Unprecedented changes in the terms of trade*

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Abstract

The development of Asia exposed many commodity-exporting economies to unprecedented changes of their terms of trade. We set up a small open economy model to estimate the magnitude and timing of breaks in the long-run level and variance of the terms of trade. The model’s balanced growth path gives rise to wedges between tradable, headline, and non-tradable rates of inflation and between the growth rates of investment and aggregate output as a result of multiple productivity trends that trigger off drifts in relative prices as in the data. Using Australian unfiltered aggregate data, we find evidence of an increase in the long-run level and volatility of the terms of trade. Single-equation structural break tests point to a larger increase in the long-run level of the terms of trade. But inferences in general equilibrium rely on many observables that also respond to shifts in the long-run level of the terms of trade.

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1 Introduction

Over the past two decades, the increasing development of Asia has exposed many economies to a surge in global commodity prices. There have been commodity price booms before, but this one, fueled by an unprecedented era of high growth in China, has been by far the largest and most persistent. In response, policy makers, market economists and the press in these economies have entertained the idea that commodity prices have reached a ‘new normal’, with a permanently higher level and greater volatility.\footnote{See Bernanke [2008], Bloxham et al. [2012], Stevens [2011], Tasker [2013] and Yellen [2011].} Understanding whether the terms of trade of commodity-exporting economies have reached a ‘new normal’ and its implications for these economies has become of first order importance.\footnote{The implications of commodity price booms are likely to differ among economies that produce commodities and those that consume them. Our focus here is on the impact of commodity price booms in commodity-exporting economies. Because for commodity-exporting economies the bulk of recent terms of trade fluctuations come from commodity prices movements, we use these terms interchangeably.}

Measuring the macroeconomic impact of unprecedented changes in the terms of trade is a considerable task because the models used to study business cycles in small open economies are typically solved and estimated around a constant unconditional mean, but the idea of a ‘new normal’ points to a shift of the unconditional mean. Moreover, changes in the terms of trade affect many relative prices that in the data trend at different rates, in particular the price of non-tradable goods and the price of investment goods. When estimating small open economy models, it is common to abstract from many of these trends and to resort to pre filtering the data somehow. While pre filtering aligns the data with the model, it comes at the cost of distorting the facts. As Beneš et al. [2009] point out, a limitation of this approach is that it treats trends and cycles separately.

In this paper we have gone to some effort to develop a unified model of the trends that we see in the data and to estimate it in the presence of shifts in the long-run level of the terms of trade. We add sector-specific productivity trends to an otherwise standard small open economy model that has nominal price frictions, imperfect exchange rate pass-through, capital accumulation and investment adjustment costs as well as a tradable, a non-tradable and a commodity-exporting sector. On the balanced growth path relative prices trend at different rates but nominal expenditure shares remain constant. In the model – as in the data – non-tradable goods prices run faster than consumer goods prices which run faster than tradable goods prices which, in turn, run faster than investment goods prices, while real investment and foreign output grow faster than domestic output. One significant contribution of this paper is to set up a small open economy that can account for these observed trends.

The long-run level of the terms of trade, among other parameters, determines the balanced growth path, so a permanent change of the long-run level of the terms of trade gives rise to a transition towards a new balanced growth path that temporarily affects relative prices. It is an empirical issue to identify trends that belong to the balanced growth path, cycles around those trends, and fluctuations that originate from a transition towards a new balanced growth path. This is what we do in this paper. One contribution is to show how these forces may be identified in the data.

Australia is an example of a commodity-exporting economy exposed to the rise of Asia.
Over the past two decades, the foreign currency price of Australia’s commodity exports increased by more than 200 per cent. As Figure 1 shows, Australia’s commodity prices are representative of world commodity prices which suggests that many other commodity exporting economies have experienced similar fluctuations as well. For these reasons, we estimate the model on Australian aggregate data.

![Figure 1: Commodity Price Indices](image)

* Deflated by US CPI; 1993-2002 average = 100

Sources: IMF; RBA

We find that the long-run level of commodity prices increased by 40 per cent in mid 2003. An increase of this magnitude is probably less than what would be inferred from visual inspection of Figure 1; it is also less than the 90 per cent increase suggested by single equation structural break tests. Our inferences, however, rely on more observables: an increase of 90 per cent is unlikely, but not because commodity prices would disagree, but because other observables series would disagree. The estimated long-run properties of the economy in our case depend on the cross-equations restrictions that rational expectations imposes on every observable series.

This is not to say, however, that the 40 per cent increase we estimate in the long-run level of commodity prices has no significant impact. In fact, some of the economic implications are these: the commodity sector’s share of exports increases from 35 to 52 per cent; the non-commodity tradable sector’s share of value added decreases from 36 to 32 per cent; inflation falls by 2 percentage points with tradable inflation and non-tradable inflation rates strongly offsetting each other; the relative price of non-tradable goods increase by 7 per cent and the real exchange appreciates permanently by 30 per cent.

3Other structural features of the Australian economy, including its monetary policy regime, have remained stable over the past two decades.
Our work builds on that of Rabanal [2009] and Siena [2014], who set up models with different productivity trends in the tradable and non-tradable sectors. We add capital accumulation with a differential trend in the productivity of the investment goods producing sector as well as a commodity-exporting goods producing sector that takes the relative price of its output as given. Our work relates to a large literature on the role of terms of trade shocks in open economies to which we cannot possibly do justice.\footnote{Instead, we point the reader to Ostry and Reinhart [1992], Bidarkota and Crucini [2000], Bleaney and Greenaway [2001], Broda [2004], Blattman et al. [2007], Jääskelä and Smith [2013], Charnavoki and Dolado [2014] and the references therein.}

Mendoza [1995] studies the contribution of terms of trade shocks to the business cycle in a calibrated open economy RBC model and finds that terms of trade shock account for 50 per cent of output fluctuations. Other papers, like Dib [2008] and Medina and Soto [2007], study the impact of terms of trade shocks in DSGE models with nominal frictions. Our work is different because we distinguish temporary shocks to the terms of trade – which has been the focus in the small open literature – from permanent shifts of the long-run level of the terms of trade.

The rest of the paper is structured as follows. Section 2 discusses the model. Many details can be found in the online appendix.\footnote{ADD LINK.} Section 3 discusses our empirical approach which involves calibration and estimation of date breaks and parameters. Section 4 describes the main results. Section 5 analyses the responses of the estimated model to temporary and permanent changes in the terms of trade. Section 6 concludes suggesting avenues for further research.

## 2 Model

We extend the standard small open economy model with nominal rigidities in the following ways.\footnote{For a version of the small open model with nominal rigidities see, for example, Gali and Monacelli [2005].} We include capital accumulation, non-tradable and commodity-exporting sectors; and we include trends, following Rabanal [2009], in sector-specific productivity technology processes to give rise to inflation differentials in steady state.

We add Rotemberg [1982] style price adjustment costs in the home-produced tradable, non-tradeable and import sectors. Sector-specific capital and investment adjustment costs restrict the economy’s ability to adjust the composition of output in response to shocks. To help the model match the empirical volatility of many observables we include intertemporal and labour market preference shocks, investment-specific productivity shocks and risk premium shocks to the uncovered interest parity equation. Since the model is large, we present the basic ingredients in the main text and leave a comprehensive presentation to the appendix.
2.1 Households

The preferences of a typical household in the small open economy are given by:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \zeta_t U(C_t, L_t)$$

where $\mathbb{E}_0$ denotes the time 0 conditional expectation, $\beta$ is the discount factor and the preference shock $\zeta_t$ follows the autoregressive process:

$$\ln \zeta_t = \rho \zeta_{t-1} + u_{\zeta,t}$$

where $0 \leq \rho < 1$ and $u_{\zeta,t}$ is an identically and independently distributed (iid) random variable with mean zero and standard deviation $\sigma_\zeta$.

The single period utility function $U$ is strictly increasing in aggregate consumption, $C_t$, decreasing in hours worked, $L_t$, and given by:

$$U(C_t, L_t) = \ln(C_t - hC_{t-1}) - \varepsilon_{L,t} \frac{L_t^{1+\nu}}{1+\nu}$$

where $h \in [0,1]$ governs the degree of external habit formation and $\nu$ controls the responsiveness of aggregate labour supply to changes in real wages. $\varepsilon_{L,t}$ is a labour supply shock that follows:

$$\ln \varepsilon_{L,t} = \rho \varepsilon_{L,t-1} + u_{\varepsilon,t}$$

where $0 \leq \rho < 1$ and $u_{\varepsilon,t}$ is an iid random variable with mean zero and standard deviation $\sigma_\varepsilon$. The functional form of the labour aggregate takes the form

$$L_t = \left[ \xi_H L_{H,t}^{1+\omega} + \xi_N L_{N,t}^{1+\omega} + \xi_X L_{X,t}^{1+\omega} \right]^{\frac{1}{1+\omega}}$$

making employment in the different sectors of the economy imperfect substitutes: $\omega$ controls the degree of substitutability between employment in different sectors.\(^7\) In Equation (3), $L_{H,t}$ stands for hours worked in the home-tradable goods producing sector, $L_{N,t}$ stands for hours worked in the non-tradable goods producing sector and $L_{X,t}$ stands for hours worked in the commodity-exporting goods producing sector. The parameters $\xi_H$, $\xi_N$ and $\xi_X$ govern the relative desirability of supplying labour to each sector.

