VALUING SURRENDER OPTIONS IN KOREAN INTEREST INDEXED ANNUITIES

Changki Kim*

* Dr. Changki Kim is Lecturer at Actuarial Studies, Faculty of Commerce & Economics, The University of New South Wales, Sydney NSW 2052 Australia. Tel: +61 2 9385 2647, Email: c.kim@unsw.edu.au

1. INTRODUCTION

- Surrender Options : Insurance policy holders can withdraw their account values from the insurance companies at any time.

  It is a right given to the policy holders.

- Surrender Behaviors and their Impacts :

– High market interest rates drive insurance policy holders to surrender their insurance contracts and find high yield alternatives in the market.

– When interest rates increase, surrender rates also increase due to the increased gap between the market interest rates and the insurance crediting rates.

– Interest rate movements affect the cash flows of assets and liabilities of insurance companies.

- Surrender Rate Models Needed :
  - Surrender rate is one of the main factors influencing the future liability cash flows.
  - Modeling appropriate surrender rates is essential in managing assets and liabilities of insurance companies.

- Factors affecting surrender rates :

  Difference between reference market rate and policy crediting rate, seasonal effect, age and gender of clients, economy growth rate, foreign exchange rate, inflation rate, policy age since issue date, and unemployment rate, etc

2. THE STRUCTURE OF INTEREST INDEXED ANNUITIES

- Many insurance companies are selling single premium deferred annuities (SPDA).

- Interest-indexed annuities (IIA) are one of the most popular SPDA products in Korea.

- The distinctive features of IIA are the surrender options and annuitization options.

- **Crediting Interest Rates of IIA**
  - Almost all contracts guarantee a minimum interest rate
  - The crediting interest rates are announced every month based on current market rates, current investment gain rates, and the expected future portfolio income gain rates.

- **Surrender Charges of IIA**
  - Many contracts credit the full premium to the account value and assess surrender charges when the policy holder surrenders.
  - The amount of surrender charges are usually from 7% decreased by 1% annually to zero over a 7 year period.

- **Death Benefits in IIA**
  - Usually the death benefit is the account value.
  - A few variations of death benefits are considered according to the companies, for example, the account value plus 10% of premium, and another 10% of premium in the case of accidental death.

- **Annuitization and Annuity Options in IIA**
  - The policy holder can choose the initial annuitization date
  - The typical types of annuity options are :

    (a) the lump sum withdrawal of the account value at the date of annuitization,

    (b) certain periods between 5 and 20 years,

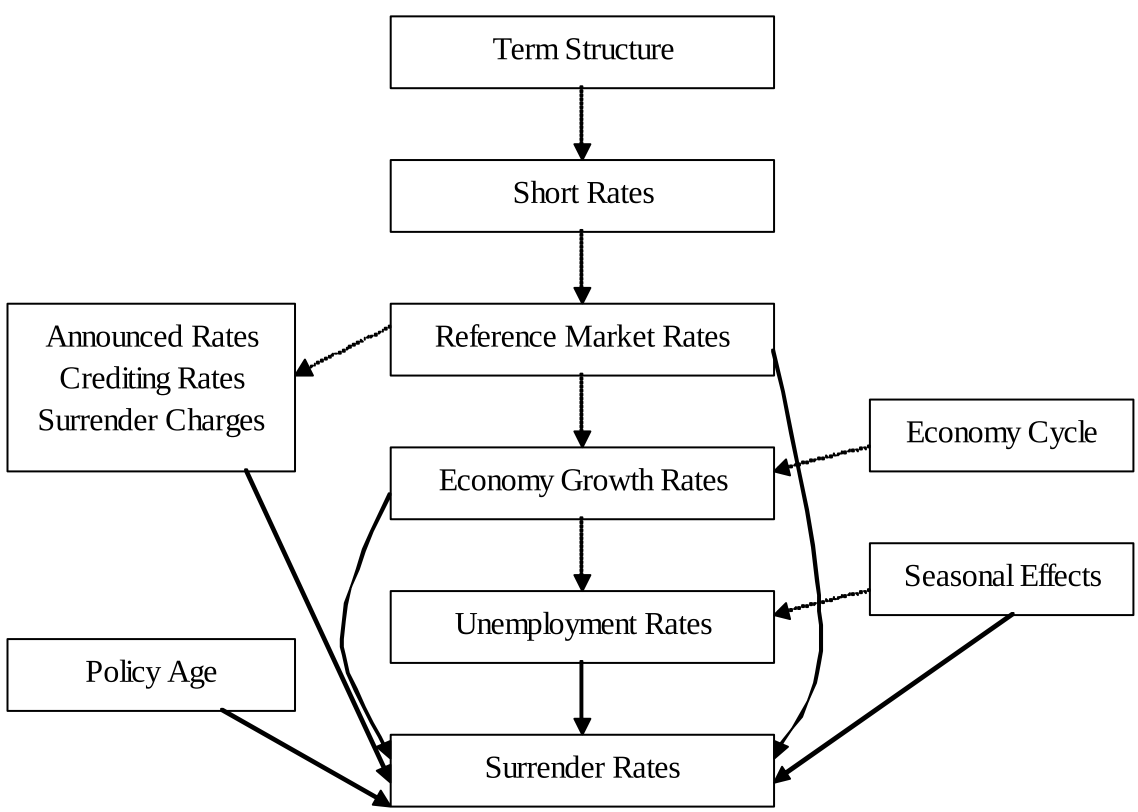
    (c) life income with a certain period of 10 years,

