Mission Coherence and Contestation Within Organisations

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Declaration

I hereby declare that this submission is my own work and any contributions or materials from other authors used in this thesis have been appropriately acknowledged. This thesis has not been previously submitted to any other university or institution as part of the requirements for another degree or award.

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October 22, 2012
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If only longer, it could last.
For all the great times, I can only say cheers.
I’ll bring the beers — I’ll bring the beers.
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ABSTRACT

This paper looks at the trade-off between hiring intrinsically motivated agents whose preferences align with those of the principal, and increasing incentives by creating a setting where agents’ motivations are conflicting. In settings where the principal’s tolerance for other missions is sufficiently high, a principal will staff his organisation with agents whose preferences do not reflect his own in equilibrium, in order to avoid this public goods problem — a reversal of the standard result in the literature. Additionally, when the principal has full control over hiring decisions, he will delegate all real authority to agents in equilibrium, regardless of whether preferences are congruent or not.
1 Introduction

In many organisational settings, it is reasonable to think that workers are motivated by factors besides monetary incentives. For example, it is odd to think of a teacher who places no value on the education of her students, or an environmental activist with no concern for the state of the environment. We say that these workers have a strong sense of mission; they care about the outcomes their organisation produces, and have strong beliefs about the best way to achieve these outcomes. Thus, it is very natural to think of mission as a tool with which managers can align incentives and resolve the standard agency problem.

However, academics in the area of public administration have noted that mission is used very differently across different organisations. Consider the National Forest Service (NFS) and the National Park Service (NPS) — two U.S. government agencies, each responsible for managing large tracts of land. In the NFS, new recruits undergo rigorous training and are immediately indoctrinated in the ways of professional forest management. Foresters are subject to frequent evaluation, to ensure performance is always in line with the overarching goal of the organisation, and those who operate outside the mission of the NFS receive “blunt and hard-hitting reports”. It is clear that the organisation is trying to induce a setting where workers’ values are perfectly aligned with that of the NFS — I term such a setting as one of mission coherence.

To compare, there is often conflict within in NPS over the proper way to manage these resources. Arguably, this is driven by an “inherent duality” in the self-stated purpose of the organisation, which is to both “conserve the scenery and the natural and historic objects” whilst at the same time “providing for the [public’s] enjoyment” of these natural resources. With proponents on both sides of this argument within the organisation, a question as simple as “Should this section of trees be cut back to enable easier access

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1Wilson., 96-97.
to camping grounds?” can be hotly debated. Where the values of agents within an organisation are competing, I say there exists mission contestation.

The purpose of this paper is to develop a theory of incentives explaining why we might observe coherence in some organisations, and contestation in others. Along the way, reference will be made to specific organisations in the literature on public administration which have received particular attention because of their mission structure, to see whether the results stemming from this theory reconcile with empirical findings.

It is worth pinning down the defining characteristics of these mission-oriented settings before we proceed. Organisations have limited resources, which can be used in the production of output. However, the organisation also requires something meaningful to spend its resources on before an outcome can be achieved (i.e. a project) — for example, an organisation devoted to aiding development in third-world countries can use its money to build a school, or to improve access to fresh water, or to set up a microfinance institution; it cannot simply “throw money” at the country to aid development. Workers are hired to develop these projects, which the organisation can then use its resources to implement.

As mentioned earlier, workers and organisations have preferences over this output — for example, if the aforementioned organisation believes that literacy rates are inhibiting development more so than poor access to credit, then it would prefer funds to go towards improving access to education. Thus, there is a clear benefit to mission coherence — when workers share the preferences of their principal, the projects they choose to develop will be more beneficial to the organisation’s cause.

The cost of coherence (equivalently, the benefit of contestation) is more subtle. Since workers care about outcomes, there exists a public goods problem, in that an implemented project will bring intrinsic benefit to all workers in

\[2\] Goodsell., 111-112.
the organisation, and not just the worker responsible for its development; thus, there is an incentive to free-ride off the efforts of other workers. To the extent that preferences are aligned in an organisation, free-riding will be more intense, since the projects workers choose to develop will be more substitutable.

The main result in this paper is that mission contestation is a mechanism which managers can exploit for incentive benefits in settings where workers are intrinsically motivated, for the reasons outlined above. I also find that the optimal degree of contestation is increasing organisation’s level of tolerance for missions other than its own. Finally, I make reference to how decision rights should be allocated within such a firm, and find that, in equilibrium, the principal will always distribute these rights to workers, so that they have formal authority and thus full control over what the organisation produces.
2 Literature Review

In her review paper on altruism, Rose-Ackerman (1997) argues that motivation should be thought of along more than one dimension. While previous work, such as that of Frey (1993), had focussed on the divide between extrinsically and intrinsically motivated agents, she puts forth that agents with a vested social interest will typically have particular beliefs about the ‘right’ way to provide social services, or to produce output. For example, while two teachers might be passionate about the education of their students (i.e. both are intrinsically motivated), they may have very different views about the best way to provide this education, or even what constitutes a good education.

Her argument was echoed in the later works of Besley & Ghatak (2005), who put forward a model in which motivated agents are characterised by their mission preferences, e.g. “doctors may have different views about the right way to treat ill patients, and teachers may want to teach different curriculums.” They find it is advantageous for principals to match with agents who share their mission preferences (i.e., they opt for mission coherence), as it economises on the need to provide pecuniary incentives, and increases welfare overall. My model borrows from this setup in that principals and agents are defined by missions, however it differs in that I consider a two-agent setting instead of a single-agent setting. Indeed, the cost of mission coherence cannot be captured in a single-agent setting, since free-riding is not an issue.

Francois (2004), however, identifies the potential for intrinsically motivated agents to free-ride by choosing to not donate their labour to a mission-oriented firm, in the hope that another agent will take up the position and produce the social good. He shows that this public goods problem can be avoided when there are unmotivated agents in the population, whose presence can encourage a motivated agent to step up and ‘make a difference’ for fear of a less motivated agent ending up with the position. My model
retains Francois’ motivated agents and explores the public goods problem in a setting where there is heterogeneity along the mission dimension instead of the motivation dimension.

Delfgauw & Dur (2005) explore how heterogeneity and intrinsic motivation interact in a model where a firm must screen a heterogeneous pool of intrinsically motivated agents in order to fill a position. They show that, while posting a lower wage will discourage less motivated agents from applying, it will also increase the chance that the position is not filled. However, like Francois (2004), they assume that agents differ in their level of intrinsic motivation, and not in their missions. Moreover, they choose to model the intrinsic motivation as “impure altruism”; thus workers derive utility directly from exerting effort, and not from the output of the firm, sidestepping the public goods problem entirely. Finally, I refrain from the topic of screening altogether.

The benefits of hiring agents with conflicting preferences have been observed before in an intrinsic motivation setting. Prendergast (2008) proposes a model in which firms prefer not to hire agents who share their exact preferences, and instead hire ‘biased’ agents who care only about some of the services the firm provides. His result, however, is driven by comparative advantages, and not the public goods problem: when the firm must produce different types of output, naturally the firm hires heterogeneous “specialised” agents who will opt to focus primarily on the task best suited to them, thereby overcoming the problems associated with poor contracting measures. Another similarity is that, as contracting measures improve, we should expect a convergence of preferences to those of the principal, as observed in my benchmark case.

Aghion & Tirole (1997) focuses on the trade-off between delegating real authority to agents in order to increase incentives, and retaining this authority to ensure that the final output coheres somewhat with the principal’s preferences. Their model, where agents must “research” projects before seek-
ing approval from the principal provides the framework for my own model. The main difference is that agents’ preferences are endogenous in my model, whereas in Aghion & Tirole, these preferences are exogenous. Moreover, I focus less on the topic of delegation (though multiple hierarchical settings are considered), and concentrate more on the problem of who the principal should hire.

Finally, since their result bears some similarity to my own, it is worth mentioning Krishna and Morgan (2001), who explore the implications of adding a second biased expert in an extension to the standard cheap talk model. They show that, when the biases are in opposite directions, it will be beneficial for the decision-maker to consult both experts and obtain more information as to the true state of the world. It should be noted that their result is not that conflicting biases is optimal, but that two biases may be better than one. Were the biases endogenous, the decision-maker would trivially opt for experts whose preferences aligned exactly with his own, which is untrue in my model.
3 Model

3.1 Setup

A risk-neutral principal (P) must hire two risk-neutral agents (A1 and A2) whose job is to develop projects on his behalf. Each agent *independently* selects a type of project to develop and an effort to exert on their own project ($e_1$ and $e_2$). The level of effort an agent chooses to exert functions as the probability that he will successfully develop his chosen project, and effort comes at a cost $c(e) = \frac{1}{2}e^2$.

The principal has limited resources, and can commit funds to only one successfully developed project, upon which payoffs will be realised. In the event that neither agent is successful, or that funds are not committed, the principal and agents receive a payoff of 0.

