International Trade, Income Distribution and Welfare

Phillip McCalman
Department of Economics
University of Melbourne

June 2013

Abstract

This paper studies the relationship between income distribution and international trade in the canonical trade setting with one change. Under the standard approach prices are a function of (constant) marginal costs and (constant) elasticities alone, implying that information on consumer income is of no value to a typical firm. To address this limitation the strategy space is expanded to include non-linear prices (i.e. potential to offer product lines). In equilibrium firms use information on the distribution of income to design a product for each income class, with associated prices that induce each group to optimally select the intended product. To achieve this outcome, some of these products are degraded relative to the first best while others exceed it. When countries with differing income distributions integrate, this has implications for the size of these distortions, influencing the gains from trade both within and across countries. The structure of prices which emerge match the systematic relationships between importer characteristics and product prices that have been documented in the literature. This is driven by firm strategy alone as preferences are assumed to be identical and homothetic across countries. The model also links factor prices to distributional characteristics and predicts a positive or a negative correlation in the change in factor prices from integration across countries depending on differences in initial income distribution.

Key Words: Intra-industry trade, monopolistic competition

JEL Classifications: F12
1 Introduction

A persistent challenge/puzzle in the international trade literature is how to model income differences. For the most part the challenge is motivated by empirical studies. Countries with similar income characteristics trade more with one another, and the same characteristics also play a role in the prices charged by exporters and paid by importers. A second dimension of this puzzle is to isolate which characteristics are important, just per capita income or is the distribution of income more generally also relevant? The challenge arises since the canonical model of international trade utilizes identical and homothetic preferences, a feature which completely suppresses issues of income difference.

To confront this challenge, the previous literature relaxed the assumption of homotheticity.\(^1\) This frees up expenditure shares to depend not just on relative prices but also income levels. However, there is another possibility that has been neglected. It could also be the case that relative prices are conditional on income. That is, firms may price discriminate. The difference between the two perspectives is subtle but important. Under non-homothetic preferences, individuals with different incomes are hardwired to make different choices. Under price discrimination individuals with different income are \emph{induced} to make different choices. Since price discrimination isn’t always based on observable differences it can be relatively inconspicuous (as we’ll see below), so it is not always apparent and certainly not straightforward to include in empirical studies. Nevertheless, it can be part of an explanation for variation in expenditure patterns with income levels, even with homothetic preferences. Moreover, price discrimination generally does have implications for welfare outcomes. The open question is whether international trade tends to enhance the positive aspects of discrimination or magnify the negative ones.

The objective of this paper is to answer this question and explore the implications of income differences both within and across countries for international trade. In contrast to the non-homothetic literature, preferences will have the standard features of being identical and homothetic for all consumers. This is done solely to highlight the central methodological contribution and clearly distinguish the analysis from the previous literature.\(^2\) The key insight that differentiates this paper from the previous literature is a focus on how a firm views and evaluates the information relating to the distribution of income in the population of consumers that it serves. In the standard analysis

---

\(^1\)Hallak (2010), Fajgelbaum et al. (2011), Świerci (2012), Markusen (2010), Mitra and Trindade (2005), Simonovska (2010), Choi et al. (2009) and Fieler (2011) to list only some of the recent contributions.

\(^2\)It should be pointed out that non-homothetic preferences may amplify the motivation to manipulate/discriminate by the firm.
the strategy space of firms is constrained to such an extent that they are only interested in the curvature of the residual demand function when formulating their optimal strategies. Moreover, with Dixit-Stiglitz preferences the elasticity of residual demand is constant and the same for all consumers in the canonical monopolistic competition setting. What this means is that even if a firm could costlessly observe the income levels of each consumer, the best they could do is implement third degree price discrimination. However, with the elasticity of demand independent of income and the same for all consumers, firms don’t change their behaviour. In fact, this information would be essentially worthless to a firm.

To begin to understand the implications of allowing firms to exploit information on a consumer’s income requires expanding the strategy space from exclusively linear prices to more general non-linear prices. However, the derivation of optimal non-linear prices depend critically on the information that the firm possesses. Rather than assuming the firm can perfectly observe a consumer’s income, as in a model of first degree price discrimination, it will be assumed that the firm knows the aggregate distribution of income but not an individual consumer’s income. More formally this is a setting where a firm implements second degree price discrimination (aka indirect discrimination). If a firm optimally chooses to exploit this information it does so through the design of a menu of options offered to a consumer (or more broadly a product line).\(^3\)

The optimal strategy of the firm and the resulting monopolistically competitive equilibrium is now not just a function of the curvature of the demand functions but also their position. As a consequence the distribution of income is a fundamental determinant of the design of the equilibrium product line. A feature of this equilibrium is that product design is distorted relative to the first best. In general products designed for low income types are below the first best, while the products targeted to the high types are above the first best.\(^4\) This implies that welfare differences are more exaggerated than income differences. The critical role of the distribution of income in this outcome immediately implies that the integration of two countries with different income distributions alters product line design and consequently welfare. If countries can be ranked in terms of income distribution, then the model produces some clear predictions. In particular countries which have a “good” distribution of income receive larger gains from trade, with these gains dispropor-\(^3\)This product line is associated with goods of different characteristics/quality and prices that induce consumers with different income to select different options from the product line. In this sense a firm offers multiple products. However, this view of multi-product firms is much narrower than usual perspective employed in the international trade literature. See for example Bernard et al. (2011).

\(^4\)Monopoly models of indirect discrimination predict the first result but not the second. See for example Maskin and Riley (1984).
tionately concentrated at the bottom of the income distribution. In this case trade reduces the distortions from indirect discrimination and the benefits are felt across the entire distribution of income. The opposite occurs in a country with a “bad” distribution of income, as trade adds to the distortions from indirect discrimination. Since these distortions are not present in the standard model of international trade they represent a new dimension of the welfare analysis.

The equilibrium price distribution that emerges from the model is also consistent with the stylized facts that have been documented in the literature between import prices and the income of an importing country. Two patterns in particular have been documented. A positive association between unit values and per capita income, and a negative association between unit values and income dispersion. Since the price paid is increasing in income (due to the dependence of price on the location of the demand curve), the positive association between per capita income and unit values arises naturally. It is also a characteristic of the optimal price distribution that it is concave in income. The concavity of the price distribution is consistent with the second empirical finding, since a transfer of income from the low end to the high end of the income distribution that leaves the mean constant involves both these types now making different choices. In particular, the low type trades down while the high type trades up, but because of the concavity of the menu, the rise in price for the high types is less than the fall in price associated with the choice of the low type. This induces a negative correlation that matches the finding of Bekkers et al. (2012).

The second empirical literature that the model connects with relates to the relationship between factor prices and product market integration. The usual Stolper-Samuelson results have been questioned since a number of studies have found a positive correlation in the change in relative factor prices across locations (Harrison et al., 2011). While mechanisms have been proposed to explain this, other studies have found a negative correlation (Amiti and Cameron, 2012). To date no model is able to account for both the positive and negative correlations for countries at seemingly similar stages of development. However, such a non-monotonic relationship is possible in this model. The central mechanism relies on the degree of heterogeneity in the population. To see this consider a simple case with two factors of production and two sectors. Suppose that in country 1 all individuals are endowed with a unit of capital and a unit of labour. In contrast, individuals in country 2 each have the same ratio of capital to labour as in country 1 but the individual endowments are strictly less than one (this shuts down the Stolper-Samuelson mechanism). Autarky in both countries is

---

characterized by an unconstrained first best outcome due to the implementation of non-linear prices in each of the homogeneous societies. Moreover, under standard assumptions each country will have the same factor prices. Now consider what happens when they integrate. The global distribution of types is a mixture of the two countries and therefore reflects heterogeneity so a distortion is introduced into the global economy due to indirect price discrimination. This causes firms in both countries to distort their production decisions in the same way which then implies that the change in relative factor prices will also be positively correlated. In contrast if two countries have relatively similar distributions to begin with, then the distortions can move in opposite directions, inducing a negative correlation in the change in factor prices across countries.

To develop these results the paper is broken into 5 sections. Section 2 constructs a quasi-linear monopolistically competitive model of indirect discrimination with two income types. This framework facilitates comparisons with the previous partial equilibrium monopoly literature and also helps build intuition for the general equilibrium analysis to follow. Section 3 considers integration between countries with different shares of the two types, while section 4 expands the model to consider an arbitrary number of types and more general comparisons across income distributions. Section 5 constructions a general equilibrium version of the model and confirms the results in this setting as well as relating prices to importing country characteristics and making comparisons across price regimes. Finally, section 5 also endogenizes the distribution of income by considering a two factor model and characterizes the behavior of factor prices when countries with different income distributions integrate.

