Sterilized Interventions and Capital Controls

Nalini Prasad*

University of Wisconsin-Madison

Abstract

Sterilized interventions in the presence of capital controls can be welfare improving for small open economies. These policies are incorporated into a standard small open economy model. When purchasing foreign bonds, households generally do not take into account the effect of their bond purchases on relative prices, such as the real exchange rate and terms of trade. In contrast, with sterilized interventions and capital controls, policymakers determine an economy’s overall level of bond holdings and explicitly take into account the effect of foreign bond purchases on relative prices. This allows policymakers to influence the real exchange rate and terms of trade and moderate movements in household labor supply; labor supply and output volatility decline. This is welfare increasing. However, capital controls imply that households face an intertemporal distortion, as their expected marginal utility of consumption is not equated across time with respect to external prices. Consumption potentially becomes more volatile. The results indicate that when the exchange rate is floating, welfare under sterilization and capital controls is increasing in the elasticity of labor supply, the elasticity of substitution between domestic and foreign goods and the share of imports in consumption.

Keywords: Small open economy, sterilization, capital controls

JEL codes: F32, F41

*I would like to thank Charles Engel, Javier Bianchi, Kamran Bilir, Menzie Chinn, Martin Fukac, Christian Gillitzer, Jun Nie and Kenneth West for helpful comments. I would also like to thank seminar participants at the Federal Reserve Bank of Kansas City, the Midwest Macroeconomics Meetings in Boulder, Colorado, the Board of the Governors of the Federal Reserve System, the University of Wisconsin-Madison, the University of Wisconsin-Whitewater and the Reserve Bank of Australia. I also wish to acknowledge the hospitality of the Federal Reserve Bank of Kansas City. Contact details: Address: William H Sewell Social Science Building, 1180 Observatory Dr Rm 7222, Madison, WI 53706. Phone: +1 608 262 3559. Email: nprasad@wisc.edu
1 Introduction

All central banks have access to sterilized interventions as a policy tool. Sterilized interventions refer to central bank interventions in foreign asset markets to influence a nation’s exchange rate without also influencing inflation.\(^1\) Surveys such as Adler and Tovari (2011) and Disyatat and Galati (2007), show that sterilized interventions are frequently used by central banks in emerging market economies. The literature also suggests that sterilized interventions tend to be more effective when a country also has capital controls in place. For example, Reinhart and Reinhart (1998) argue that a number of emerging market economies have complemented their sterilized interventions with capital controls in order to more effectively influence their exchange rates by gaining greater control over domestic interest rates (see also Adler and Tovari (2011) and Lavigne (2008)).\(^2\) While perhaps the most prominent current example of a country adopting sterilized interventions in the presence of capital controls is China, within the past two decades a number of other emerging market economies have also adopted these policies. These include Argentina, Brazil, Chile, Colombia, Indonesia, Malaysia, Korea, Peru and Thailand. Given the prevalence of sterilized intervention policies in conjunction with capital controls across numerous countries, the purpose of this paper is to provide a framework to look at the effects of these policies. In particular, this paper is interested in whether welfare can be improved in economies that implement these policies.

One reason why sterilized interventions are frequently used, is because it is a policy tool readily available to central banks; it is at the discretion of the central bank when to intervene in foreign asset markets and central banks can also decide by how much they wish to intervene by. An example that demonstrates the frequency and size of central bank interventions is provided by Brazil (Figure 1).\(^3\) The Central Bank of Brazil has frequently intervened in foreign asset markets; intervening in three out of every four months during the 2000s. The size of these interventions were also non-trivial and varied over time. These interventions accounted for around one percent of GDP on average on an annual basis and peaked at around six percent of GDP. Brazil also adopted capital controls designed to limit foreign purchases of domestic assets during part of this sample period. Figure 1 also plots the real effective exchange rate for Brazil. Comparing interventions with exchange rate movements suggests that the Central Bank of Brazil intervened in foreign asset markets to moderate movements in the exchange rate. The central bank tended to purchase foreign assets (which puts pressure on the exchange rate to depreciate) when the exchange rate was appreciating and sell foreign assets (which puts pressure on the exchange rate to appreciate) when the exchange rate was depreciating. This is consistent with the literature on sterilized interventions which finds that the motivation for central bank intervention is to reduce exchange rate volatility and to lean

---

\(^1\)&nbsp;Sterilized interventions can perhaps be best understood with an example. Say a country decides to depreciate its currency. The government would then sell its currency and purchase a foreign currency denominated asset. By taking currency out of the central bank and placing it in circulation the government increases the money base and puts upward pressure on inflation. However by undertaking sterilization, the government can unwind the effect of its exchange rate operation on the money base. In this example, it would sell bonds to domestic households to reduce inflationary pressures.

\(^2\)&nbsp;Engel (2013) contains a review.

\(^3\)&nbsp;The Asian central banks also intervened heavily in foreign exchange markets during the 2000s, consistent with the large increase in foreign exchange reserves seen in these countries. A number of these economies also implemented restrictions on capital flows during this period. However, none of the Asian central banks provide data of their foreign exchange interventions on a regular basis.
against movements, particularly appreciations, in the exchange rate (see Adler and Tovari (2011), Neely (2008) and Moreno (2005)).\(^4\)

Consistent with the data, the model outlined in this paper shows that it can be optimal for policymakers to undertake sterilized interventions to moderate movements (both appreciations and depreciations) in the exchange rate. In particular, sterilization policies in conjunction with capital market restrictions can provide policymakers with an additional policy tool. In the models studied here, policymakers will be able to influence interest rates, as is typically the case in these models. In addition, they will also be able to influence a nation’s real exchange rate and terms of trade through the foreign bonds they acquire in their sterilized interventions. Capital controls allow sterilization policies to have more bite, as it prevents private agents from undoing the effects of the central bank’s intervention. Essentially these policies break the link between foreign and domestic interest rates, providing the government with an extra policy lever.

Having access to this additional policy instrument is useful for dealing with the external distortions that result from shocks in open economies. As emphasized by Obstfeld and Rogoff (1995) and Corsetti and Pesenti (2001), a shock in an open economy, while leading to changes in domestic production, can also lead to an external distortion by changing household’s international purchasing power, through altering the terms of trade and real exchange rate. In a standard small open economy model, households make decisions about foreign bond purchases in order to smooth their consumption across time. When making decisions about bond holdings, households take prices as given and do not internalize the effects of their actions on the prices faced by others. This leads to an externality as foreign bond purchases will tend to depreciate the real exchange rate, increase import prices and reduce the terms of trade. As noted by Corsetti et al. (2011), when households do not consider the price effects of their actions, there is scope for policymakers to improve upon the competitive equilibrium by taking into account these price effects.

When a central bank undertakes sterilized interventions, it influences a country’s net foreign asset position through its choice of foreign bond holdings. From the current account identity, if policymakers are determining a nation’s net foreign asset position, this is also equivalent to them picking a country’s trade balance. The real exchange rate and terms of trade need to be consistent with this trade balance. Therefore one can think of policymakers choice of foreign bond holdings as being the same as policymakers choosing the real exchange rate and terms of trade. In turn, changes in these relative prices will affect household consumption and labor supply decisions.

This paper incorporates sterilized interventions in the presence of capital controls into a standard small open economy model. In the absence of these policies, households will supply too much, or too little labor in response to shocks. With sterilized interventions, policymakers can, through their choice of foreign bond holdings, moderate relative price movements. This allows them to engineer a shift in consumption towards (or away from) imports. By altering demand for imports policymakers can reduce volatility in household labor supply and production. This is beneficial to households. For example, by making imports cheaper policymakers can induce a relative fall in labor supply. Conversely more expensive imports encourage

\(^4\)Statements from the Central Bank of Brazil have indicated that interventions are designed to reduce exchange rate volatility. See for example discussions in the financial press such as The Wall Street Journal (http://blogs.wsj.com/economics/2013/10/31/should-brazils-central-bank-continue-currency-intervention/).
households to supply more labor. However, sterilization policies can lead to an intertemporal distortion in consumption. Since bond holding decisions are made by policymakers on behalf of households, rather than by households directly, consumption decisions no longer take into account movements in the foreign interest rate and the exchange rate. As a result consumption can become more volatile.

The results from this paper suggest that sterilized intervention and capital control policies are welfare improving; the gain to households from lower labor supply and output volatility offsets potential losses from increased consumption volatility. This is illustrated by considering two shocks: a temporary shock to technology (a shock common in the literature), and a temporary shock to the foreign interest rate (as a proxy for the fall in US interest rates seen in recent years).

In response to a temporary positive shock to technology, households increase their holdings of foreign bonds in the absence of sterilization and capital control policies in order to smooth the increase in their incomes. This leads to a decrease in the terms of trade and a depreciation of the real exchange rate, reducing the purchasing power of home country residents. Households reduce their consumption of imports as imports become more expensive. Labor supply increases as the real wage rises from the positive productivity shock. With sterilization, policymakers choose an economy’s holdings of foreign bonds on behalf of households, taking into account the effect of bond purchases on relative prices. Policymakers choose to purchase fewer foreign bonds. This allows policymakers to moderate the depreciation of the real exchange rate and the fall in the terms of trade. The more muted depreciation in the real exchange rate encourages households to shift consumption towards imports which are now cheaper. This is welfare improving, as imports allow households to reduce their labor supply, without a corresponding decrease their consumption.

In the absence of sterilization and capital controls, a negative foreign interest rate shock encourages households to borrow from overseas. Capital starts flowing into the small open economy. This leads to an appreciation in the real exchange rate and increase in the terms of trade. An exchange rate appreciation and increased borrowing enables households to reduce their labor supply. With sterilized interventions, policymakers choose to borrow less than what households would choose in the absence of these policies. The real exchange rate depreciates relative to the no intervention case. By borrowing less, policymakers moderate the decline in household labor supply enabling a greater production of exports to take advantage of the more depreciated exchange rate.

Sensitivity analysis indicates that the welfare gain under sterilization policies is generally higher in economies characterized by a higher labor supply elasticity. This is because policymakers can more easily induce a change in labor hours and reduce labor supply volatility when labor supply is more elastic. When the exchange rate is floating, welfare is higher under sterilization, the higher the elasticity of substitution between domestic and foreign goods and the greater the share of imports in consumption. Here households are more willing to alter the composition of their consumption in response to price changes engineered by policymakers. The reverse holds when the exchange rate is fixed. This is because it is harder to maintain a pegged exchange rate when the import share or the elasticity of substitution between imports and domestic goods is high; a larger movement in relative prices can lead to large shifts in consumption and labor supply but this can be inconsistent with the exchange rate remaining fixed. Finally, the gain
from sterilized interventions dissipate when home and foreign goods are perfect substitutes and when the
demand for exports is perfectly elastic. Here policymakers no longer have the ability to influence relative
prices and as a result can no longer reduce production and labor supply volatility.