Principals and agents are characterised by *missions*, which function as types in this model. I denote the principal’s mission as $m_P$ and the missions of the agents as $m_1$ and $m_2$, where a mission $m$ is any real number from the interval $[0, 1]$. Since the principal chooses who to hire, $m_1$ and $m_2$ are endogenous.

Projects, too, are defined by missions. The set of all projects is sufficiently rich such that, for each mission, there exists a corresponding project. I denote the projects developed by A1 and A2 as $\hat{m}_1$ and $\hat{m}_2$.

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3Project development can be thought to capture all necessary steps before which funds can be committed, e.g. conducting research, working out logistics, budgeting, etc.

4A more general interpretation of this constraint is that projects are mutually exclusive. This could be driven by a budget constraint of the principal, or perhaps by the nature of the projects themselves, e.g. a school cannot teach two different curricula.

5Because of this one-to-one correspondence, I will use the terms ‘mission’ and ‘project’ somewhat interchangeably throughout the model.
3.1.1 Preferences

Suppose that a project of type \( \tilde{m} \) is successfully developed and funded. Final payoffs of the principal and agents are as follows:

\[
U_P = B \left( 1 - \frac{|m_P - \tilde{m}|}{\lambda_P} \right) - w_1 - w_2
\]

\[
U_{A1} = b \left( 1 - \frac{|m_1 - \tilde{m}|}{\lambda_1} \right) - \frac{1}{2} e_1^2 + w_1
\]

\[
U_{A2} = b \left( 1 - \frac{|m_2 - \tilde{m}|}{\lambda_2} \right) - \frac{1}{2} e_2^2 + w_2
\]

\( B > 0 \) and \( b > 0 \) are the maximum intrinsic benefits the principal and agents stand to gain, respectively. Note that the functional form implies this maximum benefit is only obtained when a project of the principal/agent’s exact mission mission is implemented. Any other project will return some ‘diluted’ benefit, where the degree of dilution is increasing in the disparity of the two missions.6

\( \lambda_P, \lambda_1 \) and \( \lambda_2 \) are measures of mission tolerance, i.e. how receptive the principal and agents are to missions other than their own. I assume \( \lambda \) lies in the interval \([1, \infty]\) to ensure non-negative payoffs from project implementation, where a larger \( \lambda \) denotes a higher tolerance. Since \( P \) chooses who to hire, \( \lambda_1 \) and \( \lambda_2 \) are endogenous. Note that as mission tolerance grows arbitrarily large, the principal/agent becomes indifferent between any two projects.

Finally, \( w_1 \) and \( w_2 \) are the wages paid to \( A_1 \) and \( A_2 \). While the agents obtain benefit from the organisation’s output, I take the stance that intrinsically motivated agents should also react to monetary incentives.

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6I tentatively specify a linear dilution rate here so that the model can be easily solved, and assure the reader that no key results depend on this assumption.
3.1.2 Contracting Assumptions

I assume that effort is unobservable, and hence cannot be contracted on. Further to this, successful development of a project, while observable by the principal, is not verifiable by a court, and also cannot be contracted on.

This leaves only two contracting instruments. The first is the wage \( w \), which is paid to agents by the principal immediately upon acceptance of the contract. I assume that the principal operates under a limited liability constraint; that is, \( w \geq 0 \), and I normalise all players’ outside options to 0.

The second instrument is the allocation of decision rights within the firm. In this context, the player with decision rights is responsible for deciding which project, if any, should receive funding. I will consider the following three allocations of decision rights:

1. P retains all decision rights (henceforth, I refer to this as an autocratic setting).
2. P passes all decision rights to A1 (a dominant group setting).
3. With probability \( \frac{1}{2} \), A1 receives decision rights, and with probability \( \frac{1}{2} \), A2 receives decision rights (a representative authority setting).\(^7\)

It may not be immediately apparent why the principal would pass this authority to agents, but delegation will be shown to have incentive benefits.\(^8\) I will also show that, for any contract which specifies the principal is to retain all decision rights, there exists another where the principal passes all decision rights to agents which yields at least as much surplus for the principal.

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\(^7\)The outcome of the lottery is realised after contracts are accepted.

\(^8\)Aghion & Tirole (1997) looks more deeply at this trade-off.
3.1.3 Timing

1. P offers take-it-or-leave-it contracts to A1 and A2 of his choosing, given full access to the entire population of agents.

2. A1 and A2 each accept or refuse the contract.

3. If the contract was accepted by both, agents simultaneously select projects ($\tilde{m}_1$ and $\tilde{m}_2$) to develop and effort levels ($e_1$ and $e_2$) to exert.\(^9\)\(^10\)

4. With probability $e_1$, A1’s project is successfully developed. With probability $e_2$, A2’s project is successfully developed.

5. The player with decision rights decides which project to implement.

6. Final payoffs are realised.

3.2 Benchmark

For a benchmark, I consider a world with perfect observability and no contracting frictions. For simplicity, I also assume that agents must develop a project which corresponds to the principal’s mission. It follows that the principal will seek to formulate contracts which maximise the total surplus of all players, since he can simply extract the agents’ surpluses with $w_1$ and $w_2$ (as there is no longer a limited liability constraint).

\(^9\)If either rejects the contract, all players receive their outside option.

\(^{10}\)Thus, a strategy in the subgame is an element from the space $[0, 1]^2$. 
Take $m_1, m_2, \lambda_1, \lambda_2$ as given. What effort levels will the principal target?

$$U_P + U_{A1} + U_{A2} = (e_1 + e_2 - e_1e_2) \left( B + b \left( 2 - \frac{|m_P - m_1|}{\lambda_1} - \frac{|m_P - m_2|}{\lambda_2} \right) \right)$$

$$\frac{\partial(U_P + U_{A1} + U_{A2})}{\partial e_1} = (1 - e_2) \left( B + b \left( 2 - \frac{|m_P - m_1|}{\lambda_1} - \frac{|m_P - m_2|}{\lambda_2} \right) \right) - e_1 = 0$$

$$\frac{\partial(U_P + U_{A1} + U_{A2})}{\partial e_2} = (1 - e_1) \left( B + b \left( 2 - \frac{|m_P - m_1|}{\lambda_1} - \frac{|m_P - m_2|}{\lambda_2} \right) \right) - e_2 = 0$$

$$e_1^* = e_2^* = \frac{\theta}{1 + \theta}$$

where $\theta = \left( B + b \left( 2 - \frac{|m_P - m_1|}{\lambda_1} - \frac{|m_P - m_2|}{\lambda_2} \right) \right)$

The benchmark case clearly shows that, with perfect contracts, there is no need to hire people with different preferences. Indeed, when agents can be extracted, the principal’s surplus is strictly increasing in the degree of mission coherence within the firm; ideally, he would hire two agents who share his exact preferences so as to maximise total extractable surplus.

Once we introduce contracting frictions, the principal will no longer be able to directly control effort. However, by hiring agents with coherent or contesting preferences, he can influence their strategic interaction and thereby retain some indirect control over effort.

### 3.3 Imperfect Contracts

Suppose now that effort is unobservable, project development cannot be contracted on, and a limited liability constraint is in place. All that a contract can specify is a wage, and who is to receive decision rights. I start by considering the case where decision rights are distributed equally between the two agents.

Note that the solution concept I use in all cases is the subgame perfect Nash
equilibrium. Hence, the game is solved backwards: projects and effort levels are found as functions of agents’ missions, before turning to the principal’s problem of who to hire at the start of the game and what wages to offer them.

3.3.1 Case 1: A Representative Authority Setting

In this case, both agents are equally likely to receive decision rights. Note that this doesn’t occur until after the effort stage.

**Lemma 1:** In a representative authority setting, in equilibrium, agents choose to develop projects which correspond to their own missions.

**Proof:** See Appendix.

Given Lemma 1, we know $\hat{m}_1 = m_1$ and $\hat{m}_2 = m_2$.

**Assumption 1:** Without loss of generality, $m_2 \geq m_1$. Define $\Delta \equiv m_2 - m_1$.

Writing down the agents’ expected utilities:

$$U_{A1} = \frac{1}{2} \left( e_1 b + (1 - e_1) e_2 b \left( 1 - \frac{\Delta}{\lambda_1} \right) \right)$$

$$+ \frac{1}{2} \left( (1 - e_2) e_1 b + e_2 b \left( 1 - \frac{\Delta}{\lambda_1} \right) \right) + w_1 - \frac{1}{2} e_1^2$$

$$U_{A2} = \frac{1}{2} \left( e_2 b + (1 - e_2) e_1 b \left( 1 - \frac{\Delta}{\lambda_2} \right) \right)$$

$$+ \frac{1}{2} \left( (1 - e_1) e_2 b + e_1 b \left( 1 - \frac{\Delta}{\lambda_2} \right) \right) + w_2 - \frac{1}{2} e_2^2$$
Examining the best response functions of the agents, the public goods problem is clear; the two efforts are strategic substitutes. When one agent exerts effort, he increases the likelihood that his project will be successfully developed. But since only one project can be implemented, this in turn encourages the other agent to scale back effort and free-ride on the successful project.