2 Model

2.1 Budget Constraint

The main elements of the model are familiar from the international trade literature. Expanding firm strategy space to including non-linear pricing is the only addition to the standard model. While this naturally has implications for firm behavior, it also implies that the feasible set of options available to consumers are altered. These changes are reflected in the budget constraint. Conceptually the most direct approach is to have firms choose a general non-linear price schedule and have this presented to consumers as a set of two-part tariffs (price involves a fixed/access fee and a usage fee). From a modeling perspective this has the advantage that the fixed/access charge acts like a lump sum
tax. To see this assume that consumer type $j$ purchases a product from firm $i$ for total payment $T_{ji}$. To illustrate the notation assume that type $j$ actually buys the product designed for them. Let the two-part tariff be given by $T_{ji} = A_{ji} + p_{ji}q_{ji}$, where $A_{ji}$ is the access fee, $p_{ji}q_{ji}$ is the usage based payment. Therefore, if a consumer purchases the relevant version of each variety, then for a consumer with gross income $m_j$ their net income can be expressed as:

$$\tilde{m}_j = m_j - \sum_i A_{ji}$$

Consequently, the main modification is in relation to net income. In the standard model that uses linear prices, there is no difference between net and gross income ($\tilde{m}_j = m_j$). However, under non-linear prices net income can diverge from gross income.

2.2 Preferences

At the heart of the standard model of intra-industry trade is a representative consumer with “love of variety” preferences. A particularly convenient representation of these preferences is given by:

$$U_j = \psi_j \log(Q) + y \quad \text{where} \quad Q = \left[\sum_i q_i^\rho\right]^{1/\rho} \quad \text{and} \quad 0 < \rho < 1 \quad (1)$$

where $Q$ is an index associated with the differentiated goods sector and $y$ is produced in a perfectly competitive sector and serves as the numeraire.

While the standard approach is to assume a single representative consumer, our interest is to explore consumer heterogeneity along the income dimension. In the quasi-linear set-up these differences are modeled indirectly by assuming that higher income is associated with a lower marginal utility of income. This enters the analysis through the $\psi_j$ parameter which is interpreted as the inverse of the marginal utility of income. Although this specification is not homothetic it still has the property that when firms are restricted to use linear prices, they won’t change their optimal prices if consumer income (or $\psi_j$) becomes perfectly observable. In this case, the best a firm can do is implement third degree price discrimination. However, since each consumer type $j$ has the same $\rho$, they also have the same elasticity of demand, $\sigma = \frac{1}{1-\rho}$ in the usual monopolistically competitive set-up.

---

See Chor (2009), Antràs and Helpman (2004) and Helpman and Krugman (1989) for other applications of this quasi-linear framework in an international setting.

A general equilibrium setting is considered later in the paper.
The demand for each sector is given by:

\[ Q_j = \frac{\psi_j}{P_j}, \quad y_j = \bar{m}_j - P_j Q_j \quad \text{where} \quad P_j = \left[ \sum_i p_{ji}^{\rho} \right]^{\frac{\rho-1}{\rho}} \]

Note that the demand for a given variety is:

\[ p_{ji} = \theta_j q_{ji}^{\rho-1} \quad \text{with} \quad \theta_j = \frac{\psi_j}{Q_j^\rho} \quad (2) \]

Facing these demand curves a typical firm evaluates the surplus available in the following way:

\[ S_j(q) = \theta_j \int_0^{q_j} z^{\rho-1} dz = \frac{\theta_j q_j^{\rho}}{\rho} \]

A feature of this framework is that offers both a quantity and a quality interpretation. Since the quantity interpretation is familiar from applications of the Dixit-Stiglitz model I’ll focus on the quality perspective which is more common in the indirect discrimination literature (see for instance Tirole (1988)). The implicit assumption is that each product is purchased in a single unit and the consumer has preferences over the quality level as measured by \( q_i \). Consider for example the choice of a tablet computer. Here the quantity purchased is typically a single unit, and the margin of choice for the consumer is memory size (giga-bytes). Other examples include printers, with the dimension of choice pages per minute, or televisions and screen size. Note that this interpretation has no direct implications for the specification of the utility function. However it does have implications for how “price” is calculated. Under linear pricing the firm selects a price per unit of quality (eg giga-bytes, pages per minute, inches) and the consumer is free to select any positive quality level. As a consequence the price paid for a particular tablet computer is \( p_i q_i \). In this case higher quality products have a higher price, but it is proportionately higher. We now consider whether a firm can do better than linear pricing.

---

\( ^8 \)For example the screen size of the iPad is fixed and the dimension of choice is restricted to the memory size - 16GB, 32GB or 64GB.
2.3 Profit Maximizing Productlines

Start by considering a setting with minimal amount of heterogeneity, a world with two consumer types, a high income type ($\psi_H$) and a low income type ($\psi_L$), where $\psi_H > \psi_L$. Assume that the fraction of high income types is $\beta$ and low income types is therefore $1 - \beta$.

We make the standard assumption in the literature that the $Q$ sector is characterized by monopolistically competitive firms. We also follow the previous literature and assume a constant marginal cost, $c$, and also a firm level fixed cost, $F$.\(^\text{10}\) Since monopolistic competition assumes an absence of strategic interactions, the profit maximizing program resembles that of an indirectly discriminating monopolist (see for example Tirole (1988)). Therefore, using the surplus functions from above and the information on the distribution of types in the population, the typical firm chooses a menu of $\{(q_L, T_L), (q_H, T_H)\}$ to maximize

$$\pi = \beta(T_H - cq_H) + (1 - \beta)(T_L - cq_L) - F$$

subject to

$$\theta_H \frac{q_H}{\rho} - T_L \geq \theta_L \frac{q_L}{\rho} - T_H, \quad \theta_H \frac{q_H}{\rho} - T_H \geq \theta_H \frac{q_L}{\rho} - T_L \quad (3)$$

$$\theta_L \frac{q_L}{\rho} - T_L \geq 0, \quad \theta_H \frac{q_H}{\rho} - T_H \geq 0 \quad (4)$$

where (3) are the incentive compatibility constraints for the low and high types, respectively, while (4) are the corresponding participation constraints. In a typical non-linear pricing problem the ordering of the $\theta$’s would be enough to ensure that the single crossing property holds; implying that only two of these constraints bind, the incentive constraint for the high type and the participation constraint for the low type. However, since the $\theta_j$’s are part of an equilibrium outcome we cannot simply impose that $\theta_H > \theta_L$. Nevertheless we conjecture that this ordering holds (it is in fact satisfied in equilibrium). Under this conjecture the relevant constraints can be rewritten as:

$$T_H = \theta_H \frac{q_H}{\rho} - \theta_H \frac{q_L}{\rho} + \theta_L \frac{q_L}{\rho} \quad (5)$$

$$T_L = \theta_L \frac{q_L}{\rho} \quad (6)$$

\(9\)For an intriguing empirical analysis that relaxes the standard assumption of a representative agent see Broda and Romalis (2009).

\(10\)Returning to the iPad example, the constant marginal cost of a GB does appear to be constant. According to iSuppli estimates, the cost of the 16 GB flash memory is $16.80, $33.60 (32 GB) and $67.20 (64 GB).
This allows the profit maximization problem to be simplified to become:

$$\max_{q_L, q_H} \pi = \beta \left( q_H \frac{\phi_H}{\rho} - \theta_H \frac{q_L}{\rho} + q_L \frac{\phi_L}{\rho} - c q_H \right) + (1 - \beta)(q_L \frac{\phi_L}{\rho} - c q_L) - F \quad (7)$$

The design of the product line is then implicitly defined by:

$$\frac{\partial \pi}{\partial q_H} = \theta_H q_H^{\rho-1} - c = 0 \quad (8)$$
$$\frac{\partial \pi}{\partial q_L} = (1 - \beta)(\theta_L q_L^{\rho-1} - c) - \beta(\theta_H - \theta_L) q_L^{\rho-1} = 0 \quad (9)$$
$$\Rightarrow (\theta_L - \beta \theta_H) q_L^{\rho-1} = (1 - \beta) c \quad (10)$$

From these first order conditions we can derive a number of useful results.