Households enter the period with $K_j,t$ units of capital from sector $j \in \{H,N,X\}$, $B_t$ units of risk free bonds denominated in domestic currency and $B^*_t$ units of risk free bonds denominated in foreign currency. During the period, the household supplies labour and capital to the domestic production sector, receives profits, $\Gamma_{j,t}$, from these sectors, and pays a lump sum tax, $T_t$, to the government. The household uses its income to purchase new domestic and foreign bonds, to invest in new capital in sector $j \in \{H,N,X\}$, $I_{j,t}$, and to purchase consumption goods. The resulting flow budget constraint is given by:

$$P_t C_t + \sum_{j \in \{H,N,X\}} P_{j,t} J_{j,t} + B_{t+1} + S_t B^*_{t+1} \leq R_{t-1} B_t$$

$$+ R^F_{t-1} S_t B^*_t + \sum_{j \in \{H,N,X\}} (W_{j,t} L_{j,t} + R^J_t K_{j,t} + \Gamma_{j,t}) - T_t$$

\(^7\)When $\omega = 0$ labour is perfectly substitutable between sectors.
where $P_t$ is the consumer price index, $P_{I,t}$ is the price of the aggregate investment good and $W_{j,t}$ and $R_{j,t}$ are the wage rate and rate of return on capital in sector $j$. $R_t$ and $R^F_t$ are the rates of return on risk-free bonds in domestic and foreign currency. $S_t$ is the nominal exchange rate, the domestic price of foreign currency.

The stock of capital in sector $j$ evolves according to:

$$K_{j,t+1} = (1 - \delta) K_{j,t} + \tilde{V}_t \left[ 1 - \Upsilon \left( \frac{I_{j,t}}{I_{j,t-1}} \right) \right] I_{j,t}$$

where $\delta$ is the capital depreciation rate and $\Upsilon$ is an investment adjustment cost with the standard restrictions that in steady state $\Upsilon(\bullet) = \Upsilon'(\bullet) = 0$ and $\Upsilon''(\bullet) > 0$. $V_t$ is a process that governs the efficiency with which investment adds to the capital stock which we assume to follow,

$$\tilde{V}_t = v \left( \frac{1}{z_t} \right)^t V_t$$

where $z_t$ is the differential growth rate of real investment over the growth rate of labour-augmenting technology, $z$, and $\tilde{V}_t$ is a stationary autoregressive process that affects the marginal efficiency of investment of the form:

$$\ln V_t = \rho_V \ln V_{t-1} + u_{V,t}$$

where $0 \leq \rho_V < 1$ and $u_{V,t}$ is an iid random variable with mean zero and standard deviation $\sigma_V$. On the balanced growth path $I_{j,t}$ can be shown to grow at $z \times z_t$. The term in the left hand side of Equation (4), $\tilde{V}_t I_{j,t}$, grows at $z$. Thus, the trend in $\tilde{V}_t$ is necessary for balanced growth if real investment grows faster than real consumption, as is the case here. As the real rental rate on capital is stationary, the household’s budget constraint implies that the growth rate of real consumption, $z$, must equal the growth rate of the capital stocks. The trend in $\tilde{V}_t$ offsets the differential growth that real investment brings to the capital accumulation equations.

As explained by Schmitt-Grohe and Uribe [2003], to ensure stationarity we link the foreign interest rate to the net-foreign asset position. In particular, the interest rate on foreign bonds is given by

$$R^F_t = R^*_t \exp \left[ -\psi_b \left( S_t B^*_t \frac{NGDP^*_t}{NGDP_t} - b^* \right) + \tilde{\psi}_{b,t} \right]$$

where $R^*_t$ is the foreign interest rate, $b^*$ is the steady-state net foreign asset-to-GDP ratio and $NGDP_t$ is nominal GDP. $\tilde{\psi}_{b,t}$ is a risk-premium shock that follows the process:

$$\tilde{\psi}_{b,t} = \rho_\psi \tilde{\psi}_{b,t} + u_{\psi,t}$$

where $0 \leq \rho_\psi < 1$ and $u_{\psi,t}$ is an iid random variable with mean zero and standard deviation $\sigma_\psi$. 
2.2 Final goods producing firms

2.2.1 Final consumption goods

Final consumption goods are produced by a representative competitive firm with the technology:

\[ C_t = \left[ \gamma_{T,t} C_{T,t}^{\eta-1} + \gamma_{N,t} C_{N,t}^{\eta-1} \right]^{\frac{\eta}{\eta-1}} \]

where \( C_{N,t} \) is the output of the non-traded sector that is directed towards consumption and has price \( P_{N,t} \) while \( C_{T,t} \) is the output of the traded sector that is directed towards consumption and has price \( P_{T,t} \). The deterministic processes \( \gamma_{T,t} \) and \( \gamma_{N,t} \) ensure, as in Rabanal [2009], that expenditure shares remain stationary along the balanced growth path.\(^8\) \( C_{T,t} \) is itself a composite of domestically- and foreign-produced tradable goods produced according to the following technology

\[ C_{T,t} = \frac{(C_{H,t})^{\gamma_H}(C_{F,t})^{\gamma_F}}{(\gamma_H)^{\gamma_H}(\gamma_F)^{\gamma_F}} \]

The Cobb-Douglas specification guarantees that the expenditure shares in the tradable consumption basket remain constant. This assumption is convenient to find the normalisations to make the system stationary. Otherwise, \( \gamma_H \) and \( \gamma_F \) would have to trend to keep nominal expenditure shares constant in steady state. The trends in \( \gamma_H \) and \( \gamma_F \) together with the differential growth rate of the home-tradable producing goods, \( z_H \), and the differential growth rate of the foreign goods producing sector, \( z^* \), would determine the differential growth rate of the tradable basket, that is, \( z_T \). But to find the trends in \( \gamma_H \) and \( \gamma_F \) one must know \( z_T \).

The non-traded, home-produced traded and imported consumption goods are all bundles of a continuum of imperfectly substitutable goods:\(^9\)

\[ C_{j,t} \equiv \left( \int_0^1 C_{j,t}(i) \frac{i^{\eta-1}}{\eta} di \right)^{\frac{\eta}{\eta-1}} \]

The zero-profit condition implies that the price index of the final consumption good is given by

\[ P_t = \left[ \gamma_{T,t} P_{T,t}^{1-\eta} + \gamma_{N,t} P_{N,t}^{1-\eta} \right]^{\frac{1}{1-\eta}} \]  \hspace{1cm} (9)

and the price index of the tradable consumption good, in turn, is given by:

\[ P_{T,t} = (P_{H,t})^{\gamma_H}(P_{F,t})^{\gamma_F} \]  \hspace{1cm} (10)

2.2.2 Final Investment Goods

Final investment goods are produced by a representative competitive firm with the technology:

\[ I_t = z_t^l \frac{(I_{T,t})^{\gamma_T}(I_{N,t})^{\gamma_N}}{(\gamma_T)^{\gamma_T}(\gamma_N)^{\gamma_N}} \]

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\(^8\)See the online appendix for details about the normalisations.

\(^9\)This is also the case for investment, \( I_{j,t} \) for \( j \in \{H, N, F\} \).
where \( I_{N,t} \) is the output of the non-traded sector directed towards the production of investment, \( I_{T,t} \) is the output of the traded sector that is directed towards investment and \( z_t \) is a productivity trend that jointly with the growth rates of \( I_{T,t} \) and \( I_{N,t} \) determines the steady state growth rate of final investment, that is \( z_t^{10} \). \( I_{T,t} \) is, as before, a composite of domestically-and foreign produced tradable goods that is produced according to the technology

\[
I_{T,t} = \frac{(I_{H,t})^\gamma_H (I_{F,t})^\gamma_F}{(\gamma_H^I (\gamma_F^I)^{1/\gamma_F})^\gamma_H (\gamma_F^{1/\gamma_F})^\gamma_F}
\]

The corresponding price indices are

\[
P_t^I = (P_{T,t}^I)^\gamma_I (P_{N,t}^I)^{1/\gamma_I}
\]

and

\[
P_{T,t}^I = (P_{H,t}^I)^{1/\gamma_I} (P_{F,t}^I)^\gamma_I
\]

As the shares of non-tradable, domestically-produced tradable and imported goods in the investment and consumption composites differ, the price of final consumption goods, \( P_t \), will, in general, differ from the price of investment goods, \( P_t^I \); as will the price of tradable consumption goods, \( P_{T,t} \), differ from the price of tradable investment goods, \( P_{T,t}^I \).