**Figure 1:** Best response functions when $b = 0.5$, $\Delta = 0$ and $\lambda_A = 1$. 
As Figures 1 and 2 show, the free-riding is mitigated to an extent by the level of mission disparity $\Delta$. We can think of $\Delta$ as the degree of mission contestation within the firm. Mission contestation encourages agents to exert more effort by reducing the gains from free-riding.

Note also that effort levels are strictly decreasing in $\lambda_1$ and $\lambda_2$. Clearly, the principal can increase the gains from mission contestation by hiring those agents with low tolerance for missions other than their own.$^{11}$

Define a zealot to be an agent with $\lambda = 1$.

**Proposition 1:** When agents develop projects which correspond to their own missions, zealots will always be weakly preferred to non-zealots, all else

$^{11}$Even if $\Delta = 0$ in equilibrium, there is no downside in setting $\lambda_1$ and $\lambda_2$ to 1.
being equal.

**Proof:** See Appendix.

Given (BR1), (BR2) and Proposition 1, the Nash equilibrium in effort levels is characterised below. Moreover, it is a unique and strict equilibrium.

\[ e_1^* = e_2^* = \frac{b}{1 + b \left( 1 - \frac{\Delta}{2} \right)} \]

**Assumption 2:** To ensure interior solutions in all cases, \( b < 1 \).

Having solved the final stage of the game, we can now turn to the principal’s hiring decision.

**Lemma 2:** Hiring any pair of agents \((m_1, m_2)\) where \((m_1 - m_P)(m_2 - m_P) > 0\) is a strictly dominated strategy.

**Proof:** Consider hiring instead some pair of agents \((m_1 + \epsilon, m_2 + \epsilon)\), where \(\epsilon\) is an arbitrarily small positive number if \(m_1 < m_P\), or an arbitrarily small negative number if \(m_1 > m_P\). Since \(\Delta\) is unchanged, effort levels are unchanged. However, by Lemma 1, we know that the projects being developed are now both closer to the principal’s mission. Hence, his surplus has strictly increased.

Put simply, Lemma 2 tells us that hiring two agents where both agents’ missions lie on the same side of the principal’s mission is never optimal. We can restrict our attention to those pairs of agents where \(m_1 \leq m_P\) and \(m_2 \geq m_P\).

**Lemma 3:** In a representative authority setting, the principal is indifferent between any two pairs of agents \((m_1, m_2)\) and \((m'_1, m'_2)\) provided all of
the following conditions hold:

1. \( m_2 \geq m_P \) \& \( m'_2 \geq m_P \)
2. \( m_1 \leq m_P \) \& \( m'_1 \leq m_P \)
3. \( m_2 - m_1 = m'_2 - m'_1 \)

Proof: See Appendix.

Lemma 3 tells us that, when agents lie on either side of the principal (recall that, by Lemma 2, everything else is dominated), the principal’s surplus is determined only by \( \Delta \). This is useful, as we can now think of him as maximising his objective function with respect to a single variable.

Lemma 3 also lets us ‘assume’ that agents are equidistant from the principal for the purposes of solving the problem, since pairs are characterised by \( \Delta \) only. Of course, if \( \Delta^* > 0 \), then there exists a continuum of optimal pairs of agents, only one of which will be the symmetric solution.

We can now formally address the principal’s problem.

\[
\max_{\Delta, w_1, w_2} U_P = (e_1 + e_2 - e_1e_2)B \left( 1 - \frac{\Delta}{2\lambda_P} \right) - w_1 - w_2
\]

subject to the following constraints:

\[
e_1 \in \arg \max U_{A1} \quad \text{(IC1)}
\]
\[
e_2 \in \arg \max U_{A2} \quad \text{(IC2)}
\]
\[
U_{A1} \geq 0 \quad \text{(IR1)}
\]
\[
U_{A2} \geq 0 \quad \text{(IR2)}
\]
\[
w_1 \geq 0 \quad \text{(LLC1)}
\]
\[
w_2 \geq 0 \quad \text{(LLC2)}
\]
\[
0 \leq \Delta \leq 1
\]
Note that, since payoffs from project implementation are always non-negative, agents can always guarantee themselves a non-negative payoff by exerting zero effort. Hence, (IC1) implies (IR1), and (IC2) implies (IR2).

Since the individual rationality constraints don’t bind in equilibrium, the principal can offer wages of zero and still guarantee participation from both agents.

\[ w_1^* = w_2^* = 0 \]

**Lemma 4:** The principal’s objective function is quasiconcave with respect to \( \Delta \) over the interval \([0, 1]\).

**Proof:** See Appendix.

**Proposition 2:** In a representative authority setting, the optimal degree of mission contestation \( \Delta^* \) is given by the following:

\[
\Delta^* = \begin{cases} 
0 & \text{if } \lambda_P < \frac{2 + 3b + b^2}{2b} \\
1 & \text{if } \lambda_P > \frac{4 + 4b - b^2}{4b - 2b^2} \\
\frac{2(2b\lambda_P - 2 - 3b - b^2)}{b(2b\lambda_P - 2 - 3b)} & \text{otherwise.}
\end{cases}
\]

Moreover, \( \Delta^* \) is increasing in \( \lambda_P \).

**Proof:** See Appendix.
We observe the optimal degree of preference heterogeneity within the organisation is increasing in the principal’s degree of mission tolerance, as one might expect; when the principal is concerned less with targeting a specific type of output, he can take greater measures to counteract the public goods problem and implement more intense mission contestation by hiring agents with very different preferences. Before we can check whether this theory matches with empirical observations, we must first understand what it means for an organisation to have a high or low level of mission tolerance, i.e. $\lambda_P$.

Perhaps the most natural interpretation of $\lambda_P$ is that it reflects the landscape of interests in the organisation’s environment. Consider the example of the Occupational Safety and Health Administration (OSHA) in the United States. It is clear that both industry and labour have a vested interest in the decisions made within this organisation, but these two interest-groups should not necessarily see eye-to-eye: while workers will campaign for safer working conditions, industry may be reluctant to implement costly changes. Since the OSHA always finds an ally in either industry or labour in any decision it
makes, we can argue it is relatively *unconstrained* in terms of the actions it can take, and thus the missions it can target. Indeed, mission contestation would explain the sometimes conflicting nature of the agency’s decisions, and also its tendency to loosen regulations after introducing them.\(^{12}\)

Of course, such pressures can also originate from within the agency. Whenever management has strong views on the “right” way to accomplish the task at hand, we should expect little to no appetite for alternative methods, and thus observe perfect mission coherence. Indeed, in the example of the National Forest Service, the hierarchical controls which ensure coherence, such as a training program for new personnel and routine evaluation of old personnel, are the long-lasting efforts of the late Gifford Pinchot, who assumed control of the agency in 1898. Arguably, these controls were necessary to ensure coherence since Pinchot’s views, characterised by the need to manage forests efficiently, were not traditionally the focus of the organisation. To this day, the NFS is still dominated by a single professional culture centred around Pinchot’s views.\(^{13}\)

A similar case can be made for the Federal Bureau of Investigation. When J. Edgar Hoover accepted directorship in 1924, he radically changed the direction of the organisation from, in the words of Wilson, “actively fomenting the Red Scare” to “gathering facts about possible violations of federal laws”. To this end, Hoover cleaned out the old agents, set up a training academy, and instituted an inspection process which would ensure that all agents were *cohering* to the new mission. Later, when Congress tried to force narcotics trafficking under the umbrella of the FBI, Hoover actively resisted for fears that the Bureau would lose its reputation for “integrity and efficiency” (i.e. its mission) due to the corruption scandals which plagued the Drug Enforcement Agency.\(^{14}\)

\(^{12}\)Wilson., 81-82.

\(^{13}\)Wilson., 96-97.

\(^{14}\)Wilson., 182-183.
We should expect mission contestation to arise more readily in those organisational settings where interests are not so specific. Consider the example of the Federal Trade Commission (FTC) in the U.S., where workers are tasked with identifying firms engaging in “unfair or deceptive methods of competition”. Since no clear guidelines are provided on what actually constitutes such behaviour, we can make the case that the organisation has a high mission tolerance; officials have no strong preferences over what action is taken, as long as it is in the interests of consumer welfare. What academics have observed within the agency is intense competition between two bodies of workers: the lawyers, who are more inclined to pursuing cases where wrongdoing is clearly evident and prosecution will be swift, and the economists, who are more concerned with finding large concentrations of market power. The theory would argue that this contestation is a natural response to that fact that “bettering consumer welfare” is a goal which can be accomplished in many ways. As Wilson puts it, the two contesting missions “are not simply professional preferences about policy choices; they are competing visions... as to how best to discharge a public responsibility”.\textsuperscript{15}

\textsuperscript{15}Wilson., 61.
3.3.2 Case 2: A Dominant Group Setting

In this case, the decision of which project to implement is always granted to the same agent (henceforth the *dominant* agent). I refer to the agent without decision rights as the *outside* agent. Without loss of generality, I assume agent 1 is the dominant agent.

