Non-Marketability and the Value of Equity Based Compensation

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ABSTRACT
This paper uses the Benninga-Helmantel-Sarig (2005) framework to value employee stock options (ESOs) and restricted stocks units (RSU) in a framework which takes explicit account of employee non-diversification in addition to the standard features of vesting and forfeit of the stock options. This framework provides an endogenous explanation of early exercise of employee stock options. Incorporating non-diversification, we find that the pricing model is aligned with empirical findings of ESOs and results in lower values compare to alternative employee option pricing models such the widely-used Hull-White (2004) model. This pricing has implication for the FAS 123(R) for estimating the fair value of equity based compensation.

JEL classification: G12, G13, G32.
Keywords: Employee stock options, under pricing.

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Non-Marketability and the Value of Equity Based Compensation

1. Introduction

A large literature deals with the pricing and economic implications of employee stock options (ESOs), which are basically financial derivatives in incomplete markets and should be priced as such (Grasselli (2005)). In this paper we extend this literature to consider the impact of non-marketability on equity based compensation. Employing a model first developed by Benninga, Helmantel, Sarig (BHS, 2005), we incorporate both uncertainty and non-marketability into ESO and restricted stock units (RSU) pricing.\(^1\) Our model also allows us to take into consideration the differential access to capital markets of employees and the firm.

The ESOs valuation models can be divided to three general categories: Utility based models, lattice based models and continuous based models. The utility based models use a risk-averse utility function, and use the employee's certainty equivalent to establish early exercise. The lattice based models are usually modifications of the Cox-Ross-Rubinstein (1997) binomial model with an exogenous early exercise decision and the continuous based models are usually modification of the Black-Scholes-Merton model with exogenous early exercise decision as well.

The BHS model has two computational advantages over existing approaches to pricing ESOs. First, comparing to lattice and continuous based models, it provides an endogenous explanation of ESO early exercise. While early exercise of ESOs is widely documented,\(^2\) most ESO pricing models, such as Hull-White (2004), employ an arbitrary algorithm to explain early exercise.\(^3\) Comparing to the utility based models which also provide endogenous early exercise decision, the BHS model incorporate the utility model parameters into a single factor, and thus provide a simplified and flexible approach to describe exercise behavior and to compute the ESO value.\(^4\) The second advantage of the BHS model in pricing ESOs is that we are able to quantify the non-diversification effects. In a series of numerical examples we show that the cost of ESOs is less than that postulated by other numerical models. The implications of this for

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\(^1\) Previous literature has dealt with non-marketability by using the pricing of diversifiable risk and hedging (see, for example, Mozes 1994). The model of this paper uses an explicit pricing functional based on the binomial paradigm.

\(^2\) See, for example, Huddart and Lang (1999) and Damodaran (2005).

\(^3\) Hull and White (2004) assume that the holder of an ESO will early-exercise if the stock price \(S_t\) is a multiple \(m\) of the option exercise price \(X\). A common value for \(m\) is 2, but this is somewhat arbitrary. As we show in section 3, in our model early exercise is endogenous.

\(^4\) The utility-maximizing approach requires explicit specification of variables such as the employee risk aversion, her outside wealth, the proportion of her outside wealth comparing to her option wealth, the type of investment of her outside wealth etc. This, in addition to the computational difficulty of the utility based models, makes them very uncommon in practice.
the reporting of ESOs are significant, both economically and for the reporting of ESOs mandated by accounting regulations such as FASB 123(R). To show this, we compare between our model to the FASB mode, the Hull and White model (2004) and Brisley and Anderson model (2008), and show that our model yields lower valuation results.

The value vs. cost question has got only minor attention in the literature. From the one hand, Hall and Murphy (2002) claims that the cost to the firm is higher than the value to the employee, representing a deadweight loss to the issuing firm. As a result, for financial accounting purposes, what should matter is the company’s cost of granting an option (which is reasonably approximated by Black–Scholes) not the value of the option to the executive recipient. From the other hand, Chance (2004) and Chance and Yang (2004) claims that value equals cost, since as oppose to cash compensation, for example, granting stock options to executives provides them with incentives. This incentives provide added value to the firm (comparing to cash compensation), thus lowering the cost of the stock option granted. This discounted cost is conceptually parallel to the record of goodwill in the financial reports.5 Rubinstein (1995) and Zion and Carcache (2004) from Credit Suisse First Boston also support this view. Thus the main question relating to ESO grants is their cost to the employer and their value to the employee. Indeed, this question is the main issue of IFRS 2 and FASB 123R. To quote the latter:

IFRS 2 requires that all entities recognize an expense for all employee services received in share-based payment transactions, using a fair-value-based method that is similar in most respects to the fair-value-based method established in Statement 123 and the improvements made to it by this Statement.6

The question of measuring the expense of share-based payments such as ESOs has been dealt with in the literature in a variety of papers, both theoretical and computational.

In section 2, presents the implications of Benninga-Helmantel-Sarig (2005) model and study its implications on stock option pricing. Section 3 – theory. Section 4 implement the model and compares its implications to empirical findings. Section 5 compares various valuation models. Section 6 uses the Benninga-Helmantel-Sarig (2005) model to price restricted stocks. Section 7 uses empirical data to measure the non marketability of the pricing model. Section 8 discusses the accounting treatment of equity based compensation Section 9 concludes.

5 Ehrhardt (2004) also notes that the incentive characteristic of executive stock options has a “goodwill” nature (Chance (2004)).
6 According to statement FAS 123R (as a revision of FAS 123, Accounting for Stock-Based Compensation), it “requires a public entity to measure the cost of employee services received in exchange for an award of equity instruments based on the grant-date fair value of the award (with limited exceptions)...”.

