Are ETFs Replacing Index Mutual Funds?

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Abstract

We develop an equilibrium model to investigate whether an Exchange-Traded Fund (ETF) is a more efficient indexing vehicle than an Open-Ended Mutual Fund (OEF). We find that while flow-induced trading is costly to OEF investors, it is also beneficial to those investors who cause the flow – it is simply a zero-sum game. Indeed, the OEF structure can be viewed as providing insurance for investors with liquidity shocks, and hence is beneficial for risk averse investors. However, this liquidity insurance is not without cost – it can cause moral hazard and reduce the OEF performance. Moreover, we find that investors with higher individual liquidity needs prefer to invest via the OEF since they benefit more from the liquidity insurance. Surprisingly, the OEF structure is still viable despite the concentration of higher-liquidity-need investors in the OEF. The reason is that flow-induced trading costs depend only on the aggregate liquidity need, not on individual liquidity needs, which cancel out at the fund level. As a result, OEFs and ETFs coexist in equilibrium with different liquidity clienteles. Finally, we derive empirical predictions that ETFs are better suited for narrower and less liquid underlying indexes, and for investors with longer investment horizons.

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1 Introduction

Since the introduction of the first Exchange-Traded Fund (ETF) in the U.S. in 1993, ETFs have captured most of the growth in index mutual funds and now constitute about 40% of the index fund market share. Despite their growth, not all practitioners are convinced of their value as indexing tools. John Bogle, founder of Vanguard, has been the most vocal critic of ETFs: "if long-term investing was the paradigm for the classic index fund, trading ETFs can only be described as short-term speculation." On the other hand, Lee Kraneffuss, CEO of iShares at Barclays Global Investors, argues that ETFs are to mutual funds what compact discs were to records: "a better product with more features for less money," especially since "long-term ETF investors don’t subsidize the costs of active traders in ETFs."

The view that ETFs are a more efficient indexing vehicle is rooted in the fact that fund flows to an Open-Ended Index Mutual Fund (OEF) can be costly. In particular, whenever OEF investors purchase or redeem shares, their demand is pooled at the fund level and transacted at the daily closing price of the fund, which does not fully account for the future price impact that the fund may experience in implementing the pooled demand. Thus, there is cross-subsidization among investors – either existing investors subsidize new investors in the case of net fund inflows or the remaining investors subsidize the departing investors in the case of net fund outflows. This is the cost of the OEF structure that Kraneffuss and other ETF advocates have emphasized. This cost is also well recognized in the academic literature. For example, Edelen (1999) documents that the extra trading (and the resulting price impact) induced by fund flows can reduce the average performance of a fund by as much as 1.4% per year. Greene and Hodges (2002) find that even daily fund flows, which are more transient and less likely to cause mutual fund managers to incur transaction costs, can impose significant costs on the fund, especially for funds with large daily flows. Dickson, Shoven, and Sialm (2000) demonstrate that fund flows negatively affect mutual funds’ after-tax returns. Johnson (2004) shows that different types of investors impose different flow costs on the fund, and Christoffersen, Keim, and Musto (2007) find that index funds have even higher trading costs than active funds – despite the lack of information content of

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3 Gastineau (2004) applies Edelen (1999)’s cost estimation and concludes that the cost of providing liquidity for open-ended mutual funds can be as high as $40 billion per year.
their trades – due to the inflexibility of index funds in meeting liquidity needs.\textsuperscript{4} Overall, this literature points to significant costs of the OEF structure associated with flow-induced trading.\textsuperscript{5}

ETFs, in contrast, are designed not to have flow-induced trading costs. Like closed-ended index mutual funds (CEFs), ETFs are traded on the stock exchange with individual traders bearing their own transaction costs. The main difference from the CEFs is that ETFs allow some creation and redemption of shares via the in-kind redemption feature. That is, some authorized investors can exchange the underlying index assets for shares of the ETF or vice versa.\textsuperscript{6} This feature allows these investors to arbitrage away any price deviation between the ETF and the underlying index assets. While the in-kind redemption is similar to the creation or redemption of shares via fund flows in OEFs, it does not lead to flow-induced trading costs, since ETFs receive (or pay out) the underlying index assets directly. There is no need to purchase (or liquidate) the underlying assets or to incur any related costs.

While ETF investors can avoid the flow-induced trading costs at the fund level, they need to pay their own transaction costs when purchasing or liquidating their shares. It is therefore not clear whether on average investors are better off investing in ETFs and whether ETFs are a more efficient indexing vehicle than OEFs. In this paper, we develop an equilibrium model to compare the efficiency of ETFs and OEFs for small investors with various individual liquidity needs. We ask whether the rapid growth of the ETF industry implies a permanent structural shift in the mutual fund industry.

Our first result is that ETFs are not more efficient than OEFs – it is a zero-sum game between those who cause the fund flow and those who bear the flow-induced trading costs.

\textsuperscript{4}There is a large literature documenting the large transaction costs incurred by actively managed funds. Grinblatt and Titman (1989) find the total costs to be as high as 2.5% per year. More recently, Edelen, Evans, and Kadlec (2007) find that the annual trading costs for a large sample of equity funds are comparable in magnitude to their expense ratio. These costs are higher for larger funds, especially if the fund has larger relative trade sizes, suggesting that trading costs contribute to the diseconomies of scale for mutual funds identified in Chen, Hong, Huang, and Kubik (2004).

\textsuperscript{5}Sophisticated investors may even exploit the pricing scheme of mutual funds to design profitable trading strategies. For example, Chalmers, Edelen, and Kadlec (2001) find that on extreme days the failure to account for nonsynchronous trading in determining funds’ net asset value can result in an average one-day excess return of 0.84 percent at high beta small-cap domestic equity funds. Goetzmann, Ivkovic, and Rouwenhorst (2001) show that the issue of nonsynchronous trading is most pronounced in international mutual funds and propose a “fair pricing” mechanism that partially corrects net asset values for stale prices. Zitzewitz (2006) provides evidence that some investors may even abuse this pricing feature by trading after 4pm, and that such late trading can lead to significant shareholder welfare loss.

\textsuperscript{6}Only authorized participants can perform in-kind redemption. See Section 2 for detailed description.
Indeed, the OEF structure can be viewed as providing partial insurance against future liquidity needs: Investors pay the cost of lower average returns in exchange for better prices when they experience liquidity shocks. As long as investors are risk averse, the OEF structure is actually beneficial rather than costly to investors. However, this benefit may not be obvious to the investors, since the reported performance of OEFs can be lower than that of ETFs due to the difference in the way returns are accounted for. In particular, the flow-induced transaction costs are equally shared by all investors in the OEF and reduce reported OEF performance, whereas they are incurred only by those ETF investors with liquidity needs and do not affect reported ETF performance.

Second, we find that the liquidity insurance embedded in the OEF structure is not without cost – it can cause moral hazard issues that induce excessive trading and reduce the OEF performance. In particular, since OEF investors transact at a price that does not fully incorporate the impact of their own trading decisions, they do not internalize the price impact of their orders. As a result, they trade too aggressively given their trading needs, leading to excess trading costs for the OEF. This extra trading cost makes the liquidity insurance an ex-ante negative-sum game and leads to a lower average return for the OEF.

Third, the tradeoff between the liquidity-insurance benefit and the moral hazard cost differs for investors with different individual liquidity needs. Under the OEF structure, the moral hazard cost of excessive trading is shared evenly among all investors. Higher-liquidity-need investors benefit more from the liquidity insurance and prefer to invest via the OEFs, whereas lower-liquidity-need investors benefit less from the liquidity insurance and prefer to invest via the ETFs. As intuitive as it may seem, this result runs directly counter to John Bogle’s critique that “ETFs can only be described as short-term speculation.” Instead, we find that long-term investors may optimally choose to invest in the ETFs and that there is a viable role for ETFs in the mutual fund industry.

One might be concerned about the viability of the OEF structure given the concentration of higher-liquidity-need investors in it. Our fourth result states that the OEF structure is still viable and that OEFs and ETFs coexist in equilibrium with different liquidity clienteles. The reason behind this rather surprising result is that flow-induced trading costs depend only on the aggregate liquidity need at the fund level. Since individual liquidity needs cancel out across investors, the concentration of investors with high individual liquidity needs does not

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7This is similar to the insurance feature of bank deposit contracts in Diamond and Dybvig (1983).
lead to higher aggregate liquidity need or higher flow-induced trading costs for the OEF. This result highlights a potential pitfall of the recent trend in the mutual fund industry to impose frequent-trading restrictions. Since higher frequency traders do not necessarily impose higher costs to the OEF yet they benefit much more from the liquidity insurance provided by the OEF structure, they are the best clientele for OEFs. If they are deterred from investing in the OEFs by frequent trading restrictions, OEFs may lose a significant client base.\footnote{This result is based on the assumption that higher-liquidity-need investors do not have higher exposure to the systematic liquidity risk. If there is reason to believe that frequent traders are more likely to be reacting to market conditions or are more correlated in their trading decisions, then they may impose higher costs to OEFs and some forms of frequent trading restrictions may be optimal.}

Finally, our model provides a framework to assess the effectiveness of the OEF and the ETF structures under different circumstances and to make predictions regarding the long-term trend in the growth of the mutual fund industry. One prediction of the model is that if investors have more correlated liquidity needs, the OEF is expected to have larger unbalanced demand from its investors. The price impact is higher, making the OEF less attractive relative to the ETF as an indexing vehicle. As a result, we expect to see a smaller size of the OEF industry in equilibrium. This prediction is actually verifiable. For example, it is reasonable to assume that investors in a narrower index (such as a bio-tech index) are more likely to have correlated liquidity needs compared to investors in a broader market index (such as the S&P 500 index). Hence, we expect to have more ETFs in narrower indexes than in broader market indexes. Similarly, less liquid underlying indexes are likely to generate a higher price impact and larger tracking error for OEFs, making them less ideal candidates for OEF investing as well. These patterns are largely consistent with the growth pattern of ETFs in the market: Many new ETFs track indexes that have less number of stocks, high industry concentration, high volatility, and low liquidity.

Despite the phenomenal growth of the ETF industry, there is limited academic research on ETFs, mostly due to this sector’s short history. Elton, Gruber, Comer, and Li (2002) study SPDR, the first ETF that tracks the S&P 500 index, and document that it underperforms relative to both the underlying index and to other OEFs tracking the same index. Poterba and Shoven (2002) look at the tax implications of ETFs in general and the performance of SPDR in particular. They find that although in theory ETFs can be more tax efficient, in reality SPDR ETF performs slightly worse than the Vanguard S&P 500 both in before-tax
and after-tax returns. We add to this literature by placing ETFs in a broader context, to understand their impact on the structure of the mutual fund industry. Agapova (2006) compares fund flows into index mutual funds and ETFs and finds that their coexistence can be partially explained by a clientele effect.

Our paper is most related to Chordia (1996) and Chen, Goldstein, and Jiang (2007), who consider the cross-subsidization induced by the pricing scheme of the OEF structure. Chordia (1996) is the first to point out that this cross-subsidization can be viewed as a form of insurance against liquidity shocks along the line of Diamond and Dybvig (1983). His focus, however, is quite different. He shows that funds optimally hold more cash in order to meet redemption demands and that load and redemption fees can help alleviate liquidity impacts. Chen, Goldstein, and Jiang (2007) show that the cost of this cross-subsidization can manifest into a “bank run” in which investors flee the fund for fear of bearing the liquidity cost imposed by others’ withdrawing from the fund. We show that both features are present: While the liquidity insurance feature embedded in OEFs is beneficial, especially for higher-liquidity-need investors, it can induce moral hazard issues and reduce the performance of ETFs. As a result, lower-liquidity-need investors may prefer the ETF structure.