**Lemma 5:** *In a dominant group setting, in equilibrium, agents choose to develop projects which correspond to their own missions.*

**Proof:** See Appendix.

Given Lemma 5, we know $\tilde{m}_1 = m_1$ and $\tilde{m}_2 = m_2$. Writing down the agents’ expected utilities:

$$U_{A1} = \left( e_1 b + (1 - e_1) e_2 b \left( 1 - \frac{\Delta}{\lambda_A} \right) \right) + w_1 - \frac{1}{2} e_1^2$$

$$U_{A2} = \left( (1 - e_1) e_2 b + e_1 b \left( 1 - \frac{\Delta}{\lambda_A} \right) \right) + w_2 - \frac{1}{2} e_2^2$$

Recall Proposition 1: when agents develop projects of their own mission, zealots will always be weakly preferred to non-zealots.

$$\lambda_1^* = \lambda_2^* = 1$$

Since the utility functions are strictly concave, we can take first order conditions to find best responses.
\[ e_1^*(e_2) = b (1 - e_2 (1 - \Delta)) \quad \text{(BR3)} \]

\[ e_2^*(e_1) = b (1 - e_1) \quad \text{(BR4)} \]

**Figure 4:** Best response functions when \( b = 0.5, \Delta = 0 \) and \( \lambda_A = 1 \).
Figures 4 and 5 show the best response functions where $\Delta = 0$ and $\Delta = 1$ respectively. We observe that Figure 4 is identical to Figure 1 from the representative authority case, since the allocation of decision rights has no impact in a setting of perfect mission coherence. We also observe that, as $\Delta$ increases, $A_1$ exerts more effort due to his authority over $A_2$, and $A_2$ less effort (in response to $A_1$’s increase).

The reason that $e_1^*(e_2)$ is constant in the case of $\Delta = 1$ is that $A_1$ receives no benefit whatsoever if $A_2$’s project is successfully implemented, since their missions are extreme opposites. Thus, from his perspective, the game reduces to a simple decision problem of maximising $e_1 b - \frac{1}{2}e_1^2$, to which the solution is $b$.

Solving (BR3) and (BR4) simultaneously yields a strict and unique Nash
equilibrium in effort levels:

\[
e_1^* = \frac{b - b^2 (1 - \Delta)}{1 - b^2 (1 - \Delta)}
\]

\[
e_2^* = \frac{b - b^2}{1 - b^2 (1 - \Delta)}
\]

**Lemma 6:** For any \( \Delta > 0 \), the principal always prefers to hire a dominant agent who shares the principal’s mission, provided this is feasible.

**Proof:** See Appendix.

The feasibility clause is necessary since, depending on the principal’s mission, it may not be possible to hire a dominant agent who shares this mission while still targeting some \( \Delta > 0 \). Consider a setting where \( m_P = 0.5 \), and the principal wishes to implement contestation, where \( \Delta = 0.6 \). Clearly this will not be possible if the dominant agent’s mission perfectly coheres with that of the principal.

In order to fully explore this setting, Lemma 6 suggests we normalise \( m_P \) to 0; this enables the principal to reap the full benefits of a hierarchical allocation of decision rights.

We can now formally address the principal’s problem.

\[
\max_{\Delta, w_1, w_2} U_P = e_1 B + (1 - e_1) e_2 B \left( 1 - \frac{\Delta}{\lambda_P} \right) - w_1 - w_2
\]
subject to the following constraints:

\[ e_1 \in \arg \max U_{A1} \quad \text{(IC3)} \]
\[ e_2 \in \arg \max U_{A2} \quad \text{(IC4)} \]
\[ U_{A1} \geq 0 \quad \text{(IR3)} \]
\[ U_{A2} \geq 0 \quad \text{(IR4)} \]
\[ w_1 \geq 0 \quad \text{(LLC3)} \]
\[ w_2 \geq 0 \quad \text{(LLC4)} \]
\[ 0 \leq \Delta \leq 1 \]

As before, since agents can guarantee themselves non-negative payoffs by exerting no effort, (IC3) implies (IR3), and (IC4) implies (IR4). So the principal need not offer wages to ensure participation, and will not in equilibrium, as they do not impact the strategic interaction between agents.

\[ w_1^* = w_2^* = 0 \]

**Lemma 7:** The principal’s objective function is quasiconvex with respect to \( \Delta \) over the interval \([0, 1]\).

**Proof:** See Appendix.

**Proposition 3:** In a dominant group setting, the optimal degree of mission contestation \( \Delta^* \) is given by the following:

\[ \Delta^* = \begin{cases} 
0 & \text{if } \lambda_P < \frac{1 - 2b^2 + b^4}{b - b^2 + b^4} \\
1 & \text{otherwise.}
\end{cases} \]
Moreover, $\Delta^*$ is increasing in $\lambda_P$.

**Proof:** See Appendix.

![Figure 6: $\Delta^*$ as a function of $\lambda_P$ when $b = 0.5$](image)

As in the representative authority case, I find the gains from mission contestation are strictly higher for higher levels of mission tolerance. This is reassuring, as it suggests that contestation does not rely on a specific allocation of decision rights between agents; thus the dominant group case serves as a robustness test of my results from the representative authority case.\(^\text{16}\)

\(^{16}\)A more formal approach would involve leaving the decision rights general, e.g. “with probability $p$, A1 receives decision rights, and with probability $1 - p$, A2 receives decision rights”. This approach was attempted, but the nature of the objective function relied too heavily on $p$ (the problem has gone from quasiconcave to quasiconvex between cases 1 and 2), and as such a general solution couldn’t be found.
3.3.3 Case 3: An Autocratic Setting

The final case to be considered is an autocratic setting, where the principal always has the final say in deciding which project should receive funding. The principal retaining decision rights immediately poses a problem for mission contestation, since agents will now have a strategic incentive to pander to the preferences of the principal, and produce projects which do not necessarily correspond to their own preferences. This pandering effect can serve to drive projects together, increasing the substitutability of output, and the incentive to free-ride.

Instead of fully characterising the solution to this case, I will first illustrate the pandering effect with a simple proposition, and briefly outline the intuition behind why it can lead to suboptimal outcomes. I will then formally prove that the principal can never achieve a higher surplus in this setting than what he could have otherwise achieved by distributing decision rights to agents.

**Proposition 4:** Take two zealots with missions $m_1$ and $m_2$, where $m_1 < m_P$, $m_2 > m_P$, and $m_P - m_1 = m_2 - m_P = \frac{\Delta}{2}$. In an autocratic setting, there is no equilibrium where these agents develop projects which correspond to their own missions.

**Proof:** Assume to the contrary that such an equilibrium exists, where $A_1$ plays $(m_1, e_1)$ and $A_2$ plays $(m_2, e_2)$. Since effort is initially costless, we must also assume that $e_1 > 0$ and $e_2 > 0$. As projects are equidistant from the principal, we can assume he randomly selects a project in the event that both agents are successful.\(^{17}\) Denote $A_1$’s utility if he plays according to specified strategy as $U_{A_1}$.

\(^{17}\)The proof goes through no matter what the principal’s decision rule is in this situation; at least one agent will always want to deviate.
\[ U_{A1} = e_1(1 - e_2) b + e_2(1 - e_1) b(1 - \Delta) + e_1 e_2 \left( \frac{1}{2} b + \frac{1}{2} b(1 - \Delta) \right) - \frac{1}{2} e_1^2 \]

Holding the strategy of A2 constant, A1 has a strict incentive to deviate to the strategy \((m_1 + \epsilon, e_1)\) for some arbitrarily small \(\epsilon > 0\). He incurs the same cost of effort, but in the event both agents are successful, his project will always be selected by the principal since it is now closer. Denote his utility under the deviation as \(U_{A1}'\).

\[
U_{A1}' = e_1(b - \epsilon) + e_2(1 - e_1) b(1 - \Delta) - \frac{1}{2} e_1^2
\]

\[
U_{A1}' - U_{A1} = \frac{1}{2} b \Delta e_1 e_2 - e_1 \epsilon
\]

\[
\epsilon < \frac{1}{2} b \Delta e_2 \Rightarrow U_{A1}' > U_{A1}
\]

So \((m_1, e_1)\) is not a best response to \((m_2, e_2)\), contradicting our initial assumption that the strategy profile \(\{(m_1, e_1), (m_2, e_2)\}\) constituted a Nash equilibrium. QED.