### 2.3 Intermediate goods producing firms

There are four intermediate good producers: commodity firms, non-tradable firms, domestic tradable firms and importing firms. We describe each in turn.

#### 2.3.1 Commodity-exporting firms

Commodity firms produce an homogeneous good in a perfectly competitive market using the Cobb-Douglas production function:

\[
Y_{X,t} = A_t \tilde{Z}_{X,t} (K_{X,t})^{\alpha_X} (Z_t L_{X,t})^{1-\alpha_X}
\]

where \( Z_t \) is a labour-augmenting technology shock, common to all producing sectors, whose growth rate, \( z_t = Z_t / Z_{t-1} \), follows the process below

\[
\ln z_t = (1 - \rho_z) \ln z + \rho_z \ln z_{t-1} + u_{z,t}
\]

and \( z > 1 \) determines the trend growth rate of real GDP, \( Y_t \). \( 0 \leq \rho_z < 1 \) and \( u_{z,t} \) is an iid random variable with mean zero and standard deviation \( \sigma_z \). The sector-specific productivity process, \( \tilde{Z}_{X,t} \), follows

\[
\tilde{Z}_{X,t} = z_t^{\lambda_X} Z_{X,t}
\]

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\(^{10}\)Ireland and Schuh [2008] and Justiniano et al. [2011] are examples of closed economy models with a trend in the price of investment goods and a wedge between the growth rates of real investment and real output.
where \( z_X > 0 \) determines the differential growth rate, along the balanced growth path, between \( Y_{X,t} \) and \( Y_t \), that is between output in the commodity-exporting sector and real GDP. The stationary process \( Z_{X,t} \) gives rise to temporary departures from the differential trend by

\[
\ln Z_{X,t} = \rho_X \ln Z_{X,t-1} + u_{X,t}
\]

In Equation (13) \( A_t \) is a stationary technology shock, also common to all sectors, that follows the process

\[
\ln A_t = \rho_a \ln A_{t-1} + u_{A,t}
\]

where \( 0 \leq \rho_a < 1 \) and \( u_{A,t} \) is an iid random variable with mean zero and standard deviation \( \sigma_A \).

Commodity producers take prices as given. These prices are set in foreign currency terms in world markets and are unaffected by domestic economic developments. Specifically, we assume that the price of commodities, in foreign currency terms, is equal to

\[
P^*_X, t = \tilde{\kappa}_t P^*_t
\]

where \( P^*_t \) is the foreign price level and \( \tilde{\kappa}_t \), which governs the relative price of commodities, follows the exogenous process:

\[
\tilde{\kappa}_t = \exp (\kappa_t) \left( \frac{z^*}{z_X} \right)^t
\]

where \( z^* \) is the differential growth rate of foreign output and \( z^*_X \) is the differential growth rate of foreign production of commodities. The drift in the relative price of commodities reflects the relative productivity growth of the commodity sector and the foreign economy.\(^{11}\) Transitory shocks to commodity prices follow

\[
\kappa_t = (1 - \rho_\kappa)\kappa + \rho_\kappa \kappa_{t-1} + u_{\kappa,t}
\]

where \( 0 \leq \rho_\kappa < 1 \) and \( u_{\kappa,t} \) is an iid shock with zero mean and standard deviation \( \sigma_\kappa \). For the stochastically detrended variables, \( \kappa \) determines the unconditional mean of the terms of trade and, in turn, is one of the determinants of the economy’s steady state. In estimation, we allow for breaks in \( \kappa \) and in \( \sigma_\kappa \), possibly occurring at different dates in the sample.

The law of one price holds for commodities, which means that their price in domestic currency terms is

\[
P_{X,t} = S_t P^*_{X,t}
\]

### 2.3.2 Non-tradeable goods producing firms

Non-tradeable firms sell differentiated products, which they produce using the Cobb-Douglas production function:

\[
Y_{N,t}(i) = A_t \tilde{Z}_{N,t} (K_{N,t}(i))^{\alpha_N} (Z_t L_{N,t}(i))^{1-\alpha_N}
\]

\(^{11}\)The productivity trend in the foreign production of commodities must equal the productivity trend in the domestic production of commodities for balanced growth, that is \( z_X = z^*_X \).
$\tilde{Z}_{N,t}$ is sector-specific productivity process that follows

$$\tilde{Z}_{N,t} = z_N^{\frac{1}{t}}Z_{N,t}$$

where $z_N > 0$ and $Z_{N,t}$ are transitory deviations from the sector-specific trend that follows

$$\ln Z_{N,t} = \rho_N \ln Z_{N,t-1} + u_{N,t}$$

(23)

where $0 \leq \rho_N < 1$ and $u_{N,t}$ is an iid random variable with mean zero and standard deviation $\sigma_N$. Firms can only change prices at some cost, following a Rotemberg [1982] pricing mechanism:

$$\frac{\psi_N}{2} \left( \frac{P_{N,t}(i)}{\Pi^N P_{N,t-1}(i)} - 1 \right)^2 P_{N,t}Y_{N,t}$$

where $\psi_N$ governs the size of the price adjustment cost and $\Pi^N$ is the steady state inflation rate of non-tradable goods prices.

Aggregate non-tradable output is defined by the Dixit-Stiglitz aggregator:

$$Y_{N,t} \equiv \left( \int_0^1 Y_{N,t}(i)^{\frac{N-1}{N}} di \right)^{\frac{1}{N-1}}$$

### 2.3.3 Domestic tradeable goods producing firms

Domestic tradable firms produce differentiated products using the Cobb-Douglas production function

$$Y_{H,t}(i) = A_t \tilde{Z}_{H,t} (K_{H,t}(i))^{\alpha_H} (Z_t L_{H,t}(i))^{1-\alpha_H}$$

(24)

$Z_{H,t}$ is a stationary sector-specific TFP shock that follows

$$\tilde{Z}_{H,t} = z_H^{\frac{1}{t}}Z_{H,t}$$

where $z_H > 0$ and $Z_{H,t}$ are temporary deviations from that trend,

$$\ln Z_{H,t} = \rho_H \ln Z_{H,t-1} + u_{H,t}$$

(25)

where $0 \leq \rho_H < 1$ and $u_{H,t}$ is an iid random variable with mean zero and standard deviation $\sigma_H$. Like their non-tradable counterparts, tradable firms can only change prices at some cost, following a Rotemberg [1982] pricing mechanism:

$$\frac{\psi_H}{2} \left( \frac{P_{H,t}(i)}{\Pi^H P_{H,t-1}(i)} - 1 \right)^2 P_{H,t}Y_{H,t}$$

where $\psi_H$ governs the size of the price adjustment cost and $\Pi^H$ is the steady state inflation rate of domestic-tradable goods prices. Domestic tradable output, $Y_{H,t}$, is an aggregate of the output of each of the domestic tradable firms,

$$Y_{H,t} \equiv \left( \int_0^1 Y_{H,t}(i)^{\frac{H-1}{H}} di \right)^{\frac{1}{H-1}}$$

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12We assume that these price adjustment costs do not affect the cash flow of firms, but only affect their objective function (see De Paoli et al. [2010] for a discussion of this approach). Therefore, they do not appear in the resource constraint or net export equations. Assuming instead that these adjustment costs are real costs would yield equivalent results as quadratic terms do not appear in the linearized system.
2.3.4 Importing Firms

Importing firms purchase foreign good varieties at the price $\varsigma S_t P^*_t$ and sell them in the domestic market at price $P_{F,t}(i)$. The parameter $\varsigma$ represents a subsidy to imported firms, funded by lump-sum taxation. We set the subsidy equal to $\varsigma = (\theta_F - 1)/\theta_F$, thereby ensuring that markups in this sector are zero in equilibrium.

Importing firms can only change prices at some cost, following a Rotemberg [1982] pricing mechanism

$$\frac{\psi_F}{2} \left( \frac{P_{F,t}(i)}{\Pi^F P_{F,t-1}(i)} - 1 \right)^2 P_{F,t} Y_{F,t}$$

2.4 Foreign sector, net exports and the current account

Following Gertler et al. [2007], we postulate a foreign demand function for domestically produced tradable goods, $C^*_{H,t}$, of the form

$$C^*_{H,t} = \gamma^*_{H,t} \left( \frac{P_{H,t}}{S_t P^*_t} \right)^{-\eta^*} \bar{Y}^*_t$$

Foreign output, $\bar{Y}^*_t$, follows the non-stationary process

$$\bar{Y}^*_t = Z_t(z^*) Y^*_t$$

where $z^* > 0$ captures the differential steady state growth rate between real GDP in the domestic and the rest of the world. Transitory deviations from foreign trend growth are captured by $Y^*_t$ which follow

$$\ln Y^*_t = \rho^*_Y \ln Y^*_t + u^*_{Y,t}$$

where $0 \leq \rho^*_Y < 1$ and $u^*_{Y,t}$ is an iid random variable with mean zero and standard deviation $\sigma^*_Y$. Foreign inflation is assumed to follow

$$\ln \Pi^*_t = (1 - \rho^*_\Pi) \ln \Pi^* + \rho^*_\Pi \ln \Pi^*_{t-1} + u^*_\Pi,t$$

and the foreign interest rate follows

$$\ln R^*_t = (1 - \rho^*_R) \ln R^* + \rho^*_R \ln R^*_{t-1} + u^*_R,t$$

where $0 \leq \rho^*_\Pi < 1$ and $0 \leq \rho^*_R < 1$ and the iid shocks $u^*_{\Pi,t}$ and $u^*_R,t$ have zero mean and standard deviations $\sigma^*_\Pi$ and $\sigma^*_R$.