Literature Review

A number of authors have looked at how shocks in open economies can influence a nation’s purchasing
power (see Benigno and Benigno (2003), De Paoli (2009), Faia and Monacelli (2004) and Tille (2001)).
These papers have tended to focus on how monetary policy can affect a nation’s terms of trade. Similarly,
Corsetti et al. (2010) show how optimal monetary policy in an open economy needs to address both
output gaps and inflation as well as movements in the real exchange rate, the terms of trade and current
account balances. Others have focused on how monetary policy choices and the terms of trade are affected
by different exchange rate regimes (see for example, Devereux and Engel (2003) and Corsetti (2006)).
Sterilized interventions, however, allow policymakers to more directly influence relative prices through
picking an economy’s bond holdings. Additionally, capital controls allow policymakers to move interest
rates away from that implied by uncovered interest parity to encourage (or discourage) consumption.

There is very little work which models sterilized interventions in a general equilibrium setting. Exceptions
are Benes et al. (2013) and Kumhof (2010), who emphasize the effects of sterilized interventions through
the portfolio balance channel. In contrast, this paper looks at how sterilized interventions affects a nation’s
international purchasing power.

There is a growing literature on capital controls, with a number of authors looking at how capital controls
can be used to manipulate intertemporal prices. Costinot et al. (2011) look at how capital taxes can
be used to manipulate the interest rate to encourage consumption in periods when the marginal utility
of consumption is high. Farhi and Werning (2012) primarily focus on fixed exchange rate regimes and
show that capital controls tend to be more effective when employed against temporary shocks. Similarly,
Schmitt-Grohe and Uribe (2012) show that capital taxes can improve welfare in a small open economy
characterized by downward nominal wage rigidity and an exchange rate peg. Jeanne (2012) shows how
capital controls can allow a country to manipulate its exchange rate. This paper is closely related to
De Paoli and Lipinska (2013), who using a two country framework find that restricting capital flows
through taxes lowers international risk sharing but can benefit countries by reducing output fluctuations.
A key difference between these papers and this one is that previous work has focused on capital taxes.
This paper uses sterilized interventions. Although capital taxes and sterilized interventions can lead to
similar outcomes, there are differences in the ability of policymakers to implement each of these policies.
The imposition of capital taxes often requires legislation and institutions need to be set up to administer
and collect capital taxes. In contrast, as mentioned earlier, sterilized interventions is a policy tool that is
readily available to and frequently used by central banks. Central banks can more easily vary the amount
of intervention they undertake, while in practice it may be more difficult to frequently change tax rates.

Another focus in the literature has been on whether or not restricting capital flows can be effective during
times of crises when collateral constraints bind (see Benigno et al. (2012), Bianchi and Mendoza (2010),
Policymakers may wish to impose restrictions on capital flows to prevent over borrowing. Despite the model here having quite a different setup to these papers, some of the results are qualitatively similar; there is an incentive for policymakers to moderate the amount an economy borrows or lends in international financial markets.

This paper is also related to the large empirical literature on sterilized interventions. The empirical literature highlights the large number of countries that have adopted these policies. Sterilization policies have been evaluated in terms of the extent to which capital flows can be sterilized and the exchange rate influenced. The results from these studies have been mixed and have generally focused on the very short-run effects of interventions (see Dominguez and Frankel (1993) and Sarno and Taylor (2001) for a review). However, as noted above, there is evidence that sterilization policies are more effective at influencing the exchange rate when the capital account is closed. Similarly capital control policies were generally judged to be effective if they either reduced capital flows, gave the government greater monetary policy autonomy or changed the composition of capital flows towards those with longer maturities. The results of the empirical studies have been varied (see for example Edwards (2007), Magud et al. (2011) and Reinhart and Montiel (1999)). In contrast to these studies, this paper uses welfare to measure the effectiveness of capital control and sterilization policies.

The rest of this paper is organized as follows. The model with (and without) sterilization is laid out in Section 2. Section 3 highlights the decisions of policymakers in the presence and absence of sterilized interventions. Solution techniques and the model’s parameter values are outlined in Section 4, as are the model’s results. Sensitivity analysis is undertaken in Section 5. Section 6 looks at the case of a fixed exchange rate and Section 7 concludes.

2 Model

There is a small open economy populated by infinitely lived households and monopolistically competitive firms. There exists a foreign and domestic currency denominated bond. In the baseline model, households have access to the foreign (and domestic) bond and the exchange rate is freely floating. This model is then modified to incorporate a situation in which the government undertakes sterilized interventions. Here policymakers impose capital controls and determine foreign bond holdings for the economy so as to maximize household welfare. The government then sterilizes its foreign bond purchases (or sales) and in doing so passes its bond holdings onto households in the form of a domestic bond.

Note that the capital market does not need to be completely closed for sterilized intervention policies to work. Households could be allowed to trade foreign bonds, but they must face some constraint on how many foreign bonds they hold. This is discussed in more detail in Section 2.2.
2.1 Baseline model

Households

Households consume a single homogenous good, $C_t$, which is made up of a domestically produced good, $C^H_t$, and an imported good, $C^F_t$:

$$C_t = \left[ (1 - \omega)^{\frac{1}{\sigma}} (C^H_t)^{\frac{\sigma-1}{\sigma}} + \omega^{\frac{1}{\sigma}} (C^F_t)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} $$

(2.1)

where $\omega$ refers to the share of foreign goods in final consumption and $\eta$ refers to the elasticity of substitution between domestic and foreign goods. Let $P^H_t$ be the price of the domestically produced good and $P^F_t$ the price of the foreign good. The price of the final consumption good, $P_t$, is given by:

$$P_t = \left[ (1 - \omega)(P^H_t)^{1-\eta} + \omega(P^F_t)^{1-\eta} \right]^{\frac{1}{1-\eta}} $$

(2.2)

Households gain utility from consumption and leisure. Income is derived from previous purchases of the internationally traded bonds, $B^F_t$, a domestic currency denominated bond, $B^H_t$, and labor supply $N_t$. It is assumed that the domestic currency bond is in zero net supply amongst home households. Household’s maximize the expected value of their lifetime utility:

$$E_t \sum_{j=0}^{\infty} \beta^{t+j} \left( C_{t+j}^{1-\sigma} - \frac{N_{t+j}^{1+\phi}}{1 + \phi} \right) $$

(2.3)

The household’s budget constraint is given by:

$$P_tC_t + S_t B^F_t + B^H_t = S_t \Phi_{t-1} R^F_{t-1} B^F_{t-1} + R^H_{t-1} B^H_{t-1} + W_t N_t + D_t $$

(2.4)

where $S_t$ is the exchange rate, $W_t$ is the wage rate, $D_t$ refers to profits and lump sum taxes, $R^H_t$ is the domestic interest rate and $\Phi_t$ represents a premium on foreign asset returns above the foreign risk free rate, $R^F_{t-1}$. This premium is increasing in foreign borrowings, specifically:

$$\Phi_t = \exp(-\phi_B (\frac{S_t B^F_t}{P^H_t} - \frac{SB^F_t}{P^H_T})) $$

(2.5)

where $\frac{SB^F_t}{P^H_T}$ is the steady state level of real foreign bond purchases. Following Schmitt-Grohe and Uribe (2003), this premium exists so that the model can be solved using standard linearization techniques around a stationary steady state.

The household’s first order conditions are:

$$\frac{1}{P_tC_t} = \beta E_t \frac{S_{t+1} \Phi_t R^F_{t+1}}{S_t P_{t+1} C_{t+1}^{\sigma}} $$

(2.6)
\[
\frac{1}{P_t C_t^\sigma} = \beta E_t \frac{R_t^H}{P_{t+1} C_{t+1}^\sigma}
\]  
(2.7)

\[
\frac{W_t}{P_t} = C_t^\sigma N_t^\phi
\]  
(2.8)

Trade in the internationally traded bond, leads to the following modified uncovered interest parity condition:

\[
R_t^H E_t \left( \frac{1}{P_{t+1} C_{t+1}^\sigma} \right) = \Phi_t R_{t+1}^{F^*} E_t \left( \frac{S_{t+1}}{P_{t+1} C_{t+1}^\sigma} \right)
\]  
(2.9)

This condition implies that movements in the domestic interest rate are tied to movements in the foreign interest rate via expected movements in the exchange rate.

Firms

There are two types of domestic firms in the model: firms that produce intermediate goods and firms that combine the intermediate goods into a domestic consumption good. An imported good is produced by a foreign firm.

Final good firms

Domestic final good firms costlessly assemble a continuum of intermediate goods, \( Y_{it}^H \), indexed by \( i \in [0, 1] \), into a single final consumption good \( Y_t^H \). Individual intermediate goods are assembled according to a CES technology: \( Y_t^H = (\int_0^1 Y_{it}^H \left( \frac{1}{\varepsilon} - di \right)^{1-\varepsilon})^{\frac{1}{1-\varepsilon}} \). Demand for an individual intermediate good is given by:

\[
Y_{it}^H = Y_t^H \left( \frac{P_{it}^H}{P_t^H} \right)^{-\varepsilon}
\]  
(2.10)

where \( P_{it}^H = (\int_0^1 (P_{it}^H)^{1-\varepsilon} di)^{\frac{1}{1-\varepsilon}} \) is the price of the final domestically produced consumption good and \( P_{it}^H \) is the price of intermediate good \( i \).

Intermediate good firms

Each intermediate goods firm has access to a constant returns to scale technology given by:

\[
Y_{it}^H = A_t N_{it}
\]  
(2.11)

where \( A_t \) refers to total factor productivity. The (log) of total factor productivity is assumed to follow an AR(1) process.
Prices are set in a Calvo fashion. Each period a fraction $\theta$ of firms cannot change their prices and maintain prices such that $P^H_{it} = P^H_{it-1}$. The remaining $1 - \theta$ firms can reset their prices optimally to $\tilde{P}_{it}$.

The producers problem is to maximize discounted profits:

$$\max_{\tilde{P}_{it}} \mathbb{E}_t \sum_{j=0}^{\infty} (\beta \theta)^j v_{t+j} (\tilde{P}_{it} Y^H_{it+j} - P^H_{t+j} M_{t+j} Y^H_{it+j})$$

subject to demand (equation (2.10)), where $v_t = \frac{U'(C_t)}{P^H_t}$ is part of the household’s stochastic discount factor and $M_t = \frac{W_t}{A_t P^H_t}$ is the firm’s real marginal cost of production.

Aggregating over firms, the price of the domestically produced good is a weighted average of prices of firms that can optimally reset their prices and those that cannot:

$$P^H_t = \left[(1 - \theta)(\tilde{P}_t)^{1-\varepsilon} + \theta(P^H_{t-1})^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$

(2.12)

**Foreign Firms**

The foreign component of the consumption good is produced by a competitive firm that converts foreign homogenous output one for one into $C^F_t$. The foreign firm produces at the point where the price it receives for its good is equal to the marginal cost of producing that good: $P^F_t = S_t P^F_*$, where $P^F_*$ is the foreign currency price of the foreign good.

**The external sector**

Some of the domestically produced good is exported to the rest of the world. It is assumed that foreign demand for exports, $X_t$, is given by:

$$X_t = \bar{X} \left(\frac{P^X_t}{P^F_*}\right)^{-\eta_f} Y^F_*(-1)$$

(2.13)

where $P^X_t$ refers to the foreign country price of exports, $Y^F_*$ is foreign country output, $\bar{X}$ is a constant and $\eta_f$ is the elasticity of demand for exports. Exports are produced by a competitive firm that converts the domestic homogenous good one for one into exports. They produce at the point where price is equal to marginal cost: $S_t P^X_t = P^H_t$.

