Macroeconomic Drivers of Crude Oil Futures Risk Premia

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Abstract

We derive an affine term structure model to assess how macroeconomic risks drive risk premia on short and long-term crude oil futures. New to the literature, we construct the term structure of convenience yields to obtain insight into the theory of storage. While the short-term convenience yield is related to contemporaneous crude oil scarcity, the slope of the curve reflects anticipated changes in the availability of future physical oil. Macroeconomic risks are unspanned by both the risk free and convenience yield term structures. Both the unspanned macroeconomic risks and the slope of the convenience yield curve are important drivers of the oil futures risk premia.

Keywords: Convenience yields; futures trading strategies; prices of risk; term structure model; unspanned risks.

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1 Introduction

Elevated and volatile crude oil prices have become chronic features of the international economy and continue to preoccupy policymakers, financial analysts, and the broader public. The run-up in the price of oil between 2003 and 2008 and its persistently high level since 2009 have occurred against a backdrop of the “financialization” of global commodity markets. On the sell side, financial institutions have become more actively involved in commodity derivatives markets, including trading longer maturity futures contracts (e.g., Büyükşahin et al. 2008 and Spector 2013). On the buy side, increased investor interest has resulted in large quantities of financial capital flowing into these markets during the past decade (e.g., Büyükşahin and Harris 2011 and Alquist and Gervais 2013). As a result, a number of papers have claimed that oil prices have become disconnected from macroeconomic fundamentals (e.g., Juvenal and Petrella 2011 and Tang and Xiong 2012).¹

In this paper, we use a dynamic affine term structure model to reveal the macroeconomic determinants of risk premia in the crude oil futures market. The key methodological innovation of this paper is to construct and analyze the term structure of convenience yields, which is only possible because of the substantial increase in the trading of longer-maturity oil futures. We are thus able to use prices from liquid futures contracts and the standard no-arbitrage relationship to construct a term structure of convenience yields out to one year’s maturity. We argue that the convenience yield can be interpreted as the yield on a (synthetic) zero-coupon “oil bond,” which is priced in a fraction of a barrel of oil today and pays off a barrel of oil at maturity. Our model contains both the dollar bond market (i.e., the U.S. term structure) and the oil bond market (i.e., the convenience yield term structure), and thus allows us to highlight the different macroeconomic risks in the two markets.

An examination of the term structure of convenience yields reveals new insight into the theory of storage and its relationship to risk premia in oil futures. We start by showing that the cross section of convenience yields can be explained using the familiar level, slope, and curvature principal components. The level component is related to the short-term convenience yield which has long played a central role in the analysis of commodity futures markets (see, among others, Kaldor 1939; Working 1949; Brennan 1958; Telser 1958; Fama and French 1987 and 1988; and Pindyck 1994). In the oil market, the short-term convenience yield captures the marginal net benefit of holding physical oil inventories today. We verify that the level component plays this role in our data. We also show, however, that there is an important slope component in the term structure of convenience yields. The slope component is related to expected changes in the availability of future physical oil. It is thus important to model the entire convenience yield curve to understand the market’s expectation of relative oil scarcity.² This distinguishes our work

¹See Bassam, Kilian, and Mahadeva (2012) for a review of this literature.
²The relationship between futures prices and anticipated changes in inventories was initially discussed in Weymar (1966). However, this paper has largely been ignored in the subsequent literature which has
from other analyses which have focused either on the basis (i.e., the difference between the risk free and convenience yield curves) or solely on the short-term convenience yield.

Next we analyze the factor structure of the cross section of expected returns in the oil bond market. Similar to the results in the dollar bond literature (e.g., Cochrane and Piazzesi 2005), level risk is the only priced factor in the cross section of oil bond expected returns. Given our earlier results, we interpret this factor as compensation for inventory risk. Variation in the price of inventory risk is driven in part by the convenience yield slope component.

Using these results, the risk premium on an oil futures contract can be decomposed into three parts: a (level) risk premium in the oil bond market, a (level) risk premium in the dollar bond market, and a risk premium related to taking a position in spot oil. We relate each of the distinct risk premia to macroeconomic fundamentals (e.g., inflation and real growth). An important finding is that the macroeconomic risks are unspanned: i.e., while the macroeconomic state variables are required in the physical representation of the state vector (because they forecast future convenience yields), they do not help explain the current cross-section of convenience yields or bond yields beyond the effects already captured by the principal components. It is well known that unspanned macroeconomic risks are important for explaining the time-series variation in the expected returns on U.S. Treasury bonds. To the best of our knowledge, we are the first to show that unspanned risks are important for explaining expected excess returns in the crude oil futures market as well.

We impose all of these empirical findings onto a Gaussian dynamic term structure model (GDTSM) of the futures market that contains the convenience yield curve, the risk-free curve and the spot price of oil. The model contains a number of innovations relative to existing models. First, we show how to use the traditional no-arbitrage relationship to construct the stochastic discount factor for oil bonds. We can therefore price the oil and dollar bonds in a theoretically consistent way. Second, we capture the cross sectional variation of both the dollar and oil bond term structures by using principal components as state variables. As a result, the model has very small pricing errors. Third, we restrict the prices of risk by imposing that only level risks are priced in each of the oil and dollar bond markets. Fourth, we impose the unspanning restrictions on the macroeconomic variables. Finally, the model is estimated using the two-step procedure of Diez de los Rios (2013a) which carefully accounts for the well known small-sample biases present in the estimation of term structure models (e.g., Bauer, Rudebusch and Wu 2012).

We use the model to examine the drivers of expected returns on crude oil futures.
In particular, we analyze the expected returns from a holding strategy that goes long in the oil futures contract, holds it until maturity, and then sells at the spot price for investment horizons up to one year (e.g., Szymanowska et al. 2013). The expected returns to this strategy can be decomposed into three components. The risk premium on a spot position is captured by the expected return to the short roll strategy, which goes long in a sequence of 1-month futures contracts (e.g., Mou 2013). The oil bond term premium is the expected return to an oil bond spread strategy that holds a long term oil bond until maturity, financed by selling a sequence of one-month oil bonds. The dollar bond term premium is defined in a similar manner.

Our results show that the unspanned macroeconomic risks and the slope of the convenience yield curve are importance drivers of the oil futures risk premia. To make this point we construct model-based variance decompositions of the expected returns. As the maturity of the contract increases, expected holding returns are increasingly driven by news about unspanned macroeconomic risks and the slope of the convenience yield term structure. For example, news about the slope component accounts for approximately 45 per cent of the variation of the holding return on a one-year futures contract. Our approach thus ties the market’s expectation of the future availability of physical oil to futures risk premia. News about the (unspanned) macroeconomic risks accounts for a further 15 per cent of the variation in one-year holding returns.

Unspanned macroeconomic risks also play a large role in the returns to the short roll strategy, accounting for 45 per cent of the variation in rolling a spot position for one year. News about the slope component and macroeconomic risks together account for approximately 70 per cent of the variation of the one-year oil bond term premium. Importantly, the variance decompositions of the oil bond term premium are very different from those of the dollar bond term premium highlighting the advantage of splitting the basis into its two components.


Another set of papers has examined the macroeconomic sources of futures return variation. Bailey and Chan (1993) show that the futures-spot spreads of several commodities are related to macroeconomic risk factors, which they interpret as evidence of macroeco-

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4Some papers that predate the recent period of financialization include Breeden (1980); Carter, Rausser, and Schmitz (1983); Chang (1985); Fama and French (1987); Bessembinder (1992); Schwartz (1997); Miltersen and Schwartz (1998); and de Roon, Nijman, and Veld (2000). Rouwenhorst and Tang (2012) review the literature.
nomic variables driving a common commodity risk premium. Erb and Harvey (2006), Gorton and Rouwenhorst (2006), and Szymanowska et al. (2013) relate inflation to a cross-section of commodity futures returns. Gargano and Timmermann (2012) show that money supply and industrial production can predict commodity returns. Gospodinov and Ng (2013) construct factors from a broad cross section of commodity convenience yields that forecast inflation. Hong and Yogo (2012) show that future’s open interest is correlated with macroeconomic activity. Yang (2013) shows that a factor that can forecast futures returns is related to aggregate investment shocks.

Another recent strand of the literature examines the relationship between inventories and returns. Dincerler, Khokher and Simin (2005) show that convenience yields are related to detrended contemporaneous inventories in crude oil, copper and natural gas markets. They also show that inventory withdrawals predict futures returns. Alquist and Kilian (2010) use a simple model to show that the basis can be interpreted as reflecting uncertainty about future net oil supplies. Gorton, Hayashi and Rouwenhorst (2012) use a model to relate the theory of storage to time-varying risk premia. They show that the expected return on a broad cross section of commodities are systematically related to the level of physical inventories of the commodities. David (2013) shows how the roll return on crude oil futures can be forecast by inventories and exploration and development activity of energy firms.

The papers that are closest to ours use affine term structure models of the futures curve. Casassus and Collin-Dufresne (2005) develop a three-factor model of commodity spot prices, convenience yields, and interest rates that allows for time-varying risk premia. Hamilton and Wu (2013b) build an affine model of the futures curve, while Le and Zhu (2013) examine gold lease rates. We build on the insight of these papers by constructing the term structure of convenience yields and showing how it is related to current and future levels of crude oil inventory. The model incorporates time variation in the prices of risk arising from movements in both convenience yields and macroeconomic variables. A key contribution of the paper is to show that an economically significant component of these macroeconomic risks are orthogonal to the current term structures of convenience and dollar bond yields and therefore unlikely to show up in portfolio sorts of the basis.

The remainder of the paper is organized as follows. Section 2 introduces the idea of the convenience yield and presents the trading strategies we use to study the determinants of oil futures risk premia. Section 3 then provides a preliminary analysis of the data and the relationship between the macroeconomic variables and the oil futures and dollar bond risk premia. The asset pricing model is presented in section 4, while the estimation procedure is discussed in section 5. Section 6 reports the main empirical results. Section 7 concludes.
2 The term structure of convenience yields and trading strategies

In order to provide some background for the rest of the paper, this section discusses (i) the rationale for postulating the existence of a convenience yield in oil futures prices and (ii) the trading strategies we use to analyze the drivers of oil futures risk premia.

2.1 The dollar bond, the oil bond and the spot price of oil

We begin by discussing the representation of the U.S. Treasury bond market, which is standard. The “dollar” bond market consists of a set of \( n \)-month, zero-coupon bonds denominated in U.S. dollars with prices \( P_t^{(n)} \) for maturities \( n = 1, \ldots, N \). The bonds are default-risk free by assumption. The log yield of the dollar bond is:

\[
y_t^{(n)} = -\frac{1}{n} \log P_t^{(n)}.
\]

The 1-month (log) excess return on a bond with maturity \( n \), \( r_{P_{t+1}}^{(n)} \), is the capital gain associated with buying an \( n \)-month bond today and selling it one month later, financing the position at the short-term interest rate, \( y_t^{(1)} \):

\[
r_{P_{t+1}}^{(n)} \equiv \log \left[ \frac{P_t^{(n+1)}}{P_t^{(n)}} \right] - y_t^{(1)} = ny_t^{(n)} - (n - 1)y_t^{(n-1)} - y_t^{(1)},
\]

for \( n = 2, \ldots, N \).

The oil market consists of spot oil with price \( S_t \) and a set of futures contracts with prices \( F_t^{(n)} \) that mature at time \( t + n \). The 1-month excess return on a spot purchase of a barrel of oil is:

\[
r_{S_{t+1}} \equiv \log \left( \frac{S_{t+1}}{S_t} \right) + \delta_t^{(1)} - y_t^{(1)} = \Delta S_{t+1} + \delta_t^{(1)} - y_t^{(1)},
\]

where \( s_t = \log S_t \). The return to holding a physical barrel of oil equals the capital gain in the price of spot oil, \( \Delta S_{t+1} \), plus the convenience yield (net of storage costs), \( \delta_t^{(1)} \), associated with having access to physical oil for one month. In this way, the convenience yield is analogous to the dividend paid by a stock (also see Pindyck 1993).\(^5\)

Postulating the existence of a convenience yield in the crude oil market is quite natural given the way that the oil market operates. Holding stocks of oil is intrinsically valuable to oil refineries because of the operational flexibility they provide. Due to technological constraints, a refinery has a strong incentive to hold stocks to optimize its output of petroleum products (National Petroleum Council 2004). The value that the refinery assigns to being able to expand its product mix can be represented as a convenience yield.

\(^5\)This analogy breaks down somewhat because the convenience yield can be negative. During periods when the level of oil inventories is high, the convenience yield can be less than zero as the marginal benefit of storing oil is less than the marginal cost.
(e.g., Considine 1997). In addition, the capital investments required to establish a crude oil refinery are much longer lived than the horizon over which a refinery makes plans about storage and production. Adjusting crude oil inventories rather than the capital stock is a key way for a refinery to change its variable costs. Similarly, convenience yields arise endogenously as a consequence of the interaction of supply, demand and storage decisions (e.g., Routledge, Seppi, and Spatt 2000). In particular, periods of relative scarcity of oil are related to high convenience yields, a point to which we return below.