Net exports are given by:

$$NX_t = P_{H,t} C^*_{H,t} + P_{X,t} Y_{X,t} - P_{F,t} (C_{F,t} + I_{F,t})$$

and so the current account equation is given by:

$$S_t B^*_t = R^F_{t-1} S_t B^*_t + NX_t$$
2.5 Monetary Policy

The domestic central bank follows a Taylor rule that responds to deviations of output growth and inflation from their steady-state levels

\[
\ln \left( \frac{R_t}{R} \right) = \rho_r \ln \left( \frac{R_{t-1}}{R} \right) + (1 - \rho_R) \left[ \phi_\pi \ln \left( \frac{\Pi_t}{\Pi} \right) + \phi_y \ln \left( \frac{Y_t}{zY_{t-1}} \right) \right] + u_{R,t} 
\]

(32)

where \( \Pi_t = P_t/P_{t-1} \) is the inflation rate in terms of final consumption goods prices and \( \Pi \) is the central bank’s inflation target.

2.6 Market Clearing

Market clearing for investment goods requires that production of these goods equals to quantity demanded by the three domestic production sectors

\[ I_t = I_{h,t} + I_{n,t} + I_{x,t} \]  

(33)

For the non-tradable, domestic tradable and import sectors, market clearing requires that the quantity produced equals the quantity demanded:

\[ Y_{N,t} = C_{N,t} + I_{N,t} \]  

(34)

\[ Y_{H,t} = C_{H,t} + C_{H,t}^t + I_{H,t} \]  

(35)

\[ Y_{F,t} = C_{F,t} + I_{F,t} \]  

(36)

Nominal GDP is defined as:

\[ NGDP_t = P_{N,t} Y_{N,t} + P_{H,t} Y_{H,t} + P_{X,t} Y_{X,t} \]  

(37)

and real GDP is defined as:

\[ Y_t = \frac{P_N}{P} Y_{N,t} + \frac{P_H}{P} Y_{H,t} + \frac{P_X}{P} Y_{X,t} \]  

(38)

2.7 Balanced Growth

A novel feature of this model as that of Rabanal [2009] is the existence of trends in aggregate as well as sector-specific productivity. Next, we describe how the variables behave along a balanced growth path and the normalisations that induce stationarity.

Along a balanced growth path aggregate variables, including GDP, consumption and the capital stock, grow at the rate of aggregate productivity, \( z \). Sectoral variables, such as output of non-tradable goods, \( Y_{N,t} \), and the quantity of these goods that enter consumption and investment baskets, that is, \( C_{N,t} \) and \( I_{N,t} \), grow at aggregate productivity adjusted by the sector specific trend. For example, in steady state the growth rate of non-tradable output is \((1 + z)(1 + z_N)\).

Balanced growth requires the shares of each sector in nominal GDP to be constant. For this to hold, the relative prices of each sector must trend by the reciprocal of the
sector-specific productivity growth rate. So, for example, the relative price of non-tradable goods to the price of consumption goods, that is, $P_{N,t}/P_t$, must grow at $(1+z_N)^{-1}$ along a balanced growth path. When bundles are Cobb-Douglas expenditure shares are constant and balanced growth is satisfied regardless of trends in relative prices because income and substitution effects are offsetting. However, in the more general CES specification for the bundles, balanced growth will be achieved provided that the weights satisfy the following processes: $\gamma_{N,t} = \frac{\gamma_N}{(1+z_N)^\eta-1}$ and $\gamma_{T,t} = \frac{\gamma_T}{(1+z_T)^\eta-1}$.

3 Empirical Strategy

The structural parameters can be thought of in two categories: those that can only determine dynamics – persistence parameters, adjustment cost parameters, policy rule parameters and standard deviations – and parameters that, in addition to influencing the dynamics, pin down the steady state. Like Adolfson et al. [2007], we calibrate most parameters that determine the steady state. The habits parameter, $h$, and the long-run level of commodity prices in the final steady state, $1 + \Delta\kappa$, however, are estimated. We estimate the remaining parameters.

3.1 Calibration

We calibrate for two reasons. First, not all parameters are well identified given the usual choice of observable variables. Second, estimation could imply a steady state at odds with the sample means in the data. We set these parameters so that the balanced growth path is in line with the first moments of the data.

The traditional approach of matching sample means seems inappropriate in our case because we postulate a possible break in the long-run level of the terms of trade, which in turn leads to changes in unconditional means. We therefore focus on matching the features of the data over the first part of the sample, before commodity prices started to rise rapidly in the early 2000s. To be precise, we calibrate the model’s parameters to match means over the period 1993 to 2002, which is a period of stability in the terms of trade. In the initial steady state, we first set $\kappa$ to 1. Before other parameters are calibrated, this choice is a normalisation. After that, of course, a change in $\kappa$ changes the steady state. For the data to be consistent with the model in estimation, the index of real commodity prices must be re-normalised to average 1 over the sub-sample.

Our calibration strategy is as follows. We calibrate the model at a quarterly frequency. We assume that the steady-state rate of labour augmenting TFP growth, $z$, is 0.0049, which matches the average growth rate of per capita GDP over our sample.13 We set the central bank’s target inflation rate, $\Pi$, equal to 0.0062. This delivers an average annual inflation rate of around 2.5 per cent, which is the middle of the central bank’s stated inflation target.

13We calibrate this parameter using the average growth over our full sample of data rather than the shorter 1993 to 2002 sample because the shorter sample featured an unusually rapid period of economic growth in Australia associated with a steep recovery from a deep recession in the early 1990s and a period of rapid productivity growth due, in part, to a series of microeconomic reforms in the 1980s. Consequently, the full-sample average is likely to better reflect the average long-run TFP growth rate.
inflation target. We set the household’s discount rate, $\beta$, equal to 0.99625. Together, these three parameters match the steady state nominal interest rate to the sample of the Cash Rate of 6(?) per cent.

We set the sector-specific productivity growth differentials, $z_N$ and $z_H$ so that the inflation rates of tradable and non-tradable goods match their rates in the data. So, we set $z_N = 0.999028$ to ensure that, on average, non-tradable prices rise faster than the CPI. And we set $z_H$ to 1.0014. Given this value and the foreign productivity growth differential, $z^* = 1.00033$, set to match the average growth rate of Australia’s major trading partners at PPP, the steady state of tradables goods inflation in the model matches that in the data.

We set the capital shares in each sector, $\alpha_N$, $\alpha_H$ and $\alpha_X$ to 0.358, 0.438 and 0.7, reflecting their average values in national accounts data. The markup parameters, $\theta_N$, $\theta_H$ and $\theta_F$ are set so that each sector has an an average markup of 10 per cent. The price adjustment costs parameters that determine the slope of Phillips curves, however, are estimated.

We set the parameter governing the elasticity of labour supply, $\eta$, we set to 2, which is a standard value in the literature. The parameter governing the willingness of workers to move between sectors, we set to 1, in line with Horvath [2000]. We set the parameters $\xi_N$, $\xi_H$ and $\xi_X$ to 100, 209 and 4167. This ensures that the share of hours worked in each sector in the model broadly matches that in the data.

The shares in the Cobb-Douglas bundles are set to match averages in the data. Thus we set, $\gamma_F$ to 0.357, $\gamma_H$ to $1 - \gamma_F$, $\gamma_N^F$ to 0.664, $\gamma_I^F$ to $1 - \gamma_N^F$, $\gamma_I^F$ to 0.828 and $\gamma_I^F$ to $1 - \gamma_I^F$.

Table 2 compares the moments implied by the model’s steady state to their empirical counterparts. At the calibrated parameters values of Table 1 the model’s steady state does quite well. Investment growth in the model’s steady state is somewhat lower than in our sample. Because investment is quite volatile and can fall significantly in recessions, it is likely that the growth rate of investment in our sample (which does not have a recession) is overstated. If we extend the sample to begin in 1990:Q1 – which includes a recession – the average growth rate of investment is 3.46. We set $z_v$ so as to match the rate of inflation of investment goods prices, which in turn, implies a growth rate of investment of 3.43 per cent.