The balance of payments of the small open economy is given by:

$$S_t B^F_t + P^F_t C^F_t = S_t \Phi_{t-1} R^F_*(-1) B^F_{t-1} + S_t P^X_t X_t$$

(2.14)

The left-hand-side of equation (2.14) represents outflows in the form of foreign bond purchases and payments for imports. The right-hand-side represents inflows from export sales and the maturity payments related to foreign bond purchases. This represents the budget constraint for the economy.
Finally define the terms of trade as \( T_t \equiv \frac{P_t^X}{P_t^F} \), and the real exchange rate as \( Q_t \equiv \frac{S_t}{P_t} \).

**Equilibrium**

Clearing in the domestic goods market implies:

\[ Y_t^H = C_t^H + X_t \tag{2.15} \]

The small open economy system of equilibrium conditions is closed by specifying a rule for monetary policy.

To keep the model as simple as possible, it is assumed that foreign output, inflation and interest rates all follow an AR(1) process.

### 2.2 Sterilization and capital controls - The modified model

The above model is now modified to incorporate sterilization and capital controls. Capital controls mean that households no longer have direct access to the foreign currency bond. Rather, the central bank determines bond holdings on behalf of households. It is assumed that foreign bond holdings are chosen optimally.

As part of its sterilized interventions, the central bank now trades domestic currency bonds with home households. It sterilizes its purchases (or sales) of foreign bonds through selling (or purchasing) domestic currency bonds to (from) households. Essentially sterilization implies that the money supply is unchanged. While there is no money in the model per se, money shows up as a liability on the central bank’s balance sheet. A zero change in the money supply can be thought of as operations that leave the liabilities of the central bank unchanged. Consider the case where the central bank purchases foreign bonds, \( S_tB_t^F \) in period \( t \). The central bank also receives interest from its purchases of foreign bonds in the previous period, \( S_t\Phi_{t-1}R_{t-1}^{F*}B_{t-1}^F \) and pays interest to domestic households who purchased domestic bonds sold by the central bank in the previous period, \( R_{t-1}B_{t-1}^H \), where the interest rate on these bonds is determined by policymakers. It is assumed that central bank profits, \( Z_t = S_t\Phi_{t-1}(R_{t-1}^{F*} - 1)B_{t-1}^F - (R_{t-1}^H - 1)B_{t-1}^H \), are remitted to households in a lump fashion at the end of each period.\(^6\) Let the central bank sell \( B_t^H \) this period, so as to keep its liabilities unchanged. Then:

\[ B_t^H = S_tB_t^F - S_tR_{t-1}^{F*}B_{t-1}^F + R_{t-1}B_{t-1}^H + Z_t \tag{2.16} \]

The household holds this \( B_t^H \) which is determined by the central bank’s purchases of foreign bonds in a given period. The household’s budget constraint is given by:

\(^6\)A number of central banks remit any profits they make back to the government on a regular basis. This includes the Central Bank of Brazil.
\[ P_tC_t + B_t^H = W_t N_t + R_{t-1} B_{t-1}^H + D_t + Z_t \]  
(2.17)

where the first order condition with respect to \( B_t^H \) is given by:

\[ \frac{1}{P_tC_t^\sigma} = \beta E_t \frac{R_t}{P_{t+1}C_{t+1}^\sigma} \]  
(2.18)

Since households can only trade domestic currency bonds, the modified uncovered interest parity condition (given by equation (2.9)) no longer holds. This implies that changes in the domestic interest rate are no longer tied to movements in the foreign interest rate.

Combining the central bank and household budget constraint (equations (2.16) and (2.17)) leads to an economy-wide budget constraint:

\[ S_t B_t^F - S_t R_{t-1}^F B_{t-1}^F = W_t N_t + D_t - P_tC_t \]  
(2.19)

This constraint is the same as the balance of payments condition from the baseline model (given by equation (2.14)). However, previously households were choosing foreign bond holdings, now the central bank is choosing bond holdings for the economy as a whole.

In this modified model, policymakers determine the economy’s foreign bond holdings. Through its sterilization operations, the central bank sells domestic bonds to households to leave its balance sheet unchanged. The interest rate on this domestic bond is determined by a rule set by the government. This in turn determines consumption through the Euler equation (2.18). Therefore consumption and bond holdings are pinned down in the budget constraint (equation (2.17)), leaving prices and output to adjust to bring about equilibrium.

The model with sterilized interventions and capital controls assumes that households cannot directly hold foreign currency bonds. Although, this assumption might seem restrictive, the results from the model would still hold if households were allowed to purchase foreign currency bonds but faced a binding constraint on how many bonds they could purchase. This is because what matters for policymaker’s ability to influence relative prices is aggregate bond holdings (rather than what is held by households and the central bank individually). If households faced a constraint on their foreign bond purchases then policymakers could add to or unwind household’s bond purchases to achieve the desirable level of bond holdings for the economy as a whole. The assumption that households face a constraint on their foreign bond purchases does not seem unreasonable, especially given that capital controls tend to be more prevalent in emerging market economies (see Chinn and Ito (2008)). In other words, the countries that commonly use sterilized interventions also tend to be the ones that impose restrictions on capital flows. In addition, the empirical literature suggests that sterilized interventions are more effective when a nation has capital account restrictions (see Adler and Tovari (2011), Disyatat and Galati (2007) and Engel (2013)). Appendix C outlines a version of the model in which households face a constraint on how many foreign
bonds they can hold. Appendix C also shows how a model in which households face a binding constraint on how many foreign bonds they can hold is equivalent to the modified model outlined above. To keep the results from the model as clear as possible, in the main body of the text it is assumed that capital controls stop households from holding foreign bonds. This prevents having to aggregate household’s and policymaker’s foreign bond holding decisions.

3 Policy in the Baseline and Modified Model

Policy in both the baseline and modified models are set optimally a la Ramsey, assuming commitment.\textsuperscript{7} In the baseline model, policymakers only have access to the interest rate to maximize household welfare. In the modified model, policymakers have access to another policy instrument. Policymakers are also able to determine an economy’s aggregate bond holdings by undertaking sterilized interventions. Note that in all the analysis that follows, the steady state distortion to labor that arises from monopolistic competition has been eliminated through an appropriate subsidy to firms.

Both the baseline and modified model involve two important decisions: how many foreign bonds should be purchased and how should interest rates be set? While these decisions happen simultaneously, it is perhaps easiest to think about them individually.

3.1 The choice of foreign bond holdings

In the baseline model, foreign bond holdings are determined by households according to the Euler equation with respect to foreign bonds (equation (2.6)). Here households choose foreign bond holdings so as to equate their expected marginal utility of consumption across time, taking into account the foreign interest rate and expected future movements in the exchange rate. When determining bond holdings, households take prices as given. As will be shown below, the purchase or sale of foreign bonds will effect relative prices such as the terms of trade and real exchange rate. Households, however do not internalize these price effects. This point has been emphasized by Corsetti et al. (2011). The authors argue that policymakers can potentially improve welfare by internalizing the effect of foreign bond purchases on relative prices. This is essentially what occurs in the modified model.

Under the modified model, policymakers choose the path of foreign bond holdings that maximize household utility subject to the equilibrium conditions outlined in Section 2.2. By taking into account these constraints policymakers are directly incorporating how foreign bond holdings affect relative prices and how these prices in turn affect household labor supply and consumption decisions. Indeed under the modified model, one can think of the government’s sterilization policy as picking prices, such as the real exchange rate and terms of trade, through their choice of foreign bond holdings. The next proposition states this.

\textsuperscript{7}This paper makes use of the algorithm developed by Levin et al. (2006) to compute the Ramsey planner’s first order conditions.
**Proposition 1** When policymakers determine foreign bond holdings under capital controls and sterilization, this is equivalent to policymakers also determining the trade balance.

**Proof** Rewriting the balance of payments equation under the modified model, equation (2.19) becomes:

\[
B_t^F - \Phi_{t-1} R^F_{t-1} B_{t-1}^F = P_t^X X_t - \frac{P_t^F C_t^F}{S_t}
\]

(3.1)

The left hand side of equation (3.1) represents flows related to foreign bond purchases or sales, while the right hand side of this equation represents the trade balance. Hence by simple accounting, picking bond holdings, \(B_t^F\) today, is also equivalent to policymakers determining the trade balance.

The real exchange rate and terms of trade have to be consistent with this trade balance. The next proposition states the relationship between foreign bond holdings and price movements.

**Proposition 2** Consider a shock to domestic technology that leads to foreign bonds being purchased, then if \(\eta > 1\), as real foreign bond purchases increase, import prices rise, the terms of trade fall and the real exchange depreciates.

**Proof** See Appendix D.

The assumption that \(\eta > 1\) implies that home and foreign goods are substitutes. This assumption is relaxed in later sensitivity analysis, but is maintained for the moment for analytical convenience. A domestic technology shock was chosen because this forms the basis for later comparisons between the baseline and modified model.\(^8\) This proposition implies that policymakers choice of bond holdings maps through to a particular real exchange rate, terms of trade and import price level. So if households in the baseline model choose different bond holdings to what policymakers would choose in the modified model, then relative prices would be different across these two models. Differences in relative prices leads to different consumption and labor supply across the two models. It is these differences that matter for welfare.

### 3.2 The interest rate decision

Interest rates are set optimally in both models following Ramsey policy. In the baseline model, since households have access to foreign bonds, the modified uncovered interest parity condition (given by equation (2.9)) holds. Hence policymakers take into account households desire to smooth consumption through foreign bond purchases when setting interest rates.

The Ramsey planner in the modified model does not have to take into account the household Euler equation with respect to foreign bonds. To see this, consider the Ramsey planner’s first order conditions from the baseline model.\(^9\) Let the \(\lambda\)'s represent the various the multipliers on the model’s constraints. Lower case

---

\(^8\)This proposition also holds for other shocks including, for example, shocks to the foreign interest rate and foreign output.

\(^9\)Note here we have assumed that the risk premium is equal to one. This makes the calculations a little easier. In any case, in the numerical analysis that follows, the risk premium is set so as to have little influence on the model’s dynamics.
variables reflect the fact that prices have been normalized by the price of the domestically produced good, while s represents changes in the exchange rate. \( b^F \) represents real foreign bond holdings. Appendix B outlines the Ramsey problem in full. Only three first order conditions are shown below, because these are the only ones that differ between the baseline and modified model.

\[
[\partial C_t] : \frac{1}{C_t^\sigma} - \lambda_{5,t} \frac{\sigma}{C_t^{1+\sigma}} + \lambda_{5,t-1} \frac{\sigma s_t R^F_{t-1}}{C_t^{1+\sigma}} + \lambda_{12,t} (\omega - 1) p^\eta_t + \lambda_{13,t} \omega p^\eta_t (p^F)^{1-\eta} = 0
\]

\[
[\partial s_t] : -\lambda_{11,t} \frac{\pi^F_t}{\pi_t} - \lambda_{13,t} \frac{b^F_{t-1} R^F_{t-1}}{\pi_t^H} - \lambda_{5,t-1} \frac{R^F_{t-1}}{\pi_t C_t^\sigma} = 0
\]

\[
[\partial \pi_t] : \lambda_{9,t} + \lambda_{11,t} \frac{s_t \pi^F_t}{\pi_t^2} + \lambda_{5,t-1} \frac{s_t R^F_{t-1}}{C_t^\sigma \pi_t^2} = 0
\]

\( \lambda_{5,t} \) represents the Lagrangean multiplier on the Euler equation for foreign bond holdings. The terms with the braces underneath them represent the difference between the baseline model and the modified model. These terms are present in the baseline model but are not present in the modified model. Hence when setting interest rates, the Ramsey planner in the baseline model respects a household’s desire to intertemporally smooth consumption. The Ramsey planner in the modified model does not. This causes an intertemporal distortion in consumption as households are not equating their expected marginal utility of consumption across time with respect to external prices. But this does give policymakers the ability to move domestic interest rates away from that implied by their foreign counterpart.

