Credit Spreads with Dynamic Debt \(^1\)

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Abstract

Credit Spreads with Dynamic Debt

We provide a framework to analyze debt where there is a latent option to alter the underlying principal. Specifically, we extend the Merton (1974) model for static debt guarantees in a setting with dynamic debt, where leverage can be ratcheted up as well as written down through pre-specified policies. We show that for many dynamic debt covenants, ex-ante credit spread term structures may be derived in closed-form using modified barrier option mathematics, a class of exotic derivatives that are activated or de-activated upon accessing a pre-determined barrier. We observe that principal write-down covenants decrease the magnitude of credit spreads but increase the slope of the credit curve, transforming downward sloping curves into upward sloping ones. On the other hand, ratchet covenants increase the magnitude of ex-ante spreads without dramatically altering the slope of the credit curve. Overall, explicitly modeling this latent option to alter debt leads to term structures of credit spreads that are more consistent with observed empirics.

Keywords: Credit spreads; dynamic debt; ratchet; restructure; guarantee; barrier options.
1 Introduction

Predicting and pricing the likelihood of default is important to investors, lenders, and debtors alike, and accordingly, a substantial body of work attempts to model and price risky debt claims, and to determine related credit spreads. Beginning with Black and Scholes (1973) and Merton (1974), standard structural models start with a riskless claim, subtracting out the value of a fixed debt guarantee, which represents the value of the borrower’s option to default. Empirically, however, firms that issue debt, actively manage their debt structure and levels, and debt rarely remains fixed. This paper models in closed form, using barrier options, the magnitude of and changes to ex-ante spreads when accounting for the fact that debt is dynamically updated under flexible rules.

Changes to a firm’s debt levels over time may arise as the firm decides to issue new debt or to recall existing debt. Changes to a firm’s leverage may also result from negotiated decisions with existing debt holders to alter major contract terms, such as the principal amount, maturity, and associated debt covenants. Empirical evidence suggests that value-enhancing restructurings take place in cases of actual as well as technical default (Nini, Smith, and Sufi (2011)). Furthermore, evidence suggests that debt is renegotiated even in the absence of financial distress, as new information is realized about the firm’s prospects and credit quality (Roberts and Sufi (2009a); Roberts and Sufi (2009b); Garleanu and Zwiebel (2009)).

The notion of dynamic debt is just as true on household balance sheets as it is for firms, whereby individual borrowers ratchet up the debt supported by home equity, via second mortgages or cash-out refinancings. Specifically, a small percentage of cash-out refinancings can result in a substantial increase in the cost of the debt guarantee, and if these ratchets occur in a correlated fashion across borrowers, as was the case in the financial crisis, a large quantum of systemic credit risk can be generated by fairly small increases in leverage (Khandani, Lo, and Merton (2009)).

In sum, debt is commonly altered after issuance, and this dynamic evolution of debt levels introduces additional uncertainty into ex-ante credit spreads on the current debt of a firm. Thus, a natural question arises as to how economically important is this latent option to restructure. Our purpose is to provide a framework to analyze debt, explicitly modeling the option to alter the underlying principal, and to explore what insights we gain from incorporating this feature.

Specifically, traditional structural models (on fixed debt) predict upward sloping yield
curves on higher quality debt, but downward sloping yield curves on very risky debt. However, empirical evidence suggests that yield curves are mostly upward sloping, and that this pattern applies to both coupon-paying and zero-coupon debt of varying credit qualities (Huang and Zhang (2008)), including high risk debt (Helwege and Turner (1999); Huang and Zhang (2008)). Thus, traditional structural models do not capture empirically observed features of credit spread curves.

In the classic Merton (1974) framework with static zero-coupon debt, the debt guarantee is priced by a plain vanilla put option on the underlying firm with a strike equal to the current debt principal, the value of which can be translated into the credit spreads on the firm’s debt. To this model, we add features that allow debt to be ratcheted up or written down. That is, we allow for a possible increase in a firm’s debt level (i.e., a ratchet) in response to increases in underlying firm value; we also allow for a possible decrease in its debt level (i.e., a write down) that replaces debt principal with equity in response to decreases in underlying firm value, a process also referred to as “de-leveraging”.

Specifically, we show that, extensions of the static debt Merton model to debt guarantees (and hence spreads) on dynamic debt can be derived analytically using barrier options, a class of exotic derivatives that are activated or de-activated upon accessing a pre-determined barrier. Intuitively, if the underlying assets increase sufficiently in value, then the firm can use the extra collateral to support more debt, thereby ratcheting the debt to firm value ratio upward and increasing the price of the debt guarantee. That is, once the underlying firm value appreciates to an upper barrier, the original put option on debt is knocked out and replaced by another put at a higher strike representing the increased level of debt. Thus, in contrast to the plain vanilla Merton put representing the guarantee on non-renegotiable debt, the value of a guarantee on debt with the option to ratchet is decomposed into two single-barrier options: an up-and-out put option to capture the guarantee on the original level of debt, and an up-and-in put option to capture the new guarantee at the increased debt level.

Analogously, write downs may occur when the underlying assets decrease substantially in value, and the put option to default becomes deep in-the-money. To stave off default, lenders can swap debt principal for equity to make the default option less profitable to exercise from the borrower’s standpoint. Thus, the value of a guarantee on debt that may subsequently

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1This is now a prevalent practice in the mortgage markets, supported by government regulation (e.g., see the HAMP-PRA scheme). A recent example in the case of sovereign debt is the forgiveness of principal on Greek debt.
be reduced can be expressed as the sum of two single-barrier options: a down-and-out put option struck at the original debt level, and a down-and-in put option struck at the reduced debt level.

Under this framework, we obtain closed-form solutions for the ex-ante value of a guarantee and the corresponding credit spreads, explicitly modeling the latent option to either ratchet or write down debt after issuance. We also extend this pricing model to allow for various combinations of possible ratchets and write downs. Although the resulting barrier-option representation of the debt guarantee in such a setting is much more complex than in the single ratchet or single write-down cases, the solutions are analytical and lead to intuitive shapes of the term structures of credit spreads. These results may also be extended recursively to more complicated repetitive opportunities to alter debt.

Overall, this parsimonious extension of the static debt structural model in closed-form using barrier options results in more realistic term structures of credit spreads. The main results of our analyses are as follows:

1. **Level effect**: (a) Debt guarantee prices and credit spreads increase with ratchets and decrease with principal write-down features. (b) The ratchet effect is more pronounced for medium-debt firms than for high-debt firms, because ratchets occur at lower leverage. Similarly, the write-down effect is more pronounced for high-debt firms (than for medium-debt firms).

2. **Slope effect**: For high-debt firms, accounting for the write-down feature removes the downward bias in the slope of the yield curve, matching empirical evidence presented by Helwege and Turner (1999) and Huang and Zhang (2008).

Ours is not the first paper to extend the classic Merton (1974) structural model for risky debt; other studies departing from this traditional paradigm include Longstaff and Schwartz (1995), who extend the structural class of models to default with the additional feature of stochastic interest rates; Leland (1994) and Leland and Toft (1996), who consider credit spread term structures under the choice of optimal capital structure and debt maturity with taxes and an endogenous bankruptcy barrier; Goldstein, Ju, and Leland (2001), who allow for possible increases in future debt levels; and Collin-Dufresne and Goldstein (2001), who examine credit spreads under a mean-reverting capital structure in a setting where leverage is a stochastic process continuously tracking a pre-determined target.