### 3.2 Estimation

We use Bayesian methods, as is common in the estimated DSGE literature. Our case, however, is non standard because we allow for structural change and jointly estimate two sets of distinct parameters: the structural parameters of the model, $\vartheta$, that have continuous support and the dates of structural changes, $T = (T_\kappa, T_{\sigma_\kappa})$ that have discrete support; $T_\kappa$ is the date break in the mean and $T_{\sigma_\kappa}$ is the date break in the variance of

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14 The data appendix provides additional detail regarding our classification of industries into tradable, non-tradable and commodity-exporting.

15 See An and Schorfheide [2007] for a description of these techniques.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Household discount factor</td>
<td>0.99625</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Capital depreciation rate</td>
<td>0.005</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Labour supply parameter</td>
<td>2</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Intersectoral labour supply elasticity</td>
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<tr>
<td>$\xi_N$</td>
<td>Constant on non-tradable labor supply</td>
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</tr>
<tr>
<td>$\xi_H$</td>
<td>Constant on tradable labour supply</td>
<td>209</td>
</tr>
<tr>
<td>$\xi_X$</td>
<td>Constant on commodities labour supply</td>
<td>4167</td>
</tr>
<tr>
<td>$\psi_b$</td>
<td>Risk premium</td>
<td>0.001</td>
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<tr>
<td>$\gamma_N$</td>
<td>Non-tradables consumption weight</td>
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<tr>
<td>$\gamma_H$</td>
<td>Home-produced tradables weight</td>
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</tr>
<tr>
<td>$\gamma^I_N$</td>
<td>Non-tradables investment weight</td>
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</tr>
<tr>
<td>$\gamma^I_H$</td>
<td>Home-produced tradables investment weight</td>
<td>0.172</td>
</tr>
<tr>
<td>$\gamma^*_H$</td>
<td>Determinant of foreign demand</td>
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</tr>
<tr>
<td>$\eta$</td>
<td>Elasticity of substitution</td>
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</tr>
<tr>
<td>$\eta^*$</td>
<td>Elasticity of substitution</td>
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</tr>
<tr>
<td>$z$</td>
<td>Steady-state TFP growth</td>
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</tr>
<tr>
<td>$z_v$</td>
<td>Investment growth rate differential</td>
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<tr>
<td>$z_N$</td>
<td>Non-tradables growth differential</td>
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<td>$z_H$</td>
<td>Home tradable growth differential</td>
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<td>$z_X$</td>
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<tr>
<td>$z^*$</td>
<td>Foreign growth differential</td>
<td>1.00033</td>
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<tr>
<td>$\alpha_N$</td>
<td>Capital share in non-tradables</td>
<td>0.358</td>
</tr>
<tr>
<td>$\alpha_H$</td>
<td>Capital share in tradables</td>
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<tr>
<td>$\alpha_X$</td>
<td>Capital share in commodities</td>
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<td>$\Pi$</td>
<td>Domestic inflation target</td>
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<td>$\Pi^*$</td>
<td>Foreign inflation target</td>
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<tr>
<td>$\theta_N$</td>
<td>Markup in non-tradables</td>
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<tr>
<td>$\theta_H$</td>
<td>Markup in home tradables</td>
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</tr>
<tr>
<td>$\theta_F$</td>
<td>Markup in imports</td>
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<tr>
<td>$b^*$</td>
<td>Steady state net foreign assets</td>
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Notes:
Table 2: Steady State Properties of the Model

<table>
<thead>
<tr>
<th>Target</th>
<th>Average</th>
<th>1993 - 2013</th>
<th>1993 - 2002</th>
<th>Model</th>
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<tbody>
<tr>
<td><strong>Macro Aggregates (annual per cent)</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Per capita output growth</td>
<td>1.96</td>
<td>2.64</td>
<td>1.96</td>
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</tr>
<tr>
<td>Per capita investment growth</td>
<td>4.31</td>
<td>4.39</td>
<td>3.43</td>
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<tr>
<td>Inflation</td>
<td>2.63</td>
<td>2.50</td>
<td>2.50</td>
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<tr>
<td>Tradable inflation</td>
<td>1.65</td>
<td>2.09</td>
<td>2.09</td>
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</tr>
<tr>
<td>Non-tradable inflation</td>
<td>3.41</td>
<td>2.86</td>
<td>2.86</td>
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<tr>
<td>Investment deflator inflation</td>
<td>1.29</td>
<td>1.09</td>
<td>1.09</td>
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<tr>
<td><strong>Expenditure (per cent of GDP)</strong></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Consumption</td>
<td>74.6</td>
<td>75.8</td>
<td>72.7</td>
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<tr>
<td>Investment</td>
<td>26.3</td>
<td>24.7</td>
<td>27.3</td>
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<tr>
<td>Exports</td>
<td>19.5</td>
<td>19.3</td>
<td>19.7</td>
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<tr>
<td><strong>Consumption basket (per cent of consumption)</strong></td>
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<tr>
<td>Non-tradable consumption</td>
<td>55.2</td>
<td>53.4</td>
<td>53.4</td>
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<td>Home tradable consumption</td>
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<td>30.0</td>
<td>30.0</td>
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<tr>
<td>Imported tradable consumption</td>
<td>17.5</td>
<td>16.6</td>
<td>16.6</td>
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<tr>
<td><strong>Investment basket (per cent of investment)</strong></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-tradable investment</td>
<td>67.4</td>
<td>66.4</td>
<td>66.4</td>
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<tr>
<td>Home tradable investment</td>
<td>2.6</td>
<td>5.7</td>
<td>5.8</td>
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<tr>
<td>Imported investment</td>
<td>30.0</td>
<td>27.9</td>
<td>27.8</td>
<td></td>
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<tr>
<td><strong>Exports (per cent of exports)</strong></td>
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<tr>
<td>Resource exports</td>
<td>42.1</td>
<td>34.6</td>
<td>34.6</td>
<td></td>
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<tr>
<td>Other exports</td>
<td>57.9</td>
<td>65.4</td>
<td>65.4</td>
<td></td>
</tr>
<tr>
<td><strong>Employment (per cent of hours worked)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-tradable</td>
<td>67.6</td>
<td>65.0</td>
<td>64.3</td>
<td></td>
</tr>
<tr>
<td>Home tradable</td>
<td>31.1</td>
<td>34.0</td>
<td>33.2</td>
<td></td>
</tr>
<tr>
<td>Mining</td>
<td>1.3</td>
<td>1.0</td>
<td>2.5</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Model ratios calculated at initial regime with $\bar{\kappa} = 1.0$ and $h = 0.5$. 
commodity prices. We set the trimming parameter for both date breaks to 25 per cent of the sample. This implies, in our case, the minimum length of a segment to be 20 observations.

Next, we describe how we construct the joint posterior density of $\vartheta$ and $T$:

$$p(\vartheta, T|Y) \propto L(Y|\vartheta, T)p(\vartheta, T),$$

where $Y \equiv \{ y_{t}^{obs} \}_{t=1}^{T}$ is the data and $y_{t}^{obs}$ is a $n_{obs} \times 1$ vector of observable variables. The likelihood is given by $L(Y|\vartheta, T)$, the priors for the structural parameters and the date breaks are taken to be independent, so that $p(\vartheta, T) = p(\vartheta)p(T)$. We use a flat prior for $T$ so $p(T) \propto 1$, which is proper given its discrete support. Kulish and Pagan [2012] discuss how to construct $L(Y|\vartheta, T)$ in models with forward-looking expectations and structural changes. Appendix C describes the posterior sampler.

We estimate the model using quarterly Australian macroeconomic data for the period 1993Q1 to 2013Q4. The starting date coincides with the start of inflation targeting in Australia and represents a period over which the macroeconomic policy environment has been broadly stable.

Our data series includes aggregate and sectoral variables and foreign variables. The aggregate data include real GDP, investment, consumption, net exports, hours worked, the cash rate, trimmed mean inflation and the percentage change in the nominal exchange rate. The national accounts variables and hours are all expressed in per capita terms and seasonally adjusted. Output, investment, consumption and hours all enter in percentage changes, while net exports enters as a share of nominal GDP. We also include two sectoral variables in the model: the inflation rate of non-tradable goods and the ratio of nominal non-tradable consumption to aggregate nominal consumption. The foreign variables that we include in the model are output growth, interest rates and inflation. We take the growth rate of the Australian major trading partner GDP series constructed by the Reserve Bank of Australia as the measure of foreign output growth. For interest rates we use the average of the policy rates in the US, the Euro area and Japan.\textsuperscript{16} For the foreign inflation rate, we use the trade-weighted average inflation rate of Australia’s major trading partners. The 14 series we use in estimation are shown in Figure 2. Appendix A contains a complete description of the data sources, calculations and transformations.

We add measurement error in estimation as is standard in the literature. We calibrate the variance of the measurement errors to 5 per cent of the variance of each observable series. Macroeconomic data are measured with noise and the economic concepts in the model do not always match the measures in the data. Take real output for example. Real output is constructed using chain volume measures which are subject to revision whenever the reference period is changed. The model measure of real output, however, is constructed at steady state relative prices so the reference period does not change. Steady state relative prices change when the long-run level of the terms of trade changes, and our measurement equations change as well, this is not equivalent to chain-volume linking.