4 A Numerical Example

To illustrate the differences between the baseline and modified model two shocks are considered. Firstly, a positive transitory shock to domestic productivity, as this is a shock typically used in the literature. The size of this shock is normalized to 0.01. Secondly, to proxy a shock similar to the decline in US interest rates observed since the financial crisis in 2008, a negative transitory shock to the foreign interest rate is considered.\(^{10}\) A 50 basis point cut in the foreign interest rate is studied.

The model is solved around a zero inflation, zero net foreign asset steady state using standard linearization techniques. It is assumed that the risk premium on foreign bond holdings is zero at steady state.

4.1 Calibration

The parameter values chosen are relatively standard. The labor supply parameter, \( \phi \), and the coefficient of relative risk aversion, \( \sigma \) are set equal to 2 and 1 respectively, following the work on emerging market

\(^{10}\)Typically a cut to foreign interest rates coincides with weak foreign output and inflation. However, in order to more clearly show the transmission of the foreign interest rate shock, it is just assumed that only the foreign interest moves, with foreign output and inflation being unchanged. In any cases, both a cut in foreign interest rates and a drop in foreign output would lead to households borrowing from overseas.
economies in Elekdag et al. (2006). $\beta$ is set at 0.99, implying a steady state interest rate of 4 per cent. This is also the steady state interest rate for the rest of the world. It is assumed that the majority of the final consumption good is composed of imports, with $\omega$ set equal to 0.6, consistent with the high level of imports in many South-east Asian nations. While this import share is high for countries in Latin America, later sensitivity analysis looks at lower values of the import share. The elasticity of demand for domestic intermediate goods, $\varepsilon$, is set at 6. Similarly, the elasticity of substitution between home and foreign goods, $\eta$, is set at 6, implying that these goods are substitutes. The elasticity of demand for home exports, $\eta_f$, is set equal to 1.5. These parameter values follow Christiano et al. (2010). $\bar{X}$ is set so that foreign output is equal to one in the steady state. For the pricing parameters, it is assumed that prices are unchanged for a year on average implying $\theta$ is 0.75. Others who have used this parameter value include Devereux et al. (2006) in their work on monetary policy in emerging market economies. $\phi_{BF}$, which determines the risk premium on foreign borrowings, is set equal to 0.005 so as to have only a small effect on the model. This value is between that used by Justiniano and Preston (2010) and Schmitt-Grohe and Uribe (2003). The persistence of each stochastic process is set equal to 0.9, following Monacelli (2004).

4.2 Comparing the baseline and modified model: Impulse responses

A domestic productivity shock

Figure 2 shows the impulse responses for selected variables from a transitory positive shock to domestic productivity. The red lines show the impulse responses under the baseline model, the black lines those from the modified model. There are two sets of lines for each color. The lines with markers on them represent the impulse responses from the baseline and modified models when prices are sticky. The lines without markers show the impulse responses when prices are flexible, where following Adolfson et al. (2007) flexible prices are defined as the case where $\theta = 0.01$. These two sets of lines largely lie on top of one another. This indicates that when the exchange rate is freely floating, the same results hold regardless of whether prices are sticky or not.

Since the results are similar with flexible or sticky prices, the analysis that follows concentrates on the sticky price case. In both the baseline and modified model, output and consumption rise in response to the technology shock. Labor supply increases as a higher marginal product for labor, from the technology shock, leads to higher wages. In response to this increase in productivity, households in the baseline model smooth their consumption through the purchase of foreign bonds. Increased output and purchases of foreign bonds contribute to a depreciation of the real exchange rate and a fall in the terms of trade. The depreciation of the exchange rate causes an increase in import prices, making the consumption of imports less attractive. Imports fall while consumption of the domestically produced good increases. Conversely, exports rise as the terms of trade falls. In both models, interest rates fall in order to encourage more consumption from the increased level of output. This fall in interest rates also helps to stabilize prices. Note that in the first few periods, domestic prices are better stabilized in the modified model compared with the baseline model, owing to the fact that interest rates in the modified model can be moved away from what is implied by foreign interest rates and the exchange rate.
In the modified model, policymakers incorporate the effect of foreign bond purchases on the real exchange rate and terms of trade when determining bond holdings. Policymakers choose to purchase fewer bonds than what households themselves would purchase in the absence of capital controls. Fewer bond purchases lead to more muted movements in relative prices (which also follows from Proposition 2). Import prices rise by a smaller amount, the real exchange rate depreciates by a smaller amount and the terms of trade decline by less, compared with the baseline model. The effect of this is to increase consumption in the near term. Firstly, the smaller rise in import prices (and smaller depreciation in the real exchange rate) moderates the fall in imports in the modified model compared with the baseline model. By dampening the rise in import prices, policymakers can engineer a switch in consumption towards imports. Secondly and more mechanistically, when households spend a smaller amount of their incomes on bond purchases then more can be spent on consumption. In line with this, labor supply is lower under the modified model compared with the baseline model. This is because lower foreign bond holdings and lower import prices under the modified model mean that households can increase their consumption through a greater absorption of imports without having to increase their labor supply.

A foreign interest rate shock

Impulse responses from a negative transitory shock to the foreign interest rate are shown in Figure 3. As was the case with the productivity shock, assumptions about price stickiness do not make a material difference to the results. As before, the analysis concentrates on the sticky price case.

A fall in the foreign interest rate leads to capital inflows into the small open economy in both models. In the baseline model, lower foreign interest rates encourage households to borrow from overseas. Increased borrowing leads to an appreciation of the real exchange rate and an increase in the terms of trade in the baseline model. Uncovered interest parity implies a fall in the domestic interest rate. Consumption rises as capital inflows increase household income, while the appreciation of the real exchange rate makes imports cheaper. The more appreciated exchange rate, however, leads to a fall in imports. Labor supply falls as cheaper imports cause consumers to substitute consumption away from the domestically produced good and an appreciation in the exchange rate reduces demand for the home good in the export market.

With sterilized interventions policymakers, rather than households determine an economy’s bond holdings. Here policymakers choose a smaller level of borrowings than what households would choose in the absence of these policies. A smaller level of borrowing leads to a smaller appreciation in the real exchange rate and a smaller increase in the terms of trade. Since the domestic interest rate is no longer tied to the foreign interest rate, policymakers choose to moderate the decline in the home interest rate. The effect of these price changes is to lower household consumption relative to the baseline model; a lower level of borrowing, a smaller fall in the domestic interest rate and a more depreciated real exchange rate discourages consumption. Policymakers are able to alter the composition of consumption. A smaller real exchange rate appreciation causes consumption to be shifted from imports to the domestically produced good.

Through moderating the price effects of the foreign interest rate shock, policymakers also moderate the decline in household labor supply. This encourages domestic production as household demand for the
domestically produced good increases with a more depreciated exchange rate. The production of exports also rise to take advantage of the relative decline in the terms of trade.

4.3 Welfare

The above impulse responses showed that sterilization policies allowed policymakers to manipulate relative prices to shift consumption and labor supply through time. Here the effects of these policies on household welfare are evaluated. Household utility is re-written in recursive form as: \( Welf_t = U(C_t, N_t) + \beta E_t Welf_{t+1} \). Welfare is then computed by taking a second order approximation of this function and the whole model about the steady state.

The results in Tables 1 and 2 show that welfare is higher under sterilized interventions and capital controls in response to both a domestic productivity and a foreign interest rate shock. In the presence of a productivity shock and sticky prices a household in the baseline model would need to receive an increase in their consumption of around 0.0019 percent each period on average to be as well off as a household under the modified model. With foreign interest rate shocks (and sticky prices), a household in the baseline model would need to receive an increase of 0.0207 percent in their consumption each period to get the same level of utility as a household in an economy with sterilized interventions and capital controls. These results also hold when prices are flexible, with welfare only increasing by a small amount from the elimination of pricing frictions. This result is consistent with that from other studies such as Gali and Monacelli (2005), who find that the gains from stabilizing prices tend to quantitatively small. This highlights the fact, that under a floating exchange rate, the benefit to households from policymakers being able to optimally manipulate relative prices tend to be greater than that from eliminating price frictions.

The welfare gain from sterilized interventions across both shocks, comes from policymakers being able to reduce labor supply volatility. When taking into account the effect of foreign bond purchases on relative prices, policymakers choose to lend or borrow less than what households would in the absence of these policies. As Tables 1 and 2 indicate, this leads to more muted movements in relative prices. This is consistent with the stated motivations of central banks in the empirical literature: central banks use sterilized interventions to reduce exchange rate volatility and lean against large movements in their exchange rates (see Adler and Tovari (2011)). Smaller movements in prices in the modified model dampens movements in household labor supply. For example in response to a positive technology shock, cheaper imports enabled households to moderate the rise in their labor supply without reducing their consumption. Similarly with a negative foreign interest rate shock, a smaller movement in relative prices attenuated the fall in household labor supply. This encouraged households to work more, relative to the baseline model, to increase exports and output. Since output is a function of labor supply (and technology), less volatile labor supply also implies less volatile output in the modified model.

While labor supply is less volatile, consumption is not necessarily less volatile under sterilized interventions and capital controls. The reason for this is that policymakers in the modified model do not consider households’ desire to smooth consumption with respect to external prices. As Table 1 indicates, in the presence of a productivity shock, consumption is more volatile under sterilized interventions and capital
controls. In the presence of a foreign interest rate shock (Table 2), household consumption is less volatile under the modified model as relative price movements moderate household demand.

As an aside, note that the model here assumes that the government chooses foreign bond holdings through sterilized interventions when capital controls are imposed. The results from the modified model would also hold if the government instead allowed households to trade foreign bonds but then taxed capital flows. This is because the problem facing the Ramsey planner is the same in both situations. Deviations from uncovered interest parity in the modified model under a positive productivity shock are shown in Figure 4. These deviations are defined as: \( R^H_t E_t \left( \frac{1}{S_{t+1} P_{t+1}} \right) - \frac{\Phi_t R^F_t}{S_t} E_t \left( \frac{S_{t+1}}{P_{t+1} S_{t+1}} \right) \). If policymakers allowed households to purchase foreign bonds and then imposed a tax on capital flows that generated the same wedge between domestic and foreign interest rates and expected movements in the exchange rate as shown in Figure 4, then all of the above results would hold.