First, combining these equations tells us about the relative features of the optimal design a firm offers. That is

$$\phi_{ LH} \equiv \frac{q_L}{q_H} = \max \left\{ 0, \left( \frac{(\theta_L - \beta \theta_H)}{(1 - \beta) \theta_H} \right)^{\frac{1}{\rho-1}} \right\} \quad (11)$$

Second, we can use (8) and (9) to derive the value function and solve for the average characteristic consistent with zero profit. Consider the resulting profit function:

$$\pi = \frac{c}{(\sigma - 1)}(\beta q_H + (1 - \beta) q_L) - F$$

Free entry (i.e. $\pi = 0$) implies that $q = \beta q_H + (1 - \beta) q_L = \frac{F}{c}(\sigma - 1)$.\textsuperscript{11}

A third result relates to the derivation of two-part tariffs from the general non-linear price schedule. Start with the low type. From the participation constraint:

$$T_L = \theta_L \frac{q_L}{\rho} = \theta_L \frac{\phi_H}{\rho} \frac{q_H}{\rho} = \theta_L \frac{\phi_H}{\rho} \frac{q_H}{\rho} + \frac{c q_H}{\rho}$$

Somewhat pre-empting the discussion of equilibrium, this expression can be further simplified by

\textsuperscript{11}This has a straightforward interpretation when $q_j$ is a measure of quality. For example the tablet targeted to the high type might have 64 GB while the low type might have 16 GB, but zero profits involve the average product sold to have $\beta 64 GB + (1 - \beta) 16 GB$. 
using the symmetry of output for each type in equilibrium to derive the \( \theta' \)'s (where \( \bar{\phi}_{LH} = \frac{\psi_L}{\psi_H} \)):

\[
\frac{\theta_L}{\theta_H} = \frac{\psi_L \ nq_H^0}{nq_L \ \psi_H} = \frac{\bar{\phi}_{LH}}{\phi_{LH}}
\]

(12)

This implies that the total payment required from a low type is simply:

\[
T_L = \bar{\phi}_{LH} \frac{cq_H}{\rho}
\]

(13)

Next derive the marginal price:

\[
p_L = \theta_L \ q_L^{\rho-1} = \frac{\theta_L}{\theta_H} \phi_{LH}^{\rho-1} \theta_H \ q_H^{\rho-1} = \frac{\bar{\phi}_{LH}}{\phi_{LH}} c
\]

(14)

Using (13) and (14) the fixed fee is:

\[
A_L = T_L - p_L q_L = \bar{\phi}_{LH} \frac{cq_H}{\rho} - \bar{\phi}_{LH} cq_H = \bar{\phi}_{LH} \frac{cq_H}{\rho} (1 - \rho)
\]

This has the implication that the total payment can be decomposed into a fixed and variable component as a function of \( \rho \): \( A_L = (1 - \rho)T_L \) and \( p_L q_L = \rho T_L \).

A similar exercise can be conducted for the high type.

\[
T_H = (1 - (\phi_{LH}^\rho - \bar{\phi}_{LH})) \frac{cq_H}{\rho}
\]

Since the variable price is given by \( c \), the fixed fee is:

\[
A_H = T_H - cq_H = (1 - \rho - (\phi_{LH}^\rho - \bar{\phi}_{LH})) \frac{cq_H}{\rho}
\]

Note that a comparison of the marginal prices reveals discrimination since \( p_L = \frac{\bar{\phi}_{LH}}{\phi_{LH}} c > p_H = c \), even though there is no incentive to implement third degree price discrimination since both types have the same elasticity of demand.

### 2.4 Equilibrium: Closed Economy Results

We’ll start by looking for a symmetric equilibrium in which all firms design the same generic product lines for their uniquely differentiated variety. In such an equilibrium the location of any residual
demand curve is given by:

\[ \theta_j = \frac{\psi_j}{nq_j^\rho} \]  

(15)

Using (8) and (15) this implies the high type’s product is designed to satisfy:

\[ q_H = \frac{\psi_H}{nc} \]  

(16)

To derive the characteristics of the low types product, \( q_L \), use (10):

\[
\left( \frac{\bar{\phi}_{LH}}{\phi_{LH}^\rho} - \beta \right) \theta_H q_L^{\rho-1} - (1 - \beta)c = 0
\]

so whether or not a product is offered to the low income type turns on the sign of \( \bar{\phi}_{LH} - \beta \phi_{LH}^\rho \). This sign can determined by using (11) and (12) to implicitly define the equilibrium relative product design:

\[ \beta \phi_{LH}^\rho + (1 - \beta) \phi_{LH} = \bar{\phi}_{LH} \]  

(17)

Notice that when \( \beta = 0 \), then \( \phi_{LH} = \bar{\phi}_{LH} = \frac{\psi_L}{\psi_H} \), and more generally \( \frac{d\phi_{LH}}{d\beta} < 0 \). Since the \( \phi_{LH} \) that solves (17) is strictly positive, it follows that any equilibrium also involves \( q_L > 0 \) (i.e. \( q_L = \phi_{LH}q_H > 0 \)).

With free entry, the associated zero profit condition uniquely defines the average characteristic of a firm as \( q = \frac{E}{c}(\sigma - 1) \). Given \( \phi_{LH} \) this implies that:

\[ q = \beta q_H + (1 - \beta)q_L = (\beta + (1 - \beta)\phi_{LH})q_H = (\beta + (1 - \beta)\bar{\phi}_{LH})q_H \]

The last equality carries the implication that since \( \bar{\phi}_{LH} \geq \phi_{LH} \), then \( q_H \geq \bar{q}_H \) (where the latter is the first best product for the high types). So the high type typically receives a product better than the first best. These conditions allow \( q_H \) to be expressed as:

\[ q_H = \frac{q}{\beta + (1 - \beta)\phi_{LH}} \]

(18)
Using (16) and (18)

\[ n_{2nd} = \frac{(\beta + (1 - \beta)\phi_{LH})\psi_H}{F(\sigma - 1)} \] (19)

This implies that the distortion not only influences the relative design of the products but also the number of varieties available. Specifically the deficit in the number of varieties relative to the first best is given by:

\[ n_{1st} - n_{2nd} = \frac{\psi_H}{F(\sigma - 1)}(1 - \beta)(\bar{\phi}_{LH} - \phi_{LH}) \]

As a consequence, the differential is non-monotonic in \( \beta \). In particular, when \( \beta = 0 \), the differential equals zero since \( \bar{\phi}_{LH} = \phi_{LH} \); that is, there is no distortion. The differential also equals zero when \( \beta = 1 \); in this case there is a distortion since \( \bar{\phi}_{LH} > \phi_{LH} \) but it has no practical effect since the weight on low types in the economy is zero.

A notable feature of the prices in section 2.3 is that they are ultimately a function of \( q_H \). The symmetric \( q_H \) can be written in two equivalent ways:

\[ q_H = \frac{\psi_H}{n c} = \frac{\psi_L}{\bar{\phi}_{LH} n c} \]

We can utilize the symmetric nature of the equilibrium to characterize the equilibrium set of prices. For the low type:

\[ n T_L = \frac{\psi_L}{\rho} \]

\[ n p_L q_L = \psi_L, \quad n A_L = \frac{(1 - \rho)}{\rho} \psi_L \]

For the high type:

\[ n T_H = \frac{\psi_H}{\rho} - \left( \phi_{LH}^p - \bar{\phi}_{LH} \right) \frac{\psi_H}{\rho} \]

\[ n p_H q_H = \psi_H, \quad n A_H = \frac{(1 - \rho)}{\rho} \psi_H - \left( \phi_{LH}^p - \bar{\phi}_{LH} \right) \frac{\psi_H}{\rho} \]

A comparison across types makes it clear that the main difference stems from the fixed fee, which is lower than full extraction for the high type due to the possibility of switching to the outside option.
(the low types product). It is also worth pointing out that the fixed/access fee only impacts the numeraire good since there are no income effects in the quasi-linear specification. We’ll return to this issue in the extension to a general equilibrium setting.

Given this equilibrium, we make the following comparisons to some interesting benchmarks.

**PROPOSITION 1.** In a setting where love of variety motivates product differentiation and firms engage in monopolistic competition and employ an indirect price discrimination mechanism, then in contrast to the monopoly outcome both types are always served in equilibrium.