In the absence of arbitrage opportunities, the cost-of-carry relationship implies a term structure of convenience yields. The price of an oil futures contract that expires in \( n \) months satisfies:

\[
f_t^{(n)} - s_t = n y_t^{(n)} - n \delta_t^{(n)}
\]

where \( f_t^{(n)} = \log F_t^{(n)} \), \( f_t^{(n)} - s_t \) is the basis, and \( \delta_t^{(n)} \) is the \( n \)-month convenience yield (net of storage costs) associated with having access to physical oil for the life of the contract. The term structure of convenience yields measures the implicit benefit of physical storage over different horizons. We postulate that (3) holds continuously because of the presence of investors who simultaneously trade in the oil futures and U.S. Treasury bond markets, and thereby ensure that the two markets are fully integrated with each other. If this condition were violated, some firms would be able to earn riskless profits. Given the liquidity of the WTI futures and dollar bond markets, the absence of arbitrage is a plausible assumption during the period in question.

It is important to recognize that the no-arbitrage relationship (3) holds for oil forwards, not oil futures. However, the empirical literature shows that the differences between the prices of forwards and futures on a variety of commodities are small (Chow, McAleer and Sequeira, 2000). In addition, we use the results of the model constructed below to show that, under the assumption of monthly marking to market, the root-mean-squared price difference between the prices of oil forwards and oil futures is less than one cent (see appendix). We thus treat equation (3) as a maintained hypothesis throughout the paper.

Given equation (3), we can interpret the \( n \)-month convenience yield as the discount rate associated with a synthetic “oil bond”. This bond, which is denominated in barrels of oil and delivers a barrel of oil at maturity \( n \) months from now, has price \( O_t^{(n)} \):

\[
O_t^{(n)} = \exp \left[ -n \delta_t^{(n)} \right] = \frac{F_t^{(n)} P_t^{(n)}}{S_t}.
\]

The oil bond can be replicated by taking long positions in both the futures contract and the dollar bond, and a short position in the spot oil market. The 1-month excess return for holding the oil bond maturing at time \( t + n \) can be expressed in terms of the convenience yield at different maturities:

\[
ro_{t+1}^{(n)} \equiv \log \left[ \frac{O_{t+1}^{(n-1)}}{O_t^{(n)}} \right] = \delta_t^{(1)} = n \delta_t^{(n)} - (n - 1) \delta_{t+1}^{(n-1)} - \delta_t^{(1)},
\]

(5)
Finally, using the cost-of-carry relation in equation (3), we have that the one-month (excess) return to investing in an \( n \)-month futures contract is the sum of the returns on the spot oil contract and on the oil bond net of the return on the dollar bond:

\[
rf^{(n)}_{t+1} \equiv \log \left( \frac{F^{(n-1)}_{t+1}}{F_t^{(n)}} \right) = r_s t+1 + r_o^{(n)} t+1 - r_p^{(n)} t+1.
\] (6)

2.2 Futures trading strategies

As the oil futures market has become increasingly financialized, market participants have implemented a number of trading strategies using longer term futures contracts (Spector 2013). This development permits us to examine the behavior of oil futures risk premia using the excess returns to investing in oil futures contracts over multiple periods. There are two advantages to examining longer-term strategies. First, there is a great deal of information in the term structure of convenience yields, as we discuss in more detail below. In particular, we use the predictions from the theory of storage to interpret the \( n \)-month convenience yields as the risk-adjusted expectation of future relative scarcity. The effects of movements in longer-term convenience yields will show up in the returns on longer-term futures contracts. Second, the alternative trading strategies may exhibit different sensitivities to the macroeconomic risk factors. Thus, investors can use the futures trading strategies to select their desired exposures to the inflation, growth and oil shock risks.

We start with a simple holding period return, \( rf^{(n)}_{Hold,t\rightarrow t+n} \) that results from buying an \( n \)-month oil futures contract at time \( t \) and holding it until maturity at time \( t+n \):

\[
rf^{(n)}_{Hold,t\rightarrow t+n} = s_{t+n} - f_t^{(n)} = \sum_{j=0}^{n-1} rf^{(n-j)}_{t+j+1} = \sum_{j=0}^{n-1} r_s t+j+1 + \sum_{j=0}^{n-1} r_o^{(n-j)} t+j+1 - \sum_{j=0}^{n-1} r_p^{(n-j)} t+j+1.
\] (7)

where the last equality follows from equation (6). Consequently, holding an oil futures contract until maturity gives the investor exposure to three components of risk over an \( n \)-month horizon: returns on the spot position in oil, on the oil bond and on the dollar bond.

Investors can undertake a short roll strategy by purchasing a sequence of 1-month futures contracts for \( n \) months (Szymanowska et al. 2013):

\[
rf^{(n)}_{ShortRoll,t\rightarrow t+n} = \sum_{j=0}^{n-1} rf^{(1)}_{t+j+1} = \sum_{j=0}^{n-1} r_s t+j+1.
\] (8)

where the last equality follows from the fact that the excess return to a 1-month oil or dollar bond is zero. We note that this strategy is the first component of the holding period return (7). The excess return to this strategy is the sum of the \( n \) 1-month excess returns to spot oil – that is, the sum of the expected 1-month change in the price of oil plus the 1-month convenience yield less the 1-month interest rate. Given the persistence in the
level of the nominal oil price, the expected return of the short-roll strategy is dominated by variation in the 1-month convenience yield at short horizons. Over longer investment horizons, the analysis below reveals a larger role for the macroeconomic drivers of expected returns.

The second and third components in equation (7) are related to spread strategies in the oil and dollar bond markets. Specifically, we define the oil bond spread strategy as:

$$\rho(n)_{\text{Spread}, t-t+n} = \sum_{j=0}^{n-1} \rho(n-j)_{t+j+1} = \log \left( \frac{1}{O_t^{(n)}} \right) - \sum_{j=0}^{n-1} \delta^{(1)}_{t+j}$$

where the last equality follows from equation (5) which implies that \( \rho(n)_{\text{Spread}, t-t+n} \) can be interpreted as the return to buying an \( n \)-month oil bond financed by rolling a sequence of 1-month oil bonds. Similarly, we can define the dollar bond spread strategy as:

$$\rho(p)_{\text{Spread}, t-t+n} = \sum_{j=0}^{n-1} \rho(p-j)_{t+j+1} = \log \left( \frac{1}{P_t^{(n)}} \right) - \sum_{j=0}^{n-1} \gamma^{(1)}_{t+j}$$

where the returns represents the gains or losses associated with holding a long-term dollar bond until maturity, financed by selling a sequence of 1-month dollar bonds. The expected returns to these two strategies yield the oil and dollar bond term premia (e.g., Cochrane and Piazzesi, 2008). Examining the macroeconomic sources of expected returns associated with these two trading strategies therefore shows what macroeconomic variables are important in explaining the term premia in the oil and dollar bond markets.

We note that all of these are zero investment strategies so we can interpret their expected returns as risk premia. Below we analyze the main drivers of the risk premia through the lens of a dynamic term structure model.

3 Preliminary Analysis

3.1 Data

The cost-of-carry equation (3) relies on the premise that the spot and futures markets are linked together in a way consistent with the absence of arbitrage opportunities. For this reason, we limit the sample to the period between April 1989 and March 2012 and focus on the monthly prices of West Texas Intermediate (WTI) futures contracts traded on the NYMEX and CME exchanges. These contracts are the most liquid in the world and are fully physically deliverable, making them a natural choice for examining the dynamics of the convenience yield. During the sample period, liquid futures markets existed for maturities up to 12 months. To compute the spot price, we select the futures contract closest to delivery and use the observation on the last trading day before delivery. We follow this procedure to get the price of oil that is closest to the spot price. Further details are provided in the appendix.
Table 1a shows the summary statistics for the spot and futures prices. Over our sample period, the oil futures curve has been flat with an average difference of only $0.28 between the spot and 1-year futures prices. Longer dated futures are approximately as volatile as shorter dated ones. The time series of the monthly price data are depicted in Figure 1a. The figure shows the spot, and 3-, 6- and 12-month futures contracts over the sample period. There is a wide variation in the nominal spot price of oil, ranging between less than $20 per barrel to more than $130 per barrel. From the figure, the tight relationship between the prices of the futures contracts and the spot price is evident.

To obtain zero-coupon U.S. Treasury bond data, we follow Adrian, Crump and Moench (2013) in using the parameters of the Nelson-Siegel-Svensson curve, estimated in Gürkaynak, Sack, and Wright (2007), to construct bond yields. We sample the yields on the same day as the oil futures prices for bonds ranging in maturity from one month to 15 years. Table 1a presents the summary statistics. The U.S. Treasury yield curve was, on average, upward sloping during the sample period, with short-term rates exhibiting greater volatility than long-term rates.

Given that the futures, spot and dollar interest rates are observable, we use the no-arbitrage relationship (3) to construct the term structure of convenience yields with maturities ranging from 1 to 12 months. The summary statistics are also shown in Table 1a. On average, the term structure of convenience yields is upward sloping with a difference between one-month and one-year yields of 105 basis points. While convenience yields are much more volatile than dollar bond yields, they are less persistent. This point is also clear from Figure 1b, which depicts the 1-, 3-, 6- and 12-month convenience yields, measured in per cent per annum. The twelve-month convenience yield is less volatile, but more persistent, than are shorter-term convenience yields. This difference in the time-series behaviour between the short-term and long-term convenience yields suggests the presence of a potentially important slope factor in the term structure of convenience yields. We also note that there are periods during which the convenience yield is low or even negative. As we show below, these are precisely the periods when physical oil is readily available and a stockout is unlikely. During such periods, the marginal benefit of storing oil is low relative to the marginal cost of storage.

To understand the differences between oil futures, convenience yields and dollar bonds, Figure 2 displays the oil futures curves (panel a), convenience yield curves (panel b), and bond yield curves (panel c), drawn for end-of-quarter observations. The spot price has an important effect on the oil futures curve and acts as a level factor in the oil market, suggesting that an explanation of oil futures prices needs to take account of it. Unlike some earlier studies, we do not find that the sensitivity of oil futures prices to changes in the spot price of oil decreases with the maturity of the contract (e.g., Bessembinder et al. 1995, and Casassus and Collin-Dufresne 2005). In fact, in comparison with the yield

\[6\] These studies, however, predate the persistent increases in the price oil observed in recent years and the changes in the shape of the oil futures curve.
curves (panel b and c), the term structure of oil futures is rather flat.

By contrast, the convenience yield curve has a funnel shape, which indicates that the sensitivity of long-term convenience yields to movements in the short-term convenience yield decays with the maturity of the oil bond. When short-term convenience yields are high, the curve tends to be downward sloping, and vice versa. Unlike the dollar bond curve, which is mainly upward sloping during our sample period (see panel c), the convenience yield curve is downward-sloping almost forty per cent of the time.

Because the central purpose of the paper is to relate oil futures risk premia to macroeconomic risks, we restrict our attention to macroeconomic variables that have been shown to predict oil prices in previous studies. Several papers argue that the price of oil should be treated as endogenous with respect to global macroeconomic conditions (e.g., Kilian 2009; Baumeister and Peersman 2012; Alquist, Kilian, and Vigfusson 2013; and Lippi and Nobili 2013). For this reason, we select the index of global real economic activity, $\text{realt}$, constructed by Kilian (2009) as a measure of global demand.\footnote{This index of global economic activity is constructed from data on dry cargo single voyage ocean freight rates to capture shifts in the demand for industrial commodities in global business markets.} As dollar bond returns are part of futures returns (6), we also include U.S. CPI inflation, $\Delta p_t$, where $p_t$ is the U.S. CPI index given that inflation is considered to be an important driver of long-term interest rates (see e.g., Wright, 2011).

The oil futures curve reveals the need to include the spot price as a state variable in the model as well. Because it is a nominal variable, however, we tie its value to the CPI index over the long run by assuming that the real price of oil ($rpo_t = s_t - p_t$) is stationary. The real price of oil thus acts as an error correction term for both the U.S. inflation rate, $\Delta p_t$, and the change in the nominal price of oil, $\Delta s_t$.\footnote{The stationarity of the real price of oil is consistent with equilibrium models that predict that the U.S. dollar oil price should follow the aggregate U.S. price level if the nominal price of oil is flexible (e.g., Gillman and Nakov 2009). Some of the previous literature has emphasized mean reversion in the nominal price of oil. For example, Schwartz (1997) posits that mean reversion arises naturally in models of commodity price determination given the effect of relative prices of the supply of the commodity, although it may take time for supply to respond to the price movement.} We therefore include the real price of oil in the set of macroeconomic variables, $m_t = (\text{realt}, \Delta p_t, rpo_t)$ and can recover the nominal price from the latter two variables. The change in the nominal price, the inflation rate and the real price of oil are treated as variables in a vector error correction model (VECM) under the physical distribution.

The summary statistics for the macroeconomic variables are presented at the bottom of Table 1a. While the real price of oil has a first-order autocorrelation statistic of 0.953, the higher order statistics indicate that is stationary, supporting our view of the long-run relationship between nominal prices and the level of the CPI index.