However, credit spreads and curves predicted by these models do not adequately match empirical observations of actual spreads and curves, as evidenced in Eom, Helwege, and
Huang (2004), who empirically test five different structural models for corporate spreads. Although the Merton model produces spreads that are too low, these newer models produce spreads that are generally far too high. For example, the Longstaff and Schwartz (1995), Leland and Toft (1996), and Collin-Dufresne and Goldstein (2001) models predict spreads that are oftentimes more than double the actual spread, even yielding estimates on safer debt in the thousands of basis points. We depart from these studies in the following ways.

First, in contrast to these models, we use barrier options to explicitly model the option to ratchet or de-leverage, whereby the option to alter debt is exercised discretely upon accessing a threshold and the debt level does not undergo continuous changes. In practice, debt levels do not change continuously, and in modeling discrete, periodic, firm value-dependent revisions in debt levels, we observe substantive differences in predicted credit spreads and curves. To preserve focus on capital structure dynamics and its effect on credit spread term structures, we ignore stochastic interest rates, the effects of which are both quantitatively and qualitatively minor (Leland and Toft (1996)).

Under this framework, we obtain credit spreads and curves that more closely match prior empirical observations, not only in the shape of the curve but also in the magnitude of the spreads. Moreover, our main features, which we have enumerated above, match model prescriptions based on recent empirical tests of existing pricing models; in particular, that “more accurate structural models must avoid features that increase the credit risk on the riskier bonds while scarcely affecting the spreads of the safest bonds” (Eom, Helwege, and Huang (2004)).

Second, structural models based on mean-reverting models of leverage do not place explicit bounds on the levels of debt the firm might carry, though by increasing the rate of mean reversion, the expected range in which the leverage lies can be controlled. In these models, sufficiently high speeds of adjustment are necessary to generate the upward sloping credit curves empirically observed on high-yield debt. But paradoxically, imposing high speeds of mean reversion results in leverage itself being less dynamic, and firms do not usually evidence such strict adherence to a target capital structure. In contrast, our leverage barrier model permits free movement of leverage within the pre-specified barriers and generates mean reverting capital structures with dynamic and periodic debt adjustments, concomitant with actual practice and consistent with the literature on bounded capital structures arising from costly readjustment, as modeled in Fischer, Heinkel, and Zechner (1989a).

The rest of the paper proceeds as follows. In Section 2 we present barrier option representations of the various ratchet and write-down combinations that may be embedded in
the debt guarantee. In Section 3, we use our closed-form debt guarantees to analyze their sensitivity to parameter values, and we examine the magnitude of and changes to ex-ante spreads when accounting for ratchets and write downs. We see that these additional features result in specific changes in credit curve level and shapes that match empirical features of spreads better than models without these features. In Section 4, we conclude and discuss.

2 Model

2.1 Stochastic Process

To begin, we specify the notation in our model. Let the face value of debt be $D$, and we assume it to be zero-coupon with maturity $T$. We employ a variant of Merton (1974) as the basis for our model. Discounting takes place at the risk free rate $r$, and we posit that the underlying firm value $V$ follows the usual risk-neutral geometric Brownian motion, i.e.,

$$dV(t) = rV(t)\ dt + \sigma V(t)\ dW(t)$$

(1)

where the standard deviation is $\sigma$, with stochasticity generated by the Weiner process increment $dW(t) \sim N(0, dt), \forall t$.

2.2 Default Guarantee and Spreads

Default is triggered at maturity $T$ if $V(T) < D$, in which case debt holders only recover $V(T)$, incurring a loss rate on default of $[1 - \frac{V(T)}{D}]$, i.e., one minus the recovery rate on default. The current price of debt at time $t = 0$ is denoted by function $B(0)$. We know from the Merton model that the price of this debt is

$$B(0) = De^{-rT} \cdot N(d_2) + V(0) \cdot N(-d_1)$$

(2)

$$d_1 = \frac{\ln(V(0)/D) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

and the credit spread, $s$, in this model is

$$s = -\frac{1}{T} \cdot \ln[N(d_2) + N(-d_1)/L(0)]$$

(3)
\[ L(t) = \frac{De^{-r(T-t)}}{V(t)} \]

Note that \( L(t) \) here is the leverage (i.e., loan-to-value) ratio of the debt in question, accounting for the time value of money. Debt is defined as being underwater when \( L(t) > 1 \). In the Merton model this is possible prior to maturity. We will consider cases where the leverage is initially high, though not underwater.

The debt guarantee in the Merton (1974) model is just the price of the plain vanilla put option to default, i.e.,

\[
G(0) \equiv P(0) = De^{-rT} \cdot N(-d_2) - V(0) \cdot N(-d_1)
\]  \hspace{2cm} (4)

And, the price of defaultable debt given above is known to be the price of riskless debt minus this guarantee, i.e., \( B(0) = De^{-rT} - G(0) \), corresponding to equation (2). The credit spread, \( S \), in this model is

\[
S = -\frac{1}{T} \ln \left( \frac{B(0)}{De^{-rT}} \right)
\]  \hspace{2cm} (5)

2.3 Deadweight default losses

We may also adjust the Merton model to accommodate deadweight losses on default, i.e., the debt holders get \( \phi V \) on default instead of \( V \), where \( \phi \leq 1 \). Hence, the default put (or guarantee value) is based on the following calculation under the risk-neutral measure:

\[
P(0) = e^{-rT} \int_{-\infty}^{D} \left[ D - \phi V(T) \right] f(V(T)) \, dV(T)
\]  \hspace{2cm} (6)

which simplifies to an adjusted put option formula:

\[
G(0) = De^{-rT} \cdot N(-d_2) - \phi V(0) \cdot N(-d_1)
\]  \hspace{2cm} (7)

This equation is the same as the usual put option formula with no change in the expressions \( N(\cdot) \) for probabilities and only a multiplicative adjustment in just one term, the second half of the Merton formula. As before, the credit spread is given by equation (5).
2.4 Modified guarantees

We now develop a framework to price debt guarantees that depart from the standard Merton model, whereby we account for debt ratchets and write downs. Formulae for various barrier options we use are provided in the Appendices, and have been modified to accommodate deadweight costs of default, where recovery rates, $\phi$, are less than 1.

2.4.1 Guarantee with Debt Ratchet

Assume that when the firm value $V$ rises to an exogenous level $D/K$, we ratchet up debt to a level $D(1 + \delta)$ and pay down equity with the proceeds, thereby enhancing leverage in the capital structure. Here, we define $K < 1$ as the $D/V$ (loan-to-value) level at which the ratchet is triggered.

To keep matters simple, we normalize the value of $V$ to 1. We may start with an initial leverage ($D/V$) ratio of 0.85 ($= D$), and if $K = 0.75$, then at $V = D/K = 0.85/0.75 = 1.133$, an appreciation in $V$ of 13.33 percent, the ratchet occurs, and debt increases by, say, 10%, i.e., $\delta = 0.10$. The price of the new guarantee, to current debt holders, is now dependent on this potential increase in debt, and may be written as a portfolio of the following barrier options:

$$G_{R, noW}(0) = P_{uo}[V, D; D/K] + \left(\frac{1}{1 + \delta} \cdot P_{ui}[V, D(1 + \delta); D/K]\right)$$  

where the first term in the subscript $\{R, noW\}$ on guarantee $G$ stands for whether debt ratches are allowed and the second term for whether principal write downs are allowed. We use this convention throughout the paper.