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2. Imperfect markets, non-diversification, and the valuation of ESOs

We use a model developed by Benninga, Helmantel, and Sarig (BHS, 2005) to represent the impact of non-diversification on pricing. The BHS model represents pricing in a binomial framework and assumes that the non-diversified consumer has too much consumption in the good states and too little consumption in the bad states of the world. The resulting state prices of a non-diversified consumer will be *lower* than the market state prices in good states and *higher* than the market prices in bad states.\(^7\)

Let \(\{q_u, q_d\}\) represent the *public* price of $1 in an up/down state world, and let \(\{p_u, p_d\}\) represent the *private* price of $1 in an up/down state world, respectively. We assume that firms use the public state prices for valuation, whereas employees use the private state prices. We assume that:

\[
\begin{align*}
q_u \cdot U + q_d \cdot D &= 1 \\
q_u + q_d &= p_u + p_d = 1/R \\
p_u &< q_u, \ p_d > q_d \\
p_u &= q_u - \delta \\
p_d &= q_d + \delta
\end{align*}
\]

where

- \(r\) – The one period interest rate; \(R = 1 + r\)
- \(u\) – The one period move-up factor (in percentage); \(U = 1 + u\)
- \(d\) – The one period move-down factor (in percentage); \(D = 1 + d\)
- \(\delta\) – The spread between the public and the private state prices (this is the diversification measure)

The use of the same state prices by both the firm and employees assumes that the employees can trade freely in all the assets in the market (i.e., can create long and short positions). Differentiating between public and private state prices allows us to drop this assumption. Essentially, we assume that—as a result perhaps of trading and hedging restrictions on option grants—risk-averse employees are restricted in their diversification and are therefore exposed to some of the firm’s specific risk. The limitations on the stock option granted to the employee and on the employee hedging activity are designated to tie the employee to firm performance.\(^8\) The technical meaning of above assumptions is that both private and public state prices assume equal access to the borrowing/lending market and hence face the same borrowing rate. However, the private price for the up state \(p_u\) is lower than the public price for the same state \(q_u\) and the private price for the down state \(p_d\) is higher than the public price for the same state \(q_d\).

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\(^7\) State prices are the marginal rates of substitution adjusted for the consumer’s state probabilities and pure rate of time preference.

\(^8\) According to Damodaran (2005) this is the most common justification given by firms for employee stock options.
If state prices are computed using the probability-adjusted marginal rates of substitution, then the condition $p_u < q_u$, $p_d > q_d$ can be interpreted as meaning that the employee would like to transfer consumption from the good state to the bad state: Relative to his optimal consumption pattern, an employee has too much consumption in the good state and too little consumption in the bad state. $\delta$ is the spread between the public and private state prices that captures the diversification measure of the employee (or more precisely, the non-diversification measure). In other words, $\delta$ represents the higher tolerance to the firm’s risk of the well-diversified investor than that of the incompletely diversified employee (BHS 2005).9

Since $p_u < q_u$ and since an employee stock option pays off in the up states, it is obvious that the private valuation of an ESO is less than the public valuation. The lemma below shows that this will also be true for restricted stock:

**Lemma 1:** If $\frac{1}{R} - p_d = p_u < q_u = \frac{1}{R} - q_d$, the employee’s valuation of the firm using his private state prices is less than the same valuation using the public state prices. That is:

$$C_{\text{Private}} \equiv p_u \cdot U + p_d \cdot D < q_u \cdot U + q_d \cdot D \equiv C_{\text{Public}}$$

Where $C_{\text{Private}}$ and $C_{\text{Public}}$ denote the private and public value of the firm, respectively.

**Proof:** We assume that $p_u < q_u$ and $p_d > q_d$. Hence, $q_u - p_u > 0$ and $p_d - q_d > 0$. Since $u > d$, we have $(q_u - p_u) \cdot U > (p_d - q_d) \cdot D$, which can be rewritten to the result desired. ||

**Valuation effects of public versus private state pricing**

Throughout the paper, we will calculate the value of employee stock options for illustrative purposes and also compare different types of valuation models. Table 1 specifies the stock option characteristics of the sample stock option we use:

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9 It is natural to suppose that $\delta$ is higher as more restrictions are added to the stock option or as the stock’s price volatility increases. Meulbroek (2001) and others have noted that increase in volatility decreases ESO value. Higher employee risks may cause changes in employee behavior such as early exercise (Huddart 1999).
<table>
<thead>
<tr>
<th>Characteristics of sample stock option</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock price $S$</td>
<td>$50</td>
</tr>
<tr>
<td>Exercise price $X$</td>
<td>$50</td>
</tr>
<tr>
<td>Contractual option life $T$</td>
<td>10 years</td>
</tr>
<tr>
<td>Vesting period</td>
<td>3 years</td>
</tr>
<tr>
<td>Annual interest rate $r$</td>
<td>5%</td>
</tr>
<tr>
<td>Volatility $\sigma$</td>
<td>35%</td>
</tr>
<tr>
<td>Expected dividend yield $k$</td>
<td>2%</td>
</tr>
<tr>
<td>Exit (forfeit) rate</td>
<td>3%</td>
</tr>
<tr>
<td>Un-diversification measure $\delta$</td>
<td>0.01</td>
</tr>
<tr>
<td>Number of subdivisions of one year $n$</td>
<td>50</td>
</tr>
</tbody>
</table>

Figure 1 shows the valuation of a plain-vanilla call using public state prices in a binomial framework (the line labeled “BS,” since this valuation converges to the Black-Scholes model) and private state pricing:
In the private versus public state pricing model, the private state price for the up-state is less than the public price for the same state: \( p_U = q_U - \delta \). Figure 2 presents the estimation of plain vanilla stock options for different values of \( \delta \). The graphs assume that both the private and public prices face the same interest rate, so that \( p_D = q_D + \delta \) and \( q_U + q_D = p_U + p_D \).
Figure 2: Comparing the price of a stock option for different values of $\delta$. Exercise price = 50; Time to expiration = 5 years; Annual interest rate = 5%; Volatility = 35%; Number of subdivisions of one year (for the binomial framework) = 50.

It is clear that the private state pricing model leads to possible early-exercise of even plain-vanilla calls on non-dividend-paying stocks. It is thus also clear that this same model will lead to endogenous early exercise for employee stock options. Figure 3 presents the incorporation of ESO characteristics along with the private state pricing.
**ESO valuation with differential access to capital markets**

The previous section dealt with the case where employees are under-diversified, but have equal access to borrowing and lending markets. In this section we assume an additional market imperfection in which the employee and the firm have an unequal access to the borrowing/lending market. In practice, small investors (non-executive employees in this case) usually face a higher borrowing/lending interest rate than investors/firms with large amounts of collaterals (Berk and DeMarzo (2007)). We hypothesize that this market imperfection will drive a higher valuation wedge between the valuation of ESOs (as seen by the firm and as seen by the employee) than in the case discussed in the previous section, of different state prices but with equal access to the borrowing/lending market.