The summary statistics for the annualized realized excess returns on the futures investment strategies are shown in panel B of Table 1. The annualized holding returns (7) are increasing on average with maturity. The strategy of holding a short maturity contract to maturity has a smaller average return and greater volatility than a similar
strategy for long maturity contracts. The average excess return on the short roll strategy (8) is large, averaging near 8.00 per cent per annum, regardless of the holding period. We note that a good portion of the holding period returns thus arises from the component related to the short roll, as does the volatility.

The oil spread returns (9) increase from an annualized return of 17 basis points for a one-month holding period up to 236 basis points for a 12 month holding period. The oil bond term premium is thus upward sloping. In contrast, the dollar bond spread returns (10) are much smaller on average, though they also increase with maturity.

3.2 Summarizing the cross-section of oil futures and bond yields

3.2.1 Dollar bond yields

The cross sections of dollar and oil bond yields can each be summarized by a principal components analysis. We start with the dollar bonds. It is well known that the first three principal components of the U.S. Treasury curve (labelled ‘level’, ‘slope’ and ‘curvature’) explain the cross-section of U.S. Treasury bond yields at a point in time (Litterman and Scheinkman, 1991). Similar results are reported in the first column of Panel A of Table 2, where we present the per cent variation in the dollar bond yield curve explained by the first $k$ principal components. The first three components of dollar bonds, denoted $b_t$, describe over 99.9 per cent of the cross-sectional variation in the data. Panel A of Figure 3 displays the factor loadings of the first three principal components, where the level, slope and curvature structures are evident.\footnote{In order to ease the interpretation of the coefficients in the estimated risk premia presented below, we have normalized the loadings on the first principal components (i.e, the level factors) of both the dollar and oil bonds to be uniformly negative.} Given that the level factor can be related to inflation expectations (e.g., Rudebusch and Wu, 2008), it makes sense that the level factor is the main driver of variation in the dollar bond yield curve, explaining about 95% of the variation of yields.

Similarly, the slope factor is usually related to economic activity because it displays a strong countercyclical pattern (e.g., Estrella and Mishkin, 1998). The dollar bond curve steepens during recessions, flattens during expansions, and usually reaches its minimum level before the start of recessions. The economic interpretation of the curvature factor is less clear.

3.2.2 Convenience yields

The number of factors required to describe the term structure of convenience yields and their economic interpretation is less well established. The results of a principal component analysis are reported in the second column of Panel A of Table 2. As in the case of the dollar bond yield curve, the first three principal components, which we label $c_t$, explain
over 99.9 per cent of the variation of the term structure of convenience yields. Similarly, Panel B of Figure 3 shows that we can also interpret these three principal components as level, slope and curvature factors. The level component is responsible for 93.3 per cent of the variation. Interestingly, the factor loadings of the level factor decrease (in absolute terms) with maturity, unlike the dollar bond yields. This finding reflects the sensitivity of the long-term convenience yields to movements in the short-term convenience yield that decays with the maturity of the bond under consideration. This feature of the data explains the funnel shape of the convenience yield curve (see Panel b, Figure 3). The second principal component loads negatively on short-maturity yields and positively on long-maturity ones. It can therefore be interpreted as a slope factor, accounting for 6.3 per cent of the variation.

To assign an economic interpretation to the principal components of convenience yields, we use the relationship between production, inventories, and consumption implied by competitive storage models (e.g., Routledge, Seppi, and Spatt 2000). In such models, convenience yields arise endogenously as the result of the interaction of supply, demand and storage decisions, and imply a negative and monotonic relationship between the convenience yield and the current level of inventory of a storable commodity (the Working 1933 curve). These models predict that periods of relative scarcity of the commodity such as oil are related to high convenience yields.\(^{10}\)

We test for the existence of a Working curve using crude oil inventory data. Figure 4 shows the OLS regression of the 1-month convenience yield on the level of inventories. It uses the oil inventory data from PADD 2, the administrative region in the United States oil distribution network where Cushing, Oklahoma (the delivery point for the WTI futures contract) is located. The data span the entire sample period during which we observe the term structure of futures prices with maturities up to 12 months (i.e., April 1989–March 2012). As is evident from this figure, there is a negative relationship between the 1-month convenience yield and the level of inventories, which we find to be statistically significant.\(^{11}\) We thus interpret the level component of the convenience yield as a proxy for current crude oil scarcity.

We also rely on the predictions of the competitive storage model to interpret the slope component of the term structure of convenience yields. In particular, an upward-sloping convenience yield curve indicates a situation in which firms assign a higher value to future inventories than they do to today’s inventories, which indicates that they expect oil to be scarcer in the future. The slope of the convenience yield curve should, therefore, predict changes in inventories. We verify this hypothesis by running a regression of future changes

\(^{10}\) This assumption is common in models of exchange-traded industrial commodities, including crude oil (see Fama and French, 1987; Fama and French, 1988; Ng and Pirrong, 1994; Pindyck, 1994; Pindyck, 2001; and Geman and Ohana, 2009).

\(^{11}\) The negative relationship was found in a number of other commodities in Gorton, Rouwenhorst and Hayashi (2012). Our result is insensitive to detrending the inventory data using the Hodrick-Prescott filter and excluding outlying observations of the convenience yield (not reported).
in inventories on a constant and the slope:

\[ \Delta I_{t+n} = \gamma_0 + \gamma_1 \left[ \delta_t^{(n)} - \delta_t^{(1)} \right] + \gamma_2 \Delta I_t + \epsilon_{t+n} \]  

(11)

where \( \Delta I_t \) is the change in the level of inventories at \( t \); and \( \delta_t^{(n)} - \delta_t^{(1)} \) is the spread between the \( n \)-month and 1-month convenience yield. We add the current change in inventories to control for the persistence in inventory changes. Table 3 reports the results from this regression for maturities of \( n = 3, 6, \) and 12 months. The estimated \( \gamma_1 \) coefficients are statistically significant and negative while the adjusted \( R^2 \)’s statistics range from 50 to 80%. Based on this evidence, we interpret the second (slope) principal component as a measure of the expected future scarcity of oil. To the best of our knowledge, we are the first to provide this intuition. As in the case of the dollar bond factors, the economic interpretation of the curvature factor is unclear.

### 3.2.3 Oil futures

Although both \( b_t \) and \( c_t \) capture the cross sectional variation in the oil and dollar bond curves, an open question is whether they also capture the cross-sectional information in the oil futures curve itself. When oil futures prices are regressed on \( s_t, b_t \) and \( c_t \), the \( R^2 \) statistics are over 99.9 per cent. This evidence indicates that the selected bond and convenience yield variables, as well as the spot price of oil, capture almost all of the cross-sectional variation in the oil futures curve, making it unnecessary to model the futures curve directly. Moreover, the final column of Panel A in Table 2 documents how much of the variation is explained by its own first \( k \) principal components. The first principal component accounts for 99.7 per cent of the cross-sectional variation in the oil futures curve, and its correlation with the spot price of oil is 99.02 per cent. Therefore, the spot price of oil plays the role of the level factor in oil futures.

Panel B in Table 2 presents the correlations between the dollar bond factors, the convenience yield factors and the change in the price of oil. By construction, the principal component analysis implies that the correlation between each of the dollar bond yield (convenience yield) factors is equal to zero. In addition, there is little overlap between the information contained in the term structure of bond yields, the term structure of convenience yields and the nominal price of oil as shown by the low correlations between the factors. For example, the largest correlation is only 0.36 (between the level of the dollar bond yield curve and the convenience yield curves). This evidence indicates that studying the two components of the basis separately reveals information about the distinct drivers of oil futures risk premia.

### 3.3 Unspanned macroeconomic variables and the structure of risk premia

The previous section showed that the principal components explain almost all of the contemporaneous cross section of dollar bond interest rates and of convenience yields
(\textbf{b}_t \text{ and } \textbf{c}_t, \text{ respectively}). The macroeconomic variables that we include in the model are therefore unlikely to improve the fit of the contemporaneous, cross-sectional data. However, these variables may still play an important role in describing the time series dynamics of changes in yields, beyond the influence that is captured by the principal components. This would imply an important source of incremental forecasting power for dollar bond, oil bond and spot oil returns that should be included in the model. Macroeconomic variables such as these are labelled “unspanned” and have been shown to be important for explaining expected return variation in the dollar bond term structure literature. In this section we confirm that they are important in both the dollar bond and oil bond term structures.

Our analysis proceeds in three steps. In the first step, we show that a large portion of the variation of the macroeconomic variables is orthogonal to both the dollar bond and convenience yield curves. Table 4 presents the $R^2$ statistics obtained from regressing the macroeconomic variables on the principal components of the dollar and oil bond yields. The low values of the statistics indicate that a substantial fraction of the variation in the macroeconomic factors is unspanned by variation in the cross-section of oil bond prices and U.S. interest rates. For example, the projection of the global real activity index on the components delivers an $R^2$ of 13.84 per cent. Little is gained if we include additional components in the regression (right-hand column of Table 3). A regression of the real price of oil on the components produces slightly larger $R^2$ statistics. With three principal components from both types of bonds the generated $R^2$ is just above 30 per cent. Including additional principal components increases it to nearly 40 per cent. Regressing the inflation rate on the components produces lower $R^2$ statistics. We also regress the change in the nominal price of oil on the principal components. The resulting low $R^2$ statistic of 15.76 per cent shows that a large portion of the variation in the nominal price of oil is also orthogonal to the dollar and oil bond yield curves.

In the second step, we examine the factor structure of expected returns on the dollar and oil bonds. Following Cochrane and Piazzesi (2008), we first regress the realized excess returns on dollar bonds (1) for $n = 2$ to 180 months on a constant, the dollar bond factors, the convenience yield factors, and the set of macroeconomic variables. This results in 179 time series of expected excess returns. Second, we obtain the principal components from the series of expected returns and examine the contribution of each component to total expected return variation. The first column of Panel A of Table 5 presents the results. The first principal component of expected bond returns drives 99.0 per cent of the variation in the cross section of expected returns on the dollar bonds. There is evidently a one factor structure in the cross section of expected bond returns (i.e., a single dollar bond risk premium), a result emphasized in Cochrane and Piazzesi (2008). Further, they observe that this factor seems to be related to compensation for the risk associated with movements in the level factor because the expected return of
longer duration bonds are more sensitive to changes in the return forecasting factor.\textsuperscript{12} We confirm this result in the first row of Panel B of Table 5 that loadings of the first factor in expected dollar bond results increase (almost linearly) with the maturity of the bond.

We conduct a similar analysis on the term structure of the convenience yields. In particular, we regress the realized excess returns on oil bonds (5) for $n = 2$ to 12 months on the same set of explanatory variables, and obtain the first principal component of the term structure of oil bond expected returns. Interestingly, the second column of Panel A of Table 5 shows a result that is very similar to that in dollar bonds. There is a predominant single factor in the cross section of expected returns (i.e., a single oil bond risk premium), accounting for 96.67 per cent of the variation. By a similar argument to that of Cochrane and Piazzesi (2008), this factor also appears to be related to compensation for risk associated with the (convenience yield) level factor given that the loadings increase with maturity (second row of Panel B of Table 5). In light of our earlier findings, this evidence suggests that there is a single (dominant) priced risk associated with the cross section of crude oil scarcity. That is, the level of current inventories is the only priced risk in the oil bond market. To the best of our knowledge, we are the first to document that there is such a “Cochrane-Piazzesi” structure in the convenience yield term structure as well.

In the third step we combine the results of the first two steps to assess the overall predictability of dollar and oil bond return factors. We also examine the expected excess return to a taking a position in spot oil (2). In the spirit of Cochrane and Piazzesi (2005), we regress the average excess return (across maturities) on the set of dollar and oil bond principal components, the spot price of oil and the macroeconomic variables. The Wald test statistics assessing the joint statistical significance of all of the regressors along with their asymptotic marginal significance levels are shown in the first column of Panel C of Table 5. The predictive variables capture significant time series variation in the dollar bond, oil bond and spot oil risk premia as shown by the very small marginal significance levels of the three test statistics. The degree of predictability is as expected with regressions of monthly asset market returns. The adjusted $R^2$ statistics range from 7.61 per cent for the spot oil risk premium to 8.38 per cent for the oil bond risk premium (third column of Panel C). These results are in line of those reported in Adrian, Crump and Moench (2013) for the case of the U.S. Treasury bond market. Also, the set of macroeconomic variables contains important information for predicting excess returns. As shown in the second column of Panel C of Table 5, we are able to reject the null hypothesis that the macroeconomic variables do not help in predicting excess returns on the three components of the futures returns.

\textsuperscript{12}We show in section 4.4 that if a bond yield loading for a given factor is constant and negative for all maturities (i.e., the level factor), then the loading of the expected holding period return with respect the risk premia of that factor is a linear function of maturity.
3.4 Summary

The preliminary analysis presented thus far reveals several new characteristics of the oil futures market. First, the basis can be decomposed into a term structure of dollar bonds and convenience yields. Both term structures can be described using three principal components. The components from the convenience yield term structure can be related to current and anticipated inventory levels. Second, the cross section of expected dollar bond excess returns and of oil bond excess returns are both well characterized by one-factor structures. As in the previous literature, the priced factor in the dollar bond market is compensation for inflation risk. We find that the priced factor in the oil bond market is compensation for inventory risk. Third, there is variation in the macroeconomic variables that is orthogonal to the cross section of dollar and oil bond yields and that is useful for predicting future excess returns on the level factors. Finally, the real price of oil can be used as an error correction term for the nominal price and the inflation rate.