Service for the low type follows directly from \( q_L > 0 \) in a symmetric equilibrium. To make a comparison with a monopolist, start with the typical single product monopoly. The main difference between monopolistic competition and a single product monopoly is associated with the interpretation of \( \theta_j \). Under monopoly, \( \theta_j \) represents a preference parameter and is therefore exogenous. From (10) it is clear that a single product monopolist would not serve the low type whenever \( \theta_L - \beta \theta_H \leq 0 \), which implies that provided \( \beta \) is sufficiently close to one the low type is not served. This result can be extended to a monopolist facing consumers who also have preferences given by (1). In this case the monopolist selects the index \( Q_j \), which under symmetry is composed of \( \text{Log} \left( n^{1/\rho q_j} \right) \) with the associated first order condition for the low type implying \( q_L = \frac{(\psi_L - \beta \psi_H)}{(1 - \beta) \rho} \). Consequently, the low-income segment of the market is only served if \( \beta \) is sufficiently small (i.e. \( \beta < \frac{\psi_L}{\psi_H} \)). This implies that the extension of the product line to always include the low type is not a reflection of the love of variety preferences but instead derives from the greater competition associated with monopolistic competition.

The second characteristic of the equilibrium worth emphasizing relates to the design of the product line. In particular, given that the aggregate characteristics of any individual firm is the same as the first best (i.e. \( q = \frac{E}{\sigma - 1} \)) but the design of the product line under monopolistic competition directs relatively more of this characteristic toward the high type (i.e. \( \bar{\phi}_{LH} > \phi_{LH} \) for \( \beta \in (0,1) \)). This implies that the high type is offered a product that is in excess of the first best while the low type is offered a product that is below the first best - that is, the equilibrium involves distortions at the top and the bottom. This contrasts with the monopoly outcome which involves no distortion at the top under the standard assumptions.

12Consumer surplus in this case is given by \( \psi_j \text{Log}(Q) \) and the objective function of the monopolist is

\[
\max_{q_L, q_H, n} \pi = \beta (\psi_H \text{Log}(q_H) - (\psi_H - \psi_L) \text{Log}(q_L) - cnq_H) + (1 - \beta)(\psi_L \text{Log}(q_L) - cnq_L) + \frac{\psi_L}{\rho} \text{Log}(n) - nF
\]

Derivation of the first order conditions is straightforward.
PROPOSITION 2. *In a model of monopolistic competition and second degree price discrimination, not only is the good designed for the low types below the efficient level (degraded), but the high type is offered a product that exceeds the efficient level (lavish).*

A final comparison to make is relative to the first best.

PROPOSITION 3. *In a model of monopolistic competition and indirect discrimination the level of welfare, relative to the first best, is non-monotonically related to \( \beta \). Moreover, the relative difference is largest for \( \beta \in (0, 1) \).*

The intuition for the non-monotonicity follows directly from the role of the distortion in shaping both the design of the product line and the number of firms. When \( \beta = 0 \) there is no distortion and consequently the first best is achieved. However, as \( \beta \) increases from zero, a typical firm distorts the design of the product line to extract rents from the high type. The inability to extract all the rents implies that the number of firms is sub-optimally low and the distorted product line also lowers welfare. So in the neighborhood of \( \beta = 0 \), increases in \( \beta \) imply that welfare rises less slowly than under the first best. At the other extreme, as \( \beta \to 1 \), the first best is once again achieved, though for a different reason. The difference is not that the product line isn’t distorted (it is), but that this distortion is not important for welfare since the weight attached to the low type goes to zero. Given that welfare coincides with the first best at the two extremes but deviates from it otherwise, it follows that the largest differential occurs at a \( \beta \in (0, 1) \). It is worth pointing out that the non-monotonicity is a characteristic of indirect price discrimination and therefore there is no counter-part non-monotonicity in the standard model of monopolistic competition with linear prices. This suggests that unlike in the standard model of intra-industry trade, differences in the distribution of income across countries can directly influence the size and distribution of the gains from trade. What are the directions of these differences and are they similar within countries? These issues are addressed in the next section.

3 Implications of International Trade

3.1 Standard Model

As a benchmark consider the welfare outcomes with preferences defined by (1) and the standard model of monopolistic competition. Since a firm sets a single per unit price \( p = \frac{c}{\rho} \) it follows that
in a symmetric equilibrium $q_{LS} = \bar{\phi}_{LH} q_{HS} = \bar{\phi}_{LH} \frac{\psi_H}{\tilde{n}_C}$, where the subscript $s$ denotes standard.

In general we can calculate welfare for each type using their indirect utility:

$$U_j = \psi_j \log(Q_j) + y_j = \psi_j \log \left( n^{1/2} q_j \right) + y_j$$

In the case of the standard model (where $\tilde{n} = n^{1-\rho}$)

$$V_{Ls} = \psi_L \log (\tilde{n}_s \bar{\phi}_{LH}) - \psi_L + m_L$$
$$V_{Hs} = \psi_H \log (\tilde{n}_s) - \psi_H + m_H$$

If this country engages in free trade with another country, then the sole mechanism for welfare change is through the number of varieties. This occurs because relative prices of each variety are not influenced by either the size of the market or the distribution of income within the market. Consequently, the proportional change in welfare for each income group is given by $\frac{dV_{js}}{\psi_j} = \dot{V}_{js} = \frac{d\tilde{n}_s}{\tilde{n}_s} = \dot{\tilde{n}}_s$. This says that the largest percentage gains from trade accrue to the country with the largest proportional increase in the number of varieties and that there is no difference in the distribution of these gains within a country.

3.2 International Trade and Indirect Discrimination

Against this benchmark consider the integration of two countries with potentially different income distributions as measured by $\beta$. Given the structure of the model, income level and therefore the distribution of income within a country is not altered by integration. Instead the focus is on the implications of the distribution of income for the consequences of integration, not the other way around which is more typical (e.g. Stolper-Samuelson theorem, though we will return to this issue in later sections). It is important to stress this difference since a firm designs its product line based on the distribution of income of its customers. In a move from autarky to free trade, the product line under autarky is designed for the local $\beta$ while under free trade the product line is designed based on the global $\beta$. To trace through the implications of this change note that under indirect
discrimination the indirect utility functions are:

\[ V_{L2} = \psi_L \log (\tilde{n}_2 \phi_{LH}) - \frac{\psi_L}{\rho} + m_L \]  
\[ V_{H2} = \psi_H \log (\tilde{n}_2) - \frac{\psi_H}{\rho} (1 - \phi^\rho_{LH} + \phi_{LH}) + m_H \] 

Welfare change in response to a change in \( \beta \) net of variety gains (i.e. \( d\tilde{V}_{j2} = \tilde{V}_{j2} - \tilde{n}_2 \)):

\[ \frac{d\tilde{V}_{L2}}{d\beta} = \frac{\phi_{LH}}{\beta} \]  
\[ \frac{d\tilde{V}_{H2}}{d\beta} = \phi^\rho_{LH} \frac{\phi_{LH}}{d\beta} \]  
\[ \frac{\tilde{V}_{L2} - \tilde{V}_{H2}}{d\beta} = (1 - \phi^\rho_{LH}) \frac{\phi_{LH}}{d\beta} \]

We can map the implications of a global \( \beta \) that differs from a local \( \beta \) by considering the following derivative:

\[ \frac{\dot{\phi}_{LH}}{\beta} = -\frac{\beta (\phi^\rho_{LH} - \phi_{LH})}{\rho \beta \phi^\rho_{LH} + (1 - \beta) \phi_{LH}} = -\frac{\phi_{LH} - \phi_{LH}}{\rho \beta \phi^\rho_{LH} + (1 - \beta) \phi_{LH}} < 0 \]  

So the standard variety gains are either amplified or nullified depending on the characteristics of the trading partner. Specifically if the local \( \beta \) is greater than the global \( \beta \) then \( \phi_{LH} > 0 \). In this case both the low and high types gain but the low type gains proportionately more. Note that the opposite occurs if the local \( \beta \) is less than the global \( \beta \). So we have a magnification result that says that the welfare of the low types is more sensitive to integration (i.e. low types welfare is more elastic with respect to trade barriers).

**PROPOSITION 4.** When countries with different \( \beta \)'s integrate the standard variety gains are modified in a way that results in differences both across and within countries.

**Across countries**

The country with the higher \( \beta \) receives additional gains from trade for all income groups as the distortion imposed on the low income group is decreased relative to autarky. The country with the lower \( \beta \) has the gains from trade for all income groups reduced as the distortion imposed on the low type is increased relative to autarky.

**Within countries**

Within the high \( \beta \) country, the low income group gains proportionately more than the high income group. Within the low \( \beta \) country, the low income group loses proportionately more than the high group.
income group.

**Proof:** These results follow directly from (22), (23), (24) and (25).

Note that the differences in \( \beta \) translate directly into differences in per capita income in this two type setting. This tends to conflate both the distribution of income and the level of income. To separate these components we now extend the model to consider a set-up with an arbitrary number of types.