Let \( \{ q_u, q_d \} \) represent the public price of $1 in an up/down state world, and let \( \{ p_u^*, p_d^* \} \) represent the private price of $1 in an up/down state world, respectively. We assume that:
where $\delta_u$ is the difference between the private and the public price in up state, $\delta_d$ is the difference between the private and the public price in down state, $R^*$ is employee's gross borrowing/lending rate and the $R$ is the firm's gross borrowing/lending rate. The technical meaning of the assumptions is that the difference between the private and the public price for the up state ($\delta_u$) is higher than the difference between the private and the public price for the down state ($\delta_d$), as presented in the following lemma.

**Lemma 2**: If $\frac{1}{R} = q_u + q_d > p_u^* + p_d^* = \frac{1}{R^*}$, the difference between the private and the public price for the up state is higher than the difference between the private and the public price for the down state. That is: $\delta_u > \delta_d$.

**Proof**: We assume that $R^* > R$, or $\frac{1}{R} - \frac{1}{R^*} > 0$. Thus, $q_u + q_d - \left( p_u^* + p_d^* \right) = \delta_u - \delta_d > 0$. ||

We hypothesize that the differential access to capital markets of employees and the firm drives a higher valuation wedge between the valuation of ESOs (as seen by the firm and as seen by the employee) than in the case discussed in the previous section, of different state prices but with equal access to the borrowing/lending market. We test this hypothesis using the following lemma.

**Lemma 3**: Assuming $R^* > 0$ and $\frac{1}{R} = p_u + p_d > p_u^* + p_d^* = \frac{1}{R^*}$, then $\delta_u > \delta > \delta_d$.

**Proof**: From $\frac{1}{R} = p_u + p_d > p_u^* + p_d^* = \frac{1}{R^*} > 0$ it follows that $p_u - p_u^* > p_d - p_d^* > 0$. Then $p_u - p_u^* \Rightarrow \delta_u > \delta$ and $p_d - p_d^* \Rightarrow \delta > \delta_u$. From transitivity, it follows that $\delta_u > \delta > \delta_d$. ||

### 3. Theoretical considerations

A paper by Bick (1987) shows that geometric Brownian motion for a stock price is compatible with a utility function if and only if the utility function exhibits constant relative risk aversion and the consumption process is multiplicative. It follows that only in the cases
described by Bick is the Black-Scholes pricing for European options underpinned by utility foundations.

In the case described in this paper, Black-Scholes is not the pricing function for European options. Hence it follows from Bick that the consumption process is not multiplicative or that the utility function is not CRRA.\textsuperscript{10}

4. Private state pricing: Numerical implications

In this section we compare the predictions of the private state pricing model to the empirical findings presented in the literature. These comparisons are intended to verify that the private state pricing model is aligned with empirical findings, indicating that the model is suitable to value ESOs.

Calculating the stock price to exercise price ratio at the exercise date

Our model allows us to calculate the implied ratio of the stock price to exercise price at the exercise date (this ratio is also known as the Hull and White (2004) multiple $M$). Assuming that the employee exercises the stock option when the private value is lower than the intrinsic value (after the vesting period), we use the firm’s stock price at this date and derive the stock price to the exercise price ratio. The following figure presents the change in the stock price to exercise price ratio as a function of $\delta$ for different stock option characteristics.

\textsuperscript{10} Note that any binomial model and any utility function necessarily give rise to a set of state prices and a (binomial) pricing function for options. However only in the case that the Bick assumptions hold (they evidently do not in the Benninga-Helmantel-Sarig case) do we get Black-Scholes.
Table 2 reports the empirical findings regarding the stock to exercise price ratio. It suggests that the implied ratio calculated using the private state pricing is within the empirical findings range. Carpenter et al. (2008b) also provide estimations on the stock/exercise price across industries. Their data also indicates on a large variation in the ratios both across and within sectors, with very high ratios reflecting the run-up in the stock market during our sample period.
### Table 2:

**Empirical data on the stock price to exercise price ratio**¹¹

<table>
<thead>
<tr>
<th></th>
<th>Average</th>
<th>Median</th>
<th>Quartile</th>
<th>Quartile</th>
<th>Sample</th>
<th>Sample period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Huddart and Lang (1996)</td>
<td>2.2</td>
<td>1.6</td>
<td>1.283</td>
<td>2.5</td>
<td>All employees</td>
<td>Late 80s – early 90s</td>
</tr>
<tr>
<td>Carpenter (1998)</td>
<td>2.75</td>
<td>2.47</td>
<td>1.15</td>
<td>8.32</td>
<td>1.42 Executives</td>
<td>1979 - 1994</td>
</tr>
<tr>
<td>Bettis, Bizjak, Lemmon (2005)</td>
<td>3.55</td>
<td>2.57</td>
<td>1.04</td>
<td>17.34</td>
<td>Corporate insiders</td>
<td>1996 - 2002</td>
</tr>
</tbody>
</table>

The empirical findings show substantial variation in stock appreciation subsequent to the option grants. Possible explanations are the variation in the samples and the difference in the sample periods as well. Using the private state pricing perspective, it can be explained as higher delta for lower-level employees: a higher delta means that the employee places higher value for the up state, and therefore will exercise earlier and will have a lower ratio of the stock to exercise price.

**Calculating the ratio of the private value to BS value at the exercise date**

Table 3 reports the empirical findings regarding the ratio of intrinsic value (the stock price minus the exercise price) to the remaining American option value. However, while Bettis et al. (2005) measure that this ratio at the exercise date, Huddart and Lang (2003) measure it for an average month, and not just in the exercise date.¹² Thus, Bettis et al. (2005) data is more relevant to our outcomes. The sample data indicates on a large variation in this ratio as well. It suggests that the implied ratio calculated using the private state pricing is within the empirical findings range.

¹¹ Hemmer, Matsunaga and Shevlin (1996) report similar empirical findings on the stock to exercise price ratio.