A number of authors such as De Paoli and Lipinska (2013), Farhi and Werning (2012) and Schmitt-Grohe

---

**Table 1: Comparing the Baseline and Modified Model: A Technology Shock**

<table>
<thead>
<tr>
<th></th>
<th>Sticky prices</th>
<th>Flexible prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption</td>
<td>1.090</td>
<td>1.085</td>
</tr>
<tr>
<td>Labor</td>
<td>0.775</td>
<td>0.733</td>
</tr>
<tr>
<td>Real exchange rate</td>
<td>0.912</td>
<td>0.911</td>
</tr>
<tr>
<td>Terms of trade</td>
<td>0.912</td>
<td>0.911</td>
</tr>
<tr>
<td>Imports</td>
<td>0.614</td>
<td>0.607</td>
</tr>
<tr>
<td>Exports</td>
<td>0.912</td>
<td>0.911</td>
</tr>
</tbody>
</table>

Welfare comparisons*  
0.0019 0.0019

*The percent of consumption a household under the baseline model would need to receive each period to get the same level of utility as a household under the modified model.

**Table 2: Comparing the Baseline and Modified Model: A Foreign Interest Rate Shock**

<table>
<thead>
<tr>
<th></th>
<th>Sticky prices</th>
<th>Flexible prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption</td>
<td>0.670</td>
<td>0.672</td>
</tr>
<tr>
<td>Labor</td>
<td>0.715</td>
<td>0.672</td>
</tr>
<tr>
<td>Real exchange rate</td>
<td>0.695</td>
<td>0.672</td>
</tr>
<tr>
<td>Terms of trade</td>
<td>0.695</td>
<td>0.672</td>
</tr>
<tr>
<td>Imports</td>
<td>0.682</td>
<td>0.672</td>
</tr>
<tr>
<td>Exports</td>
<td>0.695</td>
<td>0.672</td>
</tr>
</tbody>
</table>

Welfare comparisons*  
0.0207 0.0235

*The percent of consumption a household under the baseline model would need to receive each period to get the same level of utility as a household under the modified model.
and Uribe (2012) have looked at the effectiveness of capital taxes.\footnote{Besides capital taxes, there are other policies that could be used to implement the outcomes from sterilized interventions shown above. For example, deviations from uncovered interest parity could also be achieved by a time-varying consumption tax. In the presence of capital market restrictions a set of subsidies or taxes to intermediate good firms could induce producers to mimic the production levels achieved under sterilized interventions. However, as explained below, these policies may be difficult to implement in practice. Changes in tax rates for consumers and firms are likely to require legislative approval, making them harder to vary in real time.} However, it is likely to be harder for policymakers to implement capital taxes rather than to undertake sterilized interventions, given that the former often requires legislation. Further, as indicated by Figure 4, the optimal tax varies over time making it harder to implement in practice. For example, new legislation may need to be enacted to alter the tax rate and documentation detailing the tax would need to be changed. The implementation of sterilized interventions, on the other hand, is at the discretion of central banks. Central banks can intervene in foreign exchange markets with very little notice and can more easily vary the amount intervention they undertake. Neely (2008) conducts a survey of around 30 central banks and finds that the median amount of time it takes for a central bank to intervene in foreign exchange markets, when there have been no recent interventions, is about one hour. Most central banks can undertake interventions within two hours. If a central bank has recently intervened in foreign asset markets (or is looking to intervene) than the median response time for an intervention drops to a minute.

5 Sensitivity Analysis

The above examples showed that sterilization and capital control policies are welfare improving. Sensitivity analysis is now conducted to see how these results change as parameter values change. The benefit from these policies will depend on household’s willingness to substitute between foreign and domestic goods and their desire for imports to begin with. These factors are captured by the elasticity of substitution between domestic and foreign goods, $\eta$, and the share of imports in consumption, $\omega$. Similarly, policymakers ability to encourage (or discourage) the production of exports will depend of the price elasticity of export demand, $\eta_f$. Finally, as seen above, when relative prices change, there is a labor supply response. The size of this response depends on the elasticity of labor supply $\frac{1}{\varphi}$.

Let's start with the case in which the baseline model produces results identical to that under sterilized interventions and capital controls.

**Proposition 3** If policymakers are setting policy optimally, the model under sterilized interventions and capital controls is equivalent to that in the absence of these policies when:

i. Demand for the home country’s exports are perfectly elastic (as $\eta_f \to \infty$)

ii. Domestic and foreign goods are perfect substitutes in consumption (as $\eta \to \infty$).

**Proof** See Appendix D.

Intuitively under these two cases, the gains from sterilized interventions disappear as policymakers lose their ability to influence relative prices. When demand for exports is perfectly elastic, exports are supplied...
at the steady state export price level. In contrast when export demand is downward sloping, home country producers have some monopoly power over their exports. When this is the case, policymakers can increase (or decrease) export prices in order to influence export demand and hence influence the amount of labor required for export production. Similarly when domestically produced goods are perfect substitutes for foreign goods, both goods are sold at the same price. This means that policymakers cannot alter the composition of consumption between domestic and foreign goods.

Turning to the other parameters, since the sensitivity analysis results are similar for the productivity and foreign interest rate shock, only those from the productivity shock are presented.

Empirical studies disagree over values for the elasticity of substitution between domestic and foreign goods. Studies using micro data tend to find a relatively high elasticity (see for example, Broda and Weinstein (2006)), while macroeconomic studies point to a relatively low elasticity of substitution of between zero and 1.5 (see for example Whalley (1984) and Hooper et al. (1998)). Given this, sensitivity analysis for the elasticity of substitution is conducted over the range from zero to eight. The results from Figure 5 show that welfare is greater under sterilized interventions and capital controls for all of the above mentioned values of $\eta$. The welfare improvement in the modified model is greater the greater the elasticity of substitution between domestic and foreign goods. When the elasticity of substitution is high, by changing relative prices policymakers can induce bigger shifts in consumption towards or away from imports. This enables policymakers to engineer larger shifts in labor supply and hence reduce labor supply volatility by more.

Similarly capital controls and sterilization lead to higher welfare gains the greater the share of imports in consumption (Figure 5). So while a country like Brazil in which imports account for only a small (15 percent) share of GDP will benefit from these policies the benefit is not as great at that for economies in which imports account for a larger share of GDP (such as the economies in South-east Asia). A greater share of imports allow policymakers to more easily shift consumption to (or in some cases from) foreign good purchases. For example, in response to a positive productivity shock, by changing import prices, policymakers provided a way in which consumption could be increased without increasing labor supply. When the share of imports in final consumption is low, households are less willing to switch towards consuming more imports.

Figure 5 shows how the gain in welfare under sterilized interventions depends on estimates of the elasticity of demand for exports. Over more moderate ranges of this elasticity, the welfare gain under sterilized interventions is relatively constant.

How households respond to changes in the composition of consumption will depend on their labor supply elasticity. Macroeconomic and microeconomic studies differ in their estimates of $\phi$. Macroeconomic studies which try to match the volatility in hours in macroeconomic models to the volatility observed in the data suggest higher estimates for the labor supply elasticity. In contrast microeconometric studies point to the labor supply elasticity being between 0 to 0.5. See Chetty et al. (2012) for a discussion. The results in Figure 5 suggest that welfare is greater under capital controls and sterilization the greater the elasticity of labor supply. This is because the greater the elasticity of labor supply, the easier it is for policymakers to lower labor supply in response to a positive total factor productivity shock (and to increase labor supply.
in response to the negative foreign interest shock).

6 The Case of a Fixed Exchange Rate

A number of countries have used sterilized interventions in the presence of a fixed exchange rate. Examples include China and Malaysia. In the models presented in this paper, a fixed exchange rate limits the interest rate (and exchange rate) choices of policymakers. However, a large number of countries have had, or still maintain pegged exchange rates. The literature points to a number of reasons as to why it might be beneficial for a country to adopt a fixed exchange rate. A fixed exchange rate, can be beneficial to countries that could not otherwise commit to optimal monetary policy because of its inability in preventing policymakers from launching inflationary policies. Here a fixed exchange rate can be used as a commitment device to moderate the level and volatility of inflation (see Chari et al. (2013)). Other reasons suggested by the literature as to why a country may wish to fix its exchange rate include, insulating the domestic economy against turbulence in international financial markets and providing a country with poorly developed money markets with the liquidity of the markets of the currency to which its exchange rate is fixed (see Garber and Svensson (1995) for a discussion). Given these reasons, this section looks at the effect of sterilized intervention and capital control policies for a nation with a fixed exchange rate.

The model is largely unchanged from that outlined in Section 2. However, policymakers now choose to fix the exchange rate so that \( S_t = S_{t-1} = 1 \) for all periods. The choice of foreign bond holdings in both models is the same as outlined in Section 3.1. In the baseline model the uncovered interest parity condition holds as households have access to foreign currency bonds. When the exchange rate is fixed this implies that the domestic interest rate is tied to the foreign interest rate, \( R_t^H = \Phi_t R_t^F \). In the modified model this is no longer the case. Capital controls allow domestic interest rates to be moved away from their foreign counterpart.

6.1 A numerical example

A domestic productivity shock

Impulse responses from a transitory domestic productivity shock are shown in Figure 6. When the exchange rate is fixed, it makes a difference whether prices are flexible or sticky. This result differs from the floating exchange case, because when the exchange rate is fixed, adjustment can only happen through prices (and obviously not the nominal exchange rate). This provides more scope for interest rate policy in the economy.

As above, we focus on the sticky price case. Output and consumption rise in response to the domestic technology shock. Under the baseline model, households respond to the productivity shock by smoothing consumption through the purchase of foreign bonds. This leads a depreciation in the real exchange rate and a fall in the terms of trade. As a result of this imports fall and exports rise. A key difference between

\footnote{See also Dellas and Tavlas (2009), who review the conditions under which a country should join a currency area.}

21
the impulse responses in the baseline model under a floating versus a fixed exchange rate is the more muted responses in most real variables with fixed exchange rates, consistent with that found by Monacelli (2004). The impossibility of lowering interest rates and letting the currency move means that you do not get the same support for consumption as in the floating exchange rate case. There is also a more limited initial response in the real exchange rate as adjustment can only occur through prices and not the nominal exchange rate. Indeed domestic inflation drops by a larger amount with a fixed exchange rate, given that this is the only channel through which nominal variables can adjust.

In the modified model policymakers incorporate the effect of their bond purchases on relative prices and choose fewer foreign bonds. As with the floating exchange rate case, this leads to smaller movements in the terms of trade and the real exchange rate. Since domestic interest rates are no longer tied to foreign interest rates, monetary policy can now play a more active role. Interest rates fall in response to a positive technology shock. The combined outcome of these two effects is considerably higher consumption in the modified model compared with the baseline model. Firstly, fewer foreign bond purchases leads to a smaller depreciation of the real exchange rate and hence lower import prices compared with the baseline model. Policymakers engineer a shift in consumption towards imports. Secondly, since policymakers can decrease the domestic interest rate, this further encourages consumption of both imports and the domestically produced good in the near term. Indeed the consumption of imports and the amount spent on imports rise in the modified model compared with the baseline model. A greater consumption of imports mutes the increase in labor supply in the modified model relative to the baseline model.

A foreign interest rate shock

The impulse responses from the negative foreign interest rate shock are shown in Figure 7. Again we focus on the sticky price case. In the baseline model, a fall in foreign interest rates leads to households borrowing from overseas. The fall in the foreign interest rate is associated with an appreciation of the real exchange rate and an increase in the terms of trade, though by smaller amounts compared with the floating exchange rate case. This appreciation in the real exchange rate and the fall in the domestic interest rate leads to an increase in consumption as imports become cheaper. Labor supply falls as domestic production gets substituted by imports and demand for exports fall.