    (d) inheritance annuity.

3. MODELING SURRENDER RATES FOR INTEREST INDEXED ANNUITIES

- The variables considered are
  - (a) the difference between reference new money rates and product crediting rates
  - (b) the policy age since the contract was issued
  - (c) unemployment rates
  - (d) economy growth rates
  - (e) seasonal effects.

<u>Cascade Structure for the Surrender Rate Modeling</u>



- **Short Rates**
  - For a given term structure, the set of short rates $\{i(t,k)\}$, for t = 0,1,…… and for k=0,1,2, …, t for each t, should be simulated satisfying the following equation,

$$P(0,t) = \hat{P}(0,t)$$

  , where $P(0,t)$ = time 0 price of a zero coupon bond paying 1 at maturity t and $\hat{P}(0,t)$ = time 0 price of a zero coupon bond paying 1 at maturity t from the market.

  - For discrete time interest rate model, we may use an interest rate model such as Black-Derman-Toy(BDT) or Ho-Lee(HL) model to generate short rates ,$\{i_t$, t=0,…,T-1$\}$.

  - In valuing a stream of path-dependent cash flows, we need to pick a subset of all interest rate paths with a computationally reasonable number, K, of sample paths.

  - The set of all interest rate paths is Ω
  - The number of elements of Ω is $|\Omega|$. Then $|\Omega| = 2^{120}$ for 120 months.
  - We denote the subset of sample paths as $\hat{\Omega}$ with $|\hat{\Omega}|$ = K, $\hat{\Omega} = \{\omega_1, \omega_2, ..., \omega_K\}$.

– We denote the k-th path of $\hat{\Omega}$ as ω_k. For a path ω_k, we can define a function, ω_k, from time to state such that ω_k(t) is the state, s, at time t on path ω_k,

$$\omega_k(t) = s$$

where k = 1,2, ……, K, t = 0,1, ……, T-1, and s =0,1, ……, t.

– For notational simplicity, let us denote (t, ω_k) to be the node at time t on path ω_k

- **New Money Rates**{ $i_m(t, \omega_k)$, t= 0,1, …, T-1, and k=1,2, …, K}.

– We use the maximum of the 3 year yield rates(under the increasing term structure) and the short rates(under the decreasing term structure) as the reference new money rates.

– At node (t,k), to calculate the 3 year yield rates ,Y(t,k,t+3year), for the next ten years, we use interest rate models such as Black-Derman-Toy(BDT) or Ho-Lee(HL) models with the following formula,

$$\text{P(t,k,u)} = \sum_{j=0}^{u} A(t,k,u,j)$$

$$= \frac{1}{\{1+Y(t,k,u)\}^{u-t}}$$

where P(t,k,u) is the price at node (t,k) of the zero-coupon bond maturing at time u>t, and A(t,k,u,j) is the Arrow-Debreu price.