### 4 Extensions

#### 4.1 More than 2 Types

Assume that there are \( H \) income groups, with the groups ordered \( m_j < m_k \). To account for the variation across the \( H \) types we introduce the following notation conventions:

\[
\phi_{jk} = \frac{q_j}{q_k}, \quad \bar{\phi}_{jk} = \frac{\psi_j}{\psi_k}, \quad j \in \{1, H\} \text{ and } k = \min \{j + 1, H\}
\]

The \( H \) type case generates the following objective function for a typical firm:\(^{13}\)

\[
\max_{(q_j)} \pi = \sum_{j=1}^{H} \beta_j \left( \theta_j \frac{q_j^\rho}{\rho} - cq_j - \frac{\sum_k \beta_k (\theta_k - \theta_j) q_j^\rho}{\beta_j} \right) - F
\]

Taking first order conditions gives:

\[
\frac{\partial \pi}{\partial q_j} = \theta_j q_j^{\rho - 1} - c - \frac{\sum_k \beta_k (\theta_k - \theta_j) q_j^{\rho - 1}}{\beta_j} = 0 \quad (26)
\]

Using (26), symmetry (i.e. \( \theta_j / \theta_H = \bar{\phi}_j H / \phi_{jH} \)) and normalizing relative to the design of the high income groups product, then the equilibrium product design for the \( H - 1 \) income groups solves:

\[
\phi_{jk}^\rho \left( \sum_{k=j+1}^{H} \beta_k \right) + \beta_j \phi_{jk} \frac{\phi_{kH}}{\phi_{kH}} = \bar{\phi}_{jk} \left( \sum_{j}^{H} \beta_j \right) \quad (27)
\]

\[
\Rightarrow (\phi_{jk}^\rho - \bar{\phi}_{jk}) \left( \sum \beta_k \right) = \beta_j (\bar{\phi}_{jk} - \phi_{jk} \frac{\phi_{kH}}{\phi_{kH}}) \quad (28)
\]

\(^{13}\)Where the participation constraint is assumed to bind for the lowest type and the incentive constraints bind for the other types.
4.1.1 Indirect Utility

Indirect price discrimination gives the following transformed indirect utility functions:

\[ \tilde{V}_j = \frac{1}{\rho} \ln \left( n^{1-\rho} \right) + \ln (\phi_{jH}) - \frac{1}{\rho} \left( \sum_{q=2}^{j} \bar{\phi}_{qj}(1 - \phi_{qg}) + \bar{\phi}_{1j} \right) + \frac{m_j}{\psi_j} \]  

(29)

where \( v = g - 1 \). Since moving from autarky to free trade means that the product line is now designed with respect to a different distribution of income, we can gain insight into the welfare implications by differentiating these surplus functions. In proportional change terms (net of variety gains):

\[ d\tilde{V}_j = \hat{\phi}_{jH} + \sum_{g=2}^{j} \bar{\phi}_{gj} \phi_{vg} \hat{\phi}_{vg} \]  

(30)

With this structure we have the following generalization of the two type case.

**PROPOSITION 5.** The Home country’s gains from trade are most likely to be augmented for all income groups, with the largest gains for the lowest income groups, if:

1. The likelihood ratio of Home’s income distribution dominates the likelihood ratio of the Foreign country;

2. The marginal utility of income is bounded to be strictly positive.

**Proof:**

From (30) it is clear that if the distortion is reduced for all types (i.e. \( \hat{\phi}_{jk} > 0 \), \( \forall j,k \)), then all types gain. This same condition implies the size of the gains are inversely related to income. From (30):

\[ d\tilde{V}_j - d\tilde{V}_j = (1 - \phi_{jk})\hat{\phi}_{jk} + \sum_{g=2}^{j} (\bar{\phi}_{gj} - \bar{\phi}_{gk}) \phi_{vg} \hat{\phi}_{vg} \]

To determine how the vector of distortions change, totally differentiate (27) to get:

\[ \text{sign} \left\{ \hat{\phi}_{jk} \right\} = \text{sign} \left\{ \left( \phi_{jk}^H \hat{\phi}_{jk} \right) \sum_k^{H} \beta_k \left( \hat{\beta}_j - \hat{\beta}_k \right) - \hat{\phi}_{kH} \beta_j \phi_{jk} \phi_{kH} \right\} \]

Note that the first term on the right hand side is positive under likelihood ratio dominance. To see this, note that distribution \( A \) is said to dominate \( B \) in terms of the likelihood ratio if \( \frac{\beta_A^{\mathcal{X}}}{\beta_B^{\mathcal{X}}} \leq \frac{\beta_A^{\mathcal{Y}}}{\beta_B^{\mathcal{Y}}} \)
for all $j < k$. This implies $\frac{\beta_B^j}{\beta_A^j} = \frac{\beta_A^j + \Delta \beta_j}{\beta_A^j} \geq \frac{\beta_B^k}{\beta_A^k} = \frac{\beta_A^k + \Delta \beta_k}{\beta_A^k}$. This implies the following ordering of proportional changes: $\hat{\beta}_j \geq \hat{\beta}_k$ for all $j < k$. Hence the sign of the first term is positive.

To ensure that the sign overall is positive, the size of the second term needs to be disciplined. Under the assumption that the marginal utility of income is bounded, this implies that as income gets larger, $\bar{\phi}_{kH} \to 1$. In this case, $\hat{\phi}_{kH} \to 0$. ■

To build intuition consider $H = 3$. For the three type case, the set of all possible income distributions can be represented by an equilateral triangle with an altitude of unity. Any point in this ternary plot represents a specific distribution of income. An easy way to determine the implied distribution of income is to examine the shortest distances from the point of interest to each of the three sides. Any such distance gives the fraction of the population with the income of the opposite corner.\textsuperscript{14}

The claim in the proposition relies on the likelihood ratio dominance (LRD). In the three type case we can identify all the distributions where this condition applies relative to a given income distribution (that is the likelihood ratio increases with income). LRD implies that $\hat{\beta}_1 \geq \hat{\beta}_2 \geq \hat{\beta}_3$. Since there is an adding up constraint, we know that if $\hat{\beta}_1 > 0$ then $\hat{\beta}_3 < 0$ but $\hat{\beta}_2 \leq 0$. So tracing out the different possibilities for $\hat{\beta}_2$ defines the set of distributions that satisfy LRD. The boundaries of that set are mapped by considering the extremes (i) $\hat{\beta}_2 = \hat{\beta}_3$ and (ii) $\hat{\beta}_2 = \hat{\beta}_1$. An example is depicted below in Figure 1 where the hatched area represents the distributions dominated by $A$.

With this background we can now address the claims in the proposition. A sufficient condition for a consistent ranking is that $\hat{\phi}_{12} > 0$ and $\hat{\phi}_{23} \geq 0$. This ensures the following differences are positive:

$$d\hat{V}_1 - d\hat{V}_2 = (1 - \phi_{12}^p)\hat{\phi}_{12} > 0$$  \hspace{1cm} (31)

$$d\hat{V}_2 - d\hat{V}_3 = (1 - \bar{\phi}_{23})\phi_{12}^p\hat{\phi}_{12} + (1 - \phi_{23}^p)\bar{\phi}_{23} > 0$$  \hspace{1cm} (32)

The change in product line design induced by distributional differences are determined by:

$$\hat{\phi}_{23} = \frac{\beta_3(\hat{\beta}_2 - \hat{\beta}_3)(\phi_{23}^p - \bar{\phi}_{23})}{\beta_3 p \phi_{23}^p + \beta_2 \bar{\phi}_{23}}$$  \hspace{1cm} (33)

\textsuperscript{14}This decomposition follows from Viviani’s theorem.
As move down this line, \( \beta_3 = \beta_2 < 0 \) and \( \beta_1 > 0 \)

As move down this line, \( \beta_1 = \beta_2 > 0 \) and \( \beta_3 < 0 \)

Figure 1: LRD

\[
\hat{\phi}_{12} = \frac{(\beta_2(\hat{\beta}_1 - \hat{\beta}_2) + \beta_3(\hat{\beta}_1 - \hat{\beta}_3))(\phi_{12}^u - \overline{\phi}_{12})}{(1 - \beta_1)\rho \phi_{12}^u + \beta_1 \phi_{12}} - \frac{\dot{\phi}_{23} \beta_1 \phi_{12} \dot{\phi}_{23}}{(1 - \beta_1)\rho \phi_{12}^u + \beta_1 \phi_{12}} \tag{34}
\]