\textsuperscript{16}Prior to the introduction of the euro, we construct this series using the German policy rate.
3.3 Priors

For the parameter of most interest to us, the long-run level of commodity prices in the final regime, we choose a uniform prior over the domain $-0.25$ to $3.5$ over the change in $\kappa$, that is, we put a flat prior over $\Delta \kappa$. Notice that, we allow the steady-state of relative commodity price to have declined by 25 per cent, increased by 350 per cent or taken any value in between. We choose the same prior for $\sigma^f_\kappa$ and $\sigma^i_\kappa$, the standard deviations of shocks to commodity prices in the two regimes. We choose loose beta distributions on the autoregressive parameters and inverse gamma distributions on the standard deviations of the shocks.\footnote{These choices are standard in the literature. See An and Schorfheide [2007].} For the parameters of the monetary policy rule we set a prior mean of 1.5 for the response of the Cash Rate to inflation and of 0.3 for the response to real output growth. These choice are in line with the literature. Table 4 summarises the priors.
4 Results

4.1 Date breaks

Figure 3 shows the cumulative posterior density of $T_\kappa$, the date break in the unconditional mean, and cumulative posterior density of $T_{\sigma_\kappa}$, the date break of the variance, that is the date break for $1 + \Delta \kappa$ and the date break for $\sigma_\kappa$. The data strongly prefer 2003:Q2 for the date break in the unconditional mean. The probability that the break occurred in this quarter is around 95 per cent and there is some probability that it occurred in the next few quarters. Our estimate of $T_\kappa$ is close to that of Gruen [2011] who dates the start of boom to 2002:Q2. Single equation Bai and Perron [1998] tests place the date break in $\kappa$ a quarter later, that is 2003:Q3. The date break in volatility, $T_{\sigma_\kappa}$, shown in the bottom panel shows, is estimated to have occurred after the increase in the unconditional mean. The posterior density for $T_{\sigma_\kappa}$ is bi-modal. It peaks in 2005:Q2 and then again in the second quarter of 2008:Q2.

Figure 3: Posterior cumulative distributions of date breaks

4.2 Structural parameters

In estimation we allow for breaks in $\kappa$ and $\sigma_\kappa$ because we want to give the model a chance to fit the data without necessarily having to resort to a change in $\kappa$. As it turns out, the data strongly prefer the specification in which both $\kappa$ and $\sigma_\kappa$ increase. The estimation
could have well chosen to explain the fluctuations in commodity prices post-2003 with an increase in volatility and no increase in the unconditional mean. Figure 4 shows the posterior distribution of $\Delta \kappa$. The density is bounded away from zero and the long-run level of commodity prices $1 + \Delta \kappa$ is estimated to have increased by around 40 per cent, with a distribution that ranges between 30 and 50 per cent. The posterior distribution significantly shrinks the uncertainty relative to the range of our uninformative prior on $\Delta \kappa$ which ranges from -25 per cent to 350 per cent.

Using the tests of Bai and Perron [1998] on the commodity price series points to a 90 per cent increase of the long-run level of commodity prices. But in forward-looking general equilibrium, a 90 per cent increase of the long-run level of commodity prices has implications which are not in the data of other observed domestic endogenous variables. Our estimate is 40 per cent and the single equation one is 90 per cent. Clearly, the other observables serve to moderate the estimated increase in the long-run mean. One may, therefore wonder, if our estimate of $\Delta \kappa$, would be any different from zero if we were to estimate our model after having removed real commodity prices from the set of observable variables. In fact, we have run this estimation and still find statistically significant evidence of a 20 per cent increase in $\Delta \kappa$. At a more general level this is an important result: cross-equations restrictions permits us to make inferences about structural breaks in unobservable series.

Figure 5 shows the posterior distribution for the ratio of the standard deviations of shocks to commodity prices in the two regimes, that is $\sigma'_\kappa / \sigma_\kappa$, where we use the notation that $\sigma'_\kappa$ corresponds to the post-break one. The distribution has no mass at unity or values below. There is no likelihood that the standard deviation of commodity price
shocks has fallen or stayed the same. In fact, the data point to the volatility of shocks to commodity prices having more than doubled. Below we study some of the macroeconomic implications of these changes, but before we briefly discuss estimates of other parameters.

Table 3 also summarises the estimates of other parameters. The monetary policy rule parameters reveal that there is persistence in the setting of the policy rate and that the policy rate responds relatively more strongly to inflation than it does to output growth. Habits and investment adjust costs are important; the data prefer some additional persistence, relative to the priors, stemming from these parameters. We also find heterogeneity in the degree of price stickiness across sectors. At the mode the slope of the Phillips curve in the tradable sector is 5 times steeper than in the non-tradable sector which is in turn steeper than in the importing goods producing sector.

The parameters of the exogenous process are reported in Table 4.

### 4.3 Estimated transitional dynamics

To get a sense of the magnitudes involved we compute the transitional dynamics implied by the posterior distribution of $\Delta \kappa$ for real commodity prices and for the price of non-tradable goods. We sample from the joint posterior distribution of date breaks and structural parameters. Then at each sampled value, we compute the path of the non-stochastic steady state: this is the path the economy would have taken in the absence of shocks but in the presence of $\Delta \kappa$ occurring at some $T_\kappa$. Notice that according to Equation (20) a change in $\kappa$ induces a slow transition towards the new long-run value because the process is persistent. In fact the contemporaneous impact is attenuated by the persistence of the process, is given by $(1 - \rho_\kappa)\Delta \kappa$. Figure 6 shows the observed real

---

Figure 5: Posterior distribution of $\sigma_\kappa^f / \sigma_\kappa^i$. 

---

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### Table 3: Prior and Posterior Distribution of Structural Parameters

<table>
<thead>
<tr>
<th>Variable</th>
<th>Prior Distribution</th>
<th>Posterior Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Shape</td>
<td>Mean</td>
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<tr>
<td><strong>Commodity Prices</strong></td>
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<tr>
<td>$\Delta \kappa$</td>
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</tr>
<tr>
<td>$\sigma_\kappa$</td>
<td>Inv. Gamma 0.1 2</td>
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</tr>
<tr>
<td>$\sigma'_\kappa$</td>
<td>Inv.Gamma 0.1 2</td>
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<td>$\rho_\kappa$</td>
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<td><strong>Frictions</strong></td>
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<tr>
<td>$Slope_H$</td>
<td>Gamma 5 3</td>
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<tr>
<td>$Slope_F$</td>
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Notes: $Slope_N = 100(\theta_N - 1)/\psi_N$, $Slope_H = 100(\theta_H - 1)/\psi_H$, $Slope_F = 100(\theta_F - 1)/\psi_F$
Table 4: Prior and Posterior Distribution of Shock Processes

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<tr>
<th>Variable</th>
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<th>Posterior Distribution</th>
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<td>Mean</td>
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<td>0.001</td>
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<td>0.011</td>
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</table>

Notes:
commodity price series we use in estimation (in levels) and the distribution of the non-
stochastic path of real commodity prices implied by our posterior estimates. Most date
breaks happen in 2003:Q2 and from that time, the grey paths are anticipated (because
the process is persistence) even when the shock to $\Delta \kappa$ in 2003:Q2 is not. After the break,
the forecasts for commodity prices in each quarter, for the most part, implies significant
falls in commodity prices. This is consistent with the Reserve Bank of Australia’s terms
of trade forecast since 2003.\(^\text{18}\) The difference between observed commodity prices and
the implied fundamental level, so to speak, is the result of an increase in the volatility
of shocks to commodity prices. What we find remarkable is that we have not insisted
that the long-run level had to change, all the variation could have been picked up by
volatility, but all observables (including commodity prices) point to an increase in the
unconditional mean.

It is interesting to look at similar implication for the price of non-tradable goods.
Although our model does not have housing sector, an important determinant of non-
tradable inflation in the data comes from the housing sector.\(^\text{19}\) Figure 7 shows the
observed relative price of non-tradables goods and the posterior distribution implied by
the transition induced by $\Delta \kappa$. Recall that because there is wedge between non-tradable
inflation and inflation, this relative price trends and the rate at which it trends is the
same for both balanced growth paths. According to Figure 7, the relative price of non-
tradables goods is well explained by the shift in the long-run level of the terms of trade.

\(^{18}\)See Figure 4 in Plumb et al. [2013].

\(^{19}\)Future research could extend the model to include a housing sector and study the implication for
housing prices.
4.4 Variance Decompositions

The estimated changes in the unconditional mean and volatility bring about changes in the contributions of the various shocks. The change in $\kappa$ changes the steady state, in particular, it changes the relative size of the sectors. For instance, the commodity sector’s share of exports increases from 35 to 52 per cent. This change alone alters the relative contribution of shocks to the business cycle, even if the standard deviation of the shocks does not change. But in our case, shocks to commodity prices become more important.