With sterilized interventions, policymakers attempt to limit the fall in household labor supply. They do this by borrowing less. A smaller sale of foreign bonds leads to more modest movements in the real exchange rate and terms of trade. The domestic interest rate falls by a smaller amount. A more depreciated real exchange rate allows policymakers to shift consumption away from imports towards the domestically produced good. Consumption is lower compared to the baseline model. This induces households to increase their labor supply leading to higher domestic production to take advantage of the lower terms of trade in the export market and increased demand in the home market.
<table>
<thead>
<tr>
<th>Sticky prices</th>
<th>Flexible prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption</td>
<td>1.207</td>
</tr>
<tr>
<td>Lab</td>
<td>0.873</td>
</tr>
<tr>
<td>Real exchange rate</td>
<td>0.898</td>
</tr>
<tr>
<td>Terms of trade</td>
<td>0.898</td>
</tr>
<tr>
<td>Imports</td>
<td>0.805</td>
</tr>
<tr>
<td>Exports</td>
<td>0.898</td>
</tr>
</tbody>
</table>

Welfare comparisons*  
0.0062 0.0019

* The percent of consumption a household under the baseline model would need to receive each period to get the same level of utility as a household under the modified model.

6.2 Welfare

The volatility of selected variables under the baseline model relative to the modified model are shown in Tables 3 and 4. The real exchange rate and terms of trade are less volatile in the modified model for both the domestic technology and foreign interest rate shock. This is expected given that policymakers choose fewer bonds under the modified model, leading to more moderate movements in relative prices. This leads to lower labor supply volatility under sterilized interventions and capital controls. In contrast, consumption is more volatile under the modified model in response to a productivity shock. Here policymakers try to re-time consumption through manipulating relative prices but do not take into account household’s desire to smooth consumption.

Despite consumption being more volatile, welfare is higher in the modified model in response to a domestic productivity shock. This owes to a fall in labor supply volatility. This increase in welfare also occurs when prices are flexible, but the improvement in welfare is greater in the sticky price case. This is because when the exchange rate is fixed, policymakers can gain more from manipulating external prices when domestic prices are sticky and interest rate policy helps to reduce the frictions associated with price stickiness. Therefore sterilized interventions and capital control policies can be more beneficial to households when governments decide to impose an exchange rate peg.

6.3 Sensitivity Analysis

Sensitivity analysis is conducted over the same four parameters as in the floating exchange rate case. Results are shown in Figure 8. Again the results from a technology shock are shown.

In contrast to the floating exchange rate case, welfare tends to be higher under capital controls and

---

13Welfare is higher when the exchange rate is floating. For a household living in an economy with sterilized interventions and capital controls, the welfare gain in terms of consumption equivalents from having a floating, rather than a fixed exchange rate is 0.0140 percent under the productivity shock and 0.0260 percent under the foreign interest rate shock.
Table 4: Comparing the Baseline and Modified Model: A Foreign Interest Rate Shock

<table>
<thead>
<tr>
<th></th>
<th>Standard deviations</th>
<th>Flexible prices</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sticky prices</td>
<td>Flexible prices</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.573</td>
<td>0.672</td>
</tr>
<tr>
<td>Labor</td>
<td>0.604</td>
<td>0.672</td>
</tr>
<tr>
<td>Real exchange rate</td>
<td>0.600</td>
<td>0.672</td>
</tr>
<tr>
<td>Terms of trade</td>
<td>0.600</td>
<td>0.672</td>
</tr>
<tr>
<td>Imports</td>
<td>0.586</td>
<td>0.672</td>
</tr>
<tr>
<td>Exports</td>
<td>0.600</td>
<td>0.672</td>
</tr>
<tr>
<td>Welfare comparisons*</td>
<td>0.0487</td>
<td>0.0235</td>
</tr>
</tbody>
</table>

*The percent of consumption a household under the baseline model would need to receive each period to get the same level of utility as a household under the modified model.

Sterilization the lower the elasticity of substitution between domestic and foreign goods (assuming these goods are substitutes) and the smaller the share of imports in consumption. In both the floating and fixed exchange rate case the benefit from capital controls and sterilization policies comes from the ability of policymakers to influence relative prices. However maintaining a fixed exchange rate places an extra constraint on policymakers. In particular, policymakers choice of foreign bond holdings, and hence the real exchange rate and terms of trade, have to be consistent with the fixed exchange rate. When the elasticity of substitution between domestic and foreign goods and the import share is high, a change in relative prices can have large effects on consumption, labor supply and output. This is fine when the exchange rate is floating, but under a peg, large movements in real and nominal variables make it harder to maintain the peg. For example, when the the import share is high, policymakers can only move the real exchange rate by a tiny amount from that implied in the baseline model. This limits the amount of expenditure switching that can be undertaken in the modified model. The welfare gain under sterilized interventions is also lower as the elasticity of substitution between domestic and foreign goods falls below one. For this region of $\eta$ it becomes difficult for policymakers to engineer a shift in consumption toward or away from imports.

The welfare gain under sterilized interventions and capital controls is decreasing in the export demand elasticity. This is for reasons similar to that contained in Proposition 3. The greater the elasticity of export demand, the smaller the influence that home country producers have over their prices.

Generally, the welfare gain from the modified model increases with increases in the elasticity of labor supply. However the results also indicate that welfare can be higher for relatively low labor supply elasticities.

7 Conclusion

The purpose of this paper was to see if sterilized interventions in the presence of capital controls could be beneficial for household welfare. This provided policymakers with an extra policy tool. In addition to be-
ing able to influence interest rates, policymakers could also affect a nation’s foreign bond holdings through their sterilized interventions. Capital controls prevented households from undoing the bond holding decisions of policymakers, consistent with the evidence in the literature that central bank interventions tend to be more effective when restrictions on capital flows are in place. Sterilized interventions allow policymakers to influence the relative prices faced by agents in the economy, leading to changes in a household’s consumption and labor supply decisions.

In general, households choose their foreign bond holdings in order to intertemporally smooth their consumption. However households do not take into account the effect of their bond holdings on the real exchange rate and terms of trade. This means that relative prices move by a greater amount compared with the case in which these price effects are internalized. Households will tend to either supply too much labor (in the case of the positive productivity shock) or too little labor (in the case of a negative foreign interest rate shock). When undertaking sterilization operations, policymakers can internalize the effect of bond purchases on relative prices. Here policymakers choose a lower level of bond holdings for the economy, leading to more muted movements in the terms of trade and real exchange rate. This is consistent with the empirical literature which finds that central banks undertake sterilized interventions to reduce exchange rate volatility (see Adler and Tovari (2011) and Moreno (2005)). In the model, more muted movements in relative prices benefit households by lowering the volatility of labor supply and output. In the case of a positive productivity shock, households benefited by being able to increase their consumption of imports as policymakers engineered a relative appreciation in the real exchange rate. Policymakers moderated the rise in labor supply. This means that households could maintain their consumption without having to work more. In the presence of a negative foreign interest rate shock, policymakers mute the decline in labor supply by engineering a relative depreciation in the real exchange rate. This encourages households to produce more exports to take advantage of the depreciated exchange rate.

The results suggest that welfare under sterilization with capital controls is generally increasing in the elasticity of labor supply. When the exchange rate is floating, welfare under these policies increases with the elasticity of substitution between domestic and foreign goods and the share of imports in consumption. The reverse tends to hold for a fixed exchange rate. The benefits from sterilized interventions and capital controls disappear when home and foreign goods are perfect substitutes and when export demand is perfectly elastic. Here policymakers can no longer influence relative prices.

The central message of this paper is that sterilized intervention and capital control policies can be welfare improving for small open economies. In contrast, there is sometimes a public view that sterilization and capital controls policies can be harmful. Sterilized interventions are sometimes seen as government interference in the determination of the exchange rate, while capital controls are seen as preventing capital from going to where its rate of return is the highest.14 What the results from this paper demonstrate is that these policies, if implemented appropriately, can be beneficial for a small open economy. In particular, policymakers have an incentive to moderate both depreciations and appreciations in a nation’s real exchange rate, consistent with the stated aim on many central banks (see Adler and Tovari (2011)).

14 Albeit, capital controls have recently been seen in a more favorable light following the IMF advocating their use in certain circumstances (see Ostry et al. (2010)).
and Moreno (2005)). In the model outlined in this paper, more muted movements in the real exchange rate benefited households by reducing the volatility of output. Therefore if a country is observed in reality to be employing sterilization and capital control policies, it should not at first glance be considered as a bad thing.

References


Ostry, Jonathan D, Atish R Ghosh, Karl Habermeier, Marcos Chamon, Mahvash S Qureshi, and Dennis BS Reinhardt (2010). Capital Inflows: The Role of Controls. IMF Staff Position Note 10/04, IMF.

Reinhart, Carmen and Peter Montiel (1999). Do capital controls influence the volume and composition of capital flows? Evidence from the 1990s. MPRA Paper 13710, University Library of Munich, Germany.


A Appendix: The Model

All price variables in the model are expressed in terms of relative prices. The price of the final consumption good, \( P_t \), and the price of the imported good, \( P^F_t \) are scaled by the price of the domestically produced good \( P^H_t \) such that 
\[
\begin{align*}
\hat{p}_t & \equiv \frac{P_t}{P^H_t} \\
\hat{p}^F_t & \equiv \frac{P^F_t}{P^H_t}.
\end{align*}
\]
Export prices, \( P^X_t \) are scaled by the price level in the foreign country, \( P^*_t \) to obtain a terms of trade price movement:
\[
\hat{p}^X_t \equiv \frac{P^X_t}{P^*_t}.
\]
The exchange rate \( S_t \) is expressed in terms of changes such that:
\[
\hat{s}_t \equiv \frac{S_t}{S_{t-1}} - 1.
\]
Bond holdings are also presented in real terms. For example, real foreign bonds are given by 
\[
\hat{b}^F_t \equiv \frac{S_t B^F_t}{P^H_t}.
\]

Equilibrium conditions in the baseline model are given below.