- **Economy Growth Rates**, { $i_{EG}(t,\omega_k)$, t= 0,1, …, T-1, and k=1,2, …, K}.

$$i_{EG}(t) = 0.00767 - 0.095883 * i_m(t) - 0.00565 * \sin(\frac{2\pi t}{30}) + 0.013263 * \cos(\frac{2\pi t}{30}) + \xi_t ,$$

- **Unemployment Rates**, { $i_{UE}(t, \omega_k)$, t= 0,1, …, T-1, and k=1,2, …, K}.

$$i_{UE}(t) = i_{UE}(t-1) * \{ 1 + 0.11840 - 4.11360\, i_{EG}(t) - 0.11440\, DV_3 - 0.20997\, DV_4 - 0.16229\, DV_5 - 0.12605\, DV_6 - 0.07518\, DV_7 - 0.10894\, DV_8 - 0.15145\, DV_9 - 0.09962\, DV_{10} \} + \varepsilon_t$$

- **Announced Rates** { $i_a(t, \omega_k)$, t= 0,1, …, T-1, and k=1,2, …, K}.

$$i_a(t) = 0.100538 + 0.002527 * \frac{-1}{i_m(t)} + \varepsilon_t$$

- **Crediting Rates of IIA** { $i_c(t, \omega_k)$, t= 0,1, …, T-1, and k=1,2, …, K}.

$$i_c(t, \omega_k) = \max\{ i_a(t, \omega_k), i_g \}$$

where the announced rate i_a (t, ω_k) is given, and the guaranteed annual interest rate i_g is,

$$i_g = 3\% \text{ annually.}$$

– In practice, considering the surrender behaviors of the policy holders, the crediting rate $i_c(\text{u}, \omega_j)$ at time u on path ω_j is dependent on the latest surrender time before time u,

$$i_c(\text{u}, \omega_j) = i_c(\text{u}, \omega_j, u^*)$$

where

$$u^* = 0, \text{ if there is no surrender before time u}$$

$$= \text{the latest surrender time before time u.}$$

– For computational purposes, we use the formula for the announced rates

$$i_a(\text{u}, \omega_j) = i_a(\text{u}, \omega_j, u^*)$$

$$= 0.100538 + 0.002527 \; \frac{-1}{0.2 i_m(u^*, \omega_j) + 0.8 i_m(u, \omega_j)} + \varepsilon_u$$

• **Surrender Charges** $Sc(t)$

The surrender charges during the year t, $Sc(t)$, are

$$Sc(t) = 7\%, \; 0 \le t < 1$$

$Sc(t)$ = 6%, 1 ≤ t < 2

$Sc(t)$ = 5%, 2 ≤ t < 3

$Sc(t)$ = 4%, 3 ≤ t < 4

$Sc(t)$ = 3%, 4 ≤ t < 5

$Sc(t)$ = 2%, 5 ≤ t < 6

$Sc(t)$ = 1%, 6 ≤ t < 7

$Sc(t)$ = 0%, 7 ≤ t < 10.

• **Surrender Rates** { $q_s(t,\omega_k)$, t= 0,1, …, T-1, and k=1,2, …, K}.

– We use the Logit Model for the IIA surrender rates, { $q_s(t,\omega_k)$, t=0,1,2, ⋯, T and k=1,2,⋯,K},

$$q_s(t,\omega_k) = \frac{1}{1+\exp(-\alpha)}$$

where

$$\alpha = \beta_0 + \beta_{UE} * i_{UE}(t,\omega_k) + \beta_{EG} * i_{EG}(t,\omega_k)$$