In contrast to a plain vanilla put, $P_{uo}[V, D; D/K]$ stands for an up-and-out put option with strike $D$ that is knocked out when $V$ rises to barrier $D/K$. Likewise, $P_{ui}[V, D(1 + \delta); D/K]$ stands for a up-and-in put option with strike $D(1 + \delta)$ that is knocked in when $V$ rises to barrier $D/K$.

Intuitively, when leverage drops to a level, $K$, that merits a ratchet, the original debt guarantee (characterized by a put option with strike $D$) is cancelled, and a new debt guarantee (characterized by a put option with strike $D(1 + \delta)$) is written to account for the new debt load. Since $P_{ui}[V, D(1 + \delta); D/K]$ represents the knocked-in guarantee on the new, increased debt level, we multiply this by $\frac{D}{D(1+\delta)} = \frac{1}{1+\delta}$ to capture the portion required to
guarantee the current liability level, $D$.

Thus, using barrier put options instead of vanilla puts, we capture the ex-ante guarantee value accounting for a possible ratchet. The exact formulae for the up-and-out and up-and-in puts is provided in [A]

### 2.4.2 Guarantee with Debt Write Down

Using barrier options, we also price debt guarantees that account for the latent option to write down debt principal and dial down leverage as the firm’s assets drop in value. That is, assume that when the underlying firm value $V$ declines to an exogenously determined lower barrier $D/M$, we write down debt, through a debt-equity exchange, reducing the loan principle to $D(1 - d)$, $0 < d < 1$. Here, $M > 1$ is the $D/V$ level at which the write down is triggered.

This new guarantee can be priced at time $t = 0$ as follows:

$$G_{noR,W}(0) = P_{do}[V; D; D/M] + \left( \frac{1}{1 - d} \cdot P_{di}[V; D(1 - d); D/M] \right)$$

(9)

In contrast to the $P_{uo}$ and $P_{ui}$ options introduced in the previous subsection, the $P_{do}$ and $P_{di}$ options represent down-and-out and down-and-in puts, whereby $P_{do}[V; D; D/M]$ represents a barrier put option with strike $D$ that is knocked out when $V$ drops to $D/M$, and $P_{di}[V; D(1 - d); D/M]$ represents a barrier put with strike $D(1 - d)$ that is knocked in when $V$ drops to $D/M$. Analogous to the previous case (where we allowed for debt ratchets), we multiply this knocked-in guarantee by a factor of $\frac{1}{1 - d}$ to capture the portion of the guarantee relevant to the current debt holders’ $D$ level.

The exact formulae for the down-and-out and down-and-in puts are provided in [A]

### 2.4.3 Guarantee with the Option to Either Ratchet or Write Down Debt

In this setting, we permit one adjustment to the original debt principal, depending on which case occurs first. If $V$ rises and touches $D/K$, then leverage is increased by ratcheting debt up to level $D(1 + \delta)$, after which no further ratchets or write downs are allowed. On the other hand, if $V$ falls to level $D/M$, then a principal write down is undertaken, and debt is reduced to $D(1 - d)$, after which no further write downs or ratchets is permitted.

Given that $D/K > V > D > D(1 + \delta)/M > D/M > D(1 - d)/M$, the debt guarantee can
be expressed as follows (in this case a fully analytic solution does not exist and we express the result as an integral):

\[
G_{RorW}(0) = P_{uo/do}[V, D; D/K, D/M] + \frac{1}{1 - d} \int_0^T f_{D/M,-D/K}(t) \cdot G(t; D/M, D(1 - d)) \, dt + \frac{1}{1 + \delta} \int_0^T f_{D/K,-D/M}(t) \cdot G(t; D/K, D(1 + \delta)) \, dt
\]

(10)

which we implement as follows:

\[
= P_{uo/do}[V, D; D/K, D/M] + \frac{1}{1 - d} \sum_{t=dt}^T G(t; D/M, D(1 - d)) \cdot [F_{D/M,-D/K}(t) - F_{D/M,-D/K}(t - dt)] + \frac{1}{1 + \delta} \sum_{t=dt}^T G(t; D/K, D(1 + \delta)) \cdot [F_{D/K,-D/M}(t) - F_{D/K,-D/M}(t - dt)]
\]

(11)

This expression has three lines with a double barrier knock-out option in line 1 (which is knocked out upon accessing either barrier, see [C]), the value of the restructuring component upon accessing \(D/M\) in line 2, and the value of the ratchet component upon accessing \(D/K\) in line 3. The expressions within are defined as follows:

1. \(f_{H_1,-H_2}(t)\) represents the first-passage probability that \(V_t\) has accessed \(H_1\) for the first time, but has not touched \(H_2\). \(F_{H_1,-H_2}(t)\) represents the corresponding cumulative density function. In our implementation, we define (approximate) \(dt\) by one-month intervals (i.e., \(dt = 1/12\)). This discounted first-passage density function is the same as the special case of a one-touch double barrier binary option with a payoff of $1 (see [E]).

2. \(G(t; V, D)\) in line 2 represents the unmodified debt guarantee from equation (4), which is characterized by a plain vanilla put based on underlying asset value \(V\) and debt level \(D\). Here, we set \(\{V = D/M; D = D(1 - d)\}\), since the debt issue is written down to level \(D(1 - d)\) if the firm value drops to \(D/M\), which leaves us with the following expression:

\[
P[D/M, D(1 - d), T - t]
\]
3. Again, \( G(t; V, D) \) in line 3 represents the unmodified debt guarantee from equation (4), which is characterized by a plain vanilla put based on underlying asset value \( V \) and debt level \( D \). Here, we set \( \{ V = D/K; \ D = D(1 + \delta) \} \), since the debt issue is ratcheted up to level \( D(1 + \delta) \) if the firm value rises to \( D/K \), which leaves us with the following expression:

\[
P[D/K, D(1 + \delta), T - t]
\]

Ultimately, whether \( G_{RorW}(0) \) is greater than or less than the original, unmodified debt guarantee, \( G(0) \), depends on the gap between \( K \) and \( M \), as well the extent to which debt is ratcheted (i.e., \( \delta \)) when the underlying firm value accesses the upper barrier versus the extent to which it is reduced (i.e., \( d \)) when the firm value accesses the lower barrier.

2.4.4 Guarantee with the Option to Write Down After Ratcheting

We now explore how the value of the ratcheted debt guarantee (presented in section 2.4.1) changes if we account for the option to write down debt principal after it has been ratcheted. That is, assume that when the firm value \( V \) increases and hits an exogenously determined upper barrier \( D/K \), we ratchet debt, increasing the loan principle to \( D(1 + \delta) \). Then, if \( V \) subsequently falls to level \( D(1 + \delta)/M \), we write down the ratcheted debt issue, reducing the loan principal to \( D(1 + \delta)(1 - d) \).