To see the role of LRD in determining a consistent ranking of gains within a country with income distribution \( A \), consider any point on the diagonal boundary. Along this boundary, \( d\beta_1 = -(d\beta_2 + d\beta_3) > 0 \) and \( \hat{\beta}_2 = \hat{\beta}_3 < 0 \). Since this implies \( \hat{\phi}_{12} > 0 \) while \( \hat{\phi}_{23} = 0 \), all types clearly benefit in this case (i.e. (31) and (32) are satisfied). Now consider the vertical boundary. In this case \( \hat{\beta}_1 = \hat{\beta}_2 > 0 \), which implies \( \hat{\phi}_{23} > 0 \). To ensure that \( \hat{\phi}_{12} > 0 \), requires that \( \hat{\phi}_{23} \) is relatively small, which can be guaranteed if both \( \psi_2 \) and \( \psi_3 \) are relatively similar (i.e. the inverse of the marginal utility in both cases is close to the upper bound). Under these conditions the system (31) and (32) is positive. Consequently when \( A \) and \( B \) can be ranked according to the LRD and the marginal utility of income is bounded, then not only do all income groups gain in \( A \) but the gains are proportionally greater further down the income distribution. Conversely, \( B \) has exactly the opposite experience. This implies both a consistent ranking within and across countries.
5 General Equilibrium

The previous results were all derived in a partial equilibrium setting. An advantage of this perspective is that it allows a straightforward comparison with the previous indirect discrimination literature which examines a monopoly setting. However, many international trade issues require a general equilibrium model. To extend the analysis in this direction we now consider a utility function that allows for an endogenous marginal utility of income.

For simplicity we only consider two types in this section. Upper tier utility is assumed to be Cobb-Douglas:

\[ U = \left( \frac{Q}{\gamma} \right)^{\gamma} \left( \frac{Y}{1-\gamma} \right)^{1-\gamma}, \quad \gamma \in (0, 1) \]

The demand for each sector is then given by:

\[ Y_j = (1 - \gamma) \bar{m}_j, \quad Q_j = \frac{\gamma \bar{m}_j}{P_j} \quad \text{where} \quad P_j = \left[ \sum_i p_{ji}^{\rho} \right]^{\frac{\rho-1}{\rho}} \]

While \( \rho \) and \( \gamma \) represent standard preference parameters, the main modification is in relation to net income. The divergence between \( \bar{m}_j \) and \( m_j \) under non-linear prices means that both sectors are influenced by the strategy adopted by firms in the \( Q \) sector, while this is not the case under linear pricing.

Demand for an individual variety, \( i \), targeted to consumers with income level \( j \), is given by:

\[ p_{ji} = \theta_j q_{ji}^{\rho-1} \quad \text{with} \quad \theta_j = \frac{\gamma \bar{m}_j}{Q_j^\rho} \quad (35) \]

5.1 One factor model: Equilibrium

Since a typical firm assumes that it is too small to influence the marginal utility of income, the characterization of firm behavior is exactly the same as under quasi-linear preferences. The main difference arises in the derivation of the industry equilibrium since income effects are now present in the model. The derivation of the industry equilibrium is where we pick-up the analysis.

A notable feature of optimal firm behavior is that it can be characterized as a function of \( q_H \).
Note that the symmetric $q_H$ can be written in two equivalent ways:

$$q_H = \frac{\gamma(m_H - nA_H)}{nc} = \frac{\gamma(m_L - nA_L)}{nc\phi_{LH}}$$

where $\phi_{LH} = \frac{\bar{m}_L}{m_H}$

Since these expressions are a function of the fixed fees we can further refine the calculation of these fees. Start with the fixed fee for the low type:

$$A_L = \frac{(1 - \rho)(1 - \gamma)}{\rho_mL} n$$

$$\Rightarrow m_L - nA_L = \frac{\rho_mL}{\gamma + (1 - \gamma)\rho}$$

So in equilibrium net income for the low type is a constant fraction of total income.

The fixed fee for the high types is:

$$A_H = \frac{(1 - \rho - \phi_{LH}^H)m_H}{(1 + \rho\tilde{\gamma} - \phi_{LH}^H)n} + \frac{\rho m_L}{\gamma(1 + \rho\tilde{\gamma})(1 + \rho\tilde{\gamma} - \phi_{LH}^H)n}$$

$$\Rightarrow m_H - nA_H = \frac{\rho((1 + \rho\tilde{\gamma}) - \alpha)m_H}{\gamma(1 + \rho\tilde{\gamma})(1 + \rho\tilde{\gamma} - \phi_{LH}^H)}$$

where $\alpha = \frac{m_L}{m_H}$

In contrast to the low type, the net income of the high type is a function of the distortion, $\phi_{LH}^H$ and therefore determined by the parameters of the model, including $\beta$.

Imposing symmetry and solving (11) implicitly defines $\phi_{LH}$:

$$\left(\beta + (1 - \beta)\frac{\alpha}{(1 + \rho\tilde{\gamma})}\right)\phi_{LH}^H + \left(1 - \frac{\alpha}{(1 + \rho\tilde{\gamma})}\right)(1 - \beta)\phi_{LH} = \alpha$$

(36)

### 5.2 Indirect Utility

Welfare is given by indirect utility:

$$U_L = \left(\frac{n^{1-\rho}}{c}\right)\gamma \phi_{LH}^H \gamma \frac{\rho m_L}{\gamma(1 + \rho\tilde{\gamma})}$$

(37)

$$U_H = \left(\frac{n^{1-\rho}}{c}\right)\gamma \phi_{LH}^H \gamma \frac{\rho m_L}{\gamma(1 + \rho\tilde{\gamma})}$$

(38)
To make progress note that

\[ \hat{\phi}_{LH} = \frac{\bar{m}_L}{\bar{m}_H} = \frac{(1 + \rho \hat{\gamma} - \phi_{LH}^p)\alpha}{(1 + \rho \hat{\gamma}) - \alpha} \]

So

\[ \hat{\phi}_{LH} = \frac{-\rho \phi_{LH}^p}{(1 + \rho \hat{\gamma} - \phi_{LH}^p)} \hat{\phi}_{LH} \]

This implies the proportional change in welfare is:

\[ \hat{u}_L = \gamma \hat{n} + \gamma (\hat{\phi}_{LH} - \hat{\phi}_{LH}) \]
\[ = \gamma \hat{n} + \gamma \left( \frac{1 + \rho \hat{\gamma} - (1 - \rho)\phi_{LH}^p}{(1 + \rho \hat{\gamma} - \phi_{LH}^p)} \right) \hat{\phi}_{LH} \]

(39)

The high type also gains when the low type gains:

\[ \hat{u}_H = \gamma \hat{n} - \hat{\phi}_{LH} \]
\[ = \gamma \hat{n} + \frac{\rho \phi_{LH}^p}{(1 + \rho \hat{\gamma} - \phi_{LH}^p)} \hat{\phi}_{LH} \]

(40)

So

\[ \hat{u}_L - \hat{u}_H = \gamma (1 - \phi_{LH}^p)(1 + \rho \hat{\gamma}) \frac{\hat{\phi}_{LH}}{(1 + \rho \hat{\gamma} - \phi_{LH}^p)} \]

(41)

The sign of this difference is given by:

\[ \frac{\hat{\phi}_{LH}}{d\beta} = -\frac{((1 + \rho \hat{\gamma}) - \alpha)(\phi_{LH}^p - \phi_{LH})}{\rho \phi_{LH}^p(\beta(1 + \rho \hat{\gamma}) + (1 - \beta)\alpha) + ((1 + \rho \hat{\gamma}) - \alpha)(1 - \beta)\phi_{LH}} < 0 \]

(42)

Since the system (39)-(42) has the same properties as (22) - (25), it follows that proposition 4 also applies in a general equilibrium setting.

5.3 Same old gains?

The general equilibrium setting facilities comparisons with other extensions that have been proposed to the standard model. One prominent development is the introduction of firm heterogeneity. However, Arkolakis et al. (2012) show that while firm heterogeneity may influence the welfare attributable to various dimensions within the model, the aggregate gains from trade are unaltered...
by the details of the model under surprisingly broad conditions. This naturally raises the question whether the addition of consumer heterogeneity is subject to the same constraint, and if not, how much difference could indirect discrimination make?

Using (37) and (38) the ratio of indirect utilities for both income groups is given by:

\[
\frac{V^F}{V^A} = \left( \frac{n_F}{n_A} \right)^{\frac{\gamma}{\sigma}} \left( \frac{\phi_F \bar{\phi}_A}{\phi_F \bar{\phi}_A} \right)^{\tau}
\]

\[
\frac{V^F_H}{V^A_H} = \left( \frac{n_F}{n_A} \right)^{\frac{\gamma}{\sigma}} \left( \frac{\bar{\phi}_F}{\bar{\phi}_A} \right)
\]

The first term in each expression conforms to the standard aggregate gains from trade identified by Arkolakis et al. (2012). As is apparent, these gains are augmented by product design terms associated with consumer heterogeneity.