Another related literature investigates the interaction between investor behavior and the organizational structure of mutual funds. Nanda, Narayanan, and Warther (2000) show that the existence of investor clienteles with differing liquidity and marketing needs gives rise to a variety of open-ended fund structures that differ in the average return delivered to investors. Massa (1997) suggests that the vast number of funds offered to investors can be seen as marketing strategies used to exploit investor heterogeneity and that market forces may induce a sub-optimal number of mutual funds and categories. Christoffersen and Musto (2002) show that the price sensitivity of individual investors affects the pricing scheme of mutual funds. Our paper complements this literature by introducing a new investment vehicle (an ETF) with a different trading mechanism and studying investor choice between the new and the existing vehicles in equilibrium.

This paper also relates to the literature on the structure of the mutual fund industry. The efficiency of the open-ended structure has been much debated. As discussed earlier, a large literature documents the cost of the OEF structure associated with flow-induced trading. Separately, a smaller, mainly theoretical literature, suggests a potential benefit of the OEF structure when managers have ability. For example, Berk and Green (2004) show that when
investors vote with their feet in open-end mutual funds, they can efficiently align money flow with managerial talent. Stein (2005) further argues that when open-ending is the only creditable signal of managerial ability, it becomes the dominant structure in the mutual fund industry despite its apparent inefficiency in allowing skilled managers to take advantage of arbitrage opportunities (e.g., Shleifer and Vishny (1997)). Combining the two literatures, one is tempted to conclude that the OEF structure is costly in meeting liquidity needs while it might be necessary for rewarding superior abilities. We add to the debate by showing that, even in the absence of ability, the OEF structure has its merits relative to the ETF structure.

On the practical front, our analysis suggests that the rush in the industry to develop active ETFs – in addition to presenting the technical difficulty of revealing the portfolio holding for in-kind redemption while maintaining anonymity – may be ill motivated.

Finally, we contribute to the literature on Closed-Ended Mutual Funds. While the earlier literature largely relates the discount/premium of CEFs to irrational behavior of investors and the cost of arbitrage (see, for example, Lee, Shleifer, and Thaler (1991) and Pontiff (1996)), recently more effort has been devoted to explaining the discount/premium using rational models with managerial ability and liquidity demands (see, for example, Chay and Trzcinka (1999), Berk and Stanton (2007) and Cherkes, Sagi, and Stanton (2007)). Viewing ETFs as closed-ended index funds, we establish a benchmark at which neither managerial ability nor a discount/premium exists. Our framework, which captures the main difference between the OEF and ETF structures, can provide a starting point for understanding the difference between the ETF and CEF structures and in turn shed light on the phenomenal growth of ETFs and the stagnation of CEFs (which are active ETFs except for the in-kind redemption feature).

The remainder of the paper proceeds as follows. Section 2 describes the ETF industry and how it has evolved. Section 3 sets up the model and Section 4 describes the equilibrium. Section 5 discusses the properties Finally, Section 6 concludes.

2 The growth of the ETF industry

2.1 Differences between ETFs and OEFs

ETFs are closed-ended investment companies that can be traded at any time throughout the course of the day on the secondary market. ETFs try to replicate stock market indexes. Contrary to regular index OEFs, ETFs are required to hold all the stocks that comprise the
The first ETF was introduced on the Toronto Stock Exchange in 1990. The first U.S. ETF was introduced in 1993 by State Street Global Advisors – SPDR (Standard & Poor’s Depository Receipts, or “Spiders”), which tracks the S&P 500 index – and it is still the largest ETF by market cap. Most stock exchanges around the world now offer ETFs.

ETFs are closed-ended and not open-ended mutual funds and hence have two main differences from regular open-ended mutual funds. First, they are traded on the secondary market and therefore can be traded during the day and not only once a day. Second, if there is an inflow of money to the fund, the fund does not directly create new shares. However, contrary to standard closed-ended mutual funds, new shares can be created (or redeemed). The creation and redemption of shares in ETFs is also different from that of OEFs. Rather than the fund manager dealing directly with shareholders, certain parties, such as institutional investors, who have entered into a contract with the fund (called Authorized Participants (APs)) will create a basket of shares replicating the index that the fund tracks, and deliver them to the fund in exchange for ETF shares. A basket, or creation unit, consists of anywhere from 10,000 ETF shares to 600,000 ETF shares. ETF shares are then sold and resold freely among investors on the open market. If an investor accumulates a sufficient amount of ETF shares, the investor can exchange one full creation unit of ETF shares for a basket of the underlying shares of stock. The ETF creation unit is then redeemed, and the underlying stocks are delivered out of the fund. One of the advantages of this creation/redemption process for the fund investors is that institutional investors cover the dealing costs in purchasing the required shares to make up the portfolio. This mechanism allows institutional investors to take advantage of arbitrage opportunities when the price of the ETF deviates from its Net Asset Value (NAV). It is also the main distinction between an ETF and a regular closed-ended mutual fund, which results in a very small discount/premium between the price and the NAV. Thus, none of the regular discount/premium characteristics discussed in the literature with regard to closed-ended mutual funds apply in any substantial way to ETFs.

ETFs and OEFs also have different tax advantages. Whenever an OEF realizes a capital gain that is not balanced by a realized loss, the mutual fund must distribute the capital gains to shareholders by the end of the quarter. This can happen when stocks are added to and removed from the index, or when a large number of shares are redeemed. These gains
are taxable to all shareholders. In contrast, ETFs are not redeemed by holders (instead, holders simply sell their ETFs on the stock market, as they would a stock), so investors generally only realize capital gains when they sell their own shares. However, there are some potential taxation drawbacks to ETFs. First, ETFs have to hold the exact mirror image of an index, while OEFs do not. Hence, around changes in the composition of an index, the ETF may have to sell existing stocks in order to re-balance its holdings. OEFs do not have to hold the mirror image of the index, and hence have more flexibility around index changes. Second, ETFs often trade their shares more rapidly to maintain a high cost basis of their underlying shares, which can result in ETF dividends failing to be classified as qualified dividends since the underlying shares don’t satisfy IRS requirements. This can be a substantial drawback since one’s ordinary tax rate may be significantly higher than the 15% tax charged on qualified dividends.

Note that ETFs are structured either as a mutual fund or as a Unit Investment Trust (UIT). A UIT is a U.S. investment company offering a fixed portfolio of securities that have a finite life. UITs are assembled by a sponsor and sold through brokers to investors.

2.2 Data

We use several different data sets in generating our sample:

First, we use CRSP’s Survivor-Bias Free U.S. Mutual Funds Database, from which we extract a list of the index mutual funds (OEFs). We define a fund to be an index mutual fund if the word index (or any derivative of it such as Ind, Idx, and so on) shows up in its name. Whenever in doubt, we check the prospectus of the fund using EDGAR, the SEC Filings and Forms Database, in order to verify that indeed it is a pure index fund. This step of verification from EDGAR is necessary, as some mutual funds can track an index closely (and have the word index in their name) and yet not be an index fund. For example, self described “enhanced index” funds essentially track an index but knowingly change some weights of stocks they believe will over- or under-perform the index, and hence “enhance” the index. If the prospectus is not clear that the fund performs only the task of tracking an index, we err on the side of caution and drop the fund from our sample. We then merge the different share classes of each fund into one entity. As has been recognized in the literature, the same mutual fund can have several almost identical share classes. The different share classes usually differ either in their fee structure (i.e., front-end load, back-end load, no load,
and so on) or in their distribution channel (i.e., investor class, institutional class, and so on). We merge the different share classes into one OEF entity by value weighting the different classes based on their Total Net Asset (TNA). The resulting sample is comprised of 296 OEFs over the time span of 1992-2006. These funds are managed by 131 different families. Interestingly, these 296 funds track only 63 different indexes.

Second, we use Bloomberg in order to generate a list of ETFs. We collect the ticker and CUSIP of each ETF, we also find out what fund family is sponsoring it. In order to verify that we have captured the full span of ETFs, we check the websites of the respective sponsoring families and ETF-dedicated websites (such as ETFConnect.com). The resulting sample is comprised of 320 ETFs over the time span of 1992-2006. We restrict both our ETF and OEF samples to start in 1992 due to the fact that there were no ETFs before then. This allows us to capture the entire current growth of ETFs from their creation to 2006. We exclude any ETF that existed less than one year, as we would not have enough information about it. These 320 ETFs are sponsored by 23 separate families, and they track 268 different indexes. Thus, the final sample consists of 296 OEFs and 320 ETFs over 1992-2006, tracking 289 different indexes.

Third, we collect information about the ETFs and OEFs such as assets under management, number of shares outstanding, creation and deleting date, from CRSP and CRSP Mutual Fund merging by ticker and year.

Last, in order to analyze the indexes tracked by these funds, we need their historical compositions. These compositions are not available for all the indexes. Hence, we employed a proxy for their composition. We use Thomson Financials’s CDA/Spectrum Mutual Funds Holding and CDA/Spectrum Institutional (13f) Holdings. For each index we find all the OEFs and ETFs that track it, since they have to report their holdings on a quarterly basis. Based on the holdings we calculate the variables relating to volatility and composition of the index.

2.3 Trends in the ETF industry

As we discuss in the introduction, while ETFs were introduced in the U.S. only in 1993, they have since grown at an exponential rate from one ETF to more than 300 in 2006.

Figure 1(a) shows the significant increase in money invested in ETFs between the early 1990s and the present. As one can see, the growth in ETFs was clearly slow between 1993
Figure 1: **Summary statistics of the ETF and the OEF industries.** In all panels, the dotted and the solid lines refer to OEFs and ETFs, respectively. Panel (a) reports the dollar amount invested (in billion dollars), panel (b) reports the number of funds, panel (c) reports the number of indexes tracked, and panel (d) reports the average total net assets of the funds (in million dollars).
and 2000. This is for two main reasons. First, ETFs were a new investment vehicle, and as such were not yet very familiar to investors. Second, there were few ETFs available, making it a relatively exotic investment vehicle. Figure 1(b) depicts the relative number of ETFs available compared to OEFs over time. By year 2000, there were only about 20 ETFs available covering the major indexes, while there were more than 150 OEFs available. A direct comparison of the number of available ETFs to the number of available OEFs, however, is slightly misleading. In the early years each new ETF tracked a different index; even now, there are few ETFs that track exactly the same index. OEFs, in contrast, work somewhat differently. Mutual funds depend on their respective distribution channels, and not all mutual funds are available to all investors. For example, an investor with a Vanguard account can only invest in Vanguard mutual funds. Investors in retirement accounts follow even stricter rules and have limited offerings that depend on the company administering the account. The result is that in the past 40 years each mutual fund company (or at least the larger ones with their own distribution channels) has come to offer funds that are very similar to those of other mutual fund families. In the case of index mutual funds many families offer essentially identical funds that track the same index. For example, there are more than a dozen OEFs that track the S&P 500, while not long ago there was only one ETF. Even now there are only two ETFs tracking this index. Thus, while there are more OEFs than ETFs until very recently, OEFs are tracking fewer indexes.

Figure 1(c) shows the large increase in the number of different indexes tracked by ETFs in the past seven years. Comparing Figure 1(c) to 1(a), we can see that part of the success of ETFs and the apparent market share they seem to have captured from OEFs is correlated with the huge increase in the number of indexes that are being tracked. This connection is related to our theoretical argument that the characteristics of the underlying indexes tracked may be related to the increased interest in these indexes. As Figure 1(a) shows, in the past 10 years, while ETFs have grown at a faster pace than OEFs, increasing their relative importance in the marketplace, they have also offered new indexes as investment vehicles at an exponential rate. Together, these figures imply that, indeed, there could be more to the rise in ETFs than the mere fact that they offer an advantage to small investors compared to traditional index OEFs.