In this instance, the debt guarantee is priced as follows (the first subscript “\( \text{RthenW} \)” below now denotes the allowance for a debt ratchet and subsequent write down):

\[
G_{\text{RthenW, noW}}(0) = P_{uo}[V, D; D/K] \\
+ \frac{1}{1 + \delta} P_{ui,do}[V, D(1 + \delta); D/K, D(1 + \delta)/M] \\
+ \frac{1}{(1 + \delta)(1 - d)} P_{udi}[V, D(1 + \delta)(1 - d); D/K, D(1 + \delta)/M]
\]

where the \( P_{ui,do} \) represents an up-in/down-out put (see D) that is knocked in when the firm value accesses the upper barrier and is subsequently knocked out if the firm value then depreciates and accesses the lower barrier, and the \( P_{udi} \) represents an up-down-in put that is knocked in only if the underlying firm value accesses the upper barrier then subsequently accesses the lower barrier (priced in B)
2.4.5 Guarantee with the Option to Ratchet After Write Down

We also analyze how the value of the restructurable debt guarantee (presented in section 2.4.2) changes if we account for the option to ratchet debt after it has been written down. In this case, when the firm value $V$ decreases to an exogenously determined lower barrier $D/M$, we write down debt, decreasing the loan principle to $D(1-d)$. Then, if $V$ subsequently rises to level $D(1-d)/K$, we ratchet the reduced debt issue, increasing the loan principal to $D(1+\delta)(1-d)$.

In this instance, the debt guarantee is priced as follows (the second subscript “WhenR” below now denotes the allowance for a principal write down and subsequent ratchet):

$$
G_{noR,WhenR}(0) = P_{do}[V,D;D/M] + \frac{1}{1-d} P_{di,uo}[V,D(1-d);D(1-d)/K,D/M] + \frac{1}{(1+\delta)(1-d)} P_{dai}[V,D(1+\delta)(1-d);D(1-d)/K,D/M]
$$

$P_{di,uo}$ represents a down-in/up-out put (formulated in $D$) that is knocked in when the firm value accesses the lower barrier, $D/M$, and is subsequently knocked out if the firm value then appreciates and accesses the upper barrier, $D(1-d)/K$. $P_{dai}$ represents a down-up-in put (formulated in $B$) that is knocked in only if the firm value accesses the lower barrier then subsequently accesses the upper barrier.

2.4.6 Guarantee allowing Ratchet after Write Down or vice versa

Finally, we price a debt guarantee accounting for both debt ratchets and write downs. Specifically, we assume that the debt can either be written down (and then ratcheted thereafter if applicable), or ratched (then written down thereafter if applicable).

Formally, if the firm value $V$ falls to level $D/M$, we write down the principal, reducing the debt to level $D(1-d)$. After that if $V$ rises to level $D(1-d)/K$, then we ratchet up debt to $D(1+\delta)(1-d)$. On the other hand, if $V$ rises and hits an upper barrier $D/K$, we ratchet up debt to a level $D(1+\delta)$. Then, if the firm value subsequently falls to level $D(1+\delta)/M$, we write down the debt principal to level $D(1+\delta)(1-d)$.

Given that $D/K > V > D > D(1+\delta)/M > D/M > D(1-d)/M$, the debt guarantee can be expressed as follows (in this case a fully analytic solution does not exist and we express
the result as an integral):

\[
G_{RthenW, WthenR}(0) = P_{uo/do}[V, D; D/K, D/M] \\
+ \frac{1}{1-d} \int_0^T f_{D/M, -D/K}(t) \cdot G_{R,noW}(t; D/M, D(1-d)) \, dt \\
+ \frac{1}{1+\delta} \int_0^T f_{D/K, -D/M}(t) \cdot G_{noR,W}(t; D/K, D(1+\delta)) \, dt
\] (14)

which we implement as follows:

\[
G_{RthenW, WthenR}(0) = P_{uo/do}[V, D; D/K, D/M] \\
+ \frac{1}{1-d} \sum_{t=dt}^T G_{R,noW}(t; D/M, D(1-d)) \cdot [F_{D/M, -D/K}(t) - F_{D/M, -D/K}(t - dt)] \\
+ \frac{1}{1+\delta} \sum_{t=dt}^T G_{noR,W}(t; D/K, D(1+\delta)) \cdot [F_{D/K, -D/M}(t) - F_{D/K, -D/M}(t - dt)]
\] (15)

This expression has three lines with a double barrier knock-out option in line 1 (which is knocked out by accessing either barrier, and is formulated in [C], the value of the writedown-then-ratchet component upon accessing \(D/M\) in line 2, and the value of the ratchet-then-writedown component upon accessing \(D/K\) in line 3. The expressions within are defined as follows:

1. \(f_{H_1, -H_2}(t)\) represents the first-passage probability that \(V\) has accessed \(H_1\) for the first time, but has not touched \(H_2\). \(F_{H_1, -H_2}(t)\) represents the corresponding cumulative density function. In our implementation, we define \(dt\) by one-month intervals (i.e., \(dt = 1/12\)). This (present-valued) first-passage density function is analogous to holding the special case of a one-touch double barrier binary option with a payoff of $1 (see [E]).

2. \(G_{R,noR}(t; V, D)\) represents the modified debt guarantee with ratchets from equation [8], which is characterized by a combination of single barrier down-out and down-in put options. Here, we set \(\{V = D/M; D = D(1-d)\}\), since the debt issue is written down and decreased to level \(D(1-d)\) if the firm value decreases to \(D/M\). This new debt guarantee is subsequently knocked out if the firm value later increases to level \(D(1-d)/K\), whereby the debt issue (which is now at level \(D(1-d)\) is ratcheted to level \(D(1+\delta)(1-d)\), thus knocking in a new guarantee. This sequence leaves us with
the following expression:

\[ P_{uo}[D/M, D(1 - d), T - t; D(1 - d)/K] \]
\[ + P_{ui}[D/M, D(1 + \delta)(1 - d), T - t; D(1 - d)/K] \]

3. \( G_{noR,R}(t; V, D) \) represents the modified debt guarantee with write downs from equation (9), which is characterized by a combination of single barrier down-out and down-in put options. Here, we set \( \{V = D/K; D = D(1 + \delta)\} \), since the debt issue is ratcheted and increased to level \( D(1 + \delta) \) if the firm value increases to \( D/K \). This new debt guarantee is subsequently knocked out if the firm value later drops to level \( D(1 + \delta)/M \), whereby the debt principal (which is now at level \( D(1 + \delta) \)) is written down to level \( D(1 + \delta)(1 - d) \), thus knocking in a new guarantee. This sequence leaves us with the following expression:

\[ P_{do}[D/K, D(1 + \delta), T - t; D(1 + \delta)/M] \]
\[ + P_{di}[D/K, D(1 + \delta)(1 - d), T - t; D(1 + \delta)/M] \]