To gain some insight into the potential impact of changes in product line design consider the following parameter values: \( \rho = \frac{1}{2}, \gamma = \frac{1}{5}, \alpha = \frac{1}{2} \). The largest positive effect is for a low income consumer in a predominately rich country that integrates into a low income world (\( \beta_A = 1 \) and \( \beta_F = 0 \)), \( \left( \frac{\bar{\phi}_F}{\bar{\phi}_A} \right)^{\gamma} = 1.40 \), while the result for a high income consumer in the same country is \( \frac{\bar{\phi}_F}{\bar{\phi}_A} = 1.10 \). These outcomes contrast with those for a consumer in a relatively low income country integrating into a relatively high income world (\( \beta_A = 0 \) and \( \beta_F = 1 \)). In this setting, a low income consumer only derives gains from increased product variety and suffers from inferior product design: \( \left( \frac{\bar{\phi}_F}{\bar{\phi}_A} \right)^{\gamma} = 0.70 \). The impact of inferior product design also reduces the gains available to high income groups in the same country \( \frac{\bar{\phi}_F}{\bar{\phi}_A} = 0.90 \). While these numbers are purely illustrative, they do suggest that the implications of indirect discrimination can have a relatively large impact on the gains from trade. Moreover, this is a setting where the aggregate gains from trade are thought to be robust to the details of market structure - a presumption that is clearly sensitive to pricing behavior.

### 5.4 Comparing Prices Across Destination Markets

At this point it is instructive to revisit the interpretation of the CES model and its capacity to account for observed patterns in the data. A typical finding is that unit values are positively associated with per capita income. When the quantity interpretation is adopted it is clear that the per unit price does not vary with the per capita income of the destination market since the mill price is \( \frac{c}{\rho} \). However, if we adopt the quality interpretation, that is \( q \) is an index of quality rather than quantity, then a systematic relationship between per capita income and price does arise. The price per unit of quality is \( c \frac{q}{\rho} \), but since only one unit is consumed, the price (or unit value) is the total payment (or
what is typically considered revenue), $\xi_p q_i$. Since locations with higher per capita income purchase higher quality products, this naturally translates into a positive correlation between unit value and per capita income.

While the standard model can be reinterpreted to account for the positive correlation between unit values and per capita income, a second finding cannot. Bekkers et al. (2012) document a negative relationship between within country inequality and per unit import prices. Since the standard model uses a proportional pricing rule, the price of the average product is the same as the average price of the set of products sold. Hence, there is no relationship between unit values and income inequality. However, such a negative relationship does arise under indirect discrimination. The prices associated with the optimal product line have the feature that they are increasing and concave in consumer type. Consider the prices in the three type case. By construction we know that $T_H > T_M > T_L$ and that $T_j' = p_j$. Therefore, the concavity of the price schedule follows from the marginal price declining in income:

$$p_L = \frac{\bar{\phi}_{LH}}{\phi_{LH}} c > p_M = \frac{\bar{\phi}_{MH}}{\phi_{MH}} c > p_H = c$$

Consequently, a negative correlation between average price and income dispersion can be constructed.

Imagine two countries purchasing products off the above menu. Country 1 is populated entirely by middle types (no dispersion) and country 2 is populated by an equal number of low and high types. For expositional purposes assume $m_M = \frac{m_L + m_H}{2}$. Since the menu is concave in type it follows that $T_M > \frac{T_L + T_H}{2}$. This confirms a negative correlation between average transaction price and income dispersion, holding average income constant.

5.5 Comparing Price Regimes

While the distribution of the gains from trade have common characteristics in both a partial and general equilibrium setting, the extension to a general equilibrium environment also helps to clarify another interesting comparison. This relates to the welfare ranking across price regimes and is summarized by the following proposition:

**PROPOSITION 6.** For any $\beta \in (0, 1)$ there exists a $\gamma$ where the welfare from linear pricing is greater than the welfare from indirect discrimination.

The logic behind this result is straightforward. Start by considering a world where $\gamma = 0$. This
describes a setting where the only sector is the $Y$ sector, and consequently welfare is first best. As $\gamma$ increases then the weight given to the $Q$ sector increases in the economy, and the welfare comparison across pricing regimes depends on the distortion imposed by each regime. Under linear pricing this distortion arises from the fact that the price is a constant mark-up over marginal costs. Under indirect discrimination, the size and impact of the distortion is a function of $\beta$. In particular, provided $\gamma > 0$ and $\beta \in (0, 1)$ then indirect discrimination will always deliver a welfare outcome less than the first best. However, as $\gamma \to 1$, the deviation from the first best under linear pricing is reduced and in the limit goes to zero (i.e. the correct relative prices are achieved in a single sector model involving only the $Q$ sector). Since the distortion disappears under linear pricing but persists under indirect discrimination, it follows that there exists a $\gamma$ where linear prices deliver a higher welfare outcome. The intuition is captured in Figure 2.

![Figure 2: Welfare comparison](image)

5.6 Two factors

Up to this point the distribution of income has been fixed by the relative endowments across types in a single factor model. However, the distribution of income can also be endogenized. To explore this issue consider a two factor model, labor ($L$) and capital ($K$), which receive factor payments of $w$ and
respectively. We make the typical assumptions relating to technology in the two sectors, with the additional simplifying assumption about the costs functions in the $Q$ sector: let the marginal costs be denoted by $c_Q(w,r)$, and the fixed costs in the industry by $F_{c_Q}(w,r)$ - the marginal and fixed costs have the same factor intensity (an example of this framework is presented in the appendix).

The standard formulation of this model couches discussion of income distribution in terms of the Stolper-Samuelson theorem. To isolate the new mechanism we’ll consider a situation where both countries have the same relative endowment and two types of individuals within their borders - high income and low income types. The low types have the same relative endowments as the high types, just a lower level of each factor. The only difference across countries is the relative frequency of each type in the local population. This completely shuts down the Stolper-Samuelson mechanism.

Building on the intuition from the previous section consider two extreme situations, one where all types are low types and another where all types are high types. In both of these cases, the homogeneity in the population means that a typical firm has no incentive to distort any products sold since there is no heterogeneity in the population. Consequently when $\beta = 0$ and $\beta = 1$, the first best is achieved and relative factor prices are also first best. Now consider adding a small amount of heterogeneity in each case. The optimal response of the firm is to offer a product line that involves a degraded product being offered to the low type. However, the overall distortion will be relatively small - though the reason is different in each instance. If $\beta = 0$, then adding some high types only causes a small distortion to the design of the product since the market is dominated by the low types. Starting at $\beta = 1$, the distortion imposed on the low type is relatively large, but the aggregate impact is relatively small since there are so few of them in the population. In both cases the deviation from the first best is small but in the same direction - too few resources are devoted to the differentiated goods sector. This occurs not because the scale of any individual firm is too small, but because too few firms enter in equilibrium. That is, consumer heterogeneity undermines the ability of a firm to extract rents relative to the first best. With the differentiated goods sector smaller than the first best, then the demand for the factor used intensively in that sector will also be lower than the first best, which will cause the relative return to that factor to be lower as well.

This discussion also implies that the relationship between the aggregate distortion and $\beta$ will be non-monotonic. That is, the aggregate distortion is composed of two elements; the degraded product and the frequency of the low types in the population. These forces are negatively related, which implies there must exist a $\beta^* \in (0,1)$ where the aggregate distortion is maximized. Since
the distortion in non-monotonic so is the deviation of relative factor prices from the first best. The relationship between factor prices and $\beta$ is depicted in Figure 3.

Consequently the largest impact on relative factor prices is associated with integration of two countries with very different income distributions. Moreover, they will have a change in relative factor prices that are positively correlated. If both countries have $\beta$'s on the same side of $\beta^*$, then the change in relative factor prices will be negatively correlated. What is clear is that differences in income distribution can give rise to a rich set of possibilities. These are summarized in the following proposition.

**PROPOSITION 7.** When two countries with same relative endowments integrate, then the response of factor prices are determined by the ranking of the $\beta$'s across countries and in relationship to $\beta^*$. If both $\beta$'s are strictly less than or strictly greater than $\beta^*$, then factor price change across the countries will be negatively correlated. If the $\beta$'s are separated by $\beta^*$, then factor prices will be positively correlated.