Though this impressive growth of ETFs can be partially attributed to the diversity of offerings in terms of the number of indexes covered, Figure 1(d) indicates that the average
ETF has comparable size to the average OEF. Admittedly, since there are many OEFs tracking each index and the decision of small investors to invest in them depends on their respective distribution channels, this is far from conclusive. It does indicate, however, that the growth process of this relatively new industry is a compilation of several simultaneous trends: The intrinsic growth of existing ETFs, the introduction of new ETFs covering existing indexes, and the offering of ETFs covering new indexes that are not offered by traditional OEFs. This evidence highlights the importance of the question at the root of this paper, namely, whether the apparent increase in the size of the ETF industry is going to eventually alter the landscape of the delegated portfolio management industry and, in particular, of the existing mutual fund structure.

2.4 The Growth of the ETF Industry

Since the universe of ETFs and OEFs has been quickly changing, and since the growth rate has been so substantial and different between OEFs and ETFs, we describe the time trends in the data in further detail in Table 1. Though introduced in the 1970s, by the 1990s index OEFs were still not very popular. In 1991 there were only 34 OEFs offered by 19 families managing about 11 billion dollars. ETFs entered the market in 1993 when there was a large inflow in index OEFs. As mentioned in Section 2, though introduced in 1993, ETFs took several years to get exposure. In 1996 assets under management in OEFs had increased to more than 91 billion dollars and ETFs had 3 billion under management. This fact is thoroughly discussed in Svetina and Wahal (2008) and ICI (2008). This relationship, however, is reversed when looking at the number of indexes tracked by ETFs and OEFs. The 296 OEFs track only 63 different indexes, while the 320 ETFs track 268 different ETFs. Similar to the argument about the level of competition in the two markets made by Hortaçsu and Syverson (2004), we can see that ETFs have generated large market differentiation by tracking different indexes, while OEFs, protected by their distribution networks track a substantially smaller and similar list of indexes.

By 2001, the trend of growth had flipped between OEFs and ETFs. In 2001 ETFs were already growing at a rate three times faster than OEFs. The average growth rate in 2001 of index OEFs was only 7% and of ETFs was 389%. Several interesting trends can be observed in the data. Both the number of families offering OEFs and offering ETFs increased between 1996 and 2006, from 40 to 72 for the OEFs and from 2 to 23 for the ETFs. This highlights
two interesting facts. First, the rise in the interest in indexing in the late 1990s brought an increase in offering of index funds by many families. Second, relatively few families offered ETFs even though they were growing at a much faster pace than OEFs, and even though offering one or the other is not technically very different. Vanguard is an example of a family that essentially offers mainly index OEFs and yet refused for many years (under the influence of John Bogle) to also offer ETFs. These facts explain the much higher growth in assets under management per family in ETFs than in OEFs. As we mentioned earlier, OEFs are not always available to all investors depending on the family’s distribution channel, making it more difficult at times to grow. By being available to all investors with a brokerage account, ETFs have enjoyed an advantage in growth that is fairly apparent in the data.

As shown in Table 1, an important aspect is the number of indexes tracked offered by each sponsoring family. This number has steadily been increasing for ETF sponsors, from 9 per family in 1996 to more than 13 in 2001. During the same time period OEF sponsors have only marginally increased the number of different indexes covered from 2 in 1996 to 2.3 in 2001. This highlights that there might be two different forms of growth not shared by the two organizational forms that is captured by the number of covered indexes offered.

### 2.5 Indexes tracked

As we show in Table 1 a large part of the growth of the ETF industry is through the offering of many different ETFs by each family, each one tracking a different index. One of the main motivation of this paper is to better understand this growth and the reasons behind it. In order to investigate this growth, we calculate characteristics of these indexes in order to assess the nature of the ETF growth. In Table 2 we describe the characteristics of the different indexes. The diversity across different indexes is quite large, they vary from tracking more than two thousand stocks to as few as five stocks. The market capitalization of these indexes is also quite diverse, ranging from small “niche” indexes tracking a segment of 35 billion dollars to market-wide indexes tracking more than 1 trillion dollars. We look at four main characteristics of these indexes: the size of the index, the number of index changes, the liquidity of the underlying stocks, and the concentration of the industry tracked. As expected, indexes vary significantly on all these dimensions. One of the larger costs associated with tracking an index is the cost of re-balancing. The average number of stocks entering/exitng the index is 18 stocks a quarter, with an average market value of 119 billion dollars, so these
costs can be very high. The tracking error generated by re-balancing the index is higher for less liquid and smaller stocks. There is also significant variation in the required re-balancing across indexes – some indexes require almost no re-balancing while others require selling and buying more than 60 stocks in a quarter.

We use Amihud (2002)'s liquidity measure. We calculate it by downloading the measure for all stocks from Joel Hasbrouck’s website (which is described in detail in Hasbrouck (2006)) and then averaging the underlying liquidity measures of the stocks comprising the index, both as a value-weighted average and as an equal-weighted average. Again, the difference across indexes is fairly substantial. Some indexes have relatively high liquidity stocks (the S&P 500, for example) and some have fairly low liquidity (some industry specific indexes, for example). Since the liquidity of the underlying index directly affects the tracking error of the OEF, we know from our theoretical argument that the best vehicle (OEFs or ETFs) tracking a particular index is dependent on the index liquidity. We calculate the level of industry concentration using two different measures: we use a Herfindahl-Hirschman Index and the industry concentration measure developed in Kacperczyk, Sialm, and Zheng (2007). As we can see in Table 2, there is variation on this level too.

3 Model

We construct a parsimonious model that captures the main difference between OEFs and ETFs: Orders are cleared in the ETF market directly and traders bear their own trading costs, whereas orders are pooled in the OEF before being submitted to the underlying stock market and all investors in the OEF (whether or not they trade) equally share any resulting trading costs. We model agents’ trading needs by introducing heterogeneity in their risk exposures, which motivates them to trade for risk sharing.

3.1 Securities market

There are three dates, $t = 0, 1, \text{ and } 2$. A risky stock $S$ is traded in a competitive market. The stock yields a final payoff of $V$ at time 2, which is normally distributed,

$$V \sim N(\hat{V}, \sigma^2_v).$$

The per capita supply of the stock is $\bar{\theta}$. In addition, there is a short-term riskless security, which yields a constant interest rate of zero.
In addition to trading the stock directly, investors can participate in either an OEF (denoted by $F$) or an ETF (denoted by $E$). Our main objective is to compare the effectiveness of the OEF and the ETF structure in meeting liquidity needs for individual investors. To simplify analysis, we assume that the ETF is fully integrated with the underlying stock market, that is, the stock is identical to the ETF and is omitted from the setting. We use the ETF and the underlying stock interchangeably in our discussion depending on the context.\footnote{In reality, an ETF is a portfolio of stocks. It may have a tracking error relative to the underlying stock index and may have different liquidity (can be either higher or lower) from its component stocks. The possibility of arbitrage between the ETF and its underlying stocks guarantees a small tracking error and partially alleviates this concern. Moreover, the main objective of this paper is to compare the relative efficiency of OEFs and ETFs rather than to compare their absolute efficiency relative to the underlying stocks. We leave the detailed comparison between the ETF and its underlying stocks to future research.}

We model the OEF structure in the following reduced-form way: At time 0, OEF investors give their stock endowments to the fund in exchange for OEF shares. Each OEF share is normalized to be equal to one share of the stock.\footnote{In practice, to start investing in an OEF, investors pay cash and the fund purchases stocks. We assume OEFs are formed by accepting stocks directly from their investors to eliminate the impact of OEF investing on time 0 stock prices. This price impact complicates the analysis and does not affect our main message.} At time 1, any OEF investor can redeem or purchase any number of shares at a pre-specified price of $P^F_1$ per share, which is independent of the market condition at time 1. The fund can borrow or invest at the risk free rate to accommodate the fund flow. That is, if in aggregate OEF investors demand extra shares at time 1, the fund issues new shares in exchange for cash payments from its investors and then parks the money in the risk free asset; if instead the aggregate demand is a sell order, the fund borrows money and pays out to the departing investors at the price $P^F_1$. Shortly after that, at time $1_+$, the fund passes the demand to the underlying stock market. That is, they buy or sell stock shares at the market price $P^E_{1+}$ to ensure that each remaining OEF share has the same risk exposure as one share of stock. In this sense, the OEF is extremely passive—it does not optimally manage the cash position to account for potential price impacts. To the extent that $P^E_1 \neq P^F_1$, the fund makes or loses money on these transactions. The gains or losses are shared evenly by all remaining shares in the OEF, leading to a terminal payoff different from that of the stock. We term the difference the “tracking error” of the OEF.

As will become obvious later, the price difference between $P^F_1$ and $P^E_1$ is predictable at time 1. To rule out arbitrage, we assume that once investors choose to invest in an OEF, they cannot access the ETF or the stock market directly.
3.2 Agents

There is a continuum of investors in the economy with a total population weight of $1 + \mu$, of which $\mu$ fraction are stock investors and the remaining (normalized to 1) are index investors choosing between the OEF and the ETF. The focus of the paper is the decision of indexers between the two indexing vehicles. Stock investors are introduced so that there is always a viable stock market in which the OEF can transact (at time $1_+$) to implement the demand from its investors, even when the population of ETF investors goes to 0.

Each investor is endowed with $\bar{\theta}$ shares of the stock at time 0. Investor $i$ also receives a non-traded payoff $N_i$ at the terminal date 2, which is given by

$$N_i = Y_i (V - \bar{V}), \quad Y_i = Y + \epsilon_i,$$

where $V$ is the stock payoff in (1), and $Y$ and $\epsilon_i$ are mutually independent, normally distributed random variables with a mean of zero and a volatility of $\sigma_Y$ and $\sigma_i$, respectively.\(^{11}\)

We can interpret this non-traded income as a liquidity shock that is correlated with the stock. In particular, it is equivalent to an endowment shock of $Y_i$ shares of the stock. Given the correlation between the liquidity shock and the stock payoff, investors want to adjust their stock positions in order to hedge this risk, giving rise to their trading needs.\(^{12}\)

All investors are subject to the same $Y$ shock. Hence, it captures the aggregate liquidity shock. The component $\epsilon_i$ is independent across all investors and defines their individual shock. Some investors face more individual liquidity shocks than others, so we assume

$$\sigma_i = s_i \sigma, \quad s_i \sim \text{Unif}[0, 1],$$

where $\sigma$ is the maximum individual liquidity shock, and $s_i$ is the magnitude of liquidity needs for individual $i$, which is uniformly distributed over the range $[0, 1]$. Since $\epsilon_i$ is independent with bounded variance and each individual investor has zero population weight, Law of Large Number holds. Thus, for any subset $I$ of investors, individual liquidity shocks always cancel out and the total liquidity demand depends only on the aggregate shock and is proportional

---

\(^{11}\)We assume that the non-traded payoff $N_i$ is perfectly correlated with the stock payoff $V$. As long as $N_i$ and $V$ are correlated, the qualitative nature of our results is independent of the sign and the magnitude of the correlation.

\(^{12}\)Heterogeneity in endowment is merely a device to introduce the need of trading for risk-sharing purposes as in Diamond and Verrecchia (1981) and Huang and Wang (2008). Other forms of heterogeneity can also generate trading needs, such as difference in preferences or beliefs. Our modeling choice is mainly motivated by tractability.
to the population weight $\mu_I$,

$$
\int_{i \in I} \epsilon_i = 0, \quad \text{and} \quad \int_{i \in I} N_i = \mu_I Y (V - \bar{V}).
$$

(4)

All agents have identical preference, which can be described by an expected utility function over the terminal wealth. For tractability, we assume that the agent exhibits mean-variance preference. In particular, agent $i$ has the following utility function:

$$
E[W_i] - \frac{\gamma}{2} \text{Var}[W_i],
$$

(5)

where $\gamma$ is the risk aversion and $W_i$ is the agent’s terminal wealth.

### 3.3 Time line

We now describe in detail the timing of events and actions. At time 0, indexers decide whether to invest via the ETF or the OEF. The market equilibrium determines the fractions $\eta$ and $1 - \eta$ of investors who choose to invest in the ETF and the OEF, respectively. ETF investors do nothing at time 0, and OEF investors exchange all their stock shares for an equal number of OEF shares.

