

CHANGE OF MEASURE IN STOCHASTIC VOLATILITY MODELS

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Abstract. Pricing in mathematical finance often involve taking expected values under an Equivalent Martingale Measure. One of the key theorems used to ensure the existence of the EMM is Girsanov's theorem, which in turn requires that the stochastic exponential of the market price of risk process be a true martingale. In general however this condition can be hard to validate, especially in stochastic volatility models. This had led many researchers to 'assume the condition away', even though the condition is not innocuous, and nonsensical results can occur if it is in fact not satisfied. Focusing on the important Heston stochastic volatility model as an example, we will use a combination of PDE and probabilistic techniques to derive conditions that ensure the existence of EMMs. We also take care in noting the condition under which discounted stock prices under an EMM are in fact martingales and not just local martingales, and find close relationships between the two problems that we consider. As a by product we also derive a PDE method that can be used to check the martingale property of stochastic exponentials.

Key words. Equivalent Martingale Measures, Novikov's Condition, Stochastic Exponentials, Stochastic Volatility Models.