In the next section, we derive a dynamic term structure model that incorporates these features of the oil market and enables us to determine the macroeconomic sources of risk associated with the returns on trading strategies in the oil futures market.

4 Asset pricing model

This section introduces the physical dynamics of the model with the real price of oil incorporated as an error correction term, describes how to use the dollar pricing kernel to model the risk neutral dynamics of the factors, and discusses the restrictions that the unspanned risks impose on the model. The assumption that the dollar and oil bond markets are fully integrated leads to the construction of a pricing kernel for oil bonds using the kernel for dollar bonds.

4.1 The physical dynamics

The state of the global economy is described by four sets of state variables (or pricing factors): (1) the \((B \times 1)\) vector of dollar bond factors, \(b_t\); (2) the \((C \times 1)\) vector of convenience yield factors, \(c_t\); (3) the \((M \times 1)\) vector of the macroeconomic factors, \(m_t = (rea_t, \Delta p_t, rpo_t)\); (4) and the change in the (log) spot price of oil, \(\Delta s_t\). We collect the pricing factors in the vector \(x_t = (b_t', c_t', m_t', \Delta s_t)'\) and denote the dimension of \(x_t\) as \(L = B + C + M + 1\).

The dynamic evolution of these state variable is described by a VAR(1) process under the physical measure \(\mathbb{P}\) with Gaussian innovations:

\[
x_{t+1} = \mu + \Phi x_t + v_{t+1},
\]

where \(v_t \sim iid N(0, \Sigma)\). The assumption that the real price of oil is stationary imposes a cointegration relationship between the nominal spot price of oil and the price level. Thus,
equation (12) needs to be interpreted as the VAR(1) companion form representation of a VECM(1) model where both the nominal oil price \( s_t \) and the price level \( p_t \) have a unit root, and the real price of oil \( rpo_t = s_t - p_t \) acts as the error correction term in the system.

### 4.2 The stochastic discount factor and the risk-neutral measure

We choose \( b_t \) to be the first three principal components of the term structure of bond yields in order to maximize the (cross-sectional) explanatory power of the bond factors. Consistent with this choice of variables and the presence of unspanned macroeconomic risks, we postulate that the short-term (1-month) interest rate is an affine function of the bond market factors \( b_t \):

\[
y_t^{(1)} = \psi_0 + \psi_0' b_t. \tag{13}
\]

By a similar argument, we choose \( c_t \) to be the first three principal components of the term structure of convenience yields. As the risks are also unspanned in the oil market, we assume that the short-term convenience yield is an affine function of \( c_t \):

\[
\delta_t^{(1)} = \phi_0 + \phi_0' c_t. \tag{14}
\]

To price assets in the bond market, we exploit the fact that no-arbitrage implies the existence of a (dollar) stochastic discount factor (SDF) that we postulate to be exponentially affine in \( x_t \) (e.g., Ang and Piazzesi, 2003):

\[
\xi_{t+1}^s = \exp \left[ -y_t^{(1)} - \frac{1}{2} \lambda_t' \Sigma_t^{-1} \lambda_t - \lambda_t' \Sigma_t^{-1} v_{t+1} \right]. \tag{15}
\]

The prices of risk are given by \( \lambda_t = \lambda_0 + \lambda x_t \). The (strictly positive) SDF, \( \xi_{t+1}^s \), can be used to price the zero-coupon dollar bonds using the following recursive relation:

\[
P_t^{(n)} = E_t \left[ \xi_{t+1}^s P_{t+1}^{(n-1)} \right]. \tag{16}
\]

It is possible to show that solving (16) is equivalent to using risk-neutral pricing to obtain:

\[
P_t^{(n)} = E_t^Q \left[ e^{-y_t^{(1)}} P_{t+1}^{(n-1)} \right], \tag{17}
\]

where \( E_t^Q \) denotes the expectation under the risk-neutral probability measure, \( Q \). The dynamics of the entire state vector \( x_t \) can be characterized by the following VAR(1) process under \( Q \):

\[
x_{t+1} = \mu^Q + \Phi^Q x_t + v_{t+1}^Q, \tag{18}
\]

with \( v_{t+1}^Q \sim iid N(0, \Sigma) \), \( \mu^Q = \mu - \lambda_0 \), and \( \Phi^Q = \Phi - \lambda \). That is, one can price assets in this economy as if agents were risk-neutral using a risk-adjusted law of motion of the state variables that accounts for the fact that agents are not actually risk-neutral. If agents were risk-neutral, we would have \( \mu^Q = \mu \) and \( \Phi^Q = \Phi \).
The assumption of unspanned risks imposes additional structure on the dynamics of the risk neutral distribution (i.e., on $\mu^Q$ and $\Phi^Q$).\(^{13}\) In particular:

$$
\mu^Q = \begin{pmatrix}
\mu_b^Q \\
\mu_c^Q \\
\mu_m^Q \\
-\frac{1}{2}e'_L \Sigma e_L + \psi_0 - \phi_0
\end{pmatrix}, \quad \Phi^Q = \begin{pmatrix}
\Phi_{bb}^Q & 0 & 0 & 0 \\
0 & \Phi_{cc}^Q & 0 & 0 \\
\Phi_{mb}^Q & \Phi_{mc}^Q & \Phi_{mm}^Q & \Phi_{ms}^Q \\
\psi_b^T & -\phi_c^T & 0 & 0
\end{pmatrix},
$$

(19)

where the notation $e_L$ is a vector of zeros with a 1 in the $L^{th}$ position and the four rows in each matrix correspond to the four sets of state variables in $x_t$. Given these restrictions, the bond factors (first row) follow an autonomous VAR(1) process. Neither the convenience yield factors, nor the macro variables, nor the price of oil Granger cause $b_t$ under $Q$. Otherwise, no-arbitrage pricing would imply that bond yields would be affine functions of all of the state variables in $x_t$. Unspanned risks impose a similar structure on the dynamics of the convenience yield factors $c_t$ under $Q$ (second row).

The dynamics of the change in the (log) nominal price of oil $s_{t+1}$ (last row) are determined by assuming the absence of arbitrage in the oil market. We show in the appendix that no-arbitrage implies that:

$$
E^Q_t \Delta s_{t+1} = -\frac{1}{2} \text{Var}_t \Delta s_{t+1} + \left[ \rho_t^{(1)} - \eta_t^{(1)} \right].
$$

(20)

Here the expected change in the price of oil under $Q$ is equal to the convenience yield less the risk-free rate, corrected by a Jensen’s inequality term. This equation is analogous to the uncovered interest parity condition of exchange rates that holds under risk neutrality.

The combination of unspanned risks and no arbitrage implies that the price of oil has a unit root under $Q$.\(^{14}\) While this feature of the model differs from Bessembinder et al. (1995) and Cassassus and Collin-Dufresne (2005), it is needed to replicate the level factor that characterizes the oil futures curve during our sample (see Panel A of Figure 3 and Panel A of Table 2). Since oil futures are risk-neutral expectations of the future spot price of oil, the only way to rationalize the level factor is to assume that under the risk-neutral measure, future spot prices rise in parallel when the spot price of oil moves today. In fact, we show below that the dynamics of the state variables under the risk neutral distribution (18) are well-specified. The validity of this claim can be seen from the small pricing errors obtained from the model for the zero coupon bonds, the synthetic oil bonds and the oil futures contracts.

The macroeconomic variables do not affect the risk neutral distribution of the dollar and oil bond yields (i.e., the zeros imposed in the third and fourth columns of $\Phi^Q$). Consistent with our preliminary findings, this assumption implies that they do not help describe the cross section of dollar and oil bond yields once the dollar bond yield and

\(^{13}\)See, for example, Joslin, Priebsch and Singleton (2012).

\(^{14}\)Given (20), the nominal price of oil is only mean reverting if the short dollar bond yield or the short convenience yield depend on $s_{t}$. However, the assumption that there are unspanned risks rules out this possibility.
convenience yield factors are taken into account. However, this approach still allows
the macro variables to have an influence on the dynamics of these yields through their
influence on the components of the risk premium ($\lambda$).

It is also important to recognize that in the absence of a risk premium in the spot
price of oil (i.e., $E_t^Q \Delta s_{t+1} = E_t \Delta s_{t+1}$), expected changes in the nominal price of oil are
solely determined by the difference between the convenience yield and the risk free rate,
$\delta_t^{(1)} - \gamma_t^{(1)}$. If agents were risk neutral, macroeconomic variables would not have any
predictive power over the (nominal) price of crude oil. However, this is inconsistent with
the existing evidence on the predictability of oil prices (e.g., Alquist, Kilian and Vigfusson
2013) and our analysis in the previous section.

4.3 Pricing bonds and futures

Solving equation (17), the continuously compounded yield on an $n$-period zero coupon
bond at time $t$, $y_{t}^{(n)}$, is given by

$$y_{t}^{(n)} = \alpha_{y}^{(n)} + \beta_{y}^{(n)} b_t,$$

where $\alpha_{y}^{(n)} = -A_{y}^{(n)}/n$ and $\beta_{y}^{(n)} = -B_{y}^{(n)}/n$, and $A_{y}^{(n)}$ and $B_{y}^{(n)}$ satisfy the following set of
recursive relations:

$$A_{y}^{(n+1)} = A_{y}^{(n)} + B_{y}^{(n)\prime} \Phi_{bb}^{Q} + \frac{1}{2} B_{y}^{(n)\prime} \Sigma_{bb} B_{y}^{(n)} + A_{y}^{(1)},$$

$$B_{y}^{(n+1)\prime} = B_{y}^{(n)\prime} \Phi_{bb}^{Q} + B_{y}^{(1)\prime},$$

with $A_{y}^{(1)} = -\psi_0$ and $B_{y}^{(1)} = -\psi_{b}^{t}$. These recursions are the same as those found in the
literature (e.g., Ang and Piazzesi 2003). The yields depend on the bond market factors
$b_t$ only due to the assumption of unspanned risks.

On the other hand, to price the zero-coupon oil bond (4) we require a stochastic
discount factor for the oil market, $\xi_{t+1}^{oil}$:

$$O_{t}^{(n)} = E_t \left[ \xi_{t+1}^{oil} O_{t+1}^{(n-1)} \right],$$

where, under the maintained assumption that the oil and bond markets are integrated
so that no risk-free profitable arbitrage opportunities remain, the oil SDF and the dollar
SDF are related through the spot price of oil. We show in the appendix that when the
change in the nominal price of oil is affine in the set of pricing factors (which, in our case
is trivially satisfied given that $\Delta s_t$ is itself a pricing factor), the law of one price implies
that the change in the nominal price of oil, the dollar SDF and oil SDF must satisfy the
following no-arbitrage relation:

$$\Delta s_{t+1} = \log \xi_{t+1}^{oil} - \log \xi_{s_{t+1}}^{s}.$$
This expression is the oil price analogue to the exchange rate equation derived in Backus, Foresi and Telmer (2001). It implies that one of the dollar SDF, the oil SDF and the nominal price of oil is redundant and can be constructed from the other two.\textsuperscript{15}

As in Diez de los Rios (2010) and Bauer and Diez de los Rios (2012) in the context of the modeling of international term structures, we use the dollar SDF and the no-arbitrage condition (24) to recover the oil market SDF. Substituting the law of motion for the spot oil price and the definition of the dollar SDF in equation (24), the oil market SDF is exponentially affine:

\[
x_{oil, t+1} = \exp \left( -\delta_{t}^{(1)} - \frac{1}{2} \lambda_t^{oil} \Sigma^{-1} \lambda_t^{oil} - \lambda_t^{oil} \Sigma^{-1} v_{t+1} \right),
\]

with prices of risk given by \( \lambda_t^{oil} = (\lambda_0 - \Sigma e_t) + \lambda x_t \). The appendix provides the details.

Given that the oil market SDF is exponentially affine and that the short-term convenience yield is affine in the set of pricing factors, we can price oil bonds in the same way that we price dollar bonds. In particular, the continuously compounded convenience yield of a \( n \)-period oil bond at time \( t \), \( \delta^{(n)}_t \), is given by:

\[
\delta^{(n)}_t = \alpha^{(n)}_\delta + \beta^{(n)\prime}_\delta c_t,
\]

where \( \alpha^{(n)}_\delta = -A^{(n)}_\delta / n \) and \( \beta^{(n)}_\delta = -B^{(n)}_\delta / n \), where \( A^{(n)}_\delta \) and \( B^{(n)}_\delta \) satisfy a set of recursive relations that are similar to those for the dollar bond market:

\[
A^{(n+1)}_\delta = A^{(n)}_\delta + B^{(n)\prime}_\delta (\mu^Q + \Sigma_{cc}) + \frac{1}{2} B^{(n)\prime}_\delta \Sigma_{cc} B^{(n)}_\delta + A^{(1)}_\delta,
\]

\[
B^{(n+1)\prime}_\delta = B^{(n)\prime}_\delta \Phi^Q_{cc} + B^{(1)\prime}_\delta,
\]

with \( A^{(1)}_\delta = -\phi_0 \) and \( B^{(1)}_\delta = -\phi^\prime_0 \). The yields on the oil bonds are a function of the convenience yields factors \( c_t \) only due to the assumption of unspanned risks.