<table>
<thead>
<tr>
<th>Shocks</th>
<th>$Y_i = Y + \epsilon_i$</th>
<th>$V, N_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>$\theta_i^F, X_i^F$</td>
<td></td>
</tr>
<tr>
<td>1+</td>
<td>$\theta_{i1}, X_{i1}^F$</td>
<td></td>
</tr>
<tr>
<td>OEF investors</td>
<td>$\bar{\theta}, 1 - \eta$</td>
<td></td>
</tr>
<tr>
<td>ETF investors</td>
<td>$\bar{\theta}, \eta$</td>
<td></td>
</tr>
<tr>
<td>Stock investors</td>
<td>$\bar{\theta}, \mu$</td>
<td></td>
</tr>
<tr>
<td>Equilibrium</td>
<td>$\eta$</td>
<td>$P_1^F, \Delta$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$P_1^E$</td>
</tr>
</tbody>
</table>

Figure 2: The time line of the economy.

At time 1, investors in the OEF learn the aggregate liquidity shock $Y$ and their individual risk exposure $\epsilon_i$. They take as given the price $P_1^F$ and choose their optimal holding $\theta_{i1}^F$. Investor $i$ redeems $(\bar{\theta} - \theta_{i1}^F)$ shares from the OEF and receives a payment of $P_1^F (\bar{\theta} - \theta_{i1}^F)$ (or pays cash to purchase shares if $(\bar{\theta} - \theta_{i1}^F) < 0$). Aggregating over all OEF investors, the OEF experiences a total redemption of

$$
X_i^F \equiv \int_{i \in OEF} (\bar{\theta} - \theta_{i1}^F) = (1 - \eta) \bar{\theta} - \int_{i \in OEF} \theta_{i1}^F
$$

(6)
shares (or creation if \( X^F < 0 \)). Hence, the total OEF shares outstanding is \( \int_i \theta^F_{i1} \). The assets of the OEF consist of \((1 - \eta) \bar{\theta} \) shares of the stock and \(-X^F P^E_1 \) dollars of cash.

Note that the supply of OEF shares is not fixed at time 1 and is a function of the predetermined price \( P^E_1 \), since the fund can create or redeem any number of shares based on investor demand. In this sense, \( P^E_1 \) is not an equilibrium price that clears the market, but rather a contractual price that all OEF investors agree upon as a part of the OEF contract. In the model, we impose an additional restriction that the aggregate trading demand from OEF investors \((X^F)\) is zero when the aggregate liquidity shock is \( Y = 0 \) to pin down the price.

At time \( 1_+ \), the OEF passes the aggregate demand from its investors through to the underlying asset market by buying or selling stocks. It sells exactly \( X^F \) shares of the stock, independent of the transaction price \( P^E_1 \), to make sure that each of the remaining OEF shares has the same risk exposure as a share of the stock. After the transaction, the remaining assets in the OEF include \((1 - \eta) \bar{\theta} - X^F = \int_{i \in \text{OEF}} \theta^F_{i1} \) shares of the stock and \(-X^F P^E_1 + X^F P^E_1 \) dollars of cash, to be shared equally among all the remaining OEF shares. Thus, each OEF share is equivalent to a share of the stock plus a loss of \(-\Delta\), defined as

\[
\Delta = X^F (P^E_1 - P^E_1) \left( \int_{i \in \text{OEF}} \theta^F_{i1} \right).
\]

At time \( 1_+ \), in the ETF market, all stock and ETF investors participate both to unload their own shock \( Y_i \) and to accommodate the above demand \( X^F \) from the OEF. To simplify notation, we use the same time index 1 for all variables realized at \( 1_+ \). In particular, we use \( P^E_1 \) to denote the equilibrium ETF/stock price and \( \theta^E_{i1} \) to denote the holdings of an ETF investor after trading at time \( 1_+ \). There is little room for confusion, since we assume that OEF investors are not allowed to trade in the ETF. This assumption is needed to rule out arbitrage opportunities given the price difference between \( P^F_1 \) and \( P^E_1 \).

At time 2, investors liquidate their OEF or ETF holdings at prices \( P^F_2 \) and \( P^E_2 \), respectively. Obviously, \( P^E_2 = V \). The OEF has a per share loss of \( \Delta \) dollars in addition to its stock holding. Hence, \( P^F_2 = V - \Delta \). We refer to \( \Delta \) as the tracking error of the OEF relative to the ETF. Including their non-traded labor income in (2), the terminal wealth for OEF and ETF investors are, respectively,

\[
W^E_{i2} = \bar{\theta} P^E_1 + \theta^E_{i1}(V - P^E_1) + Y_i (V - \bar{V}) \quad \text{(8a)}
\]

\[
W^F_{i2} = \bar{\theta} P^F_1 + \theta^F_{i1}(V - \Delta - P^F_1) + Y_i (V - \bar{V}). \quad \text{(8b)}
\]
3.4 Discussions of the model

In this subsection, we provide additional discussions and motivations about several important features of the model.

First, since we equate ETFs with the underlying stocks, all our discussion of the benefit and the cost of the OEF structure should be interpreted as a relative statement in relation to the ETF structure. For example, the diversification benefit of indexing is common to both structures and does not affect the comparison. Also, even though historically ETFs are introduced after OEFs and the natural question to ask is whether ETFs add value, technically our model asks the opposite question of whether OEFs add value above and beyond what ETFs can offer.

Second, while our specification of the OEF and the ETF trading mechanisms captures their key difference in terms of flow-induced trading costs, we deviate from the true trading mechanisms for tractability. In practice, for most OEFs, investors submit their purchase or redemption orders throughout the day to meet their liquidity needs. All their orders are pooled and executed at the end of day Net Asset Value (NAV), calculated using the closing prices of their stock holdings at 4pm. The OEF observes preliminary signals about fund flows and may transact in the underlying stock market throughout the day in anticipation of these demands. However, since on average OEFs do not observe the full flows until several hours after the market closure, they need to execute the remaining orders during the next trading day (or even several days afterwards if the demand is large). On the other hand, ETF investors trade sequentially at different prices depending on their time of entry into the market and bear their own transaction costs.

The above description highlights two sources of flow-induced trading costs for the average OEF investors relative to ETF investors. First, given an order flow, the overall transaction costs can differ between the OEF and the ETF. The average transaction price might be worse for the OEF because of its delay in observing the full order flow or because of the (perceived) superior investment ability of the average ETF investors. Alternatively, the average price might be better for the OEF if the OEF manager can add value by forecasting the overall liquidity demand or by implementing the orders more efficiently than the average ETF investors throughout the day. Second, even if the OEF and the ETF receive the same average transaction prices, they may allocate the overall costs differently among investors. In particular, the OEF may allocate part of the costs to investors not incurring any trades.
The reason is that all orders entered before 4pm receive the end-of-day NAV, which does not incorporate the future price impact that the OEF may experience when it implements the remaining demand in the stock market the following day. From the literature on costly mutual fund flows (e.g., Edelen (1999) and Greene and Hodges (2002)), we know that the combined effects of these two sources of flow-induced trading costs are large for OEFs.

While it is hard to separate the two types of flow-induced trading costs, in the model we rule out the differential overall transaction costs by assuming a unique price $P_{1E}$ at which both the ETF investors and the OEF transact. Thus, we do not compare the absolute efficiency of the OEF and the ETF in implementing their liquidity trades. Rather, our model focuses on the relative efficiency of the two structures given their differential allocation of the overall transaction costs: The OEF structure provides partial insurance against the liquidity shock by setting a price $P_{1F}$ that is less dependent on $Y$; the ETF structure provides no insurance by trading at $P_{1E}$ directly. Although in practice the end-of-day NAV $P_{1F}$ may depend on the part of $Y$ that is revealed during the day, it is reasonable that $P_{1F}$ does not fully incorporate the price impact of $Y$, since some OEF flows are not observable even to the OEF itself till the next day. Thus, $P_{1F}$ is less dependent on $Y$ than $P_{1E}$ is and the OEF structure provides some liquidity insurance. In the model, we assume a constant $P_{1F}$ for tractability.

Third, in reality, ETF investors need to pay bid-ask spreads to the stock exchange whereas the OEF charges zero bid-ask spread by providing internal crossing of individual trades. The assumption that all ETF investors transact at price $P_{1E}$ ignores this difference and overstates the benefits of the ETF. Moreover, this assumption of a batched price $P_{1E}$ smooths the utility of all ETF investors across the different transaction prices they may receive throughout the day, further increasing the estimated benefit of the ETF structure. This assumption is conservative for our main conclusion that the OEF structure is not dominated.

Fourth, ETF is generally perceived as more tax-efficient due to its ability to pass out low-tax-basis stocks to authorized investors via the in-kind redemption feature. However, Poterba and Shoven (2002) find that although in theory ETFs can be more tax efficient, in reality SPDR ETF performs slightly worse than the Vanguard S&P 500 both in before-tax and after-tax returns. In general, ETFs become less tax-efficient when they are forced to trade frequently to accommodate index changes or to meet redemption needs.\footnote{On Dec. 10, 2008, the Rydex Inverse 2X S&P Select Sector Energy ETF paid out 87% of its total assets under management as short-term capital gains, highlighting the extreme tax inefficiency of some ETFs. Gus Sauter, CIO of Vanguard, also encourages investors not to be oversold on the ETF tax-efficiency claims in his}
tax-law change of 2003 allows a reduced tax rates (at 15%) for qualified dividends. Utilizing the in-kind redemption process reduces the holding period and may disqualify dividends for the lower tax rate.

Fifth, we abstract away some institutional details to highlight the structural difference between OEFs and ETFs. For example, an OEF can replicate an index with a small number of stocks while an ETF organized as a unit trust has to hold the whole index. Even for an ETF that is organized as an open-ended fund and that is not required to hold the whole index, more trading is needed during index membership changes given its in ability to manage the index changes through fund flows. Therefore, OEFs have an advantage in transactions costs during index changes. Moreover, we abstract away other operational costs for OEFs, like order execution, book keeping, and so on. These costs have a large fixed component and decrease with the fund size.

4 Equilibrium

We solve the equilibrium backwards in three steps. First, taking the demand from the OEF as given, we solve the optimization problem of ETF investors to derive the equilibrium price at time $1_+$. Second, we solve the trading decision of OEF investors at time 1 and the equilibrium tracking error. Third, we evaluate the utility of investing via the OEF and the ETF for investors with different liquidity needs at time 0 and determine the equilibrium fraction of investors who choose to invest in each.

4.1 Equilibrium at time $1_+$ in the ETF market

Assume the aggregate demand from OEF investors is $X^F$. The following proposition solves the optimal decision of ETF investors and the market equilibrium at time $1_+$.

**Proposition 1.** Given the population $\eta$ (and $\mu$) of ETF (and stock) investors, the demand $X^F$ from OEF investors, and the aggregate liquidity shock $Y$, the equilibrium ETF price at time $1_+$ is

$$P_1^E = \bar{V} - \gamma \sigma^2 \bar{\theta} - \gamma \sigma^2 \left( Y + \frac{X^F}{\eta + \mu} \right),$$

(9)

and the optimal holding of an ETF (and stock) investor $i$ is

$$
\theta^E_{i1} = \theta^S_{i1} = \frac{V - P^E_1}{\gamma \sigma_v^2} - Y - \epsilon_i
$$

(10a)

$$
= \bar{\theta} + \frac{X^F}{\eta + \mu} - \epsilon_i.
$$

(10b)

By construction, once the ETF investors decide to invest via the ETF, they are identical to stock investors in the model. From equation (10a), the desired holding of investor $i$ depends on the risk-return tradeoff of the stock (the first term) plus a hedging term against the liquidity shock $Y_i = Y + \epsilon_i$. Since $Y$ is the aggregate shock, it affects everyone’s desired holding and hence the equilibrium price in (9). In equilibrium, the price $P^E_1$ fully adjusts for the impact of $Y$ and investors no longer choose to unload $Y$, as (10b) indicates. In summary, the aggregate risk $Y$ affects only the price but not the trading decisions in equilibrium. All ETF and stock investors equally share the aggregate risk $Y$ and they unload only their idiosyncratic risk exposure, $\epsilon_i$, in the market. The excess demand, $X^F$, from the OEF investors, is an aggregate risk for the ETF and stock investors (with total population weight $\eta + \mu$). They equally share this risk and the equilibrium price incorporates this risk as well.