¹² In addition, Huddart and Lang (2003) estimate the option value at time t using the Barone-Adesi and Whaley (1987) formula.
Table 3:
Empirical data on the ratio of intrinsic value to the remaining American option value

<table>
<thead>
<tr>
<th>Huddart and Lang (2003):</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th>Sample</th>
<th>Sample period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>Median</td>
<td>Quartile</td>
<td>Quartile</td>
<td>S.D.</td>
<td>Sample</td>
<td>Sample period</td>
</tr>
<tr>
<td>0.7423</td>
<td>0.7915</td>
<td>0.5544</td>
<td>0.965</td>
<td>0.2308</td>
<td>All employees</td>
<td>1985 - 1994</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bettis, Bizjak, Lemmon (2005)</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th>Sample</th>
<th>Sample period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>Median</td>
<td>1st percentile</td>
<td>99th percentile</td>
<td>Sample</td>
<td>Corporate insiders</td>
<td>Sample period</td>
</tr>
<tr>
<td>0.9</td>
<td>0.84</td>
<td>0.12</td>
<td>1</td>
<td></td>
<td>Corporate insiders</td>
<td>1996 - 2002</td>
</tr>
</tbody>
</table>

5. Comparison of valuation models

Categories of ESOs valuation models

Generally, we can divide the ESOs valuation models to three categories: Utility-maximizing models, lattice models and continuous models. Utility-maximizing models assume that early exercise occurs according to a policy that maximizes the employee’s expected utility subject to hedging restrictions. Examples include Marcus and Kulatilaka (1994), Huddart (1994) and Rubinstein (1995). Lattice models are some kind of a “code name” used in financial reports to the binomial option pricing model and Monte Carlo simulations. Examples for lattice models include Hull and White (2004), Ammann and Seiz (2004) and Brisley and Anderson (2008). Continuous models are usually variants of the Black-Scholes-Merton model that incorporate ESOs restrictions. Appendix A provides a survey of the ESO pricing models from all three categories.\(^{13}\)

Comparisons of ESOs valuation models

We compare between several valuations models of ESOs: the FASB model, Hull and White (2004), Cvitanić, Wiener, and Zapatero (2007) and Brisley and Anderson (2008). We show that the value of the stock option decreases after employing the non-marketability effect.

We begin with a short presentation of the valuation models' principals:

- **Hull and White (2004):** Incorporate dividend yield, vesting period, exit rate and exogenous early exercise decision which is based on the stock price to exercise price ratio. Denoting $M$ as the stock to exercise price ratio, the employee exercises the stock option at time

\(^{13}\) According to Chance (2004) and Brisley and Anderson (2008), the need to value employee stock options caused more and more companies to move from different variants of the Black-Scholes option pricing model towards lattice models such as the HW model, since it provides lower ESOs estimations.
the HW model is one of the most popular model to value ESOs for expensing purposes under FASB 123R (Brisley and Anderson (2008)).

- **Brisley and Anderson (2008):** Incorporate dividend yield, vesting period, exit rate and exogenous early exercise decision which is based on a fixed proportion of $\mu$ from the Black-Scholes value at the exercise date. Thus, the employee exercises the stock option at time $t$ if

$$\max[S_t - X, 0] > (M-1)X.$$  

- **FASB model:** Incorporate dividend yield, vesting period and exit rate. Accounts for early exercise by setting the maturity of the stock option equal to the option’s expected life. In addition, the adjustment of the stock option to the vesting period and the exit rate is different from other comparable valuation models.

- **Cvitanić, Wiener, and Zapatero (2007):** Incorporate dividend yield, vesting period and exit rate. The early exercise barrier here is similar to the one of Hull and White's (2004), but the barrier here is decreasing (at a rate of $\alpha$) as the option approaches expiration.

In the comparison below, we use a Hull and White multiple $M$ of 2.2, a Brisley and Anderson Multiple $\mu$ of 6, and a FASB expected life of 6 years.

Figure 5: Comparison between following ESOs valuation models: Hull and White (2004); Brisley and Anderson (2008); FASB proposal and the private state pricing valuation.

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14 The International Accounting Standard Board and the Canadian Accounting Standard Board have similar rules to the FAS 123R rule. FAS 123R standard specifically cites the HW model and illustrate its use with numerical examples (U.S. SEC Staff Accounting Bulletin No. 107 also refers to the HW model) (Brisley and Anderson (2008)).
From our computations it thus follows that most existing models of ESO valuation, which are based on permutations of the Black-Scholes pricing formula at market prices, over-cost the ESO.

The non-marketability effect
Carpenter (1998) and Ammann and Seiz (2004) compare between utility maximizing models and lattice (binomial) models. Carpenter (1998) calibrated sample averages to the models and tests the results in predicting actual exercise patterns. She finds that a simple model combining the ordinary American option exercise policy with random, exogenous early exercise and forfeiture describes exercise patterns just as well as an elaborate utility maximizing model that explicitly accounts for the non-transferability of options. This result suggests that the simpler model is equally good for valuing ESOs.

Ammann and Seiz (2004) compare the following ESOs pricing models: the utility maximizing models of Kulatilaka and Marcus (1994), Huddart (1994), Rubinstein (1995); and the lattice models of Hull and White (2002), the FASB 123 model and the author's model. They perform numerical comparisons of the models and find that although the exercise scheme is determined differently in each model, the models yield similar results. They conclude that despite the different exercise policy of each model, the pricing effect is negligible if the models are calibrated to the same expected option life. The other parameters, such as the utility based parameters, are relevant only considering their impact on the expected life of the stock option.

A possible interpretation of the findings of Carpenter (1998) and Ammann and Seiz (2004) is that the non-marketability effect of ESOs is not priced in the utility maximizing models. Consider, for example, an untradeable European stock option, without vesting period and forfeit or forced exercise. Under the findings of Carpenter (1998) and Ammann and Seiz (2004) it seems that the utility maximizing models will yield in this case the same value as the lattice models (which, in this case, will be equal to the Black-Scholes value). It follows that the utility maximizing models do not incorporate non-marketability and therefore does not consider all the characteristics of ESOs.

\[\text{Ammann and Seiz (2004) model is specified in Appendix A.}\]
6. Private state pricing model: valuation of restricted stocks

Restricted stocks

Restricted stocks are awards of the company stock to employees, in which the employee’s rights in the stock are restricted until the shares vest. Once the vesting requirements are met, the employee owns unrestricted shares. There are two popular forms of restricted stocks: restricted stocks (RS) and restricted stock units (RSU). The key difference is that RSU are delivered to the employee only after the vesting and forfeit requirements are met while restricted stocks are delivered at the grant date. Thus, unlike standard restricted shareholders, RSU holders have no voting rights and usually are not entitled to receive dividend or dividend equivalents. We use the private state pricing model to value RSU.