Given the cost-of-carry relationship (3), the price of an oil future contract is given by:

\[
f^{(n)}_t = s_t + n y^{(n)}_t - n \delta^{(n)}_t
= s_t + n \left[ \alpha^{(n)}_y + \beta^{(n)\prime}_y b_t \right] - n \left[ \alpha^{(n)}_\delta + \beta^{(n)\prime}_\delta c_t \right]
= s_t - \left[ A^{(n)}_y + B^{(n)\prime}_y b_t \right] + \left[ A^{(n)}_\delta + B^{(n)\prime}_\delta c_t \right].
\]

The cross section of the (log) futures price are described by the spot price of oil, the bond market factors and the convenience yield factors. Consistent with our preliminary findings, the spot price acts as a level factor in the oil futures curve. While the macroeconomic variables are unspanned, they influence the dynamics of both the dollar and oil bonds along with the spot price of oil and hence also influence those of the oil futures curve itself.

\textsuperscript{15}As long as the nominal price of oil is affine in the set of pricing factors, this condition holds even when markets are not complete.
4.4 Expected returns

By taking a position in the oil futures market, an investor is exposed to the risks embodied in the spot oil market, in the oil bond and in the dollar bond, and thus demand compensation for these three sources of risk. The model reveals that the expected returns on the dollar bond, the oil bond and the spot price of oil are affine in the pricing factors. We now analyze each of these returns.

Substituting the expressions for the dollar bond yields in (21) into equation (1) and taking expectations, the 1-month expected excess return for holding an \( n \)-period dollar bond is given by:

\[
E_t r_{n,t+1} = \frac{1}{2} B_y^{(n-1)\gamma} \Sigma_{bb} B_y^{(n-1)} + B_y^{(n-1)\gamma} \lambda_{bb} + B_y^{(n-1)\gamma} (\lambda_{bb} b_t + \lambda_{bc} c_t + \lambda_{bm} m_t + \lambda_{ms} \Delta s_t). \tag{30}
\]

The notation \( \lambda_{bb} \) refers to the first three rows of \( \lambda \) (i.e., those associated with the dollar bond factors).

Similarly, substituting the expression for the convenience yield (26) into equation (5), the 1-month expected excess return on holding an oil bond that matures \( n \) periods from now can be expressed as:

\[
E_t r_{n,t+1} = \frac{1}{2} B_\delta^{(n-1)\gamma} \Sigma_{cc} B_\delta^{(n-1)} + B_\delta^{(n-1)\gamma} (\lambda_{cc} - \lambda_{ca}) + B_\delta^{(n-1)\gamma} (\lambda_{cb} b_t + \lambda_{cc} c_t + \lambda_{cm} m_t + \lambda_{cs} \Delta s_t). \tag{31}
\]

The notation \( \lambda_{cc} \) refers to the rows of \( \lambda \) that are associated with the oil bond factors (i.e., rows four to six). We note that each of the bond risk premia has three terms: (1) a Jensen’s inequality term; (2) a constant risk premium; and, (3) a time-varying risk premium.

The preliminary analysis presented in section 3.2 suggested that (1) both the cross section of dollar and oil bond returns were each driven by a single risk factor and (2) the exposure of expected dollar (oil) bond returns to the dollar (oil) bond risk premium factor increases (almost linearly) with the maturity of the bond under consideration. We note that our asset pricing model can accommodate these two features of the data. In particular, since the coefficients \( B_i^{(n)} = -\beta_i^{(n)} \times n \) and \( \beta_i^{(n)} \) are almost constant (and negative) for both \( i = b, \delta \), then the exposure of expected bond returns to the level risk premium, \( B_i^{(n)} \), is an increasing of the maturity of the bond. This in turn naturally translates into the restriction that only level risks are priced in each of the term structures of dollar and oil yields. These restrictions result in zeroes in the second and third rows of \( \lambda_{cc} \) as well as in the last two elements of \( \lambda_{ca} \). A similar restriction is imposed in the second and third rows of \( \lambda_{bb} \) as well as in the last two elements of \( \lambda_{b0} \) for dollar bonds.

Finally, the expected return on holding a spot position in the oil market for one month is given by:

\[
E_t r_{s,t+1} = \frac{1}{2} e_L \Sigma_{LL} e_L + \lambda_{s0} + \lambda_{sb} b_t + \lambda_{sc} c_t + \lambda_{sm} m_t + \lambda_{ss} \Delta s_t. \tag{32}
\]
The notation $\lambda_{s\ast}$ refers to the last row of $\lambda$ (i.e., the one that is related to the spot oil price). In the absence of a risk premium in the spot oil market (i.e., $\lambda_{s0} = \lambda_{s\ast} = 0$), expected changes in the nominal price of oil would be solely determined by the excess convenience yield over the risk free rate, and thus macroeconomic variables would play no role in driving the physical dynamics of the spot price of oil.

While the dollar bond factors $b_t$, the macroeconomic variables $m_t$ and the change in the spot price of oil $\Delta s_t$ do not affect the cross-section of yields, they may still help explain time-variation in risk premium depending on the coefficients in the $\lambda$ matrix. However, as the macroeconomic variables are unspanned by any of the assets, it is not possible to estimate the prices of risk associated with these variables. Thus, we do not report estimates of the elements of $\lambda_{m\ast}$ (the rows of the $\lambda$ matrix associated with the macro variables).

5 Estimation

We estimate the model using the new approach of Diez de los Rios (2013a,b). He proposes a linear estimator that exploits three features that characterize GDTSMs. First, the model has a reduced form representation whose parameters can be easily estimated via a set of ordinary least squares (OLS) regressions. Second, the no-arbitrage assumption upon which GDTSMs are built can be characterized as a set of implicit constraints between these reduced-form parameters and the parameters of interest. Third, the set of restrictions is linear in the parameters of interest.

Consequently, Diez de los Rios (2013a) proposes a two-step estimator. In the first step, estimates of the reduced-form parameters are obtained by OLS. In the second step, the parameters of the GDTSMs are inferred by forcing the no-arbitrage constraints, evaluated at the first-stage estimates of the reduced-form parameters, to be as close as possible to zero. Note that as the constraints are linear in the parameters of interest, the solution to the estimation problem is known in closed form. In fact, the estimates of the parameters of the GDTSM resemble those obtained from an OLS regression involving the reduced-form parameter estimates. In addition, it can be shown that this new linear estimator is consistent and asymptotically normally distributed.

As Bauer, Rudebusch and Wu (2012) observe, the estimates of the $\mathbb{P}$ parameters tend to underestimate the persistence of the system in finite samples. Consequently, the largest eigenvalue of $\Phi$ estimated from the VAR(1) representation under $\mathbb{P}$ in equation (12) is usually less than 1.00, with the result that expected future long-term bond yields are almost constant.

We tackle this persistence bias in three ways. First, we follow Bauer, Rudebusch and Wu (2012) and replace the reduced-form OLS estimates of the VAR(1) process for $x_t$ in equation (12) with bias-corrected estimates. Specifically, we use a recursive-design bootstrap, coupled with the adjustment suggested by Kilian (1998) in order to guarantee
that the bias-corrected estimates are stationary.\textsuperscript{16}

Second, we follow Joslin, Priebsch and Singleton (2012) and force the largest eigenvalues of $\Phi$ and $\Phi^Q$ to be the same. This restriction is motivated by the fact that the largest eigenvalue of $\Phi^Q$ needs to be close or equal to one in order to replicate the level factor.

Third, motivated by the results in Cochrane and Piazzesi (2008), we expect that restricting the prices of risk pull $\Phi$ closer to $\Phi^Q$ so that the dynamics under $\mathbb{P}$ inherit more of the high persistence that characterizes the $Q$-measure. We therefore force the dollar bond level factor to be the only priced factor in the term structure of dollar bond returns, and the oil bond level factor to be the only priced factor in oil bond returns.

Diez de los Rios (2013b) shows that the estimation of GDTSMs subject to these two types of restrictions is tractable and can still be implemented through a set of sequential linear regressions. In this paper, we exploit such tractability to provide small-sample $P$-values for the hypothesis that risk-neutrality (i.e., zero prices of risk) using a “bootstrap plus double bootstrap” procedure.\textsuperscript{17} Further details on the estimation method can be found in Diez de los Rios (2013a,b) and the appendix.

6 Results

In this section, we report the estimation results for the model derived above.

6.1 Fitted yields and risk-neutral dynamics

Table 6 presents the pricing errors obtained from the model of the dollar bonds, the oil bonds and the oil futures contracts. We report the root mean squared pricing error (RMSPE) and the mean absolute pricing error (MAPE) for all maturities over the entire sample period.

The model captures most of the cross-sectional variation in the data. The RMSPE of the dollar bonds is 4.20 basis points, while the MAPE is 2.99 basis points. The RMSPE of the oil bonds is larger, averaging 38.22 basis points while the MAPE is 24.7 basis points.

The magnitude of the pricing errors of the oil bonds is consistent with that of the dollar bonds once the higher volatility of the former is taken into account. The volatility of the convenience yields is about fifteen times larger than that of dollar bond yields (see Table 1), while the magnitude of pricing errors of the oil and dollar bonds only differ by a factor of ten.

We next examine how the model’s restrictions reduce the cross-sectional fit of the yields by comparing the model’s fit to that obtained from unrestricted OLS regressions. The regressions are projections of the yields on the principal components of the dollar

\textsuperscript{16}Our approach extends the bootstrap methods of Joslin, Singleton and Zhu (2011) and Hamilton and Wu (2012) to include a bias correction.

\textsuperscript{17}In the “bootstrap plus double bootstrap” procedure, we start by using the bootstrap method to bias correct the estimates of the GDTSM parameters under the null, but also in each one of the bootstrap replications that we run to compute the small-sample distribution of the tests.
bonds \( (b_t) \), the oil bonds \( (c_t) \) and the spot price of oil \( \Delta s_t \). The table shows small differences (3.02 basis points for oil bonds, less than one basis point for dollar bonds) between the regression based fit and that obtained from the model.

We also assess the model’s ability to explain the cross-sectional variation in the futures curve. The final line of the table reports a RMSPE of 8.51 cents and a smaller MAPE of 4.75 cents. The small sizes of the statistics are striking when we take into account the large reversal of the nominal oil price in 2008. In fact, the model captures the cross-sectional variation about as well as the unrestricted OLS regressions do, with an average difference of less than two cents.

As Cochrane and Piazzesi (2008) observe, the risk-neutral measure parameters are pinned down by the cross-section of yields. This translates into small standard errors around the estimates of the risk-neutral parameters associated with the dollar bonds and oil bonds (Table 7). The coefficients on the short-term interest rate (13) and on the short-term convenience yield (14) are precisely estimated (see panels a and c of Table 7, respectively). The \( \Phi_{bb} \) and \( \Phi_{cc} \) matrices from (19) are also precisely estimated, with only one parameter that is statistically insignificant.

The largest eigenvalue of \( \Phi_{bb} \), the matrix associated with the risk neutral dynamics of dollar bonds, is almost equal to one (0.998), a feature needed to explain the existence of the level factor in interest rates (i.e., long rates are expected future short-term interest rates under the risk-neutral measure, corrected by a Jensen’s inequality term). Hence, the persistent dynamics under \( \mathbb{Q} \) imply that innovations to the first principal component raise expected future dollar interest rates in parallel, providing an interpretation of the level factor of interest rates (see Cochrane and Piazzesi 2008).

By contrast, the convenience yields exhibit stronger mean reversion under the risk-neutral measure as the largest eigenvalue of \( \Phi_{cc} \) is 0.911. This feature is needed to explain why long-maturity convenience yields are less sensitive to changes in the level factor than short-maturity yields which, in turn, delivers the funnel shape of the convenience yield curve (see Panel B of Figure 3). Finally, it is important to recall that the spot price of oil has a unit root under \( \mathbb{Q} \), a feature needed to rationalize the level factor in oil futures.

### 6.2 Prices of risk

Estimates of the coefficients in the prices of risk for the three risk factors (the level risks in dollar and oil bonds and the spot oil risk factor) are reported in Table 8. Each element of the table shows the parameter estimate followed by the asymptotic standard error (in round brackets). We also show the asymptotic (in square brackets) and small-sample (in curly brackets) \( P \)-values, the latter derived from the bootstrap distributions explained in the appendix.