4.2 Equilibrium at time 1 in the OEF market

We now solve the OEF equilibrium at time 1 in two steps. First, taking as given the functional forms of both the price $P^F_1$ and the tracking error $\Delta$, OEF investors choose their optimal holding conditional on their liquidity shock. Second, we aggregate the demand from OEF investors and solve for equilibrium prices and the tracking error.

As stated earlier, we impose the additional restriction that the aggregate trading demand from OEF investors ($X^F$) is zero when the aggregate liquidity shock is $Y = 0$ to pin down the price. The following proposition derives the equilibrium price $P^F_1$ and individual share holdings based on this restriction.

**Proposition 2.** Assume the OEF price is $P^F_2 = V - \Delta$ at time 2, where $\Delta = \Delta_0 + \Delta_1 Y + \Delta_2 Y^2$ is the tracking error, and that the aggregate trading demand from OEF investors ($X^F$) is zero when the aggregate shock is $Y = 0$. Then the OEF price at time 1 is

$$
P^F_1 = \bar{V} - \gamma \sigma_v^2 \bar{\theta} - \Delta_0,
$$

(11)
and the optimal holding of an OEF investor $i$ is

$$
\theta^F_{i1} = \frac{\bar{V} - \Delta - P_1^F}{\gamma \sigma^2_v} - Y - \epsilon_i,
$$

(12a)

$$
= \hat{\theta} - \frac{\Delta_1 Y + \Delta_2 Y^2}{\gamma \sigma^2_v} - Y - \epsilon_i.
$$

(12b)

The holding in (12a) is similar to that of (10a), except that the tracking error $\Delta$ lowers the expected future payoff of the OEF and reduces the demand for the stock. Similar to the ETF case, ignoring the functional form of the price, investors would like to hedge both the aggregate shocks $Y$ and the idiosyncratic shock $\epsilon_i$. In contrast to the ETF case, however, the equilibrium price $P_1^F$ does not fully account for the impact of $Y$. In particular, we model a special case in which $P_1^F$ is independent of $Y$. As a result, OEF investors optimally unload their aggregate risk exposure $Y$ in equilibrium. This is in direct contrast to the case of ETF investors in Proposition 1, who hedge only the idiosyncratic risk $\epsilon_i$. The difference between the trading strategies for the OEF and the ETF investors reflects a “moral hazard” effect. Since OEF investors are guaranteed a price $P_1^F$ independent of $Y$, they have the incentive to unload their aggregate risk exposure $Y$ even though in equilibrium it is more efficient for everyone to share the aggregate risk.

We now connect the OEF and the ETF market through the definitions of OEF demand $X^F$ and the tracking error $\Delta$ in (6) and (7), respectively. The tracking error is a complicated function of $Y$. We use Taylor expansion to expand it around $Y$. As long as $Y$ is small relative to the total supply of the stock, we can drop higher-order terms. In particular, we drop terms higher than the second order and match coefficients of the $Y$ terms to derive the equilibrium in the following Proposition.

**Proposition 3.** When $Y$ is small (relative to $\bar{\theta}$),

(i) at time 2, the equilibrium OEF price is $P^F_2 = \bar{V} - \Delta$, where $\Delta$ is the tracking error and

$$
\Delta = \Delta_2 Y^2, \quad \Delta_2 \equiv (1 + \hat{\eta}) \gamma \sigma^2_v / \hat{\theta}, \quad \hat{\eta} \equiv \frac{1 - \eta}{\eta + \mu}
$$

(13)

(ii) at time 1, the equilibrium prices of the OEF and the ETF are, respectively,

$$
P^F_1 = \bar{V} - \gamma \sigma^2_v \bar{\theta}
$$

(14a)

$$
P^E_1 = \bar{V} - \gamma \sigma^2_v \bar{\theta} - \gamma \sigma^2_v (1 + \hat{\eta}) Y - \hat{\eta} \Delta_2 Y^2
$$

(14b)
(iii) At time 1, the equilibrium holdings of the OEF and the ETF investors are, respectively,

\[ \theta^F_{t1} = \bar{\theta} - \epsilon_i - Y - (1 + \hat{\eta})Y^2 / \bar{\theta} \]  

\[ \theta^E_{t1} = \bar{\theta} - \epsilon_i + \hat{\eta}Y + \hat{\eta}(1 + \hat{\eta})Y^2 / \bar{\theta}, \]  

and the aggregate demand from OEF investors is \[ X^F = (\eta + \mu)(x_1Y + x_2Y^2), \] where \[ x_1 = \hat{\eta} \] and \[ x_2 = \hat{\eta}(1 + \hat{\eta})/\bar{\theta}. \]

Note that the population weight of OEF investors is \( 1 - \eta \) and the population weight of ETF and stock investors is \( \eta + \mu \), hence, \( \hat{\eta} = (1 - \eta)/(\eta + \mu) \) in (13) defines the relative population weight of the OEF investors. From (15), OEF investors unload all of their aggregate risk exposure \( Y \) to the ETF market, while ETF investors accommodate the demand. This extra demand causes an additional impact on the price \( P^E_1 \) and leads to the OEF tracking error \( \Delta \). Although collectively costly, individual OEF investors have little incentive to internalize this cost given their guaranteed price \( P^F_1 \).

4.3 Value functions of the OEF and the ETF investors at time 0

We calculate the value functions of the OEF and the ETF investors in two steps, taking as given the fraction of investors that choose to invest in each. First, we consider the partial equilibrium effect when both the tracking error \( \Delta \) of the OEF and the order flow \( X^F \) faced by ETF investors are taken as exogenously given. Second, we consider the general equilibrium effect by incorporating the equilibrium \( \Delta \) and \( X^F \) from Proposition 3. These two steps allow us to separate the effect of individual optimization from the equilibrium price impact.

The following lemma derives the individual value functions of OEF and ETF investors taking \( \Delta \) and \( X^F \) as given. To simplify the expressions, we use the knowledge from Proposition 3 that \( \Delta_0 = \Delta_1 = x_0 = 0 \).

Lemma 1. Assume the tracking error of the OEF is \( \Delta = \Delta_2 Y^2 \) and the ETF market has a demand shock of \( X^F = (\eta + \mu)(x_1Y + x_2Y^2) \). When \( \sigma_y \) is small (relative to \( \bar{\theta} \)), for an investor anticipating an idiosyncratic liquidity shock of the size \( \sigma_i = s_i \sigma_\epsilon \), the value function of investing in the OEF and the ETF is, respectively,

\[ J^F_{i0} = \bar{\theta}V - \frac{1}{2\gamma}(1 + s_i^2 k_\epsilon + k_y)k_\theta - (1 - s_i^2 k_\epsilon)\bar{\theta}\sigma_y^2 \Delta_2 + O(\sigma_y^4), \]  

\[ J^E_{i0} = \bar{\theta}V - \frac{1}{2\gamma}(1 + s_i^2 k_\epsilon)(k_y + k_\theta) + \frac{k_y}{2\gamma}(1 - k_\theta)x_1^2 - \frac{k_y}{2\gamma}(2x_1 + x_2^2 + 2x_2\bar{\theta})s_i^2 k_\epsilon + O(\sigma_y^4), \]  

where \( O(\sigma_y^4) \) are terms of the order \( \sigma_y^4 \) or higher, \( k_y \equiv \gamma^2 \sigma_\epsilon^2 \sigma_y^2, k_\epsilon \equiv \gamma^2 \sigma_\epsilon^2 \sigma_\epsilon^2 \), and \( k_\theta \equiv \gamma^2 \sigma_\epsilon^2 \bar{\theta}^2 \).
The value function exhibits several interesting features. First, if $\Delta = 0$ and $X^F = 0$, the gain of investing via the OEF (relative to investing via the ETF, $J_{i0}^F - J_{i0}^E$) increases with the size of the individual liquidity shock $s_i$. The reason is that the liquidity insurance provided by the OEF structure is more beneficial for investors with high liquidity needs.

Second, the tracking error ($\Delta_2$) reduces the utility for OEF investors, especially for those with smaller idiosyncratic liquidity needs ($s_i$). Although this result appears similar to the common perception that investors with low liquidity needs are subsidizing others with higher liquidity needs, the intuition is different. From (4), all idiosyncratic liquidity needs cancel out at the fund level. Hence, the only liquidity need that is costly to the fund is the aggregate liquidity shock $Y$. It is important to note that since all investors have equal exposure to the aggregate liquidity shock, they contribute equally to the aggregate trading need and the resulting tracking error of the fund, whether they have high or low idiosyncratic liquidity needs.

The same price impact and tracking error, however, can affect their utility differently. Surprisingly, it is more costly for investors with low idiosyncratic liquidity needs, as indicated by the third term in (16a), $- (1 - s_i^2 k_\epsilon) \bar{\theta} \sigma_y^2 \Delta_2$. The intuition is that the price impact and tracking error depend only on the aggregate shock, which is almost perfectly correlated with the trading needs of investors with low idiosyncratic liquidity needs. Hence, almost all their trades occur at a bad time – they tend to sell when the aggregate liquidity shock pushes down the price and buy when the aggregate liquidity shock drives up the price. On the other hand, for investors with high idiosyncratic trading needs, their trading is less correlated with the aggregate shock, and they sometimes lose and sometimes gain from the price discrepancy between the OEF and the ETF and the resulting tracking error.

Third, the demand from OEF investors have dual impacts on the ETF investors. On the one hand, ETF investors make markets for the OEF investors and earn a profit by doing so. This is reflected in the third term in (16b), $\frac{k_E}{2\gamma} (1 - k_0) x_1^2$. On the other hand, the extra demand makes price $P_1^E$ more volatile and is costly especially for those with high idiosyncratic liquidity needs, as indicated by the negative term after that, $- \frac{k_E}{2\gamma} (2x_1 + x_1^2 + 2x_2 \bar{\theta}) s_i^2 k_\epsilon$.

These results suggest that individuals with high idiosyncratic liquidity needs are ill-suited to provide liquidity to others via the ETF structure because they bear a high cost of transacting directly at volatile prices and benefit more from the liquidity insurance embedded in the OEF structure. Hence, there is a natural separation between investors with high and
low liquidity needs in their preferred investment vehicle.

In Proposition 4, we incorporate the equilibrium tracking error $\Delta$ and OEF demand $X^F$ from Proposition 3 into the value functions in Lemma 1 to derive the equilibrium utility gain of investing via the OEF.

**Proposition 4.** Let $\eta$ be the fraction of investors who choose to invest via the ETF. When $\sigma_y$ is small (relative to $\bar{\theta}$), the equilibrium utility gain of investing via the OEF relative to the ETF for an investor with idiosyncratic liquidity shock $s_i$ is

$$G_i^F(s_i, \eta) \equiv J_i^F - J_i^E = \frac{(1 + \bar{\eta})^2}{2\gamma}(3s_i^2k_\epsilon - 1 - \frac{1 - \bar{\eta}}{1 + \bar{\eta}}k_\theta)k_y + O(\sigma_y^4).$$

(i) Investor $i$ invests in the OEF if and only if $G_i^F \geq 0$, otherwise, he invests in the ETF;

(ii) $G_i^F$ increases with individuals’ idiosyncratic liquidity shock, that is, $(\partial G_i^F)/(\partial s_i) > 0$.

