effects of ambiguity aversion for both self-protection and self-insurance go the same direction for a risk elimination.
2.2 Self-protection and self-insurance efforts

In this section we examine the effect of ambiguity aversion in a model that combines self-protection and self-insurance efforts, coined under the term prevention efforts. This model is more general but we will retrieve some of the effects of ambiguity aversion on self-protection and self-insurance introduced in the previous section.

The model is the following. Let denote

\[ U(e, \theta) = (1 - p(e, \theta))u(w - e) + p(e, \theta)u(w - L(e) - e) \]

the expected utility where the probability of loss is equal to \( p(e, \theta) \) in which \( e \) is the prevention effort. We assume that the probability is differentiable with respect to \( e \) and \( \theta \), with \( p_e(e, \theta) < 0 \) and \( p_\theta(e, \theta) > 0 \) (where subscripts denote derivatives). This implies

\[ U_\theta(e, \theta) = -p_\theta(e, \theta)(u(w - e) - u(w - L(e) - e)) < 0 \]

Notice that the assumption \( p_\theta(e, \theta) > 0 \) inducing \( U_\theta < 0 \) is no more innocuous; it means that the probabilities \( p(e, \theta) \) can be ranked for all \( e \).

The objective of the decision-maker is

\[ \max_e \Phi^{-1}[E_\theta \Phi[U(e, \tilde{\theta})]] \]

and the first order condition therefore equals

\[ E_\theta U_e(e, \tilde{\theta})\Phi'[U(e, \tilde{\theta})] = 0 \]

For simplicity, we assume that the second order condition is always satisfied. We want to compare the optimal prevention effort under ambiguity aversion to the one under ambiguity neutrality, which is characterized by

\[ E_\theta U_e(e, \tilde{\theta}) = 0 \]

Using standard comparative statics techniques, ambiguity aversion therefore raises the level of prevention if and only if

\[ \text{Cov}_\theta(U_e(e, \tilde{\theta}), \Phi'[U(e, \tilde{\theta})]) \geq 0 \]

namely, since \( \Phi' \) is decreasing under ambiguity aversion, if and only if \( U_e(e, \theta) \) and \( U(e, \theta) \) covary negatively in \( \theta \). Since \( U(e, \theta) \) is decreasing in \( \theta \) by assumption, ambiguity aversion raises prevention efforts if and only if \( U_e(e, \theta) \) varies
positively with $\theta$. We obtain

$$U_{\theta\theta}(e, \theta) = p_{\theta}(e, \theta)(u'(w - e) - u'(w - L(e) - e)) - p_{\theta e}(e, \theta)(u(w - e) - u(w - L(e) - e)) - p_{\theta}(e, \theta)u'(w - L(e) - e)L'(e)$$

Interestingly, this cross-derivative represents the three effects we have presented in the previous subsection.

First, the term $p_{\theta}(e, \theta)(u'(w - e) - u'(w - L(e) - e))$ is always negative under risk aversion. This term indicates that ambiguity aversion increases the marginal cost of prevention, which in turn gives a downward pressure on prevention efforts.

The second term $-p_{\theta e}(e, \theta)(u(w - e) - u(w - L(e) - e))$ represents the effect of ambiguity aversion on the marginal benefit of self-protection. The sign of this term is not clear since it depends on the sign of $p_{\theta e}(e, \theta)$. This reflects that the effect of ambiguity aversion on the marginal benefit of self-protection is positive when the efficiency of self-protection efforts is relatively high for bad utility levels (i.e., for high values of $\theta$), that is for $p_{\theta e} < 0$. When $p_{\theta e}(e, \theta) = 0$ there is no effect of ambiguity aversion on self-protection, and ambiguity aversion therefore always reduces self-protection efforts due to its positive effect on the marginal cost. This result is similar to the result obtained in the previous subsection.

The last term $-p_{\theta}(e, \theta)u'(w - L(e) - e)L'(e)$ represents the effect of ambiguity aversion on the marginal benefit of self-insurance. Under $L' < 0$, this term is always positive and under $L' < -1$ this effect is higher than the first (negative) term in absolute value. This reflects that the overall effect of ambiguity aversion on self-insurance is positive, an effect similar to the one obtained in the previous section. To conclude, we summarize the above observations into the following proposition.