The top row of Table 8 displays the results for the dollar bond level risk factor (the first row of the \( \lambda_{bb} \) matrix in equation (30)). The expected return on the factor is positively affected by its own slope principal component (with a small-sample \( P \)-value of 1.7 per
cent). Since the value of the slope is usually high during recessions and low during expansions (see Estrella and Hardouvelis, 1991, Estrella and Mishkin, 1998), upward sloping yield curves not only predict expected bond returns to increase (Fama and Bliss, 1987; Campbell and Shiller, 1991), but also deliver a dollar bond risk premium that is counter-cyclical. This finding conforms to the conclusions of standard consumption based asset pricing models (Wachter 2006). The presence of a counter-cyclical risk premium is a common feature of term structure models with unspanned macroeconomic risks, such as Joslin, Priebsch and Singleton (2012), the risk premium identified in Cochrane and Piazzesi (2008) and the global risk premium model of Bauer and Diez de los Rios (2012).

None of the convenience yields components is statistically significant in the dollar bond risk premium equation. However, the coefficient on real economic activity is negative and significant, as is the coefficient on (monthly) inflation.\textsuperscript{18} The latter finding indicates that during periods of high (low) inflation, dollar bond returns are expected to be low (high). Since the sample period is characterized by a stable Phillips curve (e.g., Coibion and Gorodnichenko 2013), inflation tends to be high when consumption is high. Both of these coefficients reinforce our finding that the dollar bond risk premium is counter-cyclical.

A new feature of this model is the role played by the real price of oil in explaining the dollar bond risk premium. The coefficient is significant and positive, which follows from the variable acting as the error correction term for nominal oil prices and the CPI index. If the real price of oil is above its long-run average, the error correction mechanism implies that future nominal oil prices are expected to decline and future CPI inflation is expected to increase. The increase in expected future inflation makes the nominal payoff on the zero coupon dollar bonds riskier and increases the bond risk premium.\textsuperscript{19}

The results for the prices of risk in the expected excess return on the level factor in the oil bonds (the first row of the $\mathbf{\lambda}_t$ matrix in equation (31)) are shown in the middle part of the Table 8. Mirroring the case of the dollar bond premium, the slope of the convenience yield curve has predictive power for future oil bond returns. As shown in Table 3, a steepening of the convenience yield curve predicts that future oil inventories are expected to decline relative to their current level. That is, when the term structure of convenience yields is upward sloping, oil inventories are plentiful today relative to the future. Because the short-run supply curve for oil is very inelastic, consumers of oil accumulate inventories during recessions (Kilian and Murphy 2013). High levels of oil inventories today, therefore, indicate low current levels of output (i.e., a recession). Thus, the slope and the resulting oil bond risk premium are counter-cyclical.

The negative and significant coefficient on the monthly change in the spot price of oil also reinforces our finding that the oil bond risk premium is counter-cyclical. Unexpected changes in the price of oil can be largely explained by the unexpected changes in demand

\textsuperscript{18}To the best of our knowledge, we are the first to use $\text{rea}_t$ in a U.S. term structure model.

\textsuperscript{19}The relationship between bond risk premia and proxies for forward looking inflation expectations has been examined by Chernov and Mueller (2012), Ang, Bekaert and Wei (2008), and Feunou and Fontaine (2011).
(Kilian 2009) and explains the procyclical behaviour of the price of oil.

Finally, the real price of oil has a significant and positive coefficient, a finding similar to that in the dollar bond risk premium. If the current real price of oil is above its long-run average, mean reversion exerts influence over the expected path of the price of oil: the real price of oil is expected to return to its equilibrium level. The real price of oil is the payoff on the oil bond from the point of view of a U.S. investor.\(^\text{20}\) Thus, investors interpret a real price of oil that is above its long-run average as a signal of lower (real) payoffs from oil bonds in the future. In equilibrium, the price of the oil bond decreases today, and the expected return increases, which compensates U.S. investors for holding the oil bond.

The bottom part of Table 8 reports the prices of risk (the \(\lambda_{rs}\) coefficients) associated with movements in the expected excess return on the spot price of oil (32). Recall that the spot return is composed of the 1-month change in the price of oil plus the 1-month convenience yield less the risk-free rate. The dollar bond principal components are not statistically significant. However, both the level and slope components of the convenience yield curve have significant negative coefficients. An increase in either of the level or slope components is associated with a decrease in the short-term convenience yield (Panel C of Table 7). Both of the negative coefficients are thus consistent with an increase in the convenience yield to holding spot oil.

The last four numbers in the table are the coefficients associated with the macroeconomic variables in the spot oil risk premium. They show that expected returns on spot oil can be predicted by global real activity with a small-sample marginal significance level of 0.098. This is due to an expected increase in the 1-month ahead price of oil. This finding is related to other evidence showing that demand shocks have been an important driver of the real price of oil. That literature shows that the real price is pro-cyclical and that unexpected changes in global real activity have been a major driver of its fluctuations (Kilian 2009). Furthermore, measures of global real activity are strong in-sample predictors of the future price of oil (Alquist, Kilian, and Vigfusson 2013). This evidence was interpreted to suggest that the spot oil price is driven by the use of oil as a physical commodity. Our results show that the spot price of oil is not only related to current and future global net demand conditions (via the convenience yield term structure factors) but also that the spot oil risk premium is related to the global demand variable.\(^\text{21}\)

The real price of oil is another macroeconomic factor that drives expected spot oil returns (last coefficient shown in Table 8 with a small-sample \(P\)-value of 0.016). As noted above, the error correction feature of the model implies that when the real oil price is above its long-run level, then the nominal price of oil is expected to decline over the next month.

\(^{20}\) The investor sells the barrel of oil for \(s_{t+h}\) dollars and uses the proceeds to buy the basket of goods at a price of \(p_{t+h}\).

\(^{21}\) Figures with time-series evolution of the different components of the estimated risk premia are reported in the appendix.
These results extend the relationship between inventories and risk premia described in other papers. Both Dincerler, Khokher and Simin (2005) and Gorton, Hayashi and Rouwenhorst (2012) show that risk premia are related to current inventories in a broad cross section of commodities. In our dynamic model, the forward looking slope variable is related to anticipated changes in inventories and captures the time variation in the expected returns of both the level risk factor in the oil bond market as well as the risk factor in the spot market.

6.3 Variance decompositions

Next we construct variance decompositions implied by the model to understand the effect of the (spanned) dollar and oil bond principal components and the (unspanned) macroeconomic variables on the risk premia of oil futures. In particular, we focus on 1-year variance decompositions of the expected returns on the futures investment strategies described in section (2.2) – the holding return in (7), the short roll in (8) and the oil and dollar bond spread strategies in (9) and (10), respectively.\(^{22}\) Alternatively, the variance decompositions can also be used to interpret the sources of news about the returns. In particular, let \(rx_{t+h}\) be the return on a particular strategy with an investment horizon of \(h\)-months. Then, by the law of iterated expectations, we have that

\[
Var_t(E_{t-12}(rx_{t+h})) = Var_t(E_{t-12}[(E_t - E_{t-12})rx_{t+h}]) \quad \text{given that} \quad E_{t-12}rx_{t+h} \text{ is known based on the information set available at time } t - 12.
\]

We focus on the proportion of the conditional variance of expected returns that is attributed to innovations in the macroeconomic variables that is orthogonal to both the dollar and convenience yield curves (i.e., the unspanned portion of the macroeconomic variables). For this reason, we use a Cholesky factorization of \(\Sigma\), (i.e., the conditional variance of the pricing factors in the VAR dynamics in equation (12)) and order the macroeconomic variables last, \(x_t = (c_t^\prime, b_t^\prime, m_t^\prime)^\prime\) (see Joslin, Priebsch and Singleton 2012).

Figure 5 shows the variance decompositions. The bottom part of each graph represents that part of the variance that can be accounted for by innovations to the three spanned oil bond factors \((c_t)\). The contribution of the first (level) principal component in the convenience yields is shown in light red while the combined contribution of the second and third components are shown in dark red. The dotted red area between the black line and the red area represents the part of the return variation that can be accounted for by news about the spanned dollar bond term structure principal components \((b_t)\) that is orthogonal to \(c_t\). Thus the total area below the black line indicates the part of the variation that is due to the two sets of spanned yield curve principal components. The difference between the black line and 100 per cent is due to the contribution of the unspanned portion of the macroeconomic factors \((m_t)\). We show the contribution of the

\(^{22}\)In the appendix we show the variance decompositions for the individual components of the futures returns: on the expected returns on the dollar bonds (equation 1), on the oil bonds (equation 5), and on a position in the spot oil market (equation 2).
unspanned components of inflation, the real price of oil and the real growth variables separately.\textsuperscript{23}

The variance decomposition of the $n$-month return to holding a $n$-month oil futures contract, $E_t r^{\text{Hold}}_{t,n}$, is shown in Panel A. The light red area shows a large influence from the first principal component of the convenience yields in explaining expected returns for holding short maturity oil futures. News about the level component accounts for approximately 75 per cent of the variation in 1-month holding returns. Thus, prices of short-dated futures respond (mainly) to news about current inventory levels. As the maturity of the futures contract lengths, the influence of the slope component of convenience yields becomes more apparent. For a 12-month futures contract, news about the term structure of convenience yields accounts for just under 80 per cent of the variation with the second and third principal components accounting for over half of that variation. News about anticipated future inventory levels is therefore an important driver of the prices of long-dated futures. These results underscore the importance of modeling the entire term structure of convenience yields in order to understand the drivers of oil futures contracts.

The influence of dollar bond term structure principal components on the expected holding returns is small once we have accounted for the effect of the convenience yield factors. The total effect of the spanned principal components is shown by the black line. The difference between 100 per cent and the black line indicates the contribution of the unspanned macroeconomic risks. As can be seen in the figure, these risks account for approximately 15 per cent of the variation for a 12-month futures contract. Unexpected changes in the real price of oil and in inflation account for small portions of the overall variation. The majority of the unspanned macroeconomic risks is news about the level of real economic activity. Returns on long dated futures are thus driven by news about the current and future state of crude oil inventory and the level of global demand for commodities.

The variance decomposition of the expected returns to the short roll strategy, $E_t r^{\text{ShortRoll}}_{t,n}$, is shown in panel B. The contribution of the level component of the convenience yields accounts for over 75 per cent of the variation in the expected returns to a 1-month roll but declines to less than 30 per cent when the roll is carried out for one year. The contributions of the second and third components account for approximately 10 per cent of the variation across all horizons. The contribution of the dollar bond components increase with maturity, reaching approximately 15 per cent of the variation for a one-year roll.

However, there is more than the immediate and future scarcity of oil driving future expected returns on this strategy. Indeed, at a 12-month horizon, unspanned macroeconomic risks account for approximately 45 per cent of the variation of the expected return

\textsuperscript{23}Recall that the VECM structure implies that the contribution of the nominal spot price of oil is zero once the real price of oil and inflation have been accounted for. See appendix for details.
to this strategy. Unexpected changes in unspanned real economic activity play a large role (approximately 20 per cent) as they affect the spot price in the future. There is also a role for the unspanned component of the real price of oil (approximately 15 per cent) due to its role as an error correction term in the VECM. Any deviation of the real price of oil from its long-run value is gradually corrected in the spot price of oil in future periods.

Panel C in Figure 5 shows the variance decomposition for the expected return on the $n$-month oil bond spread strategy, $E_t r_{\text{Spread},t+n}^{(n)}$. This strategy involves buying a $n$-month oil bond financed by selling a sequence of 1-month oil bonds. Consequently, the expected return to this strategy can be interpreted as the term premium component of long-term convenience yields. The large role of the second and third components from the convenience yield term structure is evident at longer horizons. News concerning anticipated future inventory levels is the largest contributor to risk premia on longer-run oil bonds – around 40% at the 12-month horizon. In addition, the unspanned component of the real price of oil also plays a large role as it is the payoff on the oil bond from the point of view of a U.S. investor.

For completeness, Panel D in Figure 7 shows the decomposition of the dollar bond term premia, $E_t r_{\text{Spread},t+n}^{(n)}$. Unsurprisingly, the dollar bond factors and unspanned inflation are the main contributors to the variability of expected bond returns, a result consistent with Joslin, Priebsch and Singleton (2012).

### 6.4 An alternative ordering

The variance decompositions of the futures trading strategies show an important role for the unspanned components of the macroeconomic variables. However, variation of the spanned portion of the macroeconomic variables will cause contemporaneous movements in the principal components of the dollar bonds and convenience yields. Thus, the previous decompositions do not yield a complete picture of the role of macroeconomic risks. We therefore construct variance decompositions for the futures trading strategies using the reverse ordering to the one above, namely $x_t = (m_t^0, b_t^0, c_t^0)$. Ordering the macroeconomic variables first allows us to assess the variation in the risk premia arising from the total (combined) effect of spanned and unspanned macroeconomic risks. Once again we measure the variation in the news arising over a 12 month horizon.

Figure 6 shows the results. The three bottom areas show the contributions of the three macroeconomic variables, $m_t = (rea_t, \Delta p_t, rpo_t)$. The first area is the contribution resulting from news about real growth, the second from news about the real price of oil and the third from news about inflation. The top two areas show the contributions from the dollar bond principal components (dark red) and convenience yield principal components (light red). The latter two areas arise from variation that is orthogonal to the macroeconomic variables.