Valuation of restricted stock units

Damodaran (2005a) specify three general characteristics of restricted stock that can affect value. The first is the mandatory employment during the vesting period. The second is the illiquidity of the restricted stock: The illiquidity premium depends upon the period of illiquidity, hedging / borrowing constraints and stock volatility (which increases the cost of illiquidity). The third is a performance contingency: If the employee receives the stock only if a performance condition is met (in terms of revenues or earnings, for example), the value of the restricted stock should reflect the likelihood of this scenario.

Empirically, the evidence on illiquidity discount stems from two types of studies (Damodaran 2005b). The first estimation examines restricted stocks issued by publicly traded firms. The issuance price of the restricted stock is usually lower than the market price of the stock, and the difference is attributed to the illiquidity of the restricted stock.\(^{16}\) The second estimation of the illiquidity discount is to compare the IPO stock prices of firms to the price of the same shares prior to the IPO.

Under the first estimation type (which is more relevant to our case), Maher (1976) find an average discount of 35.43% comparing to publicly traded stocks in analyzing purchases made by 4 mutual funds during 1969-1973. Silber (1991) find a median discount of 33.75% during the period 1981-1988. Johnson (1999) finds a smaller discount of 20%. One of the criticisms on these empirical findings is that the investors in those private placements may also provide other services to the firm, and the discount reflects it (Damodaran 2005b). This service difference was isolated by comparing the unregistered private placements to registered private placements of equity by the same companies, and thus provides a better measure of the marketability discount. Wruck (1989) estimated an average (median) discount of 17.6% (10.4%)\(^{16}\)

\(^{16}\) In this respect, restricted stocks are not registered with the SEC, and sold through private placements under SEC rule 144. The stocks cannot be resold in the open market for a one (two) year period from (before) 1997, and limited amounts can be sold afterwards.
and Hertzel and Smith (1993) find a median difference of 13.2% between the two placement types (and a median discount of 13.26% across all private placements). Bajaj et al. (2001) report attribute 7.2% for marketability discount after controlling for differences across firms.

Valuing restricted stocks using private and public state pricing

We can use the BHS (2005) model to estimate restricted stocks as well. Restricted stock, however, has different characteristics than ESOs, and therefore the modeling should be different. The major difference is that restricted stocks are tradable after the vesting period, whereas stock options are not. Therefore, we model restricted stock in the following manner:

During the vesting period, the stocks are restricted and also can be forfeit upon job termination. The stocks are also non tradable. Therefore, in the vesting period we use private state pricing and also assume an exogenous exit rate. After vesting, the stock is tradable and unlike stock options does not subject to forfeit of forced exercise. Thus, in this period we will use the public state prices without any other restrictions.

Our pricing methodology considers explicitly the non-marketability of the restricted stock. This feature, along with the possibility of job termination, lowers the restricted stock value. Note that if the employee would be compensated in a tradable stock, our pricing would have been equal to the stock price. Throughout the paper, we will use the charatheristics of Table 4 for illustratrative porpuses. Table 4 specifies the restricted stock characteristics:

| Table 4: |
| Characteristics of sample restricted stock | Values |
| Stock price $S$ | $50 |
| Exercise price $X$ | $50 |
| Contractual option life $T$ | 10 years |
| Vesting period | 3 years |
| Annual interest rate $r$ | 5% |
| Volatility $\sigma$ | 35% |
| Expected dividend yield $k$ | 0% |
| Exit (forfeit) rate | 3% |
| Un-diversification measure $\delta$ | 0.01 |
| Number of subdivisions of one year $n$ | 40 |

Figure ?? presents the value of restricted stock as a function of the undiversified measure $\delta$. 

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A restricted stock value relative to \( \delta \)

Figure 6: The value of restricted stock (using table 4 characteristics) relative to un-diversification measure \( \delta \).

The value of the restricted stock (at the grant date) is lower as the un-diversification measure \( \delta \) is higher during the vesting period.

7. **Estimation the non-marketability measure \( \delta \) using empirical data\(^{17} \)**

We can in principle reverse engineer empirical data about ESOs to estimate \( \delta \). Here are some methods we are currently exploring:

- Valuing \( \delta \) using the stock price to exercise ratio at the exercise date.
- Valuing \( \delta \) using the proportion of the intrinsic value compared to Black-Scholes value at the exercise date.
- Valuing \( \delta \) using the ratio of employee stock options to restricted stocks, which is around one third in most of the corporations that allow this substitution. Under this estimation, \( \delta \) is incorporated both in the ESOs as well as in the restricted stock.

\(^{17} \) This section is still incomplete.
Estimating delta using the stock price to exercise ratio
In this subsection we value δ using a given stock price to exercise price ratio. For a given stock option parameters (such as exercise price, volatility, interest rate etc.) we can calculate what should be the un-diversification measure of the employee using his preferred stock price to exercise price ratio at the grant date.

We are currently exploring the availability of empirical data to implement this method.

Estimating delta using the ESO/restricted stock ratio
Another possibility to infer δ from empirical measures is to use the substitution rate between ESOs and restricted stocks. The substitution ratio between these two equity based compensation forms is used in "reality" in business corporations and we can apply it here. For example, on December 2005, Intel announced that its employees will receive much more of their equity compensation in restricted stock rather in stock options. Intel announced that the vesting period will remain the same (four-year period), but the RSUs will be about a third fewer than they would have received in stock options. In the popular press reports, a substitution ratio between one third and half is common.

We try to infer the value of δ using the substitution ratio between ESOs to restricted stock. Since δ is a joint pricing factor in both ESOs and restricted stocks, we estimate the value of δ for a given substitution ratio. The results are presented in Figure 7.

Figure 7: Estimation of $\delta$ for a given substitution ratio of the ESO value to the restricted stock value.