The efficiency price distortion
\[
\frac{1}{\hat{p}_t} = \left( 1 - \theta \right) \left( \frac{1 - \theta (\pi^H_t)^{\varepsilon-1}}{1 - \theta} \right)^{\frac{\varepsilon}{\varepsilon - 1}} + \frac{\theta (\pi^H_t)^{\varepsilon}}{\hat{p}_t^{\varepsilon - 1}} \tag{A.1}
\]

Price setting behavior

Let the optimal price chosen by firms that are able to reset their prices be given by \( \hat{p}_i \equiv \frac{K_t}{F_t} \). \( v \) represents the subsidy to firms to remove the distortions from monopolistic competition and is set equal to \( \frac{1}{\varepsilon} \).
\[
\begin{align*}
F_t & = \frac{Y_t}{C^\sigma_t} + \beta E_t \pi^H_{t+1} F_{t+1} \\
K_t & = \frac{\varepsilon}{\varepsilon - 1} \left( 1 - v \right) \frac{Y_t N^\phi_t}{A_t} + \beta E_t \pi^H_{t+1} K_{t+1} \\
K_t & = F_t \left[ \frac{1 - \theta \pi^H_{t+1}}{1 - \theta} \right]^{1/1 - \varepsilon} \tag{A.4}
\end{align*}
\]

Household intertemporal Euler equations
\[
\begin{align*}
\frac{1}{C^\sigma_t} & = \beta E_t \frac{R^F_t \Phi_{t+1}}{C^\sigma_{t+1} \pi^H_{t+1}} \tag{A.5} \\
\frac{1}{C^\sigma_t} & = \beta E_t \frac{R_t}{C^\sigma_{t+1} \pi^H_{t+1}} \tag{A.6}
\end{align*}
\]

Relative price equations
\[
p_t = \left[ (1 - \omega) + \omega (p^F_t)^{1 - \eta} \right]^{\frac{1}{1 - \eta}} \tag{A.7}
\]
\[ p_t^F = Q_t p_t \]  \hfill (A.8)

\[ \pi_t = \pi_t^H \left[ \frac{(1 - \omega) + \omega(p_t^F)^{1-\eta}}{(1 - \omega) + \omega(p_{t-1}^F)^{1-\eta}} \right]^{\frac{1}{1-\eta}} \]  \hfill (A.9)

\[ p_t Q_t p_t^X = 1 \]  \hfill (A.10)

\[ \frac{Q_t}{Q_{t-1}} = s_t \frac{\pi_t^F}{\pi_t^H} \]  \hfill (A.11)

Aggregate resource constraint

\[ A_t \hat{p}_t N_t = (1 - \omega) p_t^0 C_t + X_t \]  \hfill (A.12)

Balance of payments

\[ b_t^F + \omega(p_t^F)^{1-\eta} p_t^0 C_t = p_t Q_t p_t^X X_t + \frac{s_t R_t^{F*} \Phi_{t-1} b_{t-1}^F}{\pi_t^H} \]  \hfill (A.13)

Risk premium equation

\[ \Phi_t = \exp(-\phi_{BF}(B_t^F - \bar{B})) \]  \hfill (A.14)

Demand for exports

\[ X_t = \bar{X} \left( p_t^X \right)^{-\eta_f} Y_t^F \]  \hfill (A.15)

All foreign variables are assumed to follow an AR(1) process.

Technology is assumed to follow an AR(1) process. The model is closed by specifying a rule for the domestic interest rate.

The modified model is the same as the above, with two exceptions. First (A.5) no longer holds. Secondly, the model is closed by specifying a rule for the domestic interest rate as well as for foreign bond holdings.

**B Appendix: Ramsey First Order Conditions**

Let \( \lambda_{l,t} \), \( l = \{1, 2, 3, ... 15\} \), be the multipliers attached to the Ramsey first order conditions. The numbers attached to each multiplier is given by the equations numbers (after the A) from the equations in the previous section.
The Ramsey first order conditions from the baseline model are given by:

\[ [\partial b_t^F] : E_t[\lambda_{13,t} + \lambda_{14,t} \frac{\phi b_t^F}{\exp(\phi b_t^F)}] = 0 \]

\[ [\partial \Phi_t] : E_t[\lambda_{14,t} - \frac{\lambda_{13,t} \Phi_t R_t^F}{\pi_t} - \frac{\lambda_{15,t} \Phi_t R_t^F}{\pi_t}] = 0 \]

\[ [\partial C_t] : \frac{1}{C_t} - \frac{\alpha}{C_t} (\lambda_{5,t} + \lambda_{6,t} - \lambda_{2,t} A_t N_t \pi_t - \lambda_{6,t-1} R_t - \lambda_{13,t-1} R_t) + p_t^H (\lambda_{13,t} (1 - \epsilon) + \lambda_{13,t} \omega (p_t^H)^{-1} - \lambda_{13,t} \omega (p_t^H)^{-1}) = 0 \]

\[ [\partial F_t] : \frac{\lambda_{2,t} - \theta \lambda_{2,t-1} (\pi_t^H)^{\epsilon-1}}{\pi_t} + \lambda_{4,t} \left( \frac{\theta (\pi_t^H)^{\epsilon-1}}{\theta - 1} \right) \frac{1}{\pi_t} = 0 \]

\[ [\partial K_t] : \frac{\lambda_{3,t} - \lambda_{3,t-1} \theta (\pi_t^H)^{\epsilon}}{\pi_t} = 0 \]

\[ [\partial N_t] : -\frac{\lambda_{12,t} A_t \pi_t \pi_t}{\pi_t^H} - \lambda_{2,t} A_t \pi_t \pi_t C_t^{-\sigma} + \lambda_{3,t} \left( \frac{\epsilon (1 - \epsilon) \pi_t \pi_t}{\pi_t^H (1 + \phi)} \right) = 0 \]

\[ [\partial \pi_t] : \lambda_{9,t} + \frac{1}{\pi_t} (\lambda_{6,t-1} R_t - \lambda_{11,t} R_t^F - \lambda_{6,t-1} R_t^F) = 0 \]

\[ [\partial p_t] : \lambda_{7,t} + \lambda_{8,t} Q_t + \eta (\omega - 1) - \lambda_{12,t} R_t^F C_t - \lambda_{13,t} Q_t \pi_t X_t - \eta \omega p_t^H (p_t^H)^{-1} C_t = 0 \]

\[ [\partial \pi_t^H] : \frac{\pi_t^H}{\pi_t} - \frac{\pi_t^H}{\pi_t} = 0 \]

\[ [\partial K_t] : \frac{\lambda_{3,t} - \lambda_{3,t-1} \theta (\pi_t^H)^{\epsilon}}{\pi_t} = 0 \]

\[ [\partial \pi_t^H] : -\theta (\pi_t^H)^{\epsilon-2} + \lambda_{3,t} K_t (\pi_t^H)^{\epsilon-1} - \frac{\theta (\pi_t^H)^{\epsilon-1}}{\pi_t^H} - \frac{\pi_t^H}{\pi_t} = 0 \]

For the modified model, the equations are the same, except that \( \lambda_{5,t} = 0 \), \( \forall t \).

**C Appendix: Allowing Households to Hold Foreign Bonds**

In this version of the model with sterilized interventions, households are allowed to hold foreign bonds, but face a constraint on the amount of foreign bonds that they can hold. This is in line with the restrictions on capital flows adopted by many emerging market economies. For simplicity it is assumed that households face a binding constraint on their foreign bond holdings. This captures a situation in which, for example,
households wanted to hold foreign bonds equal to 5 percent of GDP, but were restricted by capital controls to holding foreign bonds equal to 1 percent of GDP.

Household’s maximize the expected value of their lifetime utility:

\[ E_t \sum_{j=0}^{\infty} B_t^{t+j} \left( \frac{C_t^{1-\sigma} + \phi}{1-\sigma} \right) \]

subject to their budget constraint and a constraint on their real bond holdings:

\[ P_t C_t + S_t B_t^F + B_t^H = S_t \Phi_t R_t^{F*} B_t^F + R_t^{H} B_t^H + W_t N_t + D_t \]  (C.2)

\[ \frac{S_t B_t^F}{P_{H,t}} = \varsigma_t \]  (C.3)

Letting \( \mu_t \) denote the Lagrangean multiplier on the household’s bond holdings constraint, the households first order condition are unchanged except for equation (2.6) which becomes.

\[ \frac{S_t B_t^F}{P_{H,t}} = \beta E_t S_{t+1} \Phi_t R_t^{F*} - \frac{S_t \mu_t}{P_{H,t}} \]  (C.4)

The risk premium on foreign borrowings now depends on both household, \( B_t^F \), and government, \( B_t^{FG} \), bond holdings. Let \( B_t^{FW} = B_t^F + B_t^{FG} \) denote economy-wide foreign bond holdings. Then the risk premium on foreign bonds becomes:

\[ \Phi_t = \exp(-\phi_B F(S_t B_t^{FW} \frac{P_t^H}{P_{H,t}} - \frac{SB_t^{FW}}{P_{H,t}})) \]

The economy wide budget constraint includes both household and government bond holdings:

\[ S_t (B_t^F + B_t^{FG}) + P_t^F C_t^F = S_t \Phi_{t-1} R_{t-1}^{F*} (B_{t-1}^F + B_{t-1}^{FG}) + S_t P_t^X X_t \]

where government bond holdings are determined optimally as described in Section 3.1. The rest of the model is unchanged from that described in the main body of the text.

The impulse responses for economy-wide foreign bond holdings as a result of a positive domestic productivity shock are shown in Figure 9. Here it is assumed that households can hold 10 percent of the foreign bond holdings of policymakers, so \( S_t B_t^F = \varsigma S_t B_t^{FG} \), where \( \varsigma = 0.1 \). The results, however, are not sensitive to the exact amount of foreign bonds that households are allowed to hold, households could also for example be restricted to holding a certain amount of foreign bonds expressed as a share of GDP. The impulse responses from the modified model where households can no longer hold foreign bonds is also shown. As the impulse responses indicate, foreign bond holdings are the same in both these cases.
The Ramsey constraints that policymakers face in this model, where households can hold some foreign bonds, are the same as those faced by the Ramsey planner in the modified model in the main body of the text. To see this consider the differences in the Ramsey planner’s first order conditions in these two models. The Ramsey planner in this model has to take into account two additional equations: the constraint that households face on their foreign bond holdings, given by equation (C.3) and the household Euler equation with respect to foreign bonds, given by equation (C.4). Let \( \lambda_{15,t} \) and \( \lambda_{16,t} \) denote the multipliers on these constraints. The following equations show the Ramsey planner’s first order condition from the current model. Note variables have been normalized as detailed in Appendix A. Only those first order conditions that differ from the modified model are shown.

\[
\begin{align*}
\partial b_t^F \colon & E_t[\lambda_{13,t} + \lambda_{15,t} + \lambda_{14,t} \frac{\phi_{BF}}{\exp(b_t^F + b_t^{FG} - b_t^{G})} - \beta \lambda_{13,t+1} \frac{s_{t+1} \Phi_t R_t^F}{\pi_t} = 0 \\
\partial b_t^{FG} \colon & E_t[\lambda_{13,t} - \zeta \lambda_{15,t} + \lambda_{14,t} \frac{\phi_{BF}}{\exp(b_t^F + b_t^{FG} - b_t^{G})} - \beta \lambda_{13,t+1} \frac{s_{t+1} \Phi_t R_t^{FG}}{\pi_t}] = 0 \\
\partial \Phi_t \colon & E_t[\lambda_{14,t} + \frac{\beta \lambda_{13,t+1} \pi_t^{s_{t+1}} b_t^{FG} R_t^F}{\pi_t} - \beta \lambda_{16,t+1} \frac{s_{t+1} \Phi_t R_t^F}{\pi_t}] = 0 \\
\partial C_t \colon & \frac{1}{C_t} - \frac{\sigma}{C_t^{1+\sigma}} (\lambda_{16,t} + \lambda_{6,t} - \lambda_{2,t} A_t N_t p_t - \frac{\lambda_{6,t-1} R_t - \lambda_{17,t-1} \pi_t}{\pi_t}) = 0 \\
\partial s_t \colon & -\lambda_{11,t} \frac{\pi_t^{s_{t+1}}}{\pi_t} - \lambda_{13,t} \frac{\Phi_{t-1} R_t^{FG} R_t^{FG}}{\pi_t} - \lambda_{16,t-1} \frac{\Phi_{t-1} R_t^{FG}}{\pi_t} = 0 \\
\partial \pi_t \colon & \lambda_{9,t} + \frac{1}{\pi_t} \left( \lambda_{6,t-1} \frac{R_t}{C_t} + \lambda_{11,t} \pi_t \frac{R_t}{C_t} + \lambda_{16,t-1} \frac{\Phi_{t-1} R_t}{C_t} \right) = 0 \\
\partial \mu_t \colon & \lambda_{16,t} = 0
\end{align*}
\]

These Ramsey first order conditions are the same as those from the modified model if \( \lambda_{16,t} = 0 \), and if there is only one first order condition with respect to foreign bond holdings. From above, it is clear that \( \lambda_{16,t} = 0 \).