**Proposition 2** The marginal cost of self-protection and of self-insurance always increases under ambiguity aversion. The marginal benefit of self-protection is positive if and only if $p_{\theta e} \leq 0$, and the overall effect of ambiguity aversion on self-protection is indeterminate in general. The marginal benefit of self-insurance always increases under ambiguity aversion; moreover, under $L' \leq -1$, this positive effect is higher than the positive effect of ambiguity aversion on the marginal cost of self-insurance so that ambiguity aversion always raises self-insurance.
3 Insurance Coverage

3.1 Proportional coinsurance

In this section, we consider an ambiguity-averse agent who can purchase a coinsurance contract from a risk neutral and ambiguity neutral insurer. Thus the indemnity function is denoted by $I(L) = \alpha L$ where $\alpha$ is the coverage rate. The premium is a function of $\alpha$, $P(\alpha) = (1 + \tau)E_{\tilde{\theta}}p(\tilde{\theta})\alpha L = \alpha(1 + \tau)P_f$ where $P_f = \overline{P}L$ is the full insurance premium and $\tau$ the loading factor.

The optimal coinsurance rate is then such that:

$$\max_{\alpha} V(\alpha) = E_{\tilde{\theta}}\Phi(U(\alpha; \tilde{\theta})) = E_{\tilde{\theta}}\Phi\left(p(\tilde{\theta})u(w_l(\alpha)) + (1 - p(\tilde{\theta}))u(w_{nl}(\alpha))\right),$$

in which $w_l(\alpha) = w - (1 - \alpha)L - P(\alpha)$ is consumption in the loss state and $w_{nl}(\alpha) = w - P(\alpha)$ is consumption in the no-loss state. It is easy to see that $V'(1) < 0$ if and only if $\tau > 0$, implying that the famous Mossin (1968)’s result is preserved under ambiguity aversion. That is, if the price of insurance includes a strictly positive loading factor $\tau > 0$, then partial insurance $\alpha < 1$ is optimal.

Under ambiguity-neutrality, the first order condition of this problem can be written as

$$E_{\tilde{\theta}}U_\alpha(\alpha; \tilde{\theta}) = 0$$

where

$$U_\alpha(\alpha; \theta) = p(\theta)Lu'(w_l(\alpha)) - (1 + \tau)P_f [p(\theta)u'(w_l(\alpha)) + (1 - p(\theta))u'(w_{nl}(\alpha))].$$

$U_\alpha(\alpha; \theta)$ is the difference between the utility gain in the loss state due to the increased indemnity and the utility loss generated by the corresponding increase in the insurance premium. Because the objective function in (6) is concave in $\alpha$, condition (7) is necessary and sufficient. This condition is rewritten as

$$\overline{p}Lu'(w_l(\alpha^*)) - (1 + \tau)P_f [\overline{p}u'(w_l(\alpha^*)) + (1 - \overline{p})u'(w_{nl}(\alpha^*))] = 0. \quad (8)$$

Let $\alpha^*$ denote the optimal coinsurance rate of the ambiguity-neutral agent. Notice that as $\alpha^*$ is less than unity under a positive loading factor, we have $w_l(\alpha^*) \leq w_{nl}(\alpha^*)$.\footnote{We are aware that this assumption is too strong. Nevertheless, we let the problem of risk sharing between ambiguity averse individuals for further research.}
Let us now consider the case of ambiguity aversion. Observe first that the objective function (6) is concave in $\alpha$. In order to determine whether ambiguity aversion raises the optimal coverage rate, it is thus enough to verify that the derivative of the objective function in (6) with respect to $\alpha$ and evaluated at $\alpha^*$ is positive:

$$E_{\tilde{\theta}} \Phi'(U(\alpha^*; \tilde{\theta})) U_\alpha(\alpha^*; \tilde{\theta}) \geq 0$$

One can use the same distortion in the vector of probabilities as in the previous sections, to rewrite the last inequality as

$$\hat{p} L u'(w_l(\alpha^*)) - (1 + \tau) P_f \left[ \hat{p} u'(w_l(\alpha^*)) + (1 - \hat{p}) u'(w_{nl}(\alpha^*)) \right] \geq 0, \quad (9)$$

where $\hat{p}$ is the distorted probability of accident

$$\hat{p} = \frac{E_{\tilde{\theta}} p(\tilde{\theta}) \Phi'(U(\alpha^*, \tilde{\theta}))}{E_{\tilde{\theta}} \Phi'(U(\alpha^*, \tilde{\theta}))}. \quad (10)$$