The non-monotonicity predicted is in contrast to the existing models which all predict a monotone relationship (i.e. a positive or a negative correlation). However, the empirical literature has
documented a wide pattern of responses to trade liberalization (see Harrison et al. (2011), Amiti and Cameron (2012) and the discussion Zhu and Trefler (2005) of Freeman and Oostendorp (2008)).

6 Conclusion

The objective of this paper is to incorporate a role for the distribution of income into the standard model of trade by allowing firms to condition their behavior on consumer income. This is achieved by expanding their strategy space to include the possibility of designing product lines. This enriches firm behavior in a way that is present in the closed economy literature on firm strategy. However, in contrast to much of the previous closed economy literature, the current setting involves both within sector competition and a general equilibrium perspective. The resulting model is both tractable and provides a link between the distribution of income and the gains from trade. The link is directly related to firm strategy that indirectly discriminates between the various income classes, which in equilibrium results in a product line that differs from the first best allocation. Since the distortions have the largest negative welfare impact at the lower end of the income distribution, this is where the consequences of international integration are also most pronounced. However, trade can mitigate these welfare losses in countries with a “good” distribution of income, while amplifying them in countries with a “bad” distribution of income. Aside from the welfare impact, the model offers a perspective on a range of empirical patterns that have previously been deemed inconsistent with the standard model of international trade based on identical and homothetic preferences. In particular, a number of findings on the relationship between import prices and country characteristics are consistent with the model. Additionally, the product line approach adopted also predicts a rich set of responses for factor prices from international integration that reflects the richness documented in studies of trade and income inequality. All these results follow from enriching firm strategy, rather than altering the structure of consumer preferences.
7 Appendix

7.1 Equilibrium factor prices and firm behaviour

Technology is symmetrically asymmetric:
\[ Y = \left( \frac{L}{\eta} \right)^{\eta} \left( \frac{K}{1-\eta} \right)^{1-\eta} \]
\[ Q = \left( \frac{K}{\eta} \right)^{\eta} \left( \frac{L}{1-\eta} \right)^{1-\eta} \]
where \( \eta \neq \frac{1}{2} \)

The associated unit cost functions are:
\[ c_y(w, r) = w^\eta r^{1-\eta} \]
\[ c_q(w, r) = r^\eta w^{1-\eta} \]

Shephard’s lemma gives:
\[ \frac{\partial c_y(w, r)}{\partial w} = \eta w c_y = a_y L \]
\[ \frac{\partial c_y(w, r)}{\partial r} = (1-\eta) r c_y = a_y K \]

Using the full employment conditions:
\[ K = a_y Y + a_q Q \]
\[ L = a_y Y + a_q Q \]

This solves out as
\[ Y = \frac{1}{2\eta - 1} \left( \frac{w}{r} - \frac{(1-\eta) K}{\eta L} \right) \left( \frac{r}{w} \right)^\eta L \]
\[ Q = \frac{1}{2\eta - 1} \left( \frac{\eta K}{(1-\eta) L} - \frac{w}{r} \right) \left( \frac{r}{w} \right)^{1-\eta} (1-\eta) L \]

Market clearing in the Q sector: use \( \omega = w/r \) and \( \kappa = K/L \)
\[ (\omega + k)(2\eta - 1) = 2(\eta k - \omega(1-\eta)) \]
\[ \omega = \kappa \] (43)

To accommodate the love of variety preferences assume that a firm in the q sector has the following cost function:
\[ C(w, r, q) = F c_q(w, r) + c_q(w, r)q \]
\[ = (F + q) c_q \]

using zero profit \[ = (F + F(\sigma - 1)) c_q \]
\[ = \sigma F c_q \]

This implies the following average cost function:
\[ C(w, r, q)/q = \frac{\sigma F c_q}{F(\sigma - 1)} \]
\[ AC_q = \frac{c_q}{\rho} \]
Since \( c_q \) is the same as above, Shephard’s lemma is only slightly modified to give:

\[
\frac{\partial AC_q(w,r)}{\partial w} = \frac{\eta c_q}{w \rho} = a_{qL} \\
\frac{\partial AC_q(w,r)}{\partial r} = \frac{(1 - \eta) c_q}{r \rho} = a_{qK}
\]

Which implies the following full employment conditions:

\[
K = a_{yK} Y + a_{qK} Q \\
L = a_{qL} Y + a_{qL} Q
\]

While it is possible to solve this system in exactly the same way as before, there is enough structure in the model to determine \( n \) and \( q \) from profit maximization, zero profits and market clearing for a typical differentiated good.

### 7.2 Solving for \( n \)

Note that the equilibrium level of firm output is the same across pricing regimes. Since this is a hard constraint, the number of firms in the market must adjust to make sure that this is also the market clearing quantity. For a given residual demand curve, each pricing regime generates different profits, which implies that the number of firms will differ as well.

Under 1\textsuperscript{st} degree price discrimination, the level of output is chosen to be efficient.

\[
\beta q_H + (1 - \beta)q_L = \frac{\beta \gamma \bar{m}_H}{nc} + \frac{(1 - \beta) \gamma \bar{m}_L}{nc} \\
q = \frac{\beta \gamma \bar{m}}{nc} \\
\Rightarrow nq = \frac{\beta \gamma \bar{m}}{c} \equiv Q
\]

Under 2\textsuperscript{nd} degree price discrimination, the level of output is chosen to be efficient for the high type but not the low type.

\[
q_L = \frac{\beta \gamma \bar{m}_L}{np_L} = \frac{\beta \gamma \bar{m}_H}{nc} \phi_{LH} \quad \text{using} \quad p_L = \frac{\phi_{LH}}{\phi_{LH} c} \\
\beta q_H + (1 - \beta)q_L = \frac{\beta \gamma \bar{m}_H}{nc} + \frac{(1 - \beta) \gamma \bar{m}_H}{nc} \phi_{LH} \\
q = \frac{\beta \gamma \bar{m}_2}{nc} \\
\Rightarrow nq = \frac{\beta \gamma \bar{m}_2}{c} \equiv Q
\]

Since \( nq \) is determined as a function of \( \frac{Y}{Y_s} \) in all three cases, they also implicitly determine the size of the \( y \) sector from the resource constraints. For example, the capital constraint implies:

\[
Y_d = Y_s = \frac{K}{a_{yK}} - \frac{a_{yK} Q}{a_{yK} Q}
\]
Since the RHS is determined by other components of the system, so is the supply of $y$. Consequently, market clearing in the $y$ sector requires:

\[
\begin{align*}
(1 - \gamma)\bar{m}_y &= \frac{rK}{(1 - \eta)c_y} - \frac{\eta c_q}{\rho(1 - \eta)c_y} n_q \\
(1 - \gamma)\bar{m}_c &= \frac{rK}{(1 - \eta)} - \frac{\eta c_q}{\rho(1 - \eta)} \beta \gamma \bar{m}_q \\
(\rho(1 - \eta)(1 - \gamma) + \eta \gamma)\bar{m} = \rho r K
\end{align*}
\]

Under first degree price discrimination, straightforward but tedious calculations give:

\[
\omega = \left( \frac{\eta (1 - \gamma) + \gamma (1 - \eta)}{\rho (1 - \gamma) - \rho \eta (1 - \gamma) + \eta \gamma} \right) \kappa
\]

\[
= \left( \frac{(\gamma + (1 - \gamma)\rho) - \rho (1 - \eta)(1 - \gamma) - \eta \gamma}{\rho (1 - \gamma) - \rho \eta (1 - \gamma) + \eta \gamma} \right) \kappa
\]

Under indirect discrimination:

\[
\omega = \left( \frac{\Delta (\gamma + (1 - \gamma)\rho) - \rho (1 - \eta)(1 - \gamma) - \eta \gamma}{\rho (1 - \gamma) - \rho \eta (1 - \gamma) + \eta \gamma} \right) \kappa
\]

where

\[
\Delta = \frac{(\beta \phi_{\text{LH}}^\rho + (1 - \beta)\phi_{\text{LH}}) (\beta + (1 - \beta)\alpha_{\text{LH}})}{(\beta + (1 - \beta)\phi_{\text{LH}}) \alpha_{\text{LH}}}
\]

Note that $\Delta = 1$ when $\beta = 0$ or $\beta = 1$. However, for $\beta \in (0, 1)$ then $\Delta \geq 1$, generating the required non-monotonicity between factor prices and $\beta$. 

31
References


Świecki, Tomasz (2012) ‘Intersectoral distortions, structural change and the welfare gains from trade’