According to our results, a substitution ratio of one third between stock options to restricted stocks (as Inter uses) would result in a $\delta$ of 0.0135 for the sample ESO and restricted stock.

8. Accounting treatment of equity based compensation

The accounting standard of equity based compensation

The numerical valuation of ESOs has become a heated issue, dealt with in both IFRS 2 and FASB 123(R). Both documents are unclear on how this value is to be measured. Statement 123(R) establish the objective that the compensation cost associated from equity based compensation should be measured based on the value of employee services received in exchange for the equity instrument. Quoting FASB:

The objective of accounting for transactions under share-based payment arrangements with employees is to recognize in the financial statements the employee services
received in exchange for equity instruments issued or liabilities incurred and the related cost to the entity as those services are consumed.\textsuperscript{20}

However, FASB recognize that it is not feasible to measure directly those employee services, and as a result concluded that the measurement should be based on the fair value of the equity instruments issued. This decision, according to FASB, is consistent with the measurement basis of other forms of employee compensation including cash and pension benefit. To see the confusion inherent in the definitions, consider the following quote from FASB:

This Statement requires a public entity to measure the cost of employee services received in exchange for an award of equity instruments based on the grant-date fair value of the award (with limited exceptions). That cost will be recognized over the period during which an employee is required to provide service in exchange for the award—the requisite service period (usually the vesting period). No compensation cost is recognized for equity instruments for which employees do not render the requisite service. Employee share purchase plans will not result in recognition of compensation cost if certain conditions are met; those conditions are much the same as the related conditions in Statement 123.\textsuperscript{21}

In general, most of the literature regarding the valuation of employee stock options acts as the accounting standard: It does not distinguish between the terms value and cost, and use them interchangeably. However, one of the major disputes regarding the recognition of ESOs as an expense in the financial reports was basically a cost-value issue.\textsuperscript{22} In addition, since today stock options are recorded as an expense in the financial reports, it's important to relate to a possible distinction between those terms, as discussed by some authors. Reviewing the literature, we can try and infer the author’s view regarding the relation between value and cost. We choose, however, to refer only to the literature that relates explicitly to this issue.

**Employee stock options: Is its cost differs from its value??**

We begin with the definitions of the cost of an ESO and the value of an ESO: The value of an ESO to the employee (executive) is her certainty equivalent – it’s the risk-less cash compensation amount that the employee is willing to exchange in lieu the stock option (hereinafter: value). The cost of an ESO to the granting firm is the cash amount the firm could

\textsuperscript{21} http://www.fasb.org/st/summary/stsum123r.shtml. The fair value recognition appears also in the Australian accounting standard, AASB 2 and in Canadian accounting standard, CASB.
\textsuperscript{22} In the debate regarding the recognition of ESOs compensation in the accounting reports, the opponents claim that the widely used valuation models are not suitable to value ESOs which are not tradable. The supporters of expensing ESOs claimed that since its part of compensation, it must be expensed.
have received in case it sells this stock option to an outside investor rather then granting it to the employee (executive) (hereinafter: cost).

We start with the notion that the cost to the granting firm is bigger that the option value to the employee. Hall and Murphy (2002) argue that traditional valuation formulas such as the Black-Scholes option pricing formula provide an adequate estimation to the firm’s cost. However, since the options are granted to an undiversified and risk-averse executive, she places a lower value on the stock option. The gap between the cost and the value reflects the unattractiveness of the stock option to the executive. According to Hall and Murphy (2002), the firm should expense its economic cost. The notion that cost is bigger than value is also mentioned by Kadam et al. (2005), Ignersoll (2006) Cvitanić et al. (2007), Cai and Vijh (2004) and Carpenter (1998, 2008).

The other view, in which we support, claims that cost equals value. This notion is supported by Chance and Yang (2004) and Rubinstein (1995). Chance and Yang (2004) identify an inconsistency in Hall and Murphy (2002) line of reasoning. They argue that Hall and Murphy (2002) ignore the incentives effect provided by the stock options, and values the options strictly as an alternative to cash compensation, without any other benefit that relates to the use of stock options. Adopting Hall and Murphy (2002) line of reasoning, Change and Yang (2004) propose pareto-optimal alternatives to stock option compensation. For example, the firm could sell the stock option to an outside investor and pay the proceeds to the executive. In this case the firm’s cost shall be the same, but the executive (facing a lower value for stock options) will be better off. However, the difference between all these alternatives to ESOs grants is that the executive would not have incentive and will probably liquidate.

Chance and Yang (2004) also identify an inconsistency in the computational procedure. This procedure, under which the stock option value (calculated using risk-neutral probabilities and discounted using the risk free rate) is lower than the option’s cost. Hence, it violates the law of one price. This is not surprising, since as mentioned arbitrage opportunities exist. According to Chance and Yang (2004), this problem would not exist if value equals cost.

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23 These definitions are presented, for example, by Carpenter (1998) and Hall and Murphy (2002).
24 This cost estimation ignores complications that relates to ESOs, such as early exercise and potential forfeiture (Hall and Murphy (2002))).
25 The “unattractiveness” includes illiquidity, possible forfeit etc.
26 See also Chance (2004).
27 Three additional examples of Chance and Yang (2004) to pareto-optimal alternatives are: buying a stock option in the open market and grant it to the executive; pay the executive its cost in cash instead using stock options; or pay the executive with $N(d_1)$ stocks and a loan with a variable face value of $N(d_2)X$.
28 The term “incentives” is used in the board sense, which in addition to creating incentives to increase firm value, include also retention, sorting and attraction of employees. In this sense, stock options provide incentives also to non-executive employees. See, for example, Ittner et al. (2001), Core and Guay (2001), Oyer and Schaefer (2005) and Kedia and Mozumdar (2002).
Chance (2004) presents additional argument which respect to early exercise of ESOs. Hall and Murphy (2002) argue that early exercise reduces the cost to the firm, saving the time value of the stock option. Thus, according to this argument companies would always want executive to exercise early, reducing cost to the firm. However, companies impose vesting periods on options holders, and substitute vesting periods (which create incentive and retain employees) with costs. Hence, these incentives should be reflected in the fair value estimation of the stock options.