Proposition 4 suggests that individuals with higher liquidity needs prefer the OEF structure relative to the ETF. One might question the viability of the OEF structure in equilibrium if high-liquidity-need investors are their main clientele. Interestingly, since individual liquidity needs cancel out at the fund level, high-liquidity-need investors do not lead to more volatile fund flows or higher flow-induced trading costs. As a result, the OEF is not dominated by the ETF in equilibrium.

4.4 The equilibrium size of the ETF industry at time 0

We solve for the equilibrium size of the ETF industry. The following proposition shows the existence of equilibrium in which the marginal investors are indifferent between investing via the OEF or the ETF and derives the equilibrium size of ETFs.

**Proposition 5.** When $\sigma_y$ is small (relative to $\bar{\theta}$), let $\eta$ solves the following equation,

$$\eta = \begin{cases} 
0, & \text{if } G_i^F(0, 0) > 0; \\
1, & \text{if } G_i^F(1, 1) < 0; \\
G_i^F(\eta, \eta) = 0, & \text{o.w.,} 
\end{cases} \quad (18)$$

then all indexing investors with $s_i \leq \eta$ invests via the ETF and the rest ($s_i > \eta$) invests via the OEF. The equilibrium population of ETF investors is $\eta$.

The proof of the proposition is straightforward. When $G_i^F(0, 0) > 0$, for any investors with $s_i > 0$, we have $G_i^F(s_i, 0) \geq G_i^F(0, 0) > 0$ since $G_i^F$ increases in $s_i$. Hence, at $\eta = 0$ the
OEF is preferred by all investors, confirming that \( \eta = 0 \) is an equilibrium. Similarly, when \( G_i^F(1, 1) < 0 \), we have \( G_i^F(s_i, 1) \leq G_i^F(1, 1) < 0 \) for any \( s_i \), and ETF is preferred by everyone, thus, \( \eta = 1 \). The rest of the proof can be found in the Appendix.

While intuitive, the proposition does not give simple conditions on the underlying parameters for the equilibrium. The following corollary provides a more explicit solution.

**Corollary 1.** The equilibrium population of ETF investors can be expressed as follows:

\[
\eta = \begin{cases} 
0, & \text{if } \mu < \frac{(k_\theta - 1)}{(k_\theta + 1)}; \\
1, & \text{if } k_\epsilon < \frac{(1 + k_\theta)}{3}; \\
\frac{1}{3(1 + \mu)k_\epsilon} \left(k_\theta + \sqrt{k_\theta^2 + 3k_\epsilon(1 + \mu)^2 + 3k_\epsilon k_\theta(\mu^2 - 1)}\right), & \text{o.w.}
\end{cases}
\]  

The first case of the corollary indicates that the ETF population drops to 0 for small \( \mu \). This is a situation with very few stock investors to make markets. If the equilibrium fraction of ETF is small, the price becomes extremely volatile whenever OEF investors unloads their aggregate demand; the volatile price makes it extremely costly to trade directly in the ETF market, and everyone is better off investing via the OEF to smooth their idiosyncratic shocks. The second case states that for small \( k_\epsilon \) – which corresponds to small idiosyncratic liquidity shocks – all investors prefer the ETF structure. The reason is that the benefit of liquidity insurance provided by the OEF structure is too low to cover the moral hazard cost of inefficiently unloading the aggregate shocks. We leave the discussion of the general case to the next section.

## 5 Equilibrium properties and empirical implications

We now examine the properties of the equilibrium. In particular, we discuss the relative efficiency of OEFs and ETFs in meeting individual liquidity needs, the determinants of the OEF tracking error, the liquidity clientele for the OEFs and ETFs, and the co-existence of OEFs and ETFs in equilibrium. Finally, we lay out empirical implications regarding the equilibrium size of the ETF for different underlying indexes.

### 5.1 Efficiency of OEFs and ETFs

The main prediction of our model is that, in absence of additional frictions, the ETF structure is as efficient as the OEF structure.
Result 1. **ETFs are not more (or less) efficient than OEFs.**

For ETFs, only traders bear their own trading costs; for OEFs, all investors might share the trading costs of a few traders. The source of the costs is the differential price between the price traders receive from the fund ($P^F_1$) and the price the fund is able to execute the order in the underlying stock market ($P^E_1$). Despite of this price discrepancy, it’s a zero-sum game between those who cause the fund flow and those who bear the flow-induced trading costs.

This result runs directly counter to Kraneffuss’s claim that ETF is a better product than OEF since long-term traders do not subsidize the cost of short term traders. Instead, we show that the OEF structure can be viewed as providing partial insurance against future liquidity needs: Investors pay the cost of lower average returns in exchange for better prices when they experience liquidity shocks. This is similar to the insurance feature of bank deposit contracts in Diamond and Dybvig (1983).

As long as investors are risk averse, the OEF structure is actually beneficial rather than costly to investors. However, this benefit may not be obvious to the investors, since the reported performance of OEFs can be lower than that of ETFs due to the difference in the way returns are accounted for. In particular, the flow-induced transaction costs are equally shared by all investors in the OEF and reduce reported OEF performance, whereas they are incurred only by those ETF investors with liquidity needs and do not affect reported ETF performance.

However, the OEF structure can induce additional inefficiency. Namely, it can cause moral hazard issues that induce excessive trading and reduce the OEF performance. In particular, since OEF investors transact at a price that does not fully incorporate the impact of their own trading decisions, they do not internalize the price impact of their orders. As a result, they trade too aggressively given their trading needs, leading to excess trading costs for the OEF. This extra trading cost makes the liquidity insurance an ex-ante negative-sum game and leads to a lower average return for the OEF.

**Result 2.** The OEF structure can induce moral-hazard, i.e, excessive trading, and hence, make OEF less efficient than the ETF

- In ETF, given $P_1(Y)$, investors bear $Y$ risk and trade only $\epsilon_i$
- In OEF, given $P^F_1$, investors trade both $Y$ and $\epsilon_i$
• When $Y$ large, can manifest into a “bank run” for OEF

5.2 Liquidity clientele

**Result 3.** *Liquidity clientele: investors with high liquidity needs (short-term investors) prefer OEFs and investors with low liquidity needs (long-term investors) prefer ETFs.*

• All OEF investors share the cost of liquidity insurance

• For investors with high liquidity needs (short-term investors), the liquidity insurance provided by OEFS is more valuable
  \[\implies\text{prefer OEFs}\]

• For investors with low liquidity needs (long-term investors), the insurance benefit is lower than the cost of insurance
  \[\implies\text{prefer ETFs}\]

• (Is Bogle wrong?)

• Potential Concerns:
  
  – In practice: OEFs have short-term trading restrictions
  
  – In theory: Does the equilibrium unravel? Would all but the highest liquidity need investors prefer ETFs?

Third, the tradeoff between the liquidity-insurance benefit and the moral hazard cost differs for investors with different individual liquidity needs. Under the OEF structure, the moral hazard cost of excessive trading is shared evenly among all investors. Higher-liquidity-need investors benefit more from the liquidity insurance and prefer to invest via the OEFs, whereas lower-liquidity-need investors benefit less from the liquidity insurance and prefer to invest via the ETFs. As intuitive as it may seem, this result runs directly counter to John Bogle’s critique that “ETFs can only be described as short-term speculation.” Instead, we find that long-term investors may optimally choose to invest in the ETFs and that there is a viable role for ETFs in the mutual fund industry.

**Result 4.** *OEFs and ETFs can co-exist in equilibrium*
One might be concerned about the viability of the OEF structure given the concentration of higher-liquidity-need investors in it. Our fourth result states that the OEF structure is still viable and that OEFs and ETFs coexist in equilibrium with different liquidity clienteles. The reason behind this rather surprising result is that flow-induced trading costs depend only on the aggregate liquidity need at the fund level. Since individual liquidity needs cancel out across investors, the concentration of investors with high individual liquidity needs does not lead to higher aggregate liquidity need or higher flow-induced trading costs for the OEF. This result highlights a potential pitfall of the recent trend in the mutual fund industry to impose frequent-trading restrictions. Since higher frequency traders do not necessarily impose higher costs to the OEF yet they benefit much more from the liquidity insurance provided by the OEF structure, they are the best clientele for OEFs. If they are deterred from investing in the OEFs by frequent trading restrictions, OEFs may lose a significant client base.\footnote{This result is based on the assumption that higher-liquidity-need investors do not have higher exposure to the systematic liquidity risk. If there is reason to believe that frequent traders are more likely to be reacting to market conditions or are more correlated in their trading decisions, then they may impose higher costs to OEFs and some forms of frequent trading restrictions may be optimal.}

- High-liquidity-need investors enjoy the liquidity insurance, but do not impose higher costs to OEFs
  - High-liquidity-need have more volatile $\epsilon_i$, i.e., high $\sigma_\epsilon$
  - Given high $\sigma_\epsilon$, they prefer safer prices $P^F_1$ (background risk)
  - $\epsilon_i$ cancels at the fund level $\implies$ no additional trading costs
- No need to impose short-term trading restrictions in OEFs
- One caveat – this result does not hold if high-liquidity-need investors have higher aggregate risk exposure
  - For example, “sentiment” or “style” investors, or informed investors
5.3 The tracking error of the OEF

The tracking error of the OEF is defined in Proposition 3. From (13), $\Delta = \Delta_2 Y^2 > 0$, hence the tracking error always reduces the terminal payoff for the OEF. This is consistent with the finding in the literature that fund flows are costly for the OEFs and can reduce fund performance.

Interestingly, from (13), the coefficient $\Delta_2 \equiv (1 + \hat{\eta})\gamma\sigma_v^2/\bar{\theta}$ is strictly positive even when $\hat{\eta}$, the relative population of OEF investors, is zero. In this case, given their tiny population weight, OEF investors do not affect the demand of the ETF investors in (15b) or the equilibrium price $P^E_1$ in (14b). Yet there is still a price discrepancy between $P^E_1$ (which depends on $Y$) and $P^F_1$ (which is independent of $Y$). In particular, if $Y > 0$, the aggregate liquid shock is equivalent to an excess supply, which reduces $P^E_1$ to below $P^F_1$. At the same time, the OEF investors unload all their $Y$ risk by selling $Y$ shares of their stocks. Since they are able to sell their shares to the OEF at a price $(P^F_1)$ higher than the OEF can realize in the stock market later $(P^E_1)$, the OEF is losing money on average and the tracking error is negative. Similarly, if $Y < 0$, OEF investors buy shares from the OEF at a price lower than the OEF can purchase in the stock market later, also leading to a negative tracking error.

When $\hat{\eta} > 0$, OEF investors follow similar strategy and unload all their $Y$ risk. Given their non-trivial population size, they now affect the equilibrium demand of ETF investors and the price $P^E_1$. In particular, each ETF investor purchases $\hat{\eta}$ shares for each share sold by an OEF investor. This extra supply ($\hat{\eta}Y$ on a per capita basis), further depresses the ETF price $P^E_1$ (by an extra $\gamma\sigma_v^2\hat{\eta}Y$). With $P^F_0$ staying constant, the OEF now loses an average of $\gamma\sigma_v^2(1 + \hat{\eta})Y$ per share on all the $Y$ shares sold, leading to the tracking error $\Delta_2 Y^2$. This tracking error term further reduces the demand for OEF shares and leads to additional price impact in the ETF market, contributing to the last terms in the expressions of (14b) and (15). Note that for small $\mu$, $\hat{\eta}$ can be very large as $\eta$ approaches 0. This represents the case when the market is dominated by OEF investors. With very few ETF and stock investors to make markets, the price $P^E_1$ can be extremely sensitive to the aggregate liquidity shock. The OEF needs to pay a significant cost to unload all their $Y$ risk to the small underlying stock market, leading to significant price impacts and tracking errors.