The decomposition for the $n$-month holding period returns show that the real economic activity variable contributes approximately 5 per cent of the variation at a one-month
horizon and 20 per cent at a 12 month horizon. The largest role is reserved for the real price of oil, contributing just under one-half of the variation at a one-year horizon. There is a large difference between this result and the one obtained using the unspanned ordering in figure 5, where the real price of oil played a small role. This indicates that the spanned portion of the real oil price – the component that is captured by the projection of the real price of oil on the dollar bond and convenience yield curves – plays a large role in explaining returns on the holding strategy. The spanned portion of the real oil price is due to the effects of current and anticipated inventory levels that are related to the convenience yield principal components.

Variation in the expected return on the short roll strategy is driven in large part by total macroeconomic risks. At a one month horizon, the total macroeconomic risks account for just under 40 per cent of the variation. This figure rises to just over 50 per cent for doing a roll over a 12 month horizon. The sources of variation in the oil bond spread and dollar bond spread returns are shown in the bottom two graphs. Total macroeconomic risks account for approximately 70 per cent of the variation in the oil bond term structure at one year.

The top part of each of the graphs shows the portion of the variation arising from components of the oil bonds and dollar bonds that are orthogonal to macroeconomic variables. In the holding period returns, the influence of the dollar bond term structure is very small. However, the portion of the convenience yield curve that is orthogonal to both the macroeconomic variables and the dollar bond principal components is quite large, accounting for over 50 per cent of the variation at short horizons. At longer horizons it diminishes to 25 per cent as the role of the macroeconomic variables increases. The convenience yield principal components also have a large influence on variation of the short roll and oil bond spread expected returns. The principal components of the dollar bond term structure have a large influence on the dollar bond term premium. These results indicate that the term structures of dollar bonds and oil bonds both contain information about returns above and beyond that in (contemporaneous) macroeconomic variables.

There are three potential explanations for this additional source of variation in the oil bond principal components. First, as mentioned above, we have relied on macroeconomic variables that have been used in previous studies of the oil market or of the U.S. term structure. In particular, we have not undertaken an extensive search to find other macroeconomic sources of return variation. For example, Ludvigson and Ng (2009) use principal components from a large cross section of macroeconomic and financial variables that are able to reveal additional risk factors in the U.S. term structure. It may well be that a more extensive search would uncover additional unspanned macroeconomic risks in the crude oil term structure.

The second potential explanation is related to non-diversifiable factors that are specific to the oil market. For example, Hirshleifer (1988) argues that limited participation in the futures market would lead to an insufficient diversification of risks. Futures risk
premia would thus not depend on aggregate risks alone. In the model of Hong and Yogo (2012), gradual information diffusion and limits to arbitrage causes option returns to be related to open interest. Acharya, Lochstoer, and Ramadori (2013) argue that the constrained capital positions of speculators results in futures returns being linked to the firm-specific risks of commodity producers. Mou (2011) shows how speculators can undertake very short-term (e.g., 15 day) rolls to front run the trades of large commodity index funds. These rolls yield large Sharpe ratios. Etrula (2013) argues that the limited capital positions of financial intermediaries show up in commodity expected returns. Thus, all of these papers suggest that futures prices are removed from fundamentals due to an insufficient level of speculative capital.

The third potential explanation comes from the papers mentioned in the introduction. Juvenal and Petrella (2011) use a structural VAR to identify speculative shocks. They find that the shocks caused an increase in the spot price of oil that coincided with the 2004-2008 period of increased investments in oil derivatives. Tang and Xiong (2012) find an increased correlation among the returns on commodities that are part of the popular commodity indexes. Singleton (2013) shows that investor flow affects oil futures returns. Basak and Pavlova (2013) study the financialization of the oil market using a model with multiple goods and agents. They find that the prices of commodity futures increases with the financialization of the market. Thus, these papers suggest that the existing amount of investment capital has caused spot and futures prices to deviate from the underlying fundamentals.

While we cannot distinguish between the second and third potential explanations, our results suggest a number of cautions. First, an improved understanding of the returns to short and long-horizon speculative strategies can be obtained by using an asset pricing model. Second, the model should account for the dynamics of the convenience yield curve which differ from those of the risk-free curve. Third, any speculative position in crude oil futures will involve some combination of a spot position and positions in the oil bond and dollar bond markets. The returns on each of these positions are affected by unspanned macroeconomic risks in different ways.

7 Final remarks

In this paper, we present a dynamic affine term structure model of the crude oil futures market. Our key methodological contribution is to construct the term structure of crude oil convenience yields. The convenience yield can be interpreted as the discount rate on an oil bond that pays off a physical barrel of oil at maturity. We show that a simple principal components analysis of this curve gives insight into the theory of storage that has not been revealed before. While the level component in the convenience yield term structure is the price of the current scarcity of physical oil, the slope component reflects anticipated future scarcity, relative to today. The level risk is priced in the oil bond market and its
price of risk is driven in part by the slope component.

We show how expected return variation in the oil bond market, the dollar bond market
and a spot oil position are related to both inflation and real growth. We note that a more
extensive search among the large number of macroeconomic variables that are available
to researchers could result in additional risks being discovered. A key contribution of the
paper is to show that these macroeconomic risks are unspanned and thus unlikely to show
up in portfolio sorts.

We incorporate all of these empirical findings into a dynamic term structure model that
relies on the integrated nature of both the oil futures and U.S. Treasury bond markets. We
use the model to examine the drivers of returns on a holding period strategy, a short roll
strategy and an oil bond spread strategy. The unspanned macroeconomic risks and the
slope of the convenience yield term structure account for a large proportion of variation
in risk premia.

The results for the multi-period strategies provide us with a framework for thinking
about the relationship between macroeconomic risks and the time series of convenience
yields. We leave exploring for additional sources of risk for further work.
References


### Table 1

**Summary Statistics**

#### Panel A: Futures price of crude oil, yields and macro variables

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std Dev</th>
<th>Skewness</th>
<th>Excess Kurtosis</th>
<th>Autocorrelation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Spot price of crude oil (in US$)</strong></td>
<td>40.13</td>
<td>28.11</td>
<td>1.255</td>
<td>0.603</td>
<td>0.981</td>
</tr>
<tr>
<td><strong>Futures price of crude oil (in US$)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-month</td>
<td>40.13</td>
<td>28.19</td>
<td>1.229</td>
<td>0.487</td>
<td>0.985</td>
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<td>3-month</td>
<td>40.19</td>
<td>28.62</td>
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<td>0.987</td>
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<td>6-month</td>
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<td>29.45</td>
<td>1.173</td>
<td>0.133</td>
<td>0.990</td>
</tr>
</tbody>
</table>

**Bond yields (in % per year)**

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std Dev</th>
<th>Skewness</th>
<th>Excess Kurtosis</th>
<th>Autocorrelation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-month</td>
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<td>2.254</td>
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<td>-0.826</td>
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</tr>
<tr>
<td>3-month</td>
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<td>-0.023</td>
<td>-0.874</td>
<td>0.995</td>
</tr>
<tr>
<td>1-year</td>
<td>3.959</td>
<td>2.309</td>
<td>-0.115</td>
<td>-0.934</td>
<td>0.994</td>
</tr>
<tr>
<td>5-year</td>
<td>4.876</td>
<td>1.936</td>
<td>-0.087</td>
<td>-0.717</td>
<td>0.989</td>
</tr>
<tr>
<td>10-year</td>
<td>5.585</td>
<td>1.606</td>
<td>0.116</td>
<td>-0.670</td>
<td>0.987</td>
</tr>
</tbody>
</table>

**Convenience yields (in % per year)**

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std Dev</th>
<th>Skewness</th>
<th>Excess Kurtosis</th>
<th>Autocorrelation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-month</td>
<td>5.795</td>
<td>39.619</td>
<td>-0.637</td>
<td>10.377</td>
<td>0.487</td>
</tr>
<tr>
<td>3-month</td>
<td>6.075</td>
<td>25.705</td>
<td>-0.102</td>
<td>3.813</td>
<td>0.780</td>
</tr>
<tr>
<td>6-month</td>
<td>6.744</td>
<td>19.183</td>
<td>-0.077</td>
<td>1.543</td>
<td>0.852</td>
</tr>
<tr>
<td>1-year</td>
<td>6.847</td>
<td>13.742</td>
<td>-0.093</td>
<td>0.658</td>
<td>0.891</td>
</tr>
</tbody>
</table>

**Macro variables**

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std Dev</th>
<th>Skewness</th>
<th>Excess Kurtosis</th>
<th>Autocorrelation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$rea_t$</td>
<td>0.002</td>
<td>0.233</td>
<td>0.434</td>
<td>-0.357</td>
<td>0.957</td>
</tr>
<tr>
<td>$rpo_t = s_t - p_t$</td>
<td>0.686</td>
<td>0.098</td>
<td>-0.060</td>
<td>0.963</td>
<td>0.953</td>
</tr>
<tr>
<td>$\pi_t = \Delta p_t$ (in % per year)</td>
<td>2.701</td>
<td>3.219</td>
<td>-1.599</td>
<td>13.878</td>
<td>0.410</td>
</tr>
<tr>
<td>$\Delta s_t$ (in % per year)</td>
<td>1.705</td>
<td>36.826</td>
<td>-0.200</td>
<td>1.570</td>
<td>0.057</td>
</tr>
</tbody>
</table>

**Note**: Data are sampled monthly from April 1989 to March 2012.
Table 1
Summary Statistics (cont.)

Panel B: Annualized returns (in per cent) on $n$-month futures trading strategies

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std Dev</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>Excess Autocorrelation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Holding Strategy</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2-month</td>
<td>8.163</td>
<td>90.348</td>
<td>-1.038</td>
<td>5.438</td>
<td>0.612</td>
</tr>
<tr>
<td>3-month</td>
<td>8.599</td>
<td>75.662</td>
<td>-1.455</td>
<td>7.244</td>
<td>0.730</td>
</tr>
<tr>
<td>6-month</td>
<td>9.537</td>
<td>53.060</td>
<td>-1.599</td>
<td>5.648</td>
<td>0.858</td>
</tr>
<tr>
<td>1-year</td>
<td>10.094</td>
<td>31.322</td>
<td>-0.542</td>
<td>0.754</td>
<td>0.916</td>
</tr>
<tr>
<td><strong>Short Roll Strategy</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2-month</td>
<td>8.032</td>
<td>94.779</td>
<td>-0.930</td>
<td>4.560</td>
<td>0.615</td>
</tr>
<tr>
<td>3-month</td>
<td>7.849</td>
<td>81.618</td>
<td>-1.226</td>
<td>5.663</td>
<td>0.738</td>
</tr>
<tr>
<td>6-month</td>
<td>7.841</td>
<td>60.446</td>
<td>-1.284</td>
<td>3.742</td>
<td>0.873</td>
</tr>
<tr>
<td>1-year</td>
<td>8.168</td>
<td>42.390</td>
<td>-0.422</td>
<td>0.155</td>
<td>0.936</td>
</tr>
<tr>
<td><strong>Oil Bond Spread Strategy</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2-month</td>
<td>0.168</td>
<td>15.248</td>
<td>2.038</td>
<td>18.659</td>
<td>0.022</td>
</tr>
<tr>
<td>3-month</td>
<td>0.824</td>
<td>16.726</td>
<td>0.792</td>
<td>7.268</td>
<td>0.515</td>
</tr>
<tr>
<td>6-month</td>
<td>1.885</td>
<td>17.487</td>
<td>-0.189</td>
<td>4.337</td>
<td>0.818</td>
</tr>
<tr>
<td>1-year</td>
<td>2.364</td>
<td>17.791</td>
<td>-0.684</td>
<td>0.933</td>
<td>0.929</td>
</tr>
<tr>
<td><strong>Dollar Bond Spread Strategy</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2-month</td>
<td>0.037</td>
<td>0.125</td>
<td>1.549</td>
<td>3.806</td>
<td>0.144</td>
</tr>
<tr>
<td>3-month</td>
<td>0.074</td>
<td>0.181</td>
<td>0.924</td>
<td>0.772</td>
<td>0.559</td>
</tr>
<tr>
<td>6-month</td>
<td>0.189</td>
<td>0.334</td>
<td>0.817</td>
<td>0.273</td>
<td>0.848</td>
</tr>
<tr>
<td>1-year</td>
<td>0.438</td>
<td>0.627</td>
<td>0.539</td>
<td>-0.371</td>
<td>0.935</td>
</tr>
</tbody>
</table>

**Note:** Data are sampled monthly from April 1989 to March 2012.
**Table 2**
Factor Structure in Yields and Futures

**Panel A:** Per cent variation in term structures explained by the first $k$ PCs

<table>
<thead>
<tr>
<th>$k$</th>
<th>Bond yields</th>
<th>Convenience yields</th>
<th>Futures</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>95.03</td>
<td>93.33</td>
<td>99.74</td>
</tr>
<tr>
<td>2</td>
<td>99.80</td>
<td>99.64</td>
<td>99.99</td>
</tr>
<tr>
<td>3</td>
<td>99.97</td>
<td>99.97</td>
<td>99.99</td>
</tr>
</tbody>
</table>