We note that using the functions \( F_{H1,-H2}(t) \) described above, we may extend the analysis to more complex, ongoing capital structure formulations. For example, firms may wish to commit to a strategy where the stipulation is that if \( V \) reaches the upper boundary \( D/K \) first they will ratchet debt, but then also allow for a subsequent write down, followed by another ratchet as well. Or if \( V \) reaches the lower boundary \( D/M \) first, the firm will write down debt, but then also allow for a subsequent ratchet, followed by another write down, if applicable. The discrete-time formula for the debt guarantee under this situation is as follows:

\[ G_{RthenW,WthenR}(0) = P_{uo/doi}[V, D; D/K, D/M] \]
\[ + \frac{1}{1 - d} \sum_{t=dt}^{T} G_{RthenW,noW}(t; D/M, D(1 - d)) \cdot [F_{D/M,-D/K}(t) - F_{D/M,-D/K}(t - dt)] \]
\[ + \frac{1}{1 + \delta} \sum_{t=dt}^{T} G_{noR,WthenR}(t; D/K, D(1 + \delta)) \cdot [F_{D/K,-D/M}(t) - F_{D/K,-D/M}(t - dt)] \]

where the formulae for \( G_{RthenW,noW}(t; D/M, D(1 - d)) \) is given in Section 2.4.4 and the formula for \( G_{noR,WthenR}(t; D/K, D(1 + \delta)) \) is given in Section 2.4.5.
This summation (or integration approach), through the use of the option formula in [E] as a proxy for a discounted first-passage time density, allows recursive computation of guarantees with multiple ratchets and write downs, and supports other nested cases as needed.

3 Analysis

The notation and baseline parameters are introduced here for various ensuing numerical analyses. We normalize the underlying firm value to \( V = 1 \), and we compare two debt levels, \( D = \{0.75, 0.50\} \), representing high-leverage and medium-leverage cases, respectively. We assume an underlying asset volatility of \( \sigma = 20\% \) and a riskless rate of \( r_f = 2\% \). We begin our analyses assuming a time to maturity of \( T = 15 \) years, later considering a range of maturities to map out the entire credit curves.

A debt ratchet entails a \( \delta = 30\% \) increase in the current debt principal; analogously, a write down entails a \( d = 30\% \) decrease. Ratchets occur when \( V \) rises to a level such that \( D/V \) drops to \( K = 0.40 \); i.e., ratchets occur at an upper barrier of \( D/K \). Write downs occur when \( V \) falls such that \( D/V \) increases to \( M = 1.00 \); i.e., write downs occur at a lower barrier of \( D/M \). In cases where the debt has already been ratcheted, we assume a write-down barrier based on a decrease in leverage relative to the new level of debt. Likewise, in cases where the debt has already been written down, we assume a ratchet barrier based on an increase in leverage relative to the new level of debt.

We consider the following cases: (1) Original unmodified guarantee, with no ratchets or write downs; (2) Guarantee with ratchets, but no write downs; (3) No ratchets, but write downs are allowed; (4) Either a ratchet or a write down is allowed, but not both; (5) Ratchets are permitted with a follow-on write down, if applicable; (6) Write downs are permitted with a follow-on ratchet, if applicable; and (7) Write down followed by a ratchet is allowed, or a ratchet with a follow-on write down. A pictorial representation of a sample of these cases is provided in Figure 1.

3.1 Debt guarantees and credit spreads

In Table 1 we present the debt-guarantee prices under these seven schemes, providing an idea of the relative value of the debt guarantee structures. Generally speaking, guarantee prices and spreads increase when allowing for debt ratchets, and decrease when allowing for debt write downs.
With respect to the high-leverage \((D/V = 0.75)\) issuer, which we present in Panel A, the original, unmodified Merton model (1) yields a debt guarantee price of 0.0714, which translates to a credit spread of 92 bps. When we augment this model to allow for a debt ratchet (2), the guarantee becomes more expensive, with a corresponding spread of 97 bps. On the other hand, when we augment the original model to allow for a principal write down (3), the guarantee becomes less expensive, with a corresponding spread of 44 bps. The ratchet effect is tempered when we allow for a follow-on write down (i.e., \((5)<(2))\); likewise, the write-down effect is tempered when we allow for a follow-on ratchet (i.e., \((6)>(3))\).

Similar observations apply to the medium-leverage \((D/V = 0.50)\) issuer, which we present in Panel B. Here, the original, unmodified guarantee (1) is priced at 0.0212, which translates to a credit spread of 39 bps. The credit spread increases to 52 bps when we allow for a debt ratchet (2), and decreases to 16 bps when we allow for a principal write down (3).

Because ratchets occur at lower leverage, the ratchet effect is more pronounced for medium-leverage issuers than for high-leverage issuers, effecting a 13 bps increase in ex-ante spreads for the medium-leverage issuer (Panel B) in contrast to a 5 bps increase in spreads for the high-leverage issuer (Panel A). Analogously, the write-down effect is more pronounced for high-leverage issuers than for medium-leverage issuers, effecting a 48 bps decrease in spreads for the high-leverage issuer (Panel A) in contrast to a 23 bps decrease in spreads for the medium-leverage issuer (Panel B).

Overall, debt guarantee prices and ex-ante credit spreads stand to change substantially when considering dynamic versus static debt issues. We now proceed to explore these relations for a range of maturities, assessing the differences not only in the magnitude of spreads but also in the shape of the credit curve.

### 3.2 Credit curves

We now plot the term structures of credit spreads for the various combinations of possible debt ratchets and write downs.

Figure 2 shows credit spreads when the initial leverage ratio is \(D/V = 0.75\). We observe a classic hump-shaped curve where short-term spreads and long-term spreads are lower than medium-term spreads, under the Merton \([1974]\) model for static debt as well as under the renegotiable debt issue allowing for ratchets. In the upper plot we see, across all maturities, that spreads obtained on debt that can ratchet are always greater than or equal to those obtained from the base case Merton model; conversely, the spreads on debt that can be
written down are always lower.

Most notably, the write-down feature increases the slope of the credit curve (in the medium to long end) relative to the static-debt model, consistent with the general upward slope in yield curves that is observed empirically (Helwege and Turner (1999); Huang and Zhang (2008)). We also observe that the write-down feature brings down spreads noticeably more than the ratchet features increases them, addressing concerns that “newer models tend to severely overstate the credit risk of firms with high leverage” (Eom, Helwege, and Huang (2004)).

The lower panel of Figure 2 compares the base case to more complex combinations where both ratchets and write-downs are allowed, and the other relative comparisons noticed in Table 1 are also borne out in the plots. In all cases, the position of these curves relative to the base case and each other depend on the choice of the leverage barrier parameters \(K\) and \(M\), as well as the choice of the debt ratchet (\(\delta\)) or write-down (\(d\)) proportions at these triggers.

Figure 3 shows credit spreads when the initial leverage ratio is \(D/V = 0.50\). We observe not only that spreads are lower from the onset, but also that the credit curves are all upward sloping, whether write downs are allowed or not. In contrast to the high-leverage issuer, here, we observe that allowing for debt ratchets has an appreciable impact on credit spreads, since a medium-leverage issuer is far more likely to reach a point where the ratchet option is applicable.

We also explore the effect of deadweight costs, \((1 - \phi)\), on the level and shape of credit curves. Figures 4 (high leverage) and 5 (medium leverage) demonstrate the difference in credit curves across varying \(\phi\). As expected, we observe an increase in spreads accompanying decreases in \(\phi\). Furthermore, the recovery rates affect not only the level of spreads, but also the shape of the curves. Specifically, slopes become steeper in the short end as deadweight costs increase, suggesting that the loss on default incrementally and substantially affects the slope of the credit curve.