The pandering we observe in the above example resembles the price undercutting observed in Bertrand competition; while I am reluctant to characterise the properties of an equilibrium in the autocratic setting, it is not unreasonable to think that this undercutting could continue all the way to the principal’s mission under certain conditions.\(^{18}\)

Although this pandering might seem beneficial for the principal on the surface, recall that in the representative authority and dominant group cases, the principal had full control over what projects were developed through his

\(^{18}\)We shouldn’t always expect undercutting to take place. Consider a pair of agents where one is significantly closer to the principal than the other; the cost of undercutting for the outside agent may outweigh the benefit of stealing preference from the other.
hiring decisions. Indeed, if he wanted two agents developing his favourite project, he could achieve it by hiring two agents with his exact preferences. Moreover, effort levels would have been *higher* relative to a pandering scenario, since the agents would have been working on their favourite projects.

This raises the all-important question as to what the principal can actually *gain* in an autocratic setting, relative to the representative authority and dominant group cases.

**Proposition 5:** Assuming a pure strategy subgame perfect equilibrium exists in the autocratic setting, it cannot yield a higher surplus for the principal than what could have otherwise been achieved under a representative authority or dominant group setting.

**Proof:** Denote the missions of the agents hired in equilibrium as $m_1$ and $m_2$. Allowing for the possibility of pandering, denote the projects they develop as $\tilde{m}_1$ and $\tilde{m}_2$. Without loss of generality, assume $|m_P - \tilde{m}_1| \leq |m_P - \tilde{m}_2|$. 

**Case 1:** $|m_P - \tilde{m}_1| = |m_P - \tilde{m}_2|$

Firstly, by Lemma 1, we know this choice of projects can be replicated in a representative authority setting where the principal hires agents with missions $\tilde{m}_1$ and $\tilde{m}_2$.

Secondly, since agents develop equally appealing projects (from the principal’s perspective), effort levels must reflect the strategic consideration that, in the event they are both successful, they each have a 0.5 probability of having their project implemented. This strategic consideration is common to a representative authority setting, and thus best response functions would have the same functional form across both settings. Moreover, if there was pandering in this equilibrium (i.e. if $m_1 \neq \tilde{m}_1$ or $m_2 \neq \tilde{m}_2$), then effort levels would be strictly *higher* in the representative authority setting, since there would be higher intrinsic benefits at stake. If there was no pandering, then
the effort levels would be identical.

Finally, since the principal’s expected surplus is a determined only by efforts, projects, and wages, and the lowest possible wages are offered in the representative authority setting, it follows that the autocratic setting cannot offer a higher surplus than what could have been achieved under a representative authority setting when $|m_P - \tilde{m}_1| = |m_P - \tilde{m}_2|$.

Case 2: $|m_P - \tilde{m}_1| < |m_P - \tilde{m}_2|

Firstly, by Lemma 1, we know this choice of projects can be replicated in a dominant group setting where the principal hires agents with missions $\tilde{m}_1$ and $\tilde{m}_2$.

Secondly, since agents develop projects over which the principal has a strict preference ordering, their effort levels must reflect the strategic consideration that, in the event they are both successful, the agent with the ‘closer’ project will always receive funding. This strategic consideration also features in any dominant group setting, so best response functions would have the same functional form. Moreover, if there was pandering in this equilibrium (i.e. if $m_1 \neq \tilde{m}_1$ or $m_2 \neq \tilde{m}_2$), then effort levels would be strictly higher in the dominant group setting, since there would be higher intrinsic benefits at stake. If there was no pandering, then the effort levels would be identical.

Finally, since the principal’s expected surplus is a determined only by efforts, projects, and wages, and the lowest possible wages are offered in the dominant group setting, it follows that the autocratic setting cannot offer a higher surplus than what could have otherwise been achieved under a dominant group setting when $|m_P - \tilde{m}_1| < |m_P - \tilde{m}_2|$. QED (since cases 1 and 2 are exhaustive).

Proposition 5 reassures us that, when the principal can commit to allocating decision rights to agents, he has nothing to lose by doing so. So assuming
principals allocate decision rights optimally, the representative authority case and the dominant group case serve to illustrate a universal trade-off between mission coherence and mission contestation.
4 Extensions

In this section, I consider two separate extensions to the model. The first involves modifying the model so that the principal is restricted to hiring a single agent — this is primarily to replicate the result from Besley & Ghatak (2005) that coherence will be optimal in these settings. The second, not fully explored here, involves respecifying the functional form of the benefits so as to disentangle intrinsic motivation from mission tolerance.

4.1 A Single Agent Setting

The purpose of this extension is two-fold. Firstly, I seek to replicate the result of Besley & Ghatak (2005), which finds that, in mission-oriented settings, principals should always seek to match with an agent who shares their mission. Contestation does not arise in their model, as the principal is restricted to hiring a single agent — accordingly, there is no public goods problem.

Secondly, one might ask if the principal in such a setting would ever choose to hire a second agent, given that it leads to free-riding. In a setting where the number of agents the principal chooses to hire is endogenous, I derive a simple sufficient condition for the principal to prefer hiring multiple agents.

4.1.1 Timing

1. P offers a take-it-or-leave-it contract (essentially $w$) to A of his choosing, given full access to the entire population of agents.

2. Agent accepts or refuses contract.

3. If the contract was accepted, agent selects a project $\tilde{m}$ to develop and effort level $e$ to exert.

4. With probability $e$, the project is successfully developed.
5. If successfully developed, the principal may choose to implement the project.

6. Final payoffs are realised.

4.1.2 Benchmark

For a benchmark, I consider a world with perfect observability and no contracting frictions. For simplicity, I also assume that the agent must develop a project which corresponds to the principal’s mission. It follows that the principal will seek to formulate contracts which maximise the total surplus of both players, since he can simply extract the agent’s surplus with \( w \).

\[
U_P + U_A = eB + eb \left( 1 - \frac{|m_P - m_A|}{\lambda_A} \right) - \frac{1}{2}e^2
\]

\[
\frac{d(U_P + U_A)}{de} = B + b \left( 1 - \frac{|m_P - m_A|}{\lambda_A} \right) - e = 0
\]

\[
e^* = B + b \left( 1 - \frac{|m_P - m_A|}{\lambda_A} \right)
\]

As in the two agent case, we see first-best effort levels are increasing in the level of mission coherence within the firm.

4.1.3 Imperfect Contracts

Suppose now that effort is unobservable, project development cannot be contracted on, and a limited liability constraint is in place. All that a contract can specify is a wage \( w \).
\[ U_P = eB \left( 1 - \frac{|m_P - \hat{m}|}{\lambda_P} \right) - w \]
\[ U_A = eb \left( 1 - \frac{|m_A - \hat{m}|}{\lambda_A} \right) + w - \frac{1}{2} e^2 \]

**Lemma 8:** In the single agent case, the agent will always choose to develop the project which corresponds to his own mission.

**Proof:** Note the model specification ensures it will always be in the principal’s best interests to commit funding to a successful proposal, no matter which mission it corresponds to - that is, the marginal benefit to either party of committing funding to a proposal is always non-negative. Accordingly, the principal will always implement a successful project, and the agent has no incentive to pander. QED.

So in equilibrium:

\[ \hat{m} = m_A \]
\[ U_A = eb + w - \frac{1}{2} e^2 \]

We can now derive the optimal effort level for the agent.

\[ \frac{dU_A}{de} = b - e = 0 \]
\[ e^* = b \]

Principal’s seeks to maximise the following objective function:
\[
\max_{m_A, w} eB \left(1 - \frac{|m_P - \tilde{m}|}{\lambda_P}\right) - w
\]

subject to the following constraints:

\[
e \in \arg \max U_A \quad \text{(IC5)}
\]

\[
U_A \geq 0 \quad \text{(IR5)}
\]

\[
0 \leq m_A \leq 1
\]

\[
w \geq 0
\]

Note that (IC5) implies (IR5), since the agent can always guarantee himself a non-negative payoff by exerting no effort. Since the participation constraint does not bind, the principal offers no wage.

\[
w^* = 0
\]

\[
U_P = bB \left(1 - \frac{|m_P - m_A|}{\lambda_P}\right)
\]

Clearly, the function is strictly decreasing in \(|m_P - m_A|\) — contestation offers no benefits in this setting.

\[
m_A = m_P
\]

\[
U_P^* = bB
\]

**Proposition 6:** In the single agent case, there is perfect mission coherence.

**Proof:** See above. In the single agent case, we observe that incentives are perfectly aligned, consistent with Besley & Ghatak (2005). Heterogeneity offers no benefit in the single agent case, as there is no public goods problem which needs to be overcome.
We also observe that, even in the absence of free-riding, the agent exerts an effort level below that which maximises social surplus. This is driven by the fact he considers only the private intrinsic benefit which will result from output, and does not consider the gains the principal stands to make. Free-riding adds another ‘layer’ of inefficiency.