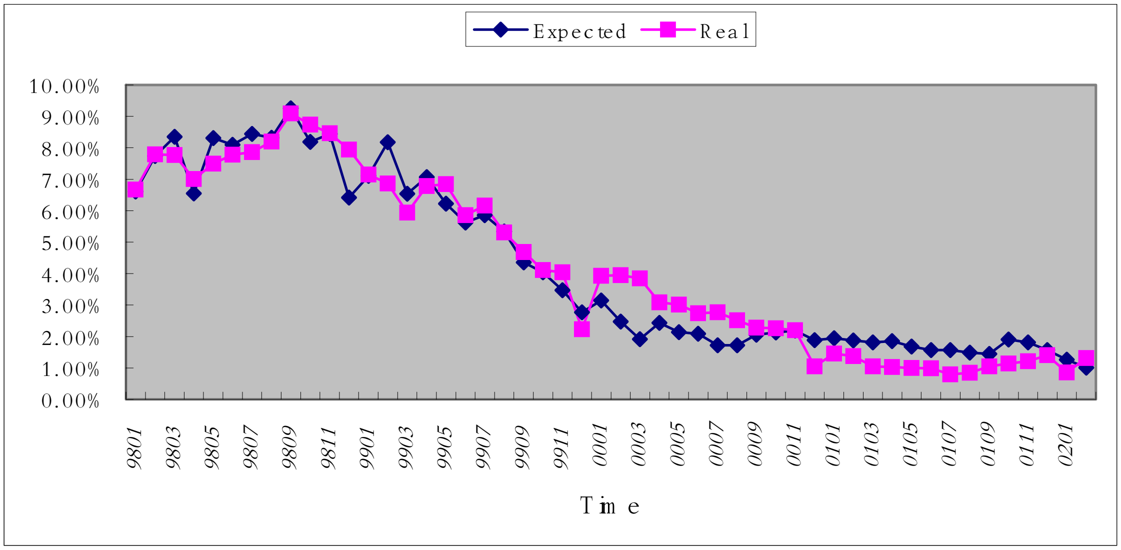
$$+ \sum_{j=0,2,4,6,8,10,12} \beta_j * \{i_m(t-j,\omega_k) - i_c(t-j,\omega_k)\} + \sum_{j=1}^{11} \beta_{month\_j} * DV_j \cdot$$

Parameter Estimates with Logit Model(IIA)

Analysis of Maximum Likelihood Estimates

Parameter	DF	Estimate	Standard Error	Chi-Square	Pr > ChiSq
Intercept	1	-6.0132	0.00617	950275.502	<.0001
DIFFLAG0	1	9.3465	0.0563	27551.1981	<.0001
DIFFLAG2	1	0.9728	0.0412	557.6077	<.0001
DIFFLAG4	1	-6.2020	0.0438	20031.9722	<.0001
DIFFLAG6	1	-2.7553	0.0399	4776.8774	<.0001
DIFFLAG8	1	1.4655	0.0390	1410.1121	<.0001
DIFFLAG10	1	0.5252	0.0389	182.5160	<.0001
DIFFLAG12	1	-1.8470	0.0468	1557.8107	<.0001
Unemploy	1	50.6348	0.1640	95314.7985	<.0001
Eco GROWTH	1	-5.3360	0.1723	959.5427	<.0001
MONTH1	1	-0.2111	0.00409	2662.3227	<.0001
MONTH2	1	-0.4199	0.00446	8867.3221	<.0001
MONTH3	1	-0.3629	0.00446	6633.6120	<.0001
MONTH4	1	0.1121	0.00415	728.9672	<.0001
MONTH5	1	0.2443	0.00408	3589.7187	<.0001
MONTH6	1	0.2961	0.00424	4879.2107	<.0001
MONTH7	1	0.2111	0.00429	2421.8388	<.0001
MONTH8	1	0.2082	0.00458	2065.2003	<.0001
MONTH9	1	0.4040	0.00452	7970.0766	<.0001
MONTH10	1	0.4919	0.00469	11024.0567	<.0001
MONTH11	1	0.3720	0.00447	6913.5047	<.0001

Real and Predicted Surrender Rates of IIA



4. VALUING THE SURRENDER OPTIONS IN INTEREST INDEXED ANNUITIES

- At time t and on path ω_k, the probability of surrender, $q_s(t,\omega_k)$, is given by

$$q_s(t,\omega_k) = \frac{1}{1+\exp(-\alpha)} ,$$

where

$$\alpha = \beta_0 + \beta_{UE} * i_{UE}(t,\omega_k) + \beta_{EG} * i_{EG}(t,\omega_k)$$

$$+ \sum_{j=0,2,4,6,8,10,12} \beta_j * \{i_m(t-j,\omega_k) - i_c(t-j,\omega_k)\} + \sum_{j=1}^{11} \beta_{month\_j} * DV_j \cdot$$

– The account value at time 0 and on path ω_k, $AV(0,\omega_k)$, is the initial single premium,

$$AV(0,\omega_k) = A'.$$

– The account value at time t and on path ω_k is given by

$$AV(\text{t},\omega_k) = AV(\text{t-1},\omega_k)\,[1+i_c(\text{t-1},\omega_k)],$$

where $i_c(\text{t-1},\omega_k)$ is the crediting rate at time t-1 and on path ω_k.

– $\Omega(\text{t},\omega_k)$ = the set of the whole paths from the node (t,ω_k)

– $\hat{\Omega}(\text{t},\omega_k)$ = a subset of sample paths of $\Omega(\text{t},\omega_k)$.