To measure the implications of these changes we compare variance decomposition at the mode for some variables of interest in two regimes: a low regime, $\kappa = 1$ and $\sigma_\kappa = .05$) and a high regime, $\kappa = 1.42$ and $\sigma_\kappa = .11$.\(^{20}\) Shocks to commodity prices and shocks to the productivity of the commodity-exporting sector explain little of the fluctuations in inflation. However, shocks to the productivity of the commodity-exporting sector explain 27 per cent of the variance of output growth in the low regime and around half of the variance post 2007. Shocks to real commodity prices explain in the high regime over 50 per cent of the variance of net exports.

Many other results are well-known in the literature but we point them out. The nominal exchange rate is mostly driven by risk-premium shocks. However, it is noticeable that the contribution of $\sigma_\kappa$ increases from half a percent to about 5 per cent. Three quarters of investment growth is explained by shocks to the marginal efficiency of investment.

\(^{20}\)We do not report variance decompositions for the high-$\kappa$ low-$\sigma_\kappa$-regime because the joint posterior implies a low probability for this regime.
Table 5: Variance Decomposition: Low-regime (%)

<table>
<thead>
<tr>
<th></th>
<th>$\pi_t$</th>
<th>$\pi_{N,t}$</th>
<th>$\Delta y_t$</th>
<th>$\Delta I_t$</th>
<th>$\Delta c_t$</th>
<th>$\Delta s_t$</th>
<th>$\tau_{F,t}$</th>
<th>$n_{x_t}$</th>
<th>$r_t$</th>
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<tbody>
<tr>
<td>$\sigma_R$</td>
<td>13.2</td>
<td>15.4</td>
<td>1.4</td>
<td>2.0</td>
<td>1.4</td>
<td>1.7</td>
<td>0.5</td>
<td>1.1</td>
<td>10.0</td>
</tr>
<tr>
<td>$\sigma_v$</td>
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<td>13.7</td>
<td>12.1</td>
<td>73.9</td>
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<td>0.1</td>
<td>0.3</td>
<td>3.1</td>
<td>14.2</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>1.5</td>
<td>2.2</td>
<td>9.3</td>
<td>1.8</td>
<td>20.1</td>
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<td>0.5</td>
<td>1.5</td>
<td>2.2</td>
</tr>
<tr>
<td>$\sigma_A$</td>
<td>0.5</td>
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<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.2</td>
</tr>
<tr>
<td>$\sigma_\zeta$</td>
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<td>2.7</td>
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<td>67.9</td>
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</tr>
<tr>
<td>$\sigma_H$</td>
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<td>2.4</td>
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<td>$\sigma_X$</td>
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<td>1.4</td>
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<tr>
<td>$\sigma^*_y$</td>
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</tr>
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</tr>
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<td>2.1</td>
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<td>0.1</td>
<td>0.5</td>
<td>3.5</td>
<td>14.8</td>
<td>0.2</td>
</tr>
</tbody>
</table>

What is interesting is that the contribution of $\sigma_\kappa$ to the real exchange rate goes from 3.5 to 27.

5 Impulse responses

5.1 Temporary shocks to the terms of trade

Figure 8 compares the response of output, consumption, investment growth and net exports to a temporary shock to commodity prices in the two estimated regimes, a low regime with a low mean and volatility of commodity prices and a high regime with a high mean and volatility. In each case, we take a one standard deviation shock so the size of the change in commodity prices in the high regime is around 120 per cent larger. To be precise, in the low regime we estimate the size of a typical shock to be around 5 per cent and in the high regime a typical shock is 11 per cent.

In both regimes, an increase in commodity prices leads to an initial increase in output, consumption and investment. The net exports-to-GDP ratio also increases. Naturally, the size of these increases is larger in the high regime, where the size of the increase in commodity prices is greater. However, the change in the responses between the two regimes is not proportional to the change in the size of the shock. In fact, it is more than proportional. For example, in the low regime a 5 per cent increase in commodity prices leads to a 0.1 percentage point in annualised output growth, from around 2 per cent to 2.1 per cent. In contrast, in the high regime an 11 per cent increase in commodity prices leads to a 0.5 percentage point increase in output growth, from around 2 per cent to almost 2.5 per cent.

The more than proportionate change in the responses reflects the fact that the increase
Table 6: Variance Decomposition: High-regime (%)

<table>
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<tr>
<th></th>
<th>$\pi_t$</th>
<th>$\pi_{N,t}$</th>
<th>$\Delta y_t$</th>
<th>$\Delta I_t$</th>
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<td>13.4</td>
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<td>2.0</td>
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<td>7.1</td>
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<td>20.1</td>
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<tr>
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<td>1.0</td>
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<tr>
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<td>0.5</td>
<td>4.6</td>
<td>27.6</td>
<td>57.8</td>
<td>2.4</td>
</tr>
</tbody>
</table>

in the mean of commodity prices changes the steady state of the model. In particular, the size of the commodity sector increases relative to the size of the traded and non-traded sectors. This change in the steady state affects the coefficients of the linearization. A larger commodity sector means that an increase in commodity prices has larger income effects and larger substitution effects. Consequently, its impact on the economy is larger.

In both regimes positive shocks to $\kappa_t$ are mildly inflationary. As the variance decompositions suggest, most fluctuations in inflation do not come from temporary changes in $\kappa_t$. To compare temporary shocks to the permanent change it is useful to set them to the same size. So we compute the impact on inflation of a 40 per cent temporary shock to $\kappa_t$. On impact annualised inflation increases by 0.1 percentage points in the low regime and by 0.15 percentage points in the high regime. But non-tradable inflation increases by 0.7 percentage points and tradable inflation falls by 0.5 percentage points. Our results are consistent with those of Chen and Rogoff [2003] and Cashin et al. [2004] who find that increases in the terms of trade increase the price of non-tradable goods.

### 5.2 A change in the long-run level of the terms of trade

A permanent change in $\kappa$ works differently. Non-tradable inflation rises and tradable inflation falls as with temporary shocks, but aggregate inflation falls. The relative responses are different. Here the response of the exchange rate is half the estimated change in $\Delta \kappa$ while with a temporary shock, the response of the exchange rate is only 5 per cent of the response in $\kappa$. A permanent change in $\kappa_t$ changes commodity prices contemporaneously by $(1 - \rho_\kappa)\Delta \kappa$, but shifts up the expected path of commodity prices as shown in Figure 6. Because the exchange rate is forward looking, it appreciates on impact on expectations of future inflows of foreign currency to the commodity sector. The price ef-
The effect is magnified further by a quantity effect, as the capacity of the commodity-exporting sector is expected to increase when the change is permanent.\footnote{Chen and Rogoff [2003] find for Australia a strong connection between exchange rates and commodity prices and Cashin et al. [2004] report evidence of a long-run relationship between the real exchange rate and real commodity prices for about 20 commodity-exporting countries.}

The larger appreciation of the exchange rate implies a stronger response of tradable inflation. This is illustrated in Figure 9 which shows the response of non-tradable inflation, tradable inflation and aggregate inflation after a permanent change in $\kappa$. The transition paths are computed for permanent changes in $\kappa_t$ taken from the estimated posterior. At the mode aggregate inflation falls 2 percentage points because the substitution effect is larger than the income effect: the appreciation of the exchange rate is sufficiently large that the deflationary impact from import prices dominates the increase in non-tradable prices that comes from rising incomes.

Another significant difference can be found in the behaviour of net exports. When the shock is temporary, consumption growth, investment growth and the net exports to GDP ratio increase. When the change $\kappa$ is permanent, consumption and investment growth increase by more and net exports initially fall; in part because commodity prices converge slowly to its new long-run level and in part because the appreciation of the exchange rate causes households to substitute away from domestic tradable goods and towards imports and because households borrow from abroad to fund the pick up investment in the non-tradable and commodity-exporting sectors.
6 Conclusion

The recent development of Asia exposed many economies to a unprecedented increases in global commodity prices. A recurrent question for these economies has been the degree to which these changes are permanent. Our objective has been to estimate the permanent change in the level and volatility of export prices and to measure the consequences with a structural open economy model. Using aggregate data for Australia, we detect a change in the long-run level and volatility of its terms of trade. We find that the long-run level of commodity-export prices increased by 40 per cent around 2003:Q2. Our estimate is less than what casual observation might suggest and less than what single equation structural break tests suggest. In forward-looking general equilibrium a change in the long-run mean of the terms of trade manifests itself in other observable series we use in estimation. More observables can ground estimates of the long-run properties of an economy. Single equation inferences are unrestricted and can yield unreliable inferences.

Although our general equilibrium estimates are less than what may be inferred otherwise, the estimated change in the long-run mean of 40 per cent has a significant (visible) impact on the economy. To name a few, it appreciates the exchange rate by 25 per cent, it reduces aggregate inflation by 2 percentage points, it decreases net exports by 2 percentage points and it increases the share of commodity in exports by 17 percentage points.