In summary, the tracking error consists of two components. The first component is due to the price discrepancy between the OEF price and the price in the ETF market when we take the ETF price as exogenous. This is the moral hazard cost of providing the liquidity
insurance. The second component of the tracking error is a “feedback effect,” driven by the price impact in the underlying stock market when OEF investors unload their aggregate risk exposure. The price impact then leads to additional tracking errors for the OEF, further decreasing investors’ demand for the OEF. This component of tracking error increases with the size of the OEF. Thus, we have the following result.

Result 5. The tracking error always reduces the OEF performance, and it is not zero even when the size of the OEF approaches zero. Moreover, the magnitude of the tracking error increases as the relative size of the OEF (η) increases.

5.4 The cross-section of the OEF demand

Our framework allows us to assess the effectiveness of the OEF and the ETF structures under different circumstances. Viewing the underlying risky asset in the model as a stock index, we can compare characteristics of different stock indexes and ask whether one index is more suited for the OEF or the ETF investors. In Figure 3, we report the equilibrium size of the ETF relative to the OEF for different indexes and investor characteristics.

Figure 3: The equilibrium size of the ETF industry (η). Panels (a) and (b) report the optimal size of ETF for different values of σ_y and σ_ε, respectively. Other than the variables being changed, the parameters are γ = 4, θ = 1, V = 0, Y = 0, σ_v = 0.3, σ_y = 0.1, σ_ε = 1.0, and µ = 1.

First, we consider the impact of aggregate liquidity shock. A more volatile aggregate liquidity shock (higher σ_y) corresponds to the case in which investors have more correlated liquidity needs. We have the following result:

In the paper we only consider the case of small Y for tractability. In practice, when Y is reasonably large, this effect can manifest into a “bank run” situation, in which the remaining investors are exceedingly concerned about this negative externality and may choose to sell shares even when they personally do not experience liquidity shocks. This situation is considered in Chen, Goldstein, and Jiang (2007).
**Result 6.** *When the aggregate liquidity shock is more volatile, it is more costly to provide liquidity insurance via the OEF structure, and the equilibrium size of the ETF is larger.*

With more volatile aggregate liquidity shock, the OEF is expected to have larger unbalanced demand from its investors. This unbalanced demand is going to translate into large excess demand in the underlying stock market, leading to a large price impact and a large tracking error. As a result, the OEF becomes less attractive relative to the ETF as an indexing vehicle, and the equilibrium ETF size is larger.

Empirically, it is reasonable to assume that investors of a narrower index (such as a bio-tech index) have more correlated liquidity needs compared to investors of a broader market index (such as the S&P 500 index). Thus, we expect to have more ETFs in narrower indexes than in broader market indexes. We proxy for the narrowness of an index using the number of stocks, the market capitalization, and the industry concentration of the index.

Next, we consider the impact of idiosyncratic liquidity shocks. A more volatile idiosyncratic liquidity shock (higher $\sigma_\epsilon$) corresponds to the case in which investors on average have large liquidity needs but these liquidity needs are not correlated. We have the following result:

**Result 7.** *When the individual liquidity shock is more volatile, the liquidity insurance provided by the OEF structure is more beneficial, and the equilibrium size of the ETF is smaller.*

- ETFs are more beneficial for narrower indexes
  - Investors have more correlated demand $\Rightarrow$ high $\sigma_y$
    $\Rightarrow$ liquidity insurance too costly $\Rightarrow$ prefer ETFs
- higher stock volatility $\Rightarrow$ riskier indexes
- ETFs are more beneficial for retirement or institutional accounts
  - Investors have lower idiosyncratic shocks $\Rightarrow$ low $\sigma_\epsilon$
    $\Rightarrow$ liquidity insurance not valuable $\Rightarrow$ prefer ETFs

Figure 3(b) indicates that if all investors in the economy face relatively low idiosyncratic shocks (low $\sigma_\epsilon$), the equilibrium size of ETF can reach 1, or there is no need for the OEF. The
reason is that investors with low idiosyncratic liquidity demand can still generate significant tracking errors as long as they are exposed to the aggregate liquidity shock. On the other hand, the liquidity insurance feature – the key benefit of the OEF structure – is not very important for them. As a result, the ETF structure dominates as an indexing vehicle.

This result points out a potential pitfall of the recent trend in the mutual fund industry to impose trading restrictions to deter high frequency trading in the OEFs. Our findings suggest that this strategy may not be efficient since high frequency traders do not pose high costs to the fund, unless there is reason to believe that their trading is more correlated with the aggregate shocks. On the other hand, those investors benefit much more from the liquidity insurance provided by the OEF structure and hence are the natural demanders of the OEFs. If they are deterred from investing in the OEFs by the trading restrictions, then OEFs can potentially lose a significant client base.

Empirically, we expect investors to use more liquid assets to meet their individual liquidity needs, hence, those indexes are likely to have a client base with larger idiosyncratic liquidity needs. Thus, Result 7 suggests that less liquid underlying indexes should have a client base with smaller idiosyncratic liquidity needs and the equilibrium size of the ETF is larger.

In addition, we can proxy for the idiosyncratic liquidity needs ($\sigma_\varepsilon$) using the characteristics of fund investors. For example, investors of retirement accounts or other institutional accounts generally have lower liquidity needs than the average individual investors. Our model suggests they may be better candidates for ETFs since they do not highly value the liquidity insurance of OEFs.

Other comparative statistics: $\sigma_v$, $\mu$, $\gamma$

In summary, we find that narrower indexes (e.g., indexes with fewer stocks or are more concentrated) and less liquid indexes are better suited for ETFs. Hence, we predict that these are the indexes in which more ETFs are introduced and, once introduced, ETFs enjoy a higher growth rate. In contrast, the corresponding OEFs should experience a bigger loss of market share to ETFs once these ETF are introduced.

6 Conclusions

Fifteen years ago, investors seeking to invest their money by tracking an index could only do so by buying an OEF. In the past decade this paradigm has changed with the introduction of ETFs that are similar to OEFs but are traded on the stock exchange. The huge success
of ETFs, seemingly at the expense of OEFs, raises a very interesting question regarding the advantage and disadvantage of the organizational form of an OEF from the investor’s point of view.

To address this question, we develop a theoretical model of equilibrium choice between these two vehicles. We show that OEFs provide cross-subsidization among investors by sharing the transaction costs for those investors experiencing large liquidity shocks, at the cost of lower average returns for the remaining investors. While it is a zero-sum game, the OEF structure provides partial insurance against future liquidity needs and is ex-ante beneficial for risk averse investors. However, the insurance feature embedded in the OEF structure can cause moral hazard issues in the form of excessive trading and increase the cost of the insurance. We also find that investors with higher liquidity needs benefit more from the liquidity insurance and hence prefer to invest via the OEF. Interestingly, the concentration of high-liquidity-need investors in the OEF does not lead to higher flow-induced trading costs since individual liquidity needs cancel out at the fund level. As a result, OEFs are not dominated by ETFs in equilibrium. Moreover, we predict that the ETF is a more suitable investment vehicle when investors have more correlated liquidity shocks or when the underlying indexes are narrower or less liquid.

This paper is the first seeking to understand the growth of the ETF industry. Our main finding is that although this industry is growing and will probably continue to grow in the future, it is not likely to totally replace the standard OEFs. ETFs are a new and exciting addition to the different investment options available both to individual and institutional investors. However, our model demonstrates that one cannot view all ETFs as equal. Though some ETFs are very similar to OEFs and offer investors a fairly identical investment vehicle (except for the liquidity cross-subsidization), other ETFs offer new options to track narrower and less liquid indexes that are not cost effective in the OEF structure.
References


A Appendix

Proof of Proposition 1

Plugging $P_2^E = V$ into the definition of terminal wealth $W_{i2}^E$ for an ETF investor in (8a) and integrating over the distribution of $V$ at given $Y_i$ and $\theta_{i1}^E$, we have

$$
E[W_{i2}^E \mid Y_i] = \bar{\theta} P_1^E + \theta_{i1}^E (\bar{V} - P_1^E) \tag{A1a}
$$

$$
\text{Var}[W_{i2}^E \mid Y_i] = (\theta_{i1}^E + Y_i)^2 \sigma^2_v \tag{A1b}
$$

Thus, the expected utility $J_1^E$ at time $1_+$, defined as the utility (5) conditional on $Y_i$ is:

$$
J_1^E \equiv \max_{\theta_{i1}^E} \left[ \bar{\theta} P_1^E + \theta_{i1}^E (\bar{V} - P_1^E) - \frac{1}{2} \gamma \sigma^2_v (Y_i + \theta_{i1}^E)^2 \right] \tag{A2}
$$

The optimal holding is calculated by solving the first-order condition with respect to $\theta_{i1}^E$,

$$
\theta_{i1}^E = \frac{\bar{V} - P_1^E}{\gamma \sigma^2_v} - Y_i. \tag{A3}
$$

The holdings for stock investors are identical to those for ETF investors. Plugging $Y_i = Y + \epsilon_i$ and (A3) into the market clearing condition, from (4), we have

$$
\int \theta_{i1}^E + \int \theta_{i1}^S = (\eta + \mu) \left( \frac{\bar{V} - P_1^E}{\gamma \sigma^2_v} - Y \right) = (\eta + \mu) \bar{\theta} + X^F. \tag{A4}
$$

Solving (A4) yields the equilibrium price $P_1^F$ in (9). The optimal holding in the proposition is obtained by substituting the equilibrium price $P_1^E$ back into (A3).

Proof of Proposition 2

The proof of (12a) is almost identical to that of Proposition 1, except that $E[P_2^F] = \bar{V} - \Delta$. We then plug the definition of $\Delta$ into individual holding (12a) and aggregate over all OEF investors:

$$
\int \theta_{i1}^F = \int \left( \frac{\bar{V} - \Delta - P_1^F}{\gamma \sigma^2_v} - Y - \epsilon_i \right) = (1 - \eta) \left( \frac{\bar{V} - \Delta_0 - \Delta_1 Y - \Delta_2 Y^2 - P_1^F}{\gamma \sigma^2_v} - Y \right).
$$

Imposing the restriction that $\int \theta_{i1}^F(Y = 0) = (1 - \eta) \bar{\theta}$ yields the expression for $P_1^F$ in the proposition. Substituting this $P_1^F$ back into (12a) to derive the expression in (12b).
Proof of Proposition 3

To calculate the tracking error, we plug the expressions of $P^E_1$, $P^F_1$ and $\theta^F_{i1}$ from Propositions 1 and 2 into the definitions of $X^F$ and $\Delta$ in (6) and (7). Since $\int_i \epsilon_i = 0$, we have

$$\Delta = \frac{((1-\eta)\bar{\theta} - \int_i \theta^F_{i1})(P^F_1 - P^E_1)}{\int_i \theta^F_{i1}}$$

(A5a)

$$= \frac{Y(\gamma\sigma^2_v + \Delta_1 + \Delta_2 Y)(-\Delta_0 + ((1 + \hat{Y})\gamma\sigma^2_v + \hat{Y}(\Delta_1 + \Delta_2 Y))Y)}{\gamma\sigma^2_v(\theta - Y) - (\Delta_1 + \Delta_2 Y)Y}$$

(A5b)

When $Y$ is small, we can use Taylor expansion to expand around $Y$ and drop higher order terms (i.e., $O(Y^3)$). Equalizing the coefficients of zero, first, and second order terms in (A5b) to those in the definition of $\Delta \equiv \Delta_0 + \Delta_1 Y + \Delta_2 Y^2$, we get three equations and three unknowns ($\Delta_0, \Delta_1, \text{and } \Delta_2$). The solution is $\Delta_0 = \Delta_1 = 0$ and $\Delta_2$ is given in (13).

Parts (ii) and (iii) are straightforward. Substituting the expressions of $\Delta_0, \Delta_1, \text{and } \Delta_2$ into (11) and (12) yields $P^F_1$ and $\theta^F_{i1}$. Using the definitions of $\Delta, X^F$ in (6) and Proposition 1, we derive the equilibrium expressions of $P^E_1$ and $\theta^E_{i1}$.