**Panel B:** Correlations

<table>
<thead>
<tr>
<th></th>
<th>$b_{1t}$</th>
<th>$b_{2t}$</th>
<th>$b_{3t}$</th>
<th>$c_{1t}$</th>
<th>$c_{2t}$</th>
<th>$c_{3t}$</th>
<th>$\Delta s_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_{1t}$</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b_{2t}$</td>
<td>0</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b_{3t}$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$c_{1t}$</td>
<td>0.36</td>
<td>-0.03</td>
<td>-0.13</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$c_{2t}$</td>
<td>-0.30</td>
<td>0.01</td>
<td>0.00</td>
<td>0</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$c_{3t}$</td>
<td>0.04</td>
<td>-0.08</td>
<td>-0.18</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$\Delta s_t$</td>
<td>0.01</td>
<td>0.01</td>
<td>-0.12</td>
<td>-0.26</td>
<td>-0.03</td>
<td>0.27</td>
<td>1</td>
</tr>
</tbody>
</table>

**Note:** Data are sampled monthly from April 1989 to March 2012.
Table 3
PADD 2 Inventory changes and the slope of the convenience yield curve

<table>
<thead>
<tr>
<th></th>
<th>$n = 3$</th>
<th>$n = 6$</th>
<th>$n = 12$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>125.27</td>
<td>168.14</td>
<td>137.43</td>
</tr>
<tr>
<td></td>
<td>(0.54)</td>
<td>(0.56)</td>
<td>(0.58)</td>
</tr>
<tr>
<td>$\delta_t^{(n)} - \delta_t^{(1)}$</td>
<td>-65.68</td>
<td>-28.48</td>
<td>-28.06</td>
</tr>
<tr>
<td></td>
<td>(4.48)</td>
<td>(2.85)</td>
<td>(5.68)</td>
</tr>
<tr>
<td>$\Delta I_t$</td>
<td>0.68</td>
<td>0.79</td>
<td>0.87</td>
</tr>
<tr>
<td></td>
<td>(17.50)</td>
<td>(24.35)</td>
<td>(33.60)</td>
</tr>
<tr>
<td>adj.-$R^2$</td>
<td>0.50</td>
<td>0.65</td>
<td>0.81</td>
</tr>
</tbody>
</table>

**Note:** Data are sampled monthly from April 1989 to March 2012. Coefficient estimates of the predictive regression of future (PADD 2) inventory changes:

$$\Delta I_{t+n} = \gamma_0 + \gamma_1 [\delta_t^{(n)} - \delta_t^{(1)}] + \gamma_2 \Delta I_t + \epsilon_{t+n}.$$  

Newey-West robust $t$-statistics are given in parentheses.
Table 4
Unspanned risks

<table>
<thead>
<tr>
<th>LHS\RHS</th>
<th>PC1-PC3</th>
<th>PC1-PC5</th>
</tr>
</thead>
<tbody>
<tr>
<td>rea&lt;sub&gt;t&lt;/sub&gt;</td>
<td>13.84</td>
<td>17.62</td>
</tr>
<tr>
<td>rpo&lt;sub&gt;t&lt;/sub&gt; = s&lt;sub&gt;t&lt;/sub&gt; - p&lt;sub&gt;t&lt;/sub&gt;</td>
<td>30.22</td>
<td>39.28</td>
</tr>
<tr>
<td>Δp&lt;sub&gt;t&lt;/sub&gt;</td>
<td>9.85</td>
<td>11.36</td>
</tr>
<tr>
<td>Δs&lt;sub&gt;t&lt;/sub&gt;</td>
<td>15.76</td>
<td>18.90</td>
</tr>
</tbody>
</table>

Note: $R^2$ (in per cent) from contemporaneous regression of LHS variables on RHS variables (i.e. a constant, the $k$ first principal components of the term structure of bond yields, and the $k$ first principal components of the term structure of convenience yields).
Table 5
Factor Structure in Risk Premia

Panel A: Per cent variation in risk premia explained by the first \( k \) PCs

<table>
<thead>
<tr>
<th>( k )</th>
<th>Dollar Bond Risk Premia</th>
<th>Oil Bond Risk Premia</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>99.00</td>
<td>96.67</td>
</tr>
<tr>
<td>2</td>
<td>99.87</td>
<td>99.47</td>
</tr>
<tr>
<td>3</td>
<td>99.97</td>
<td>99.99</td>
</tr>
<tr>
<td>4</td>
<td>99.99</td>
<td>99.99</td>
</tr>
<tr>
<td>5</td>
<td>99.99</td>
<td>99.99</td>
</tr>
</tbody>
</table>

Panel B: Factor loadings of expected returns from first principal component of dollar and oil bond risk premia

<table>
<thead>
<tr>
<th>Maturity (in months)</th>
<th>2</th>
<th>3</th>
<th>6</th>
<th>12</th>
<th>24</th>
<th>60</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dollar Bonds</td>
<td>0.0006</td>
<td>0.0013</td>
<td>0.0031</td>
<td>0.007</td>
<td>0.015</td>
<td>0.0413</td>
<td>0.0871</td>
</tr>
<tr>
<td>Oil Bonds</td>
<td>0.1017</td>
<td>0.1794</td>
<td>0.2989</td>
<td>0.3821</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Panel C: Predictive Regressions

<table>
<thead>
<tr>
<th></th>
<th>Wald Tests</th>
<th>( R^2 ) (in per cent)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Time-varying Risk Premia</td>
<td>Unspanned Risks</td>
</tr>
<tr>
<td>Dollar Bond Risk Premium</td>
<td>33.54</td>
<td>12.61</td>
</tr>
<tr>
<td></td>
<td>[&lt;0.001]</td>
<td>[0.006]</td>
</tr>
<tr>
<td>Oil Bond Risk Premium</td>
<td>22.51</td>
<td>10.49</td>
</tr>
<tr>
<td></td>
<td>[0.013]</td>
<td>[0.015]</td>
</tr>
<tr>
<td>Spot Oil Risk Premium</td>
<td>25.84</td>
<td>11.02</td>
</tr>
<tr>
<td></td>
<td>[0.004]</td>
<td>[0.012]</td>
</tr>
</tbody>
</table>

Note: Data are sampled monthly from April 1989 to March 2012.
<table>
<thead>
<tr>
<th></th>
<th>RMSPE</th>
<th>MAPE</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Affine</td>
<td>OLS</td>
<td>Difference</td>
</tr>
<tr>
<td>Convenience Yields (in bps)</td>
<td>38.22</td>
<td>35.20</td>
<td>3.02</td>
</tr>
<tr>
<td>Bond Yields (in bps)</td>
<td>4.20</td>
<td>3.26</td>
<td>0.94</td>
</tr>
<tr>
<td>Oil Futures (in US cents)</td>
<td>8.51</td>
<td>7.06</td>
<td>1.45</td>
</tr>
</tbody>
</table>

Note: Affine model fit in basis points (bps) for yields and US cents for oil futures. RMSPE gives the root mean squared pricing error, and MAPE gives mean absolute pricing error. “Affine” provides the fit of the no-arbitrage term structure model, while “OLS” provides the model fit of a regression of yields (oil futures) on a constant, the nominal (log) spot price of oil, first three principal components of the term structure of bond yields, and the first three principal components of the term structure of convenience yields. “Difference” provides the loss of fit in basis points (cents) of estimating an affine term structure model instead of unrestricted OLS regressions.
### Table 7
Cross-sectional parameters

**Panel A:** Short-term nominal bond rate, $y_t^{(1)}$

<table>
<thead>
<tr>
<th></th>
<th>$1200 \times \psi_0$</th>
<th>$\psi_b$</th>
<th>$b_{1t}$</th>
<th>$b_{2t}$</th>
<th>$b_{3t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_t^{(1)}$</td>
<td>-0.362</td>
<td></td>
<td>-0.084</td>
<td>-0.177</td>
<td>0.321</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td></td>
<td>(&lt;0.001)</td>
<td>(&lt;0.001)</td>
<td>(0.002)</td>
</tr>
<tr>
<td></td>
<td>[&lt;0.001]</td>
<td></td>
<td>[&lt;0.001]</td>
<td>[&lt;0.001]</td>
<td>[&lt;0.001]</td>
</tr>
</tbody>
</table>

**Panel B:** Risk-neutral dynamics for nominal bond factors

<table>
<thead>
<tr>
<th></th>
<th>$1200 \times \mu_b^Q$</th>
<th>$\Phi_{bb}$</th>
<th>$b_{1t}$</th>
<th>$b_{2t}$</th>
<th>$b_{3t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_{1t}$</td>
<td>-0.201</td>
<td></td>
<td>0.999</td>
<td>-0.056</td>
<td>0.134</td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td></td>
<td>(&lt;0.001)</td>
<td>(&lt;0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td></td>
<td>[&lt;0.001]</td>
<td></td>
<td>[&lt;0.001]</td>
<td>[&lt;0.001]</td>
<td>[&lt;0.001]</td>
</tr>
<tr>
<td>$b_{2t}$</td>
<td>-0.140</td>
<td></td>
<td>0.005</td>
<td>0.982</td>
<td>0.155</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td></td>
<td>(&lt;0.001)</td>
<td>(&lt;0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td></td>
<td>[&lt;0.001]</td>
<td></td>
<td>[&lt;0.001]</td>
<td>[&lt;0.001]</td>
<td>[&lt;0.001]</td>
</tr>
<tr>
<td>$b_{3t}$</td>
<td>0.198</td>
<td></td>
<td>-0.004</td>
<td>-0.003</td>
<td>0.910</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td></td>
<td>(&lt;0.001)</td>
<td>(&lt;0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td></td>
<td>[&lt;0.001]</td>
<td></td>
<td>[&lt;0.001]</td>
<td>[&lt;0.001]</td>
<td>[&lt;0.001]</td>
</tr>
</tbody>
</table>

**Note:** Data are sampled monthly from April 1989 to March 2012. Asymptotic standard errors are given in parentheses, asymptotic $p$-values in square brackets.
Table 7
Cross-sectional parameters (cont.)

**Panel C:** Short-term convenience yields, $\delta^{(1)}_t$

<table>
<thead>
<tr>
<th>$\delta^{(1)}_t$</th>
<th>$\phi_c$</th>
<th>$c_1t$</th>
<th>$c_2t$</th>
<th>$c_3t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1200 \times \phi_0$</td>
<td>$\phi_c$</td>
<td>$c_1t$</td>
<td>$c_2t$</td>
<td>$c_3t$</td>
</tr>
<tr>
<td>$\delta^{(1)}_t$</td>
<td>-0.090</td>
<td>-0.499</td>
<td>-0.708</td>
<td>0.465</td>
</tr>
<tr>
<td>(0.006)</td>
<td>(&lt;0.001)</td>
<td>(&lt;0.001)</td>
<td>(0.001)</td>
<td></td>
</tr>
<tr>
<td>[&lt;0.001]</td>
<td>[&lt;0.001]</td>
<td>[&lt;0.001]</td>
<td>[&lt;0.001]</td>
<td></td>
</tr>
</tbody>
</table>

**Panel D:** Risk-neutral dynamics for oil bond factors

<table>
<thead>
<tr>
<th>$1200 \times \mu^Q_t$</th>
<th>$\Phi^Q_{ct}$</th>
<th>$c_1t$</th>
<th>$c_2t$</th>
<th>$c_3t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_{1t}$</td>
<td>-1.128</td>
<td>0.696</td>
<td>-1.185</td>
<td>1.080</td>
</tr>
<tr>
<td>(0.029)</td>
<td>(&lt;0.001)</td>
<td>(0.001)</td>
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**Note:** Data are sampled monthly from April 1989 to March 2012. Asymptotic standard errors are given in parentheses, asymptotic p-values in square brackets.
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**Note:** Data are sampled monthly from April 1989 to March 2012. Asymptotic standard errors are given in parentheses, asymptotic \(p\)-values in square brackets, and bootstrap \(p\)-values in curly brackets.
Figure 1: Crude Oil Futures Prices and Convenience Yields

Panel a: Crude Oil Spot and Futures Prices

Panel b: Convenience Yields
Figure 2. Term Structures

Panel a: Crude Oil Futures Curves

Panel b: Crude Oil Convenience Yield Curves
Figure 2. Term Structures (cont.)

Panel c: Dollar Bond Yield Curves
Figure 3. Factor Loadings: Yields

Panel a: Dollar Bond Yields

Maturity (months)

Panel b: Convenience Yields

Maturity (months)
Figure 4. Working Curve Estimates

Inventory (thousand barrels) vs. Annualized 1-Month Convenience Yield (percent) for PADD 2.
Figure 5. One-year ahead variance decompositions of expected returns on futures trading strategies (unspanned ordering)

Panel a: Holding Strategy

Panel b: Short Roll Strategy

Panel c: Oil Bond Spread Strategy

Panel d: Dollar Bond Spread Strategy
Figure 6. One-year ahead variance decompositions of expected returns on futures trading strategies (total ordering)

Panel a: Holding Strategy

Panel b: Short Roll Strategy

Panel c: Oil Bond Spread Strategy

Panel d: Dollar Bond Spread Strategy