Notice that (10) is similar to (4), except that $\hat{p}$ now depends on the optimal insurance coverage under ambiguity neutrality. Using the same methodology as in section 2.1, we know that $\hat{p}$ is larger than $\bar{p}$; ambiguity aversion raises the perceived probability of accident. It is then intuitive that the optimal coverage rate is increased by ambiguity aversion. This is formally proved by checking that inequality (9) holds. This inequality can be rewritten as

$$\hat{p} \left[ L u'(w_l(\alpha^*)) + (1 + \tau) P_f \{ u'(w_{nl}(\alpha^*)) - u'(w_l(\alpha^*)) \} \right] \geq (1 + \tau) P_f u'(w_{nl}(\alpha^*)). \quad (11)$$

But by (8), we know that

$$\bar{p} \left[ L u'(w_l(\alpha^*)) + (1 + \tau) P_f \{ u'(w_{nl}(\alpha^*)) - u'(w_l(\alpha^*)) \} \right] = (1 + \tau) P_f u'(w_{nl}(\alpha^*)).$$

This implies that condition (11) can be rewritten as $\hat{p} \geq \bar{p}$, which holds under ambiguity aversion. This concludes the proof of the following proposition.

**Proposition 3** Consider the standard coinsurance problem with two states of nature. Ambiguity aversion always raises the insurance coverage rate.

We can infer from Gollier (2009) that this result cannot be generalized to more than two states of nature.
3.2 Optimal insurance contract

In this section, we explore the problem of the optimal insurance contract design when there are more than one possible loss level. Under ambiguity-neutrality, we know that the coinsurance contract is not optimal. As first proved by Arrow (1971), it is dominated by a contract with a straight deductible. As is intuitive, the deductible contract optimizes the risk transfer to the insurer for any insurance budget level, since it provides indemnities where the marginal utility of wealth is the largest. We are interested here in determining whether this Arrow’s result is robust to the introduction of ambiguity aversion.

In this paper, we limit the analysis to the case in which the ambiguity is only about the probability of loss. Conditional to the occurrence of a loss, the cumulative distribution function of the loss \( L \) is unambiguous. Let \( p(\theta) \) denote the ambiguous loss probability. An insurance contract stipulates an indemnity \( I(L) \) for each possible loss level \( L \). To any such indemnity schedule, there is an insurance premium which is proportional to the actuarial value of the policy:

\[
P = \overline{p}(1 + \tau)E_{\tilde{L}}I(\tilde{L}),
\]

where as before \( \overline{p} = E_{\theta}p(\tilde{\theta}) \) is the expected probability of accident and \( \tau \) is the loading factor. Conditional to \( \theta \), the policyholder’s expected utility is written as

\[
U(\theta) = (1 - p(\theta))u(w - P) + p(\theta)E_u(w - L + I(L) - P).
\]

The ambiguity-averse policyholder selects the indemnity schedule that maximizes its ex ante welfare which is measured by \( E_{\theta}\Phi(U(\tilde{\theta})) \). We first prove the following proposition.

**Proposition 4** Suppose that the policyholder is ambiguity-averse and that, conditional to the occurrence of a loss, the probability distribution of the loss is unambiguous. The only source of ambiguity is about the probability of the occurrence of a loss. Under this condition, the optimal insurance contract contains a straight deductible \( d \): \( I(L) = \max(0, L - d) \).

Proof: Suppose by contradiction that the contract \( I_0 \) that maximizes \( E_{\theta}\Phi(U(\tilde{\theta})) \) is not a straight deductible contract. It yields an insurance premium \( P_0 = \overline{p}(1 + \tau)E_{\tilde{L}}I_0(\tilde{L}) \). Let \( d_0 \) denote the deductible which yields the
same insurance premium than $P_0$:

$$p(1 + \tau)E_{\hat{L}}(\max(0, \tilde{L} - d_0)) = P_0.$$ 

But we know from Arrow (1971) that this alternative contract with a straight deductible $d_0$ dominates any other contractual form as $I_0$:

$$E_{\tilde{L}}u(w - \min(\tilde{L}, d_0) - P_0) \geq E_{\hat{L}}u(w - \hat{L} + I_0(\hat{L}) - P_0).$$