Figures 6 (high leverage) and 7 (medium leverage) demonstrate plot spread curves for all seven cases when deadweight costs of default \((1 - \phi)\) are 30% of firm value. Overall, we observe that even under high deadweight costs, credit curves of high-leverage issuers are still upward sloping when we account for the possibility of a principal write down, consistent with the findings of Helwege and Turner (1999); ?; ? and Huang and Zhang (2008).
4 Concluding Comments

We extend the Merton (1974) and Merton (1977) models by developing analytic expressions for the ex-ante pricing of debt guarantees where debt principal may be ratcheted up or written down at future dates based on changes in underlying firm value. These features can be explicitly incorporated into the pricing model using various single- and double-barrier option formulations, embedding discretely punctuated mean-reversion in capital structure and allowing debt to dynamically change.

With this framework, we are able to extend extant results in the dynamic debt literature, providing closed-form expressions for the term structure of credit spreads across many different prepackaged debt covenants and matching empirical stylized features. The model’s predicted effect of the ratchet and write-down features is consistent with recent evidence that leverage expectations have a material impact on ex-ante spreads (Flannery, Nikolova, and Öztekin (2012)). Given that the cross-sectional determinants of capital structures are similar across countries in Europe and the U.S. (Rajan and Zingales (1995)), the results in this paper apply to many firms across the world.
Appendices

A Single Barrier Option Formulae

We provide the pricing equations for single barrier options here. Note that there are 8 different possible barrier options, based on combinations of calls and puts, in or out, up or down cases. The parameter convention we use for these options is taken from Haug (2006). The following equations feed into the barrier option formulae we use in the paper (the variable \( H \) denotes the single barrier in all cases):

\[
A = \xi \phi_p V e^{(b-r)T} N(\xi x_1) - \xi \phi_c D e^{-rT} N(\xi x_1 - \xi \sigma \sqrt{T})
\]
\[
B = \xi \phi_p V e^{(b-r)T} N(\xi x_2) - \xi \phi_c D e^{-rT} N(\xi x_2 - \xi \sigma \sqrt{T})
\]
\[
C = \xi \phi_p V e^{(b-r)T} (H/V)^{2(\mu+1)} N(\eta y_1) - \xi \phi_c D e^{-rT} (H/V)^{2\mu} N(\eta y_1 - \eta \sigma \sqrt{T})
\]
\[
D = \xi \phi_p V e^{(b-r)T} (H/V)^{2(\mu+1)} N(\eta y_2) - \xi \phi_c D e^{-rT} (H/V)^{2\mu} N(\eta y_2 - \eta \sigma \sqrt{T})
\]
\[
E = D e^{-rT} [N(\eta x_2 - \eta \sigma \sqrt{T}) - (H/V)^{2\mu} N(\eta y_2 - \eta \sigma \sqrt{T})]
\]
\[
F = D [(H/V)^{\mu+\lambda} N(\eta z) + (H/V)^{\mu-\lambda} N(\eta z - 2\eta \lambda \sigma \sqrt{T})]
\]

where \( \xi, \eta \) are parameters that are set to values \{-1, +1\} depending on the type of barrier option being considered. Calls that are down-and-in or down-and-out have \( \xi = 1, \eta = 1 \); calls that are up-and-in or up-and-out have \( \xi = 1, \eta = -1 \); puts that are down-and-in or down-and-out have \( \xi = -1, \eta = 1 \); and puts that are up-and-in or up-and-out have \( \xi = -1, \eta = -1 \). The parameter \( \phi = \{\phi_p, \phi_c\} \) is one minus the deadweight loss in the firm’s value on default. Hence, if the firm has no deadweight loss on default, then \( \phi = 1 \), else \( \phi < 1 \). In the equations above, if a call is being priced then \( \phi_p = 1, \phi_c = \phi \), and if a put is being priced, then \( \phi_p = \phi, \phi_c = 1 \).

The parameter \( b \) is the cost of carry, i.e., the risk free rate plus/minus any other costs/benefits, but in the absence of dividends, we assume that \( b = r \) in all cases. The other parameters
are defined as follows:

\[
x_1 = \frac{\ln(V/D)}{\sigma \sqrt{T}} + (1 + \mu)\sigma \sqrt{T}
\]

\[
x_2 = \frac{\ln(V/H)}{\sigma \sqrt{T}} + (1 + \mu)\sigma \sqrt{T}
\]

\[
y_1 = \frac{\ln(H^2/(VD))}{\sigma \sqrt{T}} + (1 + \mu)\sigma \sqrt{T}
\]

\[
y_2 = \frac{\ln(H/V)}{\sigma \sqrt{T}} + (1 + \mu)\sigma \sqrt{T}
\]

\[
z = \frac{\ln(H/V)}{\sigma \sqrt{T}} + \lambda \sigma \sqrt{T}
\]

\[
\begin{align*}
\mu &= \frac{b - \sigma^2/2}{\sigma^2} \\
\lambda &= \sqrt{\mu^2 + \frac{2r}{\sigma^2}}
\end{align*}
\]

We define the single-barrier options we need as functions of the preceding expressions:

1. Up-and-out put: with barrier \(D/K\). The following equation holds when \(D < H\), where \(H = D/K\) is the ratchet level for \(V\) to reach.

\[P_{uo}[V,D;H] = A - C + F\]

with \(\xi = -1\) and \(\eta = -1\).

2. Up-and-in put: with barrier \(D/K\). The following equation holds when \(D < H\), where \(H = D/K\) is the ratchet level for \(V\) to reach.

\[P_{ui}[V,D;H] = C + E\]

with \(\xi = -1\) and \(\eta = -1\).

B Double Touch Barrier Options

These options are only knocked in (or knocked out) when the underlying touches the lower (upper) barrier, and then touches the upper (lower) barrier. Since there are down-up and
up-down, and calls and puts, there are four such cases. We present only the cases we apply
in the paper. Here we have two barriers, the upper barrier $H_U$, and the lower barrier $H_L$.

1. Up-down-in put: We define this double touch option as follows

\[ P_{udi}[V, D; H_U, H_L] = \frac{D}{H_U} C_{ui} \left[ \frac{H_U^2}{D}; \frac{H_U^2}{H_L}, -r \right] \] (16)

where the $(-r)$ denotes the fact that the up-and-in call $C_{ui}$ is being priced off a
stochastic process that has reverse drift than the one in equation (1), i.e., $dV(t) = -rV(t) \, dt + \sigma V(t) \, dW(t)$.

To see the equivalence of the LHS and RHS of equation (16), note that when $V$ hits
the upper barrier $H_U$, it becomes a down-and-in put ($P_{di}$), which by barrier option
symmetry (see Gao, Huang, and Subrahmanyam (2000); Haug (2006)), is equal to
the RHS of equation (16). When $V < H_U$, both RHS and LHS are not triggered
and hence have the same value, i.e., zero. But when $V = H_U$, both the RHS and
LHS become equal to the value of $P_{di}[V, D; H_L]$. See Gao, Huang, and Subrahmanyam
(2000), equations (28) and (29), for the reasoning to flip the drift of the process. They
show that barrier option symmetry results in

\[ P_{di}[V, D; H_L] = \frac{D}{V} C_{ui} \left[ \frac{V^2}{D}; \frac{V^2}{H_L}, -r \right] \] (17)

Therefore, we may write the double touch options as function of single barrier options.
Using barrier option parity we may also write

\[ P_{udo}[V, D; H_U, H_L] = P[V, D] - P_{udi}[V, D; H_U, H_L] \]

which allows us to price the “out” versions of these double touch options once we have
the pricing for the “in” version.