Proof of Lemma 1

First, we calculate the time 0 expectation and variance of $W^F_{i2}$ conditional on the realization of $Y$.

$$E[W^F_{i2} | Y] = E[E[W^F_{i2} | \epsilon_i, Y]] = \bar{\theta}V - Y(\gamma\sigma^2_v + 2\Delta_2 Y)\bar{\theta} + O(Y^3)$$

$$\text{Var}[W^F_{i2} | Y] = E[\text{Var}[W^F_{i2} | \epsilon_i, Y]] + \text{Var}[E[W^F_{i2} | \epsilon_i, Y]]$$

$$= (1 + s_i^2 k_\epsilon)\bar{\theta}(\sigma^2_v \bar{\theta} - 2\Delta_2 Y^2/\gamma) + O(Y^3)$$

We then calculate the unconditional expectation and variance:

$$E[W^F_{i2}] = E[E[W^F_{i2} | Y]] = \bar{\theta}V - 2\bar{\theta}\Delta_2 \sigma^2_y + O(\sigma^4_y)$$

$$\text{Var}[W^F_{i2}] = E[\text{Var}[W^F_{i2} | Y]] + \text{Var}[E[W^F_{i2} | Y]]$$

$$= (1 + s_i^2 k_\epsilon + k_y)k_\theta/\gamma^2 - 2(1 + s_i^2 k_\epsilon)\bar{\theta}\Delta_2 \sigma^2_y/\gamma + O(\sigma^4_y)$$

Then, $J^F_{i0} = E[W^F_{i2}] - (\gamma/2)\text{Var}[W^F_{i2}]$ yields (16a).

Similarly, we calculate the time 0 expectation and variance of $W^E_{i2}$ conditional on the
realization of $Y$.

\[
E[W_{i2}^E | Y] = E[E[W_{i2}^E | \epsilon_i, Y]] = \bar{\theta}V + \gamma \sigma_y^2(\bar{\theta}x_1Y + (x_1 + x_1^2 + \bar{\theta}x_2)Y^2) + O(Y^3)
\]

\[
\text{Var}[W_{i2}^E | Y] = E[\text{Var}[W_{i2}^E | \epsilon_i, Y]] + \text{Var}[E[W_{i2}^E | \epsilon_i, Y]]
\]

\[
= (1 + s_i^2k_\epsilon^2)\sigma_y^2(\bar{\theta}^2 + 2\bar{\theta} (1 + x_1)Y + ((1 + x_1)^2 + 2\bar{\theta}x_2)Y^2) + O(Y^3)
\]

We then calculate the unconditional expectation and variance:

\[
E[W_{i2}^E] = E[E[W_{i2}^E | Y]] = \bar{\theta}V + k_y(x_1^2 + x_2\bar{\theta})/\gamma + O(\sigma_y^4)
\]

\[
\text{Var}[W_{i2}^E] = E[\text{Var}[W_{i2}^E | Y]] + \text{Var}[E[W_{i2}^E | Y]]
\]

\[
= (x_1^2k_y + (1 + s_i^2k_\epsilon)(k_\theta + k_y((1 + x_1)^2 + 2x_2\bar{\theta}))) / \gamma^2 + O(\sigma_y^4)
\]

Then, $J_{i0}^E = E[W_{i2}^E] - (\gamma/2)\text{Var}[W_{i2}^E]$ yields (16b).

**Proof of Proposition 4**

By substituting the definitions of $\Delta_2$, $x_1$, and $x_2$ from Proposition 3 into the value functions $J_{i0}^F$ and $J_{i0}^E$ in Lemma 1, we derive

\[
J_{i0}^F = \bar{\theta}V - \frac{k_\theta}{2}\gamma \left(1 + s_i^2k_\epsilon + k_y\right) - \frac{1 + \hat{\eta}}{\gamma} (1 - s_i^2k_\epsilon k_y) + O(\sigma_y^4)
\]

\[
J_{i0}^E = \bar{\theta}V - \frac{k_\theta}{2}\gamma \left(1 + s_i^2k_\epsilon + k_y\right) - \frac{(1 + \hat{\eta})(1 + 3\hat{\eta})}{2\gamma} s_i^2k_\epsilon k_y - \frac{1 - \hat{\eta}^2}{2}\gamma (1 - k_\theta) k_y + O(\sigma_y^4).
\]

Taking the difference between $J_{i0}^F$ and $J_{i0}^E$ yields (17). Clearly, an individual investor should invest in OEF if and only if $G_i^F \geq 0$.

To prove part (ii), we take derivative of $G_i^F$:

\[
\frac{\partial G_i^F}{\partial s_i} = \frac{(1 + \hat{\eta})^2}{\gamma} 3s_i k_\epsilon k_y > 0.
\]

**Proof of Proposition 5 and Corollary 1**

We have proved the cases of $G_i^F(0, 0) > 0$ and $G_i^F(1, 1) < 0$ of Proposition 5 in the text. Using (17), we can verify that the first two conditions in (19) correspond exactly to those two conditions.

We now consider the case of $G_i^F(0, 0) \leq 0 \leq G_i^F(1, 1)$. Whenever $k_\epsilon \geq (1 + k_\theta)/3$, we can verify that the expression of $\eta$ in the third case of (19) is bounded between (0, 1) and solves $G_i^F(\eta, \eta) = 0$. 


For any $s_i > \eta$, we have $G^F_i(s_i, \eta) \geq G^F_i(\eta, \eta) = 0$, hence $s_i$ invests via the OEF. For any $s_i < \eta$, we have $G^F_i(s_i, \eta) \leq G^F_i(\eta, \eta) = 0$, hence $s_i$ invests via the ETF. Given that $s_i \in Unif[0, 1]$, the population of investors with $s_i < \eta$ is exactly $\eta$. Hence, the population of ETF investors is $\eta$. 

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Table 1: Summary Statistics - Time Trend

This table provides descriptive statistics of the full sample of OEFs and ETFs for the years 1992 to 2006. The data is based in data collected from: CRSP’s Survivor-Bias Free U.S. Mutual Funds Database, EDGAR’s SEC Filings and Forms Database, Bloomberg, textitThomson Financials’s CDA/Spectrum Mutual Funds Holding, and CRSP. OEFs are Open-Ended Index Mutual Funds. All OEF share classes are collapsed to one entity. ETFs are Exchange Traded Funds tracking indexes and trading on the secondary market. All sizes are in millions of dollars. The percentage growth numbers are winsorized at the 2% level.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable</td>
<td>OEF</td>
<td>ETF</td>
<td>All</td>
<td>OEF</td>
</tr>
<tr>
<td>Number of Funds</td>
<td>62</td>
<td>81</td>
<td>19</td>
<td>100</td>
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<tr>
<td>Number of Families</td>
<td>30</td>
<td>40</td>
<td>2</td>
<td>42</td>
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<tr>
<td>Number of Indexes</td>
<td>15</td>
<td>21</td>
<td>19</td>
<td>38</td>
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<tr>
<td>Funds per Family</td>
<td>Mean</td>
<td>1.90</td>
<td>2.00</td>
<td>9.50</td>
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<tr>
<td></td>
<td>Median</td>
<td>1</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>Std</td>
<td>1.81</td>
<td>1.89</td>
<td>10.61</td>
</tr>
<tr>
<td>Number of Indexes per Family</td>
<td>Mean</td>
<td>1.76</td>
<td>1.76</td>
<td>9.50</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>1</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>Std</td>
<td>1.55</td>
<td>1.58</td>
<td>10.61</td>
</tr>
<tr>
<td>Fund Size ($M)</td>
<td>Mean</td>
<td>281.84</td>
<td>1,037.22</td>
<td>49.91</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>88.42</td>
<td>259.64</td>
<td>6.00</td>
</tr>
<tr>
<td></td>
<td>Std</td>
<td>860.69</td>
<td>3,608.74</td>
<td>159.02</td>
</tr>
<tr>
<td>Family Size ($M)</td>
<td>Mean</td>
<td>575.63</td>
<td>2,100.36</td>
<td>474.14</td>
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<tr>
<td></td>
<td>Median</td>
<td>179.36</td>
<td>317.90</td>
<td>474.14</td>
</tr>
<tr>
<td></td>
<td>Std</td>
<td>1,780.78</td>
<td>8,731.34</td>
<td>464.77</td>
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<tr>
<td>Percentage</td>
<td>Mean</td>
<td>57.00</td>
<td>60.58</td>
<td>357.10</td>
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<tr>
<td>Fund Growth</td>
<td>Median</td>
<td>39.17</td>
<td>56.44</td>
<td>277.31</td>
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<td></td>
<td>Std</td>
<td>92.69</td>
<td>86.06</td>
<td>39.19</td>
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<tr>
<td>Percentage</td>
<td>Mean</td>
<td>68.06</td>
<td>68.60</td>
<td>39.19</td>
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<td>Family Growth</td>
<td>Median</td>
<td>51.00</td>
<td>60.07</td>
<td>39.19</td>
</tr>
<tr>
<td></td>
<td>Std</td>
<td>96.80</td>
<td>71.52</td>
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</tr>
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</table>
This table provides descriptive statistics of the indexes tracked by the ETFs and OEFs in the sample. The numbers are averages across all indexes over the entire time of the sample, 1992-2006. The indexes are sampled every quarter and the changes are recorded from one quarter to the other. The liquidity measure is based on Amihud (2002). Amihud’s Liquidity Measure (11) is the absolute value of the return of a stock divided by the absolute value of its price multiplied by its volume. We calculate the liquidity measure of each stock in the index using data downloaded from Joel Hasbrouck’s website and described in Hasbrouck (2006) and average it across the index either Equally-Weighted (EW) or Value-Weighted (VW) by the market cap of the stock. We use two industry concentration measures. First, we use the Herfindahl-Hirschman Index using either the Fama-French 10 or 48 industries classifications. We assign to each stock in the index an industry based on the firm’s SIC code. Second, we use the industry concentration following Kacperczyk, Sialm, and Zheng (2007).

<table>
<thead>
<tr>
<th>Variable</th>
<th>OEF</th>
<th>ETF</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Index Size:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Number of Stocks in Index</td>
<td>593.66</td>
<td>325.73</td>
<td>487.41</td>
</tr>
<tr>
<td>- Market Cap of Index (in $B)</td>
<td>5656.22</td>
<td>2337.15</td>
<td>4443.42</td>
</tr>
<tr>
<td><strong>Index Turnover (over a quarter):</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Number of Stocks Changing in the Index</td>
<td>13.60</td>
<td>7.47</td>
<td>11.22</td>
</tr>
<tr>
<td>- Percentage of the Stocks in Index Changing</td>
<td>2.49%</td>
<td>3.19%</td>
<td>2.74%</td>
</tr>
<tr>
<td>- Market Cap Changing in the Index (in $B)</td>
<td>378.40</td>
<td>159.23</td>
<td>299.02</td>
</tr>
<tr>
<td>- Percentage of Market Cap in Index Changing</td>
<td>10.22%</td>
<td>18.84%</td>
<td>13.34%</td>
</tr>
<tr>
<td><strong>Liquidity of Underlying Stocks:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Amihud Liquidity Measure (EW)</td>
<td>0.031</td>
<td>0.074</td>
<td>0.060</td>
</tr>
<tr>
<td>- Amihud Liquidity Measure (VW)</td>
<td>0.003</td>
<td>0.022</td>
<td>0.016</td>
</tr>
<tr>
<td><strong>Industry Concentration:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Herfindahl Index (10 industries)</td>
<td>0.23</td>
<td>0.47</td>
<td>0.32</td>
</tr>
<tr>
<td>- Herfindahl Index (48 industries)</td>
<td>0.13</td>
<td>0.32</td>
<td>0.20</td>
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<tr>
<td>- KCZ Industry Concentration</td>
<td>0.09</td>
<td>0.31</td>
<td>0.17</td>
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</table>