4.1.4 Endogenous Number of Agents

Given that the public goods problem only has bite in multiple agent settings, one may ask why the principal would ever opt to hire more than a single agent. Indeed, effort levels will necessarily decrease due to free-riding; however the total probability of at least one project being successfully developed may be higher with more agents.

Consider, for example, a principal with one perfectly aligned agent, who is given the option of hiring another perfectly aligned agent. As shown above, the probability that the agent’s project is successfully developed is currently $b$. Were he to hire the second agent, efforts would be given by the Nash equilibrium derived earlier in the paper.

$$e_1^* = e_2^* = \frac{b}{1+b}$$

$$e_1 + e_2 - e_1 e_2 = \frac{2b}{1+b} - \left( \frac{b}{1+b} \right)^2$$

Thus, a sufficient condition for the principal to prefer hiring two agents over one is simply the following:
Thus, if motivation is sufficiently low, the principal can always do better with two agents. Note that the above is by no means a necessary condition — I have assumed perfect mission coherence for simplicity. We should also expect a preference towards multiple agents in settings where the principal’s mission tolerance is high, as mission contestation imposes less cost on these principals.

4.2 Disentangling Motivation and Tolerance

This extension is motivated primarily by the lack of a clear comparative static with regards to $b$ and $\Delta^*$. Having previously conjectured that high levels of intrinsic motivation would crowd out the incentive to free-ride and thus the need for mission contestation, I was surprised to find no such relationship in my results.19

The reason I believe this occurs is because of an unintended interaction between the level of intrinsic motivation $b$ and the level of mission tolerance $\lambda$. Consider an agent’s payoff if a project $\hat{m}$ is implemented, where $\Delta \equiv |m_1 - \hat{m}|$:

$$U_{A1} = b \left(1 - \frac{\Delta}{\lambda_1}\right) - \frac{1}{2} e_1^2 + w_1$$

19Numerical examples revealed that $\Delta^*(b)$ was convex.
Note that as $\Delta$ changes, the change in the agent’s utility is a function of both $\lambda_1$ and $b$:

$$
\frac{dU_{A1}}{d\Delta} = b \left( \frac{-1}{\lambda_1} \right)
$$

Thus, a low $b$ has the same effect as a high mission tolerance, and vice versa. The modelling assumption is not completely unwarranted; it simply says that those agents who are less motivated have, ceteris paribus, weaker preferences over the set of missions. This sounds perfectly reasonable on the surface — however, it would be ideal to completely isolate the two effects from one another to enable a greater understanding of the phenomenon.

Consider instead the following utility specification:

$$
U_{A1} = b - \frac{\Delta}{\lambda_1} - c e_1^2 + w_1
$$

With the above specification, when $\Delta$ changes, the change in utility is entirely captured by $\lambda_1$. Of course, this specification brings with it its own problems. For example, in order to ensure non-negativity of project payoffs, one might be tempted to make the assumption $b > 1$ — however, this immediately results in an upper corner solution. To remedy this, we can scale the cost of effort upwards by $c$; but introduction of another parameter is never ideal, especially when the model results are almost intractable in their current state.
5 Conclusion

This paper has developed a theory of incentives capable of explaining the phenomenon of mission contestation as observed by academics in the literature on public administration. In developing this theory, we have seen that the choice between coherence and contestation is primarily determined by factors which determine the organisation’s attitude to missions other than its own. In settings where management has strong views on the best way to achieve its goals, or where the organisation is exogenously constrained in the actions it can take, we should expect the missions of workers to cohere with the overarching goal of the organisation. However, if goals are unclear and output is hard to measure, then the incentive benefits offered by mission contestation can counteract the public goods problem associated with intrinsically motivated agents.

I also find that, when the composition of the organisation is endogenous, delegation of real authority to workers will always occur in equilibrium, and pandering will never occur.

Many questions remain unanswered, however. It is not clear whether these results would go through in a setting where model specification does not guarantee worker participation, as mission contestation obviously comes at some cost to agents. Moreover, the current model specification has rendered the relationship between intrinsic motivation and mission contestation too difficult to interpret meaningfully. I hope to explore and solve both these problems in the immediate future.
6 Appendix

6.1 Proofs

Lemma 1: In a representative authority setting, in equilibrium, agents choose to develop projects which correspond to their own missions.

Claim L1A: Any mission-effort pair $(\tilde{m}, 1)$ is strictly dominated by $(\tilde{m}, 1-\epsilon)$ for some $\epsilon > 0$.

Proof of Claim L1A: Note that the marginal benefit of exerting effort is bounded below by 0 and above by $b$, the maximum benefit a project can bring. Further, we have assumed $b < 1$ with Assumption 2. By our specification of the cost of effort function, $c(e) = \frac{1}{2}e^2$, we also know that the marginal cost of exerting effort equals the effort level itself; that is, $c'(e) = e$. So when $e = 1$, the marginal cost is strictly larger than the marginal benefit, and reducing effort by some small amount $\epsilon$ will result in a strict increase in utility.

Claim L1B: Consider the reduced game obtained by deleting all strictly dominated strategies identified by Claim L1A. Take an agent with mission $m$. Any mission-effort pair $(\tilde{m}, 0)$ is strictly dominated by the mission-effort pair $(m, \epsilon)$ for some $\epsilon > 0$.

Proof of Claim L1B: For simplicity, let us consider A1. Denote A1’s expected payoff as $U_{A1}$ when he plays $(\tilde{m}, 0)$, and his expected payoff as $U_{A1}'$ when he plays $(m_1, \epsilon)$. A2’s strategy is some general $(\tilde{m}_2, e_2)$. Denote $|m_1 - \tilde{m}_2|$ as $\Delta$. 
\[ U_{A1} = e_2 b (1 - \Delta) \]
\[ U_{A1}' = \epsilon (1 - e_2) b + (1 - \epsilon) e_2 b (1 - \Delta) + \epsilon e_2 \left( \frac{1}{2} b + \frac{1}{2} b (1 - \Delta) \right) - \frac{1}{2} \epsilon^2 \]
\[ \epsilon < 2b \left( 1 - e_2 \left( 1 - \frac{\Delta}{2} \right) \right) \implies U_{A1}' > U_{A1} \]

Note that such an \( \epsilon \) will always exist in the reduced game, since all strategies where \( e_2 = 1 \) have been previously deleted.

**Claim L1C:** Consider the reduced game obtained by deleting all strictly dominated strategies identified by Claims L1A and L1B. Assuming \( m_1 \leq m_2 \), for \( A1 \), any mission-effort pair \((\tilde{m}_1, e_1)\) where \( \tilde{m}_1 < m_1 \) is strictly dominated by the mission-effort pair \((m_1, e_1)\). Similarly, for \( A2 \), any mission-effort pair \((\tilde{m}_2, e_2)\) where \( \tilde{m}_2 > m_2 \) is strictly dominated by the mission-effort pair \((m_2, e_2)\).

**Proof of Claim L1C:** (Note I only prove the result from \( A1 \)'s perspective. Of course, the same argument holds for \( A2 \).)

Firstly, the cost of effort is constant across both strategies. Secondly, if implemented, the \( m_1 \) project gives a strictly higher benefit to \( A1 \) than the \( \tilde{m}_1 \) project. Finally, if implemented, the \( m_1 \) project gives a strictly higher benefit to \( A2 \) than the \( \tilde{m}_1 \) project, which means that it is at least as likely to be implemented as the \( m_1 \) project.\(^{20}\)

**Claim L1D:** Consider the reduced game obtained by deleting all strictly dominated strategies identified by Claims L1A, L1B, and L1C. Assuming \( m_1 \leq m_2 \), for \( A1 \), any mission-effort pair \((\tilde{m}_1, e_1)\) where \( \tilde{m}_1 > m_1 \) is strictly dominated by the mission-effort pair \((m_1, e_1)\). Similarly, for \( A2 \), any mission-

\(^{20}\)That is, by selecting a project which is more appealing by both agents’ standards, he cannot hurt his chances that his project is implemented.
effort pair \((\tilde{m}_2, e_2)\) where \(\tilde{m}_2 < m_2\) is strictly dominated by the mission-effort pair \((m_2, e_2)\).

**Proof of Claim L1D:** (Note I only prove the result from A1’s perspective. Of course, the same argument holds for A2.)

Firstly, the cost of effort is constant across both strategies. Secondly, if implemented, the \(m_1\) project gives a strictly higher benefit to A1 than the \(\tilde{m}_1\) project.

Finally, for A1, there is no longer an incentive in this game to ‘undercut’ A2 and deviate to some project \(\tilde{m}_1'\) where \(|\tilde{m}_1' - m_2| \leq |\tilde{m}_2 - m_2|\). The gain to doing so would be that, if A2 received decision rights and both agents were successful in developing projects, A2 would choose to implement A1’s project. However, in order to gain this privilege, A1 would have to develop a project which is less appealing to him than the project A2 would have otherwise implemented, defeating the purpose entirely.