2. Down-up-in put: We define this double touch option as follows

\[ P_{dui}[V, D; H_U, H_L] = \frac{D}{H_L} C_{di} \left[ \frac{H_L^2}{D}; \frac{H_L^2}{H_U}, -r \right] \] (18)

This identity is analogous to the one presented in equation (16), and the same proof/logic
applies, the crux of which is that when $V = H_L$, both the RHS and LHS become equal
to the value of $P_{ui}[V, D; H_U]$. Likewise, the barrier option parity is
\[
P_{duo}[V, D; H_U, H_L] = P[V, D] - P_{dui}[V, D; H_U, H_L]
\]

Since barrier option symmetry allows us to write double barrier options as functions of single barrier options, the formulae in [X] for single barrier options that are modified for deadweight costs also apply to the prices in this appendix, and these equivalences are also adapted to the presence of deadweight default costs.

C Double Barrier Knock-Out Options

These options are knocked out when either the upper or lower barrier is hit.

Up-out / down-out puts: These options have the same payoff as a plain vanilla put given that neither barrier has been accessed prior to maturity. The pricing equation for this option is as follows:
\[
P_{uo/do}[V, D; H_U, H_L] = De^{-rT} \sum_{n=-\infty}^{\infty} A_1(n) - \phi_p Ve^{(b-r)T} \sum_{n=-\infty}^{\infty} A_2(n) \]  
(19)

where
\[
A_1(n) = \left( \frac{H^n_U}{H^n_L} \right)^{\mu_1-2} \left( \frac{H_L}{V} \right)^{\mu_2} [N(y_1 - \sigma \sqrt{T}) - N(y_2 - \sigma \sqrt{T})]
- \left( \frac{H^{n+1}_L}{H^n_U} \right)^{\mu_3-2} [N(y_3 - \sigma \sqrt{T}) - N(y_4 - \sigma \sqrt{T})]
\]
\[
A_2(n) = \left( \frac{H^n_U}{H^n_L} \right)^{\mu_1} \left( \frac{H_L}{V} \right)^{\mu_2} [N(y_1) - N(y_2)]
- \left( \frac{H^{n+1}_L}{H^n_U} \right)^{\mu_3} [N(y_3) - N(y_4)]
\]
\[
y_1 = \frac{1}{\sigma \sqrt{T}} \cdot [\ln(VH_U^{2n}/H_L^{2n+1}) + (b + \sigma^2/2)T]
\]
\[
y_2 = \frac{1}{\sigma \sqrt{T}} \cdot [\ln(VH_U^{2n}/(DH_L^{2n})) + (b + \sigma^2/2)T]
\]
\[
y_3 = \frac{1}{\sigma \sqrt{T}} \cdot [\ln(H_L^{2n+2}/(H_LV H_U^{2n})) + (b + \sigma^2/2)T]
\]
\[ y_4 = \frac{1}{\sigma \sqrt{T}} \ln \left( \frac{H_L^{2n+2}}{(D V H_U^{2n})} \right) + (b + \sigma^2/2)T \]
\[ \mu_2 = 0 \]
\[ \mu_1 = \mu_3 = \frac{2b}{\sigma^2} + 1 \]

For implementation purposes the infinite sum is taken in a smaller range from \([-5, +5]\), see the suggested implementation in [Haug (2006)]. Note that the second term in equation (19) above has been adapted for deadweight costs by the use of a multiplicative factor \(\phi_p\) defined in [\(\underline{A}\)].

## D In-Out Barrier Options

These options are knocked in upon accessing the first barrier, then knocked out upon accessing the next barrier. In-out options can be expressed as a portfolio of the previously priced single barrier and double-touch barrier options. Specifically, an up-in/down-out put can be expressed as:

\[ P_{ui,do}[V, D; H_U, H_L] = P_{udo}(V, D; H_U, H_L) - P_{uo}(V, D; H_U) \]  

(20)

Through parity relations, where \(P_{uo} = P - P_{ui}\) and \(P_{udo} = P - P_{udi}\), we may write the above expression as:

\[ P_{ui,do}[V, D; H_U, H_L] = P_{ui}(V, D; H_U) - P_{udi}(V, D; H_U, H_L) \]  

(21)

and a down-in/up-out put can be expressed as:

\[ P_{di,uo}[V, D; H_U, H_L] = P_{duo}(V, D; H_U, H_L) - P_{do}(V, D; H_L) \]  

(22)

or again, by parity

\[ P_{di,uo}[V, D; H_U, H_L] = P_{di}(V, D; H_L) - P_{dui}(V, D; H_U, H_L) \]  

(23)

The formulae here are re-expressed as functions of single and double barrier options that have been adapted for deadweight costs of default in [\(\underline{A}\) and \(\underline{B}\)] so these are already adjusted for these costs as well.
E One Touch Double Barrier Binary Options

The results here were derived in Hui (1996). Consider an option with two barriers $H_1$ and $H_2$, with $H_1 < V < H_2$, such that the option is knocked out if $V$ touches $H_2$ but instantly pays $1$ if $V$ touches $H_1$. This valuation formula forms the building block for computing the ratchet and restructure debt guarantee value. The equation is as follows:

$$P[V; H_1, H_2] = \int_0^T 1 \cdot e^{-rT} \cdot \text{Prob}[V_t = H_1 | V_t < \forall t] dt$$

$$= \left( \frac{V}{H_1} \right)^\alpha \left\{ \sum_{j=1}^{\infty} \frac{2}{j\pi} \left[ \frac{\beta - (j\pi/L)^2 \exp \left[ -\frac{1}{2} ((j\pi/L)^2 - \beta) \sigma^2 T \right]}{(j\pi/L)^2 - \beta} \right] \right\}$$

$$\times \sin \left( \frac{j\pi}{L} \ln \frac{V}{H_1} \right) + \left( 1 - \ln \frac{V}{H_1} \right)$$

where

$$L = \ln \left( \frac{H_2}{H_1} \right)$$

$$\alpha = -\frac{1}{2} (k_1 - 1)$$

$$\beta = -\frac{1}{4} (k_1 - 1)^2 - \frac{2r}{\sigma^2}$$

$$k_1 = \frac{2r}{\sigma^2}$$

Because this is a digital/binary option and the pay off on this option is $1$, the value here is the expected discount probability that the firm value $V$ touches the lower barrier $H_1$ and does not touch $H_2$. We will use this formula to derive components of the ratchet and restructure guarantee.

Note that if we want the converse option, i.e., an option with two barriers $H_1$ and $H_2$, with $H_1 > V > H_2$, such that the option is knocked out if $V$ touches $H_2$ but instantly pays $1$ if $V$ touches $H_1$, then use the same equation with $H_1$ and $H_2$ flipped.