It implies that for all $\theta$, we have

$$(1 - p(\theta))u(w - P_0) + p(\theta)E_{\tilde{L}}u(w - \min(\tilde{L}, d_0) - P_0)$$

$$\geq (1 - p(\theta))u(w - P_0) + p(\theta)E_{\hat{L}}u(w - \hat{L} + I_0(\hat{L}) - P_0).$$

Because the expected utility conditional to $\theta$ is larger with the straight deductible $d_0$ than with contract $I_0$ for all $\theta$, the former necessarily yields a larger ex ante welfare $E_{\tilde{\theta}}\Phi(U(\tilde{\theta}))$. This is a contradiction.

When the ambiguity is only about the probability of occurrence of a loss, the Arrow’s result is robust to the introduction of ambiguity aversion. We now examine the impact of ambiguity aversion on the optimal deductible. The previous section implicitly solved this question in the special case of only one possible loss, since in that case a proportional coinsurance or a straight deductible are formally equivalent. At least in that special case, the previous section can be reinterpreted as ambiguity aversion having a negative impact on the optimal deductible. We hereafter examine whether this result can be generalized to more than one possible loss level.

The decision problem can thus be rewritten as follows:

$$\max_d E_{\tilde{\theta}}\Phi(U(d; \tilde{\theta})), \quad (12)$$

with

$$U(d; \theta) = (1 - p(\theta))u(w - P(d)) + p(\theta)E_{\tilde{L}}u(w - \min(\tilde{L}, d) - P(d))$$

and $P(d) = p(1 + \tau)E_{\tilde{L}} \max(0, \tilde{L} - d)$. Let $\tilde{d}$ denote the optimal deductible under ambiguity-neutrality, and let $\overline{P} = P(\overline{d})$ denote the corresponding insurance premium. The first-order condition for $\tilde{d}$ can be written as

$$- \frac{\partial P}{\partial d} \bigg|_{d=\overline{d}} \left[ (1 - p)u'(w - \overline{P}) + pE_{\tilde{L}}u'(w - \min(\tilde{L}, \overline{d}) - \overline{P}) \right] = p(1 - F(\overline{d}))u'(w - \overline{d} - \overline{P}). \quad (13)$$

where $F$ is the cumulative distribution function of $L$. Because the objective function in (12) is concave in $d$, the optimal deductible under ambiguity aversion

aversion is smaller than $\overline{d}$ if the derivative of this objective function with respect to $d$ evaluated at $d = \overline{d}$ is negative. This is true if

\[- \frac{\partial P}{\partial d} \bigg|_{d=\overline{d}} \left[ (1 - \hat{p})u'(w - \overline{P}) + \hat{p}E_L u'(w - \min(\overline{L}, \overline{d}) - \overline{P}) \right] \leq \hat{p}(1 - F(\overline{d})) u'(w - \overline{d} - \overline{P}), \]

where $\hat{p}$ is the usual distorted loss probability defined by

$$\hat{p} = \frac{E_\theta \Phi(\overline{U}(\overline{d}; \theta)) p(\overline{\theta})}{E_\theta \Phi(\overline{U}(\overline{d}; \overline{\theta}))}.$$  

Because $U(\overline{d}; \overline{\theta})$ and $p(\overline{\theta})$ are negatively correlated and because $\Phi'$ is decreasing, we have that $\hat{p}$ is larger than $E_\theta p(\overline{\theta}) = \overline{p}$, under ambiguity aversion. Observe now that, using (13), one can rewrite (14) as follows:

$$\frac{(1 - \hat{p})u'(w - \overline{P}) + \hat{p}E_L u'(w - \min(\overline{L}, \overline{d}) - \overline{P})}{(1 - \overline{p})u'(w - \overline{P}) + \overline{p}E_L u'(w - \min(\overline{L}, \overline{d}) - \overline{P})} \leq \frac{\hat{p}}{\overline{p}},$$

or, equivalently, $\overline{p} \leq \hat{p}$. As just explained above, this inequality always holds under ambiguity aversion. This concludes the proof of the following proposition.

**Proposition 5** Under the condition of Proposition 4, the optimal deductible is reduced by ambiguity aversion.
References