Additional support for the notion that value equals cost is presented by Rubinstein (1995) and Zion and Carcache (2004) from Credit Suisse First Boston. Rubinstein (1995) mentions that if the potential for early exercise exists, there should be no difference between the value of the option to its cost. Zion and Carcache (2004) mention that ESOs are a potential claim on the firm’s future cash flows, and the best estimation of the fair value of that claim is the amount that the firm would pay to its employees to settle the obligation.

It is interesting to mention that most of the authors that support the view the cost is bigger than the value attribute this difference to incentive effects. So, basically, the argument can be narrowed to reporting of incentives: should only the cost be reported, or the reduced cost (i.e., value), which takes into account the firm's benefit from incentive of equity based compensation?29

We support Chance (2004) and Rubinstein (1995) view that the value of stock option to the employee is equal to the cost to the granting firm, since the stock options represents the alternative wage replaced by the ESO and hence the economic cost of the option (Putting differently, we disagree with the cost definition of Hall and Murphy (2002) which ignores the incentives value). A different interpretation implies sub-optimal compensation or sub-optimal cost (an mentioned by Chance and Yang (2004)): If cost is bigger than value than the firm cost from option grants is bigger than the alternative cash compensation or that the employee stock option compensation is lower than his alternative cash compensation. Thus, the rational costing of the ESO is to value the option through the eyes of the employee.

From the accounting treatment perspective, we believe that since the cost to the granting firm equals to the value of the employee, the accounting expense on ESOs should reflect it. First, recall that the objective of accounting standard is to recognize in the financial statements the employee services received in exchange for equity instruments issued. This view is expressed literally in the accounting standard itself, which demands estimation of the fair value of the ESO, and includes incentive value. Second, as mentioned by Chance (2004), conceptually it is not much different from goodwill. When a company purchases another company for more than book value, it records goodwill. In issuing an option, the company

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29 For example, Carpenter (1998) view and Rubinstein (1995) and Chance (2004) views are the opposite of each other, but they both share the same reasoning.
receives a form of goodwill from the option holder: the incentives lead to loyalty and effort over a future period. Third,

**Accounting treatment of restricted stocks**

According to the accounting standard, restricted stocks should be estimated at the grant date. The restricted stock value is treated by the firm as a compensation cost. Like ESOs, the value of the restricted stock is spread over the vesting period. According to FASB, in the case of restricted stocks the fair value shall be the market price of a share of the non-restricted stock on the grant date. The reason is that like ESOs, restrictions on transferability and hedging during the vesting period are not considered in estimating the fair value of the share based compensation. Quoting FASB:

“The fair value of a share of nonvested stock (usually referred to as a restricted stock) awarded to an employee is measured at the market price of a share of a non restricted stock on the grant date unless a restriction will be imposed after the employee has a vested right to it, in which case fair value is estimated taking that restriction into account.”

9. **Conclusion and summary**

This paper examines the valuation of employee stock options in markets typified by unequal access and lack of employee diversification. Our base-case model, Benninga-Helmantel-Sarig (2005), provides a simple framework for pricing these options.

The main results of the paper:

- The BHS model provides an endogenous explanation of early exercise of ESOs.
- Valuation of the ESO in the BHS framework shows that the cost of the options has been overestimated by most existing models.

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Appendix A

In this appendix we survey the valuation models of employee stock options. We divide the valuation models into 3 categories: Utility maximizing models, lattice models and continuous time models (most of them are variants of the Black-Scholes option pricing model). We present the adjustments for ESOs features and the early exercise decision of each valuation model.

Utility maximizing models

The utility maximizing models approach is based on the assumption that the employee checks his expected utility from holding the stock option comparing to his expected utility from early exercise subject to hedging and vesting restrictions. Early exercise occurs when the latter is higher (“expected utility role”). This approach is implemented on discrete lattice models (usually the binomial framework) or on continuous models (as variants of the Black-Scholes option pricing model). If, for example, the binomial framework is used, a ‘utility tree’ is generated instead of a ‘stock tree’, and is being used to determine when the employee will use early exercise.

The utility-maximizing approach requires explicit specification of the model variables such as the executive’s risk aversion, his outside wealth, the proportion of his outside wealth comparing to his option wealth, the type of investment of his outside wealth and the potential gain from changing his employment. Many of these factors are difficult to measure directly or to infer from publicly observable data.

Lambert, Larcker and Verrecchia (1991): one of the first utility maximization models for measuring ESOs value. It ignores the possibility that the stock options are exercised early or forfeit rate.

Kulatilata and Markus (1994): use a CRRA utility function and a binomial framework (with risk-neutral pricing), and show that ESOs are worth much less than suggested by the conventional models. Early exercise occurs according to the expected utility role.

Huddart (1994): uses the certainty-equivalent approach with a binomial tree.

Carpenter (1998): consider vesting period, forfeit rate and a random exogenous early exercise decision. The exogenous ‘stopping rate’ arrives with some fixed probability each period, and serves as a proxy for anything that causes the employee to exercise early. According to the results, this extension predicts exercise times and payoffs just as well as an elaborate utility-maximizing model that explicitly accounts for the non-transferability of options.

Detemple and Sundaresan (1999): provide a binomial framework for a utility-maximizer agent and value American style derivatives with trading restrictions. They use CRRA utility
function as an example, and allow the executive to choose investment (for his non-option wealth) and exercise strategies simultaneously.

Hall and Murphy (2002): estimate the value of an ESO to an undiversified risk-averse executive using a “certainty-equivalence” approach. They use a CRRA utility function. Early exercise occurs according to the expected utility role.

Henderson (2005): propose a continuous time utility maximization model to value stock option compensation from the executive’s perspective. The executive may invest non-option wealth in the market and risk less asset but not in the company stock itself, leaving them subject to firm-specific risk for incentive purposes.

Cai and Vijh (2004): provide a discrete time ESO valuation model based on a risk averse utility maximizer agent with CRRA utility function. The model uses two state variables to allow optimal investment of the non-option wealth of the employee in the risk-free asset and the market portfolio.

Grasselli (2005): extends the utility maximizing valuation of ESOs to the case where trading on a correlated asset is allowed. It corresponds to a situation in which the employee is forbidden to trade underlying company stock, but is allowed to trade on an index with a known correlation with the company.