Since the formula in this appendix is only used for computing first passage time density functions and does not involve payoffs, no adjustment needs to be made for deadweight costs of default.
F Sub-Homogeneity Property of Barrier Options

We state without proof an interesting property of barrier options that may be used to derive bounds in comparing some of the guarantees in this paper to others. Assume a barrier option priced using a generic function $B[V, K; H]$, where $H$ is the barrier, $V$ is the underlying, and $K$ is the strike. Irrespective of the nature of the option, i.e., put/call, or up/down, or in/out, it is the case that for $\gamma \in (0, 1)$, we have the following two inequalities:

\[
(1 + \gamma)B[V, D; H] \geq B[V(1 + \gamma), K(1 + \gamma); H(1 + \gamma)]
\]
\[
(1 - \gamma)B[V, D; H] \leq B[V(1 - \gamma), K(1 - \gamma); H(1 - \gamma)]
\]

What this means is that the barrier option is less sensitive than one-for-one. Therefore, if we increase each of $V, K,$ and $H$ by 10%, then the option value will increase by less than 10%. Likewise, if we reduce all three inputs by 10%, the option price will drop by less than 10%. We call this property the “sub-homogeneity” of barrier options. This is in contrast to vanilla (non-barrier) options that are homogeneous of degree one in the underlying and the strike.
References


Han, Bing., and Yi Zhou (2010). “Understanding the Term Structure of Credit Default Swap Spreads,” Working paper, University of Texas, Austin.


Table 1: Credit spreads and guarantee pricing for a debt issue that has a loan principal of $D = \{0.75, 0.50\}$ and where the firm value is normalized to $V = 1$. The remaining loan parameters are: $T = 15$ years, $\sigma = 0.20$, and $r_f = 0.02$. Debt ratchets entail a $\delta = 30\%$ increase in debt level when the firm appreciates in value such that $D/V$ reaches $K = 0.40$, and write downs entail a $d = 30\%$ reduction in debt level when the firm depreciates in value such that the $D/V$ reaches $M = 1.00$. Spreads are expressed in basis points. The last column in the table shows the guarantee prices when there is also a deadweight loss on default of 30%, i.e., $\phi = 0.7$.

<table>
<thead>
<tr>
<th>$D/V$</th>
<th>$G$</th>
<th>% change</th>
<th>Spread</th>
<th>change</th>
<th>$G_{\phi=0.7}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A. $D/V = 0.75$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1) original</td>
<td>0.0714</td>
<td>—%</td>
<td>92</td>
<td>—</td>
<td>0.1092</td>
</tr>
<tr>
<td>(2) ratch, no wdown</td>
<td>0.0753</td>
<td>5.38%</td>
<td>97</td>
<td>5</td>
<td>0.1162</td>
</tr>
<tr>
<td>(3) no ratch, wdown</td>
<td>0.0354</td>
<td>-50.47%</td>
<td>44</td>
<td>-48</td>
<td>0.0586</td>
</tr>
<tr>
<td>(4) ratch or wdown</td>
<td>0.0404</td>
<td>-43.50%</td>
<td>50</td>
<td>-41</td>
<td>0.0677</td>
</tr>
<tr>
<td>(5) ratch then wdown</td>
<td>0.0728</td>
<td>1.90%</td>
<td>94</td>
<td>2</td>
<td>0.1116</td>
</tr>
<tr>
<td>(6) wdown then ratch</td>
<td>0.0381</td>
<td>-46.68%</td>
<td>47</td>
<td>-44</td>
<td>0.0638</td>
</tr>
<tr>
<td>(7) ratch then wdown, or vice versa</td>
<td>0.0389</td>
<td>-45.54%</td>
<td>48</td>
<td>-43</td>
<td>0.0654</td>
</tr>
<tr>
<td>Panel B. $D/V = 0.50$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1) original</td>
<td>0.0212</td>
<td>—%</td>
<td>39</td>
<td>—</td>
<td>0.0354</td>
</tr>
<tr>
<td>(2) ratch, no wdown</td>
<td>0.0280</td>
<td>32.45%</td>
<td>52</td>
<td>13</td>
<td>0.0467</td>
</tr>
<tr>
<td>(3) no ratch, wdown</td>
<td>0.0088</td>
<td>-58.58%</td>
<td>16</td>
<td>-23</td>
<td>0.0159</td>
</tr>
<tr>
<td>(4) ratch or wdown</td>
<td>0.0200</td>
<td>-5.39%</td>
<td>37</td>
<td>-2</td>
<td>0.0349</td>
</tr>
<tr>
<td>(5) ratch then wdown</td>
<td>0.0212</td>
<td>0.33%</td>
<td>39</td>
<td>0</td>
<td>0.0354</td>
</tr>
<tr>
<td>(6) wdown then ratch</td>
<td>0.0095</td>
<td>-55.32%</td>
<td>17</td>
<td>-22</td>
<td>0.0173</td>
</tr>
<tr>
<td>(7) ratch then wdown, or vice versa</td>
<td>0.0105</td>
<td>-50.42%</td>
<td>19</td>
<td>-20</td>
<td>0.0190</td>
</tr>
</tbody>
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Figure 1: This figure provides a pictorial representation of what happens to the various debt guarantees we consider as the underlying asset appreciates or depreciates in value.
Figure 2: This figure plots the term structure of credit spreads for each of our various cases under a current leverage ratio of $D/V = 0.75$, with a target band of $K = 0.40$ and $M = 1.00$. Ratchets entail a 30% increase in debt level and write downs entail a 30% reduction in debt level. We assume an underlying asset volatility of 20% and a risk free rate of 2%.
Figure 3: This figure plots the term structure of credit spreads for each of our various cases under a current leverage ratio of $D/V = 0.50$, with a target band of $K = 0.40$ and $M = 1.00$. Ratchets entail a 30% increase in debt level and write downs entail a 30% reduction in debt level. We assume an underlying asset volatility of 20% and a risk free rate of 2%.
Figure 4: Comparing credit spreads across various deadweight costs. This figure plots the term structure of credit spreads under a current leverage ratio of $D/V = 0.75$, with a target band of $K = 0.40$ and $M = 1.00$. Ratchets entail a 30% increase in debt level and write downs entail a 30% reduction in debt level. We assume an underlying asset volatility of 20% and a risk free rate of 2%.
Figure 5: Comparing credit spreads across various deadweight costs. This figure plots the term structure of credit spreads under a current leverage ratio of $D/V = 0.50$, with a target band of $K = 0.40$ and $M = 1.00$. Ratchets entail a 30% increase in debt level and write downs entail a 30% reduction in debt level. We assume an underlying asset volatility of 20% and a risk free rate of 2%.
Figure 6: Credit spreads with 30% deadweight costs on default, i.e., $\phi = 0.70$. This figure plots the term structure of credit spreads for each of our various cases under a current leverage ratio of $D/V = 0.75$, with a target band of $K = 0.40$ and $M = 1.00$. Ratchets entail a 30% increase in debt level and write downs entail a 30% reduction in debt level. We assume an underlying asset volatility of 20% and a risk free rate of 2%.
Figure 7: Credit spreads with 30% deadweight costs on default, i.e., $\phi = 0.70$. This figure plots the term structure of credit spreads for each of our various cases under a current leverage ratio of $D/V = 0.50$, with a target band of $K = 0.40$ and $M = 1.00$. Ratchets entail a 30% increase in debt level and write downs entail a 30% reduction in debt level. We assume an underlying asset volatility of 20% and a risk free rate of 2%.