Together, claims L1A, L1B, L1C, and L1D imply that, in a representative authority setting, the unique Nash equilibrium in the project-effort game must be of the form \(\{(m_1, e_1), (m_2, e_2)\}\), where \(0 < e_1 < 1\), \(0 < e_2 < 1\), and \(m_1\) and \(m_2\) denote the agents’ missions.

**Proposition 1:** When agents develop projects which correspond to their own missions, zealots will always be weakly preferred to non-zealots, all else being equal.

**Proof of Proposition 1:** Consider the best response functions from either the representative authority case or the dominant group case. Firstly, note that effort levels are strictly decreasing in \(\lambda_1\) and \(\lambda_2\); so there is a clear upside to zealots. However, there might also be a downside: the risk in hiring agents with low mission tolerance is that, because they experience a lower surplus in equilibrium than agents with high mission tolerance due to
their specific preferences, this may lead to the principal being required to pay higher wages to ensure participation.

However, we observe that even when the principal hires agents with the minimum possible tolerance, the incentive compatibility condition still implies the individual rationality condition, and thus no wages are paid in equilibrium. So there is no downside in hiring zealots over non-zealots. QED.

**Lemma 3:** In a representative authority setting, the principal is indifferent between any two pairs of agents \((m_1, m_2)\) and \((m'_1, m'_2)\) provided all of the following conditions hold:

1. \(m_2 \geq m_P \& m'_2 \geq m_P\)
2. \(m_1 \leq m_P \& m'_1 \leq m_P\)
3. \(m_2 - m_1 = m'_2 - m'_1\)

**Proof of Lemma 3:** Firstly, because effort levels depend only on \(\Delta\), statement 3 implies that effort levels will be the same across both pairs of agents; that is, \(e_1 = e_2 = e_1' = e_2'\). Denote this common effort level as \(e\). Denote \(m_2 - m_1 = m'_2 - m'_1\) as \(\Delta\). Denote the principal’s surplus under the pair of agents \((m_1, m_2)\) as \(U_P\), and under the pair of agents \((m'_1, m'_2)\) as \(U'_P\). We have the following:

\[
U_P = (2e - e^2)B \left(1 - \frac{|m_P - m_1|}{2\lambda_P} - \frac{|m_P - m_2|}{2\lambda_P}\right)
\]

\[
= (2e - e^2)B \left(1 - \frac{\Delta}{2\lambda_P}\right)
\]

\[
U'_P = (2e - e^2)B \left(1 - \frac{|m_P - m'_1|}{2\lambda_P} - \frac{|m_P - m'_2|}{2\lambda_P}\right)
\]

\[
= (2e - e^2)B \left(1 - \frac{\Delta}{2\lambda_P}\right)
\]
QED.

**Lemma 4:** The principal’s objective function is quasiconcave with respect to $\Delta$ over the interval $[0, 1]$.

**Proof of Lemma 4:**

\[
U_P = (e_1 + e_2 - e_1 e_2)B \left( 1 - \frac{\Delta}{2\lambda_P} \right) - w_1 - w_2
\]

We know $w_1 = w_2 = 0$ and $e_1 = e_2 = \frac{b}{1 + b \left( 1 - \frac{\Delta}{2} \right)}$ in equilibrium.

\[
U_P = \left( \frac{2b}{1 + b \left( 1 - \frac{\Delta}{2} \right)} - \left( \frac{b}{1 + b \left( 1 - \frac{\Delta}{2} \right)} \right)^2 \right) B \left( 1 - \frac{\Delta}{2\lambda_P} \right)
\]

\[
\frac{dU_P}{d\Delta} = \frac{2bB(4 - 2b(-3 + \Delta + 2\lambda_P) + b^2(2 + \Delta(-3 + 2\lambda_P)))}{(-2 + b(-2 + \Delta))^3\lambda_P}
\]

\[
\frac{d^2U_P}{d\Delta^2} = -\frac{4b^2B(4 - 2b(-2 + \Delta + 2\lambda_P) + b^2(2\lambda_P + \Delta(-3 + 2\lambda_P)))}{(-2 + b(-2 + \Delta))^4\lambda_P}
\]

**Claim L4A:** The principal’s objective function is smooth and continuously differentiable over the interval $[0, 1]$.

**Proof of Claim L4A:** Trivial. By our assumptions, $0 < b < 1$ and $\lambda_P \geq 1$, when ensure non-zero denominators. It immediately follows that the function and its first derivative are continuous over this interval.

**Claim L4B:** The objective function has, at most, one stationary point in the interval $[0, 1]$.

**Proof of Claim L4B:**
\[
\frac{dU_P}{d\Delta} = 0 \implies \Delta^* = \frac{2(2b\lambda_P - 2 - 3b - b^2)}{b(2b\lambda_P - 2 - 3b)}
\]

Either this value exists and satisfies \(0 \leq \Delta^* \leq 1\), or it does not. Both imply Claim L4B.

**Claim L4C:** *Should a stationary point exist at \(\Delta^*\), where \(0 \leq \Delta^* \leq 1\), it is necessarily a local maximum.*

**Proof of Claim L4C:**

\[
\left. \frac{d^2U_P}{d\Delta^2} \right|_{\Delta=\Delta^*} = -\frac{B(2b\lambda_P - 2 - 3b)^4}{32b(b(\lambda_P - 1) - 1)^3\lambda_P}
\]

Note that, by assuming \(0 \leq \Delta^* \leq 1\), we know \(2b\lambda_P - 2 - 3b\) is non-zero, as it is the denominator of \(\Delta^*\). So the numerator of the second derivative is positive.

Also, since \(\Delta^* \leq 1\), we know that \(0 \leq 2b\lambda_P - 2 - 3b - b^2 \leq 2b\lambda_P - 2 - 3b\), because \(2 > b\). In turn, this implies that \(\lambda_P > \frac{2 + 3b + b^2}{2b}\), so the denominator of the second derivative is also necessarily positive.

Thus, \(\left. \frac{d^2U_P}{d\Delta^2} \right|_{\Delta=\Delta^*} < 0\).

Since the objective function:

1. is continuously differentiable over \([0, 1]\), and
2. has at most one stationary point in \([0, 1]\), which is necessarily a local maximum,

it immediately follows that it is quasiconcave. QED.
Proposition 2: In a representative authority setting, the optimal degree of mission contestation $\Delta^\ast$ is given by the following:

$$\Delta^\ast = \begin{cases} 
0 & \text{if } \lambda_P < \frac{2 + 3b + b^2}{2b} \\
1 & \text{if } \lambda_P > \frac{4 + 4b - b^2}{4b - 2b^2} \\
\frac{2(2b\lambda_P - 2 - 3b - b^2)}{b(2b\lambda_P - 2 - 3b)} & \text{otherwise.}
\end{cases}$$

Moreover, $\Delta^\ast$ is increasing in $\lambda_P$.

Proof of Proposition 2: Because the function is quasiconcave over $[0, 1]$, we know the following:

$$\left. \frac{dU_P}{d\Delta} \right|_{\Delta = 0} < 0 \iff \lambda_P < \bar{\lambda} \equiv \frac{2 + 3b + b^2}{2b} \implies \Delta^\ast = 0 \quad \text{(BC1)}$$

$$\left. \frac{dU_P}{d\Delta} \right|_{\Delta = 1} > 0 \iff \lambda_P > \overline{\lambda} \equiv \frac{4 + 4b - b^2}{4b - 2b^2} \implies \Delta^\ast = 1 \quad \text{(BC2)}$$

It is sufficient to show that $\Delta^\ast$ is continuous as a function of $\lambda_P$, and that the interior solution is increasing in $\lambda_P$.

Suppose $\lambda_P = \underline{\lambda}$, so that (BC1) has just been violated.

$$\Delta^\ast = \frac{2(2b\lambda_P - 2 - 3b - b^2)}{b(2b\lambda_P - 2 - 3b)} = 0$$
Thus, $\Delta^*$ is continuous at the lower boundary. Now suppose $\lambda_P = \bar{\lambda}$, so that (BC2) has just been violated.

$$\Delta^* = \frac{2(2b\bar{\lambda} - 2 - 3b - b^2)}{b(2b\bar{\lambda} - 2 - 3b)} = 1$$

Note also that the interior solution is necessarily continuous as a function of $\lambda_P$, as the violation of (BC1) implies a non-zero denominator. So $\Delta^*$ is continuous.

Now suppose $0 < \Delta^* < 1$.

$$\frac{d\Delta^*}{d\lambda_P} = \frac{4b^2}{(b(2\lambda_P - 3) - 2)^2} > 0$$

The derivative is necessarily positive since the violation of (BC1) implies a non-zero denominator.

Thus, $\Delta^*$ is monotonically increasing in $\lambda_P$. QED.

Lemma 5: In a dominant group setting, in equilibrium, agents choose to develop projects which correspond to their own missions.