Henderson (2005): proposes a continuous time utility maximization model to value stock and option compensation to the executives receiving it. The executive is able to invest in the market portfolio whilst being restricted from trading in the stock of her own company. The executive is exposed to total risk, but only firm-specific risk can not be hedged (for incentive purposes) which leads to decline in the stock option value compared to the market value.

Kadam, Lakner and Srinivasan (2005): develop a continuous time utility maximizing model for valuing ESOs. Assuming perpetual ESOs with CRRA utility function they derive optimal exercise policy and the ESOs value.

Ignersoll (2006): uses CRRA utility function to derive the subjective parameters in the option pricing problem that allow the use in the Black-Scholes model. The model can be used to determine 3 different values for each option, which Ignersoll name as the market value, the subjective value and the objective value (i.e., its cost).

Carpenter, Stanton and Wallace (2008): provide analytical results for an executive with general concave utility function under a continuous-time framework, and give conditions under which the exercise policy is completely characterized by a single stock price boundary. The optimal exercise policy varies with risk aversion, wealth and dividend.


Leung and Sircar (2009): use exponential utility function and find that incorporating the main characteristics of ESOs yield a lower value comparing to tradable stock options. In
addition, they find that job termination risk, vesting, finite maturity and non-zero interest rates are significant in valuing the ESO.

**Lattice models**

*Dennis and Randelman (2003)*: develop a model that calculates simultaneously the value of all ESOs issued by the same firm. This multiple employee option pricing model is based on an extended version the binomial model. They assume that the stock options are European-style since the model requires intensive computations.

*Hull and White (2004):* perhaps the most widely used valuation model for ESOs. The model take into account the vesting period, forfeiture rate (or exit rate) and early exercise. The early exercise decision is exogenous: It assumes that the employee will early-exercise if the stock price $S_t$ is a multiple $m$ of the option exercise price $X$.

*Ammann and Seiz (2004):* the model (called the Enhanced American Model) considers a vesting period, forfeiture rate and an exogenous early exercise decision. The early exercise takes place whenever the option’s intrinsic value is positive and the adjusted exercise value (using the factor $M^*$) is larger than the holding value—that is, $\max (S_{i,j} - M^*X, 0) \geq e^{-r\Delta t} (p f_{i+1,j+1} + (1-p) f_{i+1,j})$, where $f$ is the option value and $p$ is the up-state state price. For $M^*$ smaller (greater) than 1, the model accelerates (delays) exercise. They also incorporate a dynamic forfeit rate.

*Taylor and van Zyl (2005):* consider the possibility of hedging ESOs under the assumption that if we hedge ESO, we can priced it. The exogenous early exercise decision used in this model is according to Hull and White (2004).

*Bajaj et. Al. (2006):* the model considers a vesting period, forfeiture rate and an exogenous early exercise decision. The early exercise rule at each node is expressed as a two-dimensional matrix. The first dimension represents the ESOs remaining vested life and the second dimension represents its in-the-moneyness at that particular node (as in Hull and White (2004)). Applying this matrix, the early exercise rule predicts the fraction of outstanding exercisable stock options that will be exercised at each node.

*Brisley and Anderson (2008):* propose an alternative exogenous early exercise boundary consistent with the empirical evidence of early exercise (Huddart and Lang 1996) under which employees require a relative high multiple of stock price to exercise price in the ESO life, but later in the option life early exercise occurs at a relatively low multiple. The model also considers a vesting period and forfeit rate.

**Continuous time models (Black-Scholes based)**

*Carr and Linetsky (2000):* present a general intensity-based framework to value ESOs, which uses advances from the credit risk modeling. The early exercise or forfeiture due to voluntary or involuntary employment termination and the early exercise due to the executive’s
desire for liquidity or diversification are modeled as an exogenous point process with random intensity dependent on the stock price.

Raupach (2003): presents a pricing model that incorporates a vesting period and independent, forced termination of the contract. It enables to account for correlation between the exercise date and the stock price.

Finnerty (2005): an extension to BS model to value ESOs. Consider vesting period, forfeit rate and an exogenous early exercise decision. Finnerty treats ESOs as a portfolio of European options with a representative set of discrete exercise dates between vesting and expiration and model ESO exercise as a mean reverting stochastic process. Historical data is used to identify patterns of ESO exercise.

Change and Yang (2007): introduce a hypothetical dividend yield into the Black-Scholes model that calibrates the option price to the Hall-Murphy option value. The results are applied only to European style options.

Sircar and Xiong (2007): an extension to BS model using dynamic programming, accounting for the vesting period, resetting and reloading provisions and trading restrictions. They explore the infinity maturity limit within the framework of perpetual call options with an exercise barrier. The framework is based on asset pricing theory, so expected returns and risk premiums are required.

West (2007): develop a model where the price of the stock option obeys the Black-Scholes differential equation with two additional parameters: one controls the forfeit rate of unvested options, and the another controls the exercise rate of vested options.

Hancock, Mendoza and Merton (2005): their method based on the insight that current-period compensation expense should reflect only that part of the option value that is earned independently of the obligation of continued employment. According to most stock option plans, upon employment termination, the maturity for vested options is truncated to 90 days. Hence, at any given point in time, an employee in fact owns a 90-day option. Thus, the appropriate compensation expense in each accounting period is the value of the “extension” of the option’s maturity. In case a vesting period exists, no expense would record until the option date and then charge the value on the vesting date of a 90-day option as an expense to the final quarter before the vesting ends (another alternative is to accrual some of the option expense during the vesting period). A similar idea is presented by Bulow and Shoven (2005).

Cvitanic, Wiener and Zapatero (2007): present an ESO pricing model based on the BS framework which accounts for vesting; forfeit rate and exogenous early exercise (using a barrier). The model provides an analytic expression that can be computed directly. The early exercise decision is subject to a barrier that decays at a certain rate as the option’s maturity approaches.

Kadan, Liu and Yang (2009): develop a new Black-Scholes type closed-form valuation formula for executive stock options. The early exercise decision is consisted of the following
characteristics: first, there is a positive probability that the executive will early exercise the option on the vesting date. In addition, the forced exercise of the ESO after vesting (possibly due to liquidity shock or unexpected departure) is modeled at the first jump time of a Poisson process. Finally, the optimal exercise decision is modeled at the jump times of another independent Poisson process.
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