An important contribution of our paper is that we treat trends and cycles in a unified way. Economic theory shows that permanent changes in the terms of trade must influence many other relative prices. In the data of open economies non-tradable prices, consumer
prices, investment prices, foreign output, foreign prices, real investment and real output all trend at different rates. Because our model’s balanced growth path has these trends, we can identify the extent to which trends, cycles and breaks drive the changes in relative prices we see in the data.

Standard practice in open economies is to pre filter the data somehow. While pre filtering may align the data with the model, pre filtering also distorts the facts. There are different degrees to which pre filtering distorts the data and this would matter but the extent to which it does would depend on the application. Our approach of handling trends, breaks and cycles in the estimation of open economies improves on existing practice.

There are questions we leave for future research. We have not studied the design of optimal policy. Our model has a monetary authority, but abstracts from a fiscal one. In recent years, commodity exporting economies have designed fiscal rules (sovereign wealth funds) that respond to commodity price fluctuations (Céspedes and Velasco [2011]). Our model can be extended to study the design of fiscal policy in the presence of a shifting long-run mean in commodity prices.

One relevant characteristic of the Australian economy is that the commodities exported are not consumed domestically. Other economies, food-producing economies, that have also experienced similar unprecedented increases in their terms of trade consume a significant fraction of the goods they also export. Commodity price shocks can be expected to behave differently in these economies. Thus, extending the model along these lines and taking it to the data of other commodity-exporting economies is worthwhile. For other countries still, commodities also enter as intermediate goods in production (Kilian (2009, ADD). So, studying and estimating the impact of long-run shifts in commodity for alternative specifications is also an avenue for further research.
A Data Sources

A.1 Data used in estimation

The model is estimated using 14 macroeconomic time series. Real GDP, consumption, investment, net exports and hours worked are taken from the Australian Bureau of Statistics’ National Accounts series (ABS cat. 5206.0). All data are seasonally adjusted and measured in chain volume terms, except for the net exports-to-GDP ratio which is seasonally adjusted and measured in current prices and hours worked, which is seasonally adjusted. We convert all of the real activity series into per capita series by dividing by the Australian population, which we derive from the ABS’ GDP per capita series. Our final domestic activity series is the ratio of nominal non-traded consumption to aggregate consumption. We discuss the construction of the non-traded consumption series below.

Our measures of Australian inflation are the Consumer Price Index and Non-Tradeables Price Index, both excluding interest and tax. These series are published by the Australian Bureau of Statistics (ABS Cat. 6401.0). Our measure of the nominal exchange rate is the Australian trade weighted index and our policy interest rate is the cash rate. We source these series from the Reserve Bank of Australia (RBA) statistical tables F1 and F11.

For foreign GDP we use the index of Australia’s major trading partners’ GDP, calculated at purchasing power parity exchange rates, produced by the RBA. For foreign interest rates, we use the average policy rate in the United States, Japan and the euro area (we use German interest rates before the introduction of the euro). Finally, we calculate foreign inflation implicitly using Australian inflation and the real exchange rate index, published by the Reserve Bank of Australia.

A.2 Data used in calibration

This section describes the calculation of the sectoral data used to calibrate the model.

- ** Tradable and Non-tradable consumption:** We allocated private consumption categories in the national accounts to either tradable or non-tradable consumption to match the components of the published tradable and non-tradable consumption price indices (see ABS Cat. 6461 Appendix 2 for these categories). Tradable consumption is the sum of private Food, Cigarettes and Tobacco, Alcohol, Clothing and Footwear, Purchase of Vehicles, Communications and Recreation and Culture consumption. It also includes 24 per cent of healthcare consumption, reflecting the share of pharmaceutical products, which are tradable, in total healthcare consumption and 50 per cent of other household services. Non-tradable consumption includes Rent, Electricity, Gas & Water, Operation of Vehicles, Transport Services, Education, Hotels, Cafes & Restaurants, Insurance & Financial Services as well as Healthcare and Other Households Services not allocated to tradable consumption. We assume that the allocation of private consumption between tradable and non-tradable goods is the same as for private consumption. We measure imported consumption as the sum of consumption goods imports, 75 per cent of services imports and 25 per cent of intermediate imports.
• ** Tradable and Non-Tradable Investment:** We define tradable investment as the sum of Machinery & Equipment Investment and 36 per cent of both Non-Residential Construction and Public Investment. We base the tradable shares of construction on Table 1 of Burstein et al. [2004]. Non-tradable investment is total investment less tradable investment. We measure imported investment as the sum of capital goods imports plus 25 per cent of services imports and 75 per cent of intermediate imports.

• **Resource and Non-Resource Exports:** We use the measure of Resource Exports published in ABS Cat. 5302.0. Non-resource exports equal total exports less resource exports.

• **Hours Worked:** As hours worked data is unavailable at an industry level in Australia, we construct our measure using employment data. This will result in some inaccuracy if average hours worked varies across industries. We define tradable employment as the sum of Agriculture, Mining, Wholesale Trade, Accommodation & Food and Transport, Postal & Warehousing employment. Our measure of employment in the resources sector is Mining employment. As discussed in Plumb et al. [2013], this understates total employment in the resources sector as it excludes workers in industries that service the operations of the mining sector. Our model calibration takes account of this feature of the data. Non-tradable employment is all employment not categorized as Tradable or Mining.

B  The Posterior Sampler

To simulate from the joint posterior of the structural parameters and the date breaks, \( p(\vartheta, T | Y) \), we use the Metropolis-Hastings algorithm following a strategy similar to Kulish et al. [2014]. As we have continuous and discrete parameters we modify the standard setup for Bayesian estimation of DSGE models. We separate the parameters into two blocks: date breaks and structural parameters. To be clear, though, the sampler delivers draws from the joint posterior of both sets of parameters.

The first block of the sampler is for the date breaks, \( T \). As is common in the literature on structural breaks (Bai and Perron [1998]), we set the trimming parameter to 25 per cent of the sample size so that the minimum length of a segment has 20 observation. Within the feasible range we draw from a uniform proposal density and randomize which particular date break in \( T \) to update. This approach is motivated by the randomized blocking scheme developed for DSGE models in Chib and Ramamurthy [2010].

The algorithm for drawing for the date breaks block is as follows: Initial values of the date breaks, \( T_0 \), and the structural parameters, \( \vartheta_0 \), are set. Then, for the \( j^{th} \) iteration, we proceed as follows:

1. randomly sample which date break to update from a discrete uniform distribution with support ranging from one to the total number of breaks, in our case two.
2. randomly sample the corresponding elements of the proposed date breaks, $T_j', \text{ from a discrete uniform distribution } [T_{\text{min}}, T_{\text{max}}]$ and set the remaining elements to their values in $T_{j-1}$

3. calculate the acceptance ratio $\alpha_j^T \equiv \frac{p(\vartheta_{j-1}, T_j'|Y)}{p(\vartheta_{j-1}, T_{j-1}|Y)}$

4. accept the proposal with probability $\min\{\alpha_j^T, 1\}$, setting $T_j = T_j'$, or $T_{j-1}$ otherwise.

The second block of the sampler is for the $n_\vartheta$ structural parameters.\textsuperscript{22} It follows a similar strategy to the date-breaks-block described above - we randomize over the number and which parameters to possibly update at each iteration. The proposal density is a multivariate Student’s $t -$ distribution.\textsuperscript{23} Once again, for the $j^{th}$ iteration we proceed as follows:

1. randomly sample the number of parameters to update from a discrete uniform distribution $[1, n_\vartheta]$

2. randomly sample without replacement which parameters to update from a discrete uniform distribution $[1, n_\vartheta]$

3. construct the proposed $\vartheta_j'$ by drawing the parameters to update from a multivariate Student’s $t -$ distribution with 10 degrees of freedom and with location set at the corresponding elements of $\vartheta_{j-1}$, scale matrix based on the corresponding elements of the negative inverse Hessian at the posterior mode multiplied by a tuning parameter $\iota = 0.15$.

4. calculate the acceptance ratio $\alpha_j^\vartheta \equiv \frac{p(\vartheta_{j'}, T_j|Y)}{p(\vartheta_{j-1}, T_{j-1}|Y)}$ or set $\alpha_j^\vartheta = 0$ if the proposed $\vartheta_j'$ includes inadmissible values (e.g. a proposed negative value for the standard deviation of a shock or autoregressive parameters above unity) preventing calculation of $p(\vartheta_j', T_j|Y)$

5. accept the proposal with probability $\min\{\alpha_j^\vartheta, 1\}$, setting $\vartheta_j = \vartheta_j'$, or $\vartheta_{j-1}$ otherwise.

We use this multi-block algorithm to construct a chain of 575,000 draws from the joint posterior, $p(\vartheta, T|Y)$, throwing out the first 25 per cent as burn-in. Trace plots show that the sampler mixes well.

\textsuperscript{22}In our application $n_\vartheta = 37.$

\textsuperscript{23}The hessian of the proposal density is computed at the mode of the structural parameters.
References