Proof of Lemma 5: Assume $A_1$ is the dominant agent, and $A_2$ is the outside agent. Denote respective missions as $m_1$ and $m_2$. Note this proof will rely on claims from the proof of Lemma 1.

Claim L5A: Consider the reduced game obtained by deleting all strictly dominated strategies identified by Claims L1A and L1B (proof for Lemma 1). For $A_1$, any mission-effort pair $(\tilde{m}_1, e_1)$ where $\tilde{m}_1 \neq m_1$ is strictly dominated by the mission-effort pair $(m_1, e_1)$. 
Proof of Claim L5A: Trivial. Since A1 always chooses which project to implement, and effort is always positive in the reduced game, he strictly gains by developing his favourite project.\textsuperscript{21}

Claim L5B: Consider the reduced game obtained by deleting all strictly dominated strategies identified by Claims L1A, L1B, and L5A. For A2, any mission-effort pair $(\tilde{m}_2, e_2)$ where $\tilde{m}_2 \neq m_2$ is strictly dominated by the mission-effort pair $(m_2, e_2)$.

Proof of Claim L5B: In the reduced game, $0 < e < 1$, and $\tilde{m}_1 = m_1$. Since A2 cannot ‘undercut’ A1 and develop a project that A1 likes more than A1’s own project, it immediately follows that A2’s project will be implemented if and only if A1 fails. Accordingly, A2 should maximise his expected surplus by developing his favourite project.

Together, claims L1A, L1B, L5A, and L5B imply that, in a dominant group setting, the unique Nash equilibrium in the project-effort game must be of the form $\{(m_1, e_1), (m_2, e_2)\}$, where $0 < e_1 < 1$, $0 < e_2 < 1$, and $m_1$ and $m_2$ denote the agents’ missions. QED.

Lemma 6: For any $\Delta > 0$, the principal always prefers to hire a dominant agent who shares the principal’s mission, provided this is feasible.

Proof of Lemma 6: Consider two pairs of agents $(m_1, m_2)$ and $(m'_1, m'_2)$ where $m_2 - m_1 = m'_2 - m'_1 = \Delta > 0$. Assume $m_1 = m_P$, $m'_1 < m_P$ and $m'_2 \geq m_P$.\textsuperscript{22} Denote the principal’s surplus under each pair of agents as $U_P$ and $U_{P'}$.

\textsuperscript{21}This only holds after we delete strategies of the form $(m, 1)$. Consider if A2 played $(m_1, 1)$; A1 would gain nothing here by choosing to develop his favourite project, since A2 will always successfully develop it for him.

\textsuperscript{22}This specification simply ensures that the pair is not strictly dominated according to Lemma 2.
\[ U_P = e_1B + (1 - e_1)e_2B \left(1 - \frac{\Delta}{\lambda_P}\right) \]

\[ U_P' = e_1B \left(1 - \frac{|m_P - m'_1|}{\lambda_P}\right) + (1 - e_1)e_2B \left(1 - \frac{|m_P - m'_2|}{\lambda_P}\right) \]

Subtract \( U_P' \) from \( U_P \).

\[ U_P - U_P' = e_1B \left(\frac{|m_P - m'_1|}{\lambda_P}\right) - (1 - e_1)e_2B \left(\frac{|m_P - m'_1|}{\lambda_P}\right) \]

\[ = \left(\frac{|m_P - m'_1|}{\lambda_P}\right) (e_1B - (1 - e_1)e_2B) > 0 \]

The term on the left is positive, by our assumption that \( m'_1 < m_P \). The term on the right is positive since, for any \( \Delta > 0 \), we know \( 0 < e_2 < e_1 < 1 \), where \( e_1 \) and \( e_2 \) are the equilibrium effort levels of the dominant and outside agents respectively.

\[ e_1^* = \frac{b - b^2 (1 - \Delta)}{1 - b^2 (1 - \Delta)} \]

\[ e_2^* = \frac{b - b^2}{1 - b^2 (1 - \Delta)} \]

QED.

**Lemma 7:** The principal’s objective function is quasiconvex with respect to \( \Delta \) over the interval \([0, 1]\). 

**Proof of Lemma 7:**
\[ U_P = e_1 B + e_2 (1 - e_1) B \left( 1 - \frac{\Delta}{\lambda_P} \right) - w_1 - w_2 \]

We know \( w_1 = w_2 = 0, \) \( e_1 = \frac{b - b^2 (1 - \Delta)}{1 - b^2 (1 - \Delta)}, \) and \( e_2 = \frac{b - b^2}{1 - b^2 (1 - \Delta)} \) in equilibrium.

\[ U_P = \left( \frac{b - b^2 (1 - \Delta)}{1 - b^2 (1 - \Delta)} \right) B + \left( \frac{b - b^2}{1 - b^2 (1 - \Delta)} \right) \left( 1 - \frac{b - b^2 (1 - \Delta)}{1 - b^2 (1 - \Delta)} \right) B \left( 1 - \frac{\Delta}{\lambda_P} \right) \]

\[ \frac{dU_P}{d\Delta} = \frac{(1 - b)b B (-1 + b^2 (1 + b - b\lambda_P)) + b^3 (1 + \Delta) (\lambda_P - 1) + b(\lambda_P + 1)}{(1 - b^2 (1 - \Delta))^3 \lambda_P} \]

\[ \frac{d^2U_P}{d\Delta^2} = -\frac{2(1 - b)b^3 B (-2 + b^2 (2 + b - 3b\lambda_P)) + b^3 (2 + \Delta) (\lambda_P - 1) + b(\lambda_P + 2)}{(1 - b^2 (1 - \Delta))^4 \lambda_P} \]

Claim L7A: The principal’s objective function is smooth and continuously differentiable over the interval \([0, 1]\).

Proof of Claim L7A: Trivial. By our assumptions, \( 0 < b < 1 \) and \( \lambda_P \geq 1, \) which ensure non-zero denominators. It immediately follows that the function and its first derivative are continuous over this interval.

Claim L7B: The objective function has, at most, one stationary point in the interval \([0, 1]\).

Proof of Claim L7B:

\[ \frac{dU_P}{d\Delta} = 0 \implies \Delta^* = \frac{(1 - b)^2 (1 + b - b\lambda_P)}{b^2 (1 - b + b\lambda_P)} \]

Either this value exists and satisfies \( 0 \leq \Delta^* \leq 1, \) or it does not. Both imply Claim L4B.
Claim L7C: *Should a stationary point exist at \( \Delta^* \), where \( 0 \leq \Delta^* \leq 1 \), it is necessarily a local minimum.*

**Proof of Claim L7C:**

\[
\frac{d^2 U_P}{d \Delta^2} \bigg|_{\Delta = \Delta^*} = \frac{b^3 B (1 + b(\lambda_P - 1))^4}{8(1 - b)^2 (1 + b^2(\lambda_P - 1))^3 \lambda_P} > 0
\]

By inspection, this is positive for all \( 0 < b < 1 \), \( B > 0 \), \( \lambda_P \geq 1 \).

Since the objective function:

1. is continuously differentiable over \([0, 1]\), and
2. has at most one stationary point in \([0, 1]\), which is necessarily a local minimum,

it immediately follows that it is quasiconvex. QED.

**Proposition 3:** *In a dominant group setting, the optimal degree of mission contestation \( \Delta^* \) is given by the following:

\[
\Delta^* = \begin{cases} 
0 & \text{if } \lambda_P < \frac{1 - 2b^2 + b^4}{b - b^2 + b^4} \\
1 & \text{otherwise.}
\end{cases}
\]

Moreover, \( \Delta^* \) is increasing in \( \lambda_P \).

**Proof of Proposition 3:** Because the function is quasiconcave over \([0, 1]\), we know the following:
\[ U_P|_{\Delta=0} \geq U_P|_{\Delta=1} \iff \frac{b(2 + b)B}{(1 + b)^2} > B \left( b + \frac{(1 - b)^2b(\lambda_P - 1)}{\lambda_P} \right) \]
\[ \implies \lambda_P < \frac{1 - 2b^2 + b^4}{b - b^2 + b^4} \implies \Delta^* = 0 \]

otherwise \( \Delta^* = 1 \)

Since \( U_P|_{\Delta=0} \) is independent of \( \lambda_P \), it suffices to show that \( U_P|_{\Delta=1} \) is increasing in \( \lambda_P \).

\[ \frac{d}{d\lambda_P} \left( B \left( b + \frac{(1 - b)^2b(\lambda_P - 1)}{\lambda_P} \right) \right) = \frac{(1 - b)^2bb}{\lambda_P^2} > 0 \]

By inspection, \( U_P|_{\Delta=1} \) is increasing in \( \lambda_P \) since \( 0 < b < 1 \), \( B > 0 \), and \( \lambda_P \geq 1 \). Thus, \( \Delta^* \) is monotonically increasing in \( \lambda_P \). QED.
References