Set the first order conditions with respect to \( b_t^F \) and \( b_t^{FG} \) equal to each other. This implies that \( \lambda_{15,t}(1-\zeta) = 0 \). Since \( \zeta \neq 0 \), this means that \( \lambda_{15,t} = 0 \). This implies that the Ramsey planner’s first order condition with respect to \( b_t^F \) is the same as the first order condition with respect to \( b_t^{FG} \). These conditions collapse to:

\[
E_t[\lambda_{13,t} + \lambda_{14,t} \frac{\phi_{BF}}{\exp(b_t^F + b_t^{FG} - b_t^{G})} - \beta \lambda_{13,t+1} \frac{s_{t+1} \Phi_t R_t^F}{\pi_t}] = 0
\]

Since \( b_t^F + b_t^{FG} \) represents economy-wide foreign bond holdings, this is the same condition as that in the modified model. Thus the Ramsey constraints that policymakers face in a model where households have restricted access to foreign bonds is the same as those faced by the Ramsey planner in the modified model where only policymakers have access to foreign bonds.

D Appendix: Proof of Propositions

Proof of Proposition 2

The balance of payments equation can be re-written in terms of import prices and domestic output, where for simplicity it is assumed at \( \Phi_t = 1 \) for all periods.
\[ b_t^F = \hat{X}(p_t^F)^{\eta_f} Y_t^{F^*}(1 + \frac{\omega}{1-\omega}(p_t^F)^{1-\eta_f}) - \frac{\omega}{1-\omega}(p_t^F)^{1-\eta_f} Y_t + \frac{p_t^F R_{F^*,t}^* b_{t-1}^{F^*}}{p_{t-1} Q_{t-1} \pi_t^*} \] (D.1)

The derivative of equation (D.1) is given by:

\[
\frac{\partial b_t^F}{\partial p_t^F} = \eta_f \hat{X}(p_t^F)^{\eta_f-1} Y_t^{F^*} - \frac{\omega(1-\eta_f)}{1-\omega}(p_t^F)^{-\eta_f} Y_t - \frac{\omega}{1-\omega}(p_t^F)^{1-\eta_f} \frac{\partial Y_t}{\partial p_t^F} + \frac{p_t^F R_{F^*,t}^* b_{t-1}^{F^*}}{p_{t-1} Q_{t-1} \pi_t^*} 
\]

Then provided that \( \eta > 1 \) and \( b_{t-1}^F > 0 \) (which is assumed), the first, second and last term of equation (D.2) are positive.

A positive shock to technology, leads to an increase in output. Looking at the term \(-\frac{\omega(1-\eta_f)}{1-\omega}(p_t^F)^{-\eta_f} Y_t - \frac{\omega}{1-\omega}(p_t^F)^{1-\eta_f} \frac{\partial Y_t}{\partial p_t^F} \), we want this term to be greater than zero. Rearranging, this equation is greater than zero if \( Y_t > \frac{p_t^F}{(1-\eta_f) \frac{\partial Y_t}{\partial p_t^F}} \), as output is composed of exports and the part of the domestically produced good that is consumed at home. We know that \( \frac{\partial Y_t}{\partial p_t^F} > 0 \) and if domestic and imported goods are substitutes in consumption then \( \frac{\partial C_t^H}{\partial p_t^F} > 0 \). Since \( 1 - \eta < 0 \), then \( Y_t > \frac{p_t^F}{(1-\eta_f) \frac{\partial Y_t}{\partial p_t^F}} \), as the left hand side of this equation is positive, while the right hand side is negative. Therefore this inequality holds.

Therefore \( \frac{\partial b_t^F}{\partial p_t^F} > 0 \). Then using (A.8) and (A.10) we can show that \( \frac{\partial b_t^F}{\partial Q_{t-1}^F} > 0 \) and \( \frac{\partial b_t^F}{\partial p_{t-1}^F} < 0 \).

**Proof of Proposition 3**

Let's start with the case where foreign demand for home country exports is perfectly elastic, or in other words as \( \eta_f \rightarrow \infty \). We show that welfare under the baseline model is the same as that under sterilized interventions by showing that household consumption and labor is the same across the two models.

Here again for simplicity it is assumed that \( \Phi_t = 1 \) for all periods. Re-writing the demand for exports, equation (A.15), as: \( p_t^X = \left( \frac{X_t}{XY_t^*} \right)^\frac{1}{\eta_f} \). Then as \( \lim_{\eta_f \rightarrow \infty} p_t^X = 1 \), for all \( t \). From equations (A.7), (A.8), (A.9) and (A.10), this implies that as \( \lim_{\eta_f \rightarrow \infty} p_t = \lim_{\eta_f \rightarrow \infty} p_t^F = \lim_{\eta_f \rightarrow \infty} Q_t = 1 \) for all \( t \). Then, \( \lim_{\eta \rightarrow \infty} \pi_t = \lim_{\eta \rightarrow \infty} \pi_t^H = \lim_{\eta \rightarrow \infty} \pi_t^H = \lim_{\eta \rightarrow \infty} \pi_t = 1 \) (using equations (A.9), (A.1) and the definition of inflation).

Since \( \pi_t^H \) approaches 1, from equation (A.4) this implies that \( \lim_{\eta \rightarrow \infty} K_t = \lim_{\eta \rightarrow \infty} F_t \). Then using equations (A.2) and (A.3) this implies that as:

\[
C_t^\sigma N_t^\phi = \frac{A_t(\varepsilon - 1)}{\varepsilon(1-\tau)} \]

Subbing equation (D.3) into equation (A.10) and using the fact that \( \lim_{\eta \rightarrow \infty} \hat{p}_t = 1 \), we get:
\[ C^\alpha ((1 - \omega)C_t + \bar{XY}_t^F)^\phi = \frac{A_t(\varepsilon - 1)}{\varepsilon(1 - \tau)} \]

Since \( C \) only depends on parameters and exogenous variables, it must be the case that consumption is the same across the baseline and modified model. Plugging in the value for \( C \) into equations (D.3), implies that \( N \) only depends on parameters and exogenous variables in the model. Therefore labor supply must be the same across the two model. Since consumption and labor supply is the same in the two models, welfare is the same in both models.

Home and foreign goods are perfect substitutes, when \( \eta \to \infty \). Again we show that the baseline model is equivalent to the modified model by showing labor supply and consumption is the same across the two models. From equation (2.1) \( \lim_{\eta \to \infty} C_t = C^H_t + C^F_t \). Since home and foreign goods are perfect substitutes this implies that \( P_{H,t} = P_{F,t} \), or \( p_{F,t} = 1 \). Using equations (A.9), (A.8), and (A.10): \( \lim_{\eta \to \infty} p_t = \lim_{\eta \to \infty} p_t^X = \lim_{\eta \to \infty} Q_t = 1 \), for all \( t \). Then \( \lim_{\eta \to \infty} \pi_t = \lim_{\eta \to \infty} \pi^H_t = 1 \). Since prices and inflation are the same in both models, from the same reasoning as that contained in the previous proof, consumption and labor supply is the same across the two models.
Figure 1: Sterilized Interventions and the Exchange Rate

Brazil

%GDP

Index

Real effective exchange rate
(2000 = 100, RHS)

Interventions
(Share of annual GDP, LHS)

Notes: Positive values for interventions refer to purchases of foreign assets, negative values refer to sales of foreign assets. Upward movements of the exchange rate correspond to depreciations, downward movements to appreciations.

Sources: Central Bank of Brazil and IFS.
Figure 2: The Model With and Without Sterilized Interventions and Capital Controls

Impulse responses to a transitory positive productivity shock

Notes: The red (or lighter) lines show the impulse responses under the baseline model. The black lines show the impulse responses with sterilized interventions and capital controls. The solid lines with markers show the impulse responses under sticky prices, the dashed lines without markers show the impulse responses when prices are flexible. Impulse responses are scaled by 100.
Figure 3: The Model With and Without Sterilized Interventions and Capital Controls

Impulse responses to a transitory negative foreign interest rate shock

Notes: The red (or lighter) lines show the impulse responses under the baseline model. The black lines show the impulse responses with sterilized interventions and capital controls. The solid lines with markers show the impulse responses under sticky prices, the dashed lines without markers show the impulse responses when prices are flexible. Impulse responses are scaled by 100.
Figure 4: Deviations from Uncovered Interest Parity in the Modified Model

Impulse responses to a transitory positive productivity shock

Notes: The uncovered interest parity deviation is defined by $R^H_t E_t \left( \frac{1}{P_{t+1}C_{t+1}} \right) - \Phi_t R^F_t E_t \left( \frac{S_{t+1}}{P_{t+1}C_{t+1}} \right)$. Impulse responses are scaled by 100.
Figure 5: **Sensitivity Analysis: A Technology Shock**

Welfare gain under sterilized interventions expressed in terms of consumption equivalents

Notes: The percent of consumption a household under the baseline model would need to receive each period to get the same level of utility as a household under the modified model. Consumption equivalents have all been scaled by 100.
Figure 6: The Model With and Without Sterilized Interventions and Capital Controls: A Fixed Exchange Rate

Impulse responses to a transitory positive productivity shock

Notes: The red (or lighter) lines show the impulse responses under the baseline model. The black lines show the impulse responses with sterilized interventions and capital controls. The solid lines with markers show the impulse responses under sticky prices, the dashed lines without markers show the impulse responses when prices are flexible. Impulse responses are scaled by 100.
Figure 7: The Model With and Without Sterilized Interventions and Capital Controls: A Fixed Exchange Rate

Impulse responses to a transitory negative foreign interest rate shock

Notes: The red (or lighter) lines show the impulse responses under the baseline model. The black lines show the impulse responses with sterilized interventions and capital controls. The solid lines with markers shows the impulse responses under sticky prices, the dashed lines without markers show the impulse responses when prices are flexible. Impulse responses are scaled by 100.
Figure 8: Sensitivity Analysis: A Technology Shock and a Fixed Exchange Rate

Welfare gain under sterilized interventions expressed in terms of consumption equivalents

Notes: The percent of consumption a household under the baseline model would need to receive each period to get the same level of utility as a household under the modified model. Consumption equivalents have all been scaled by 100.
Figure 9: Allowing Households to Hold Some Foreign Bonds

Impulse responses for foreign bonds from a transitory positive productivity shock

Notes: Impulse responses are scaled by 100.