– $\hat{\Omega} = \{\omega_1, \omega_2, \ldots\ldots, \omega_K\}$,

– A finite sequence of partitions, $\{P_0, P_1, \ldots, P_t \ldots, P_T\}$, of $\hat{\Omega}$ is defined satisfying $P_0 \subseteq P_1 \subseteq \ldots \subseteq P_t \subseteq \ldots \subseteq P_T$.

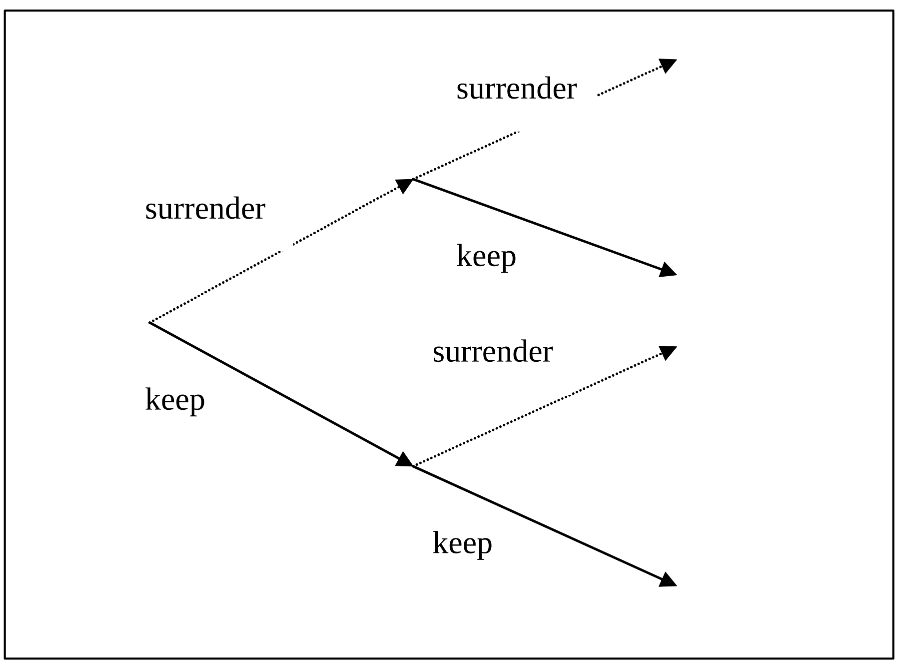
– $v_t = |P_t|$ and $P_t = \{H_1^{(t)}, H_2^{(t)}, \ldots\ldots, H_{v_t}^{(t)}\}$.

– The elements $\{H^{(t)}\}$ of P_t are called to the time-t histories.

– The element of partition P_t which contains a path ω is denoted by $H^{(t)}(\omega)$.

– The policy holder has the option to surrender at any time on any path or to keep his/her policy.

Decision Trees



– The surrendered cash value is accumulated by the reference rates(new money rates).

– And we use the crediting rates to calculate the accumulated value when the policy holder does not surrender.

– C(t, ω_k ,T, ω_j) = the set of surrender behaviors of the policy holder from time t to time T-1, given surrender at time t, on a path $\omega_j \in H^{(t)}(\omega_k)$,

$$C(t, \omega_k, T, \omega_j) = \{ e_1^{(t,\omega_k,T,\omega_j)},\ e_2^{(t,\omega_k,T,\omega_j)},\ e_3^{(t,\omega_k,T,\omega_j)},\ \ldots\ldots,\ e_{2^{T-1-t}}^{(t,\omega_k,T,\omega_j)} \},$$

where each $e_l^{(t,\omega_k,T,\omega_j)}$, l = 1,2, ……, 2^{T-1-t}, is a surrender behavior of the policy holder from time t to time T-1, given surrender at time t, on a path $\omega_j \in H^{(t)}(\omega_k)$. Here we assume that the policy holders always reinvest.

– Example of the set C(T-3, ω_k ,T, ω_j) of surrender behaviors given surrender at T-3,

$$C(T\text{-}3, \omega_k, T, \omega_j) = \{ e_1^{(T-3,\omega_k,T,\omega_j)},\ e_2^{(T-3,\omega_k,T,\omega_j)},\ e_3^{(T-3,\omega_k,T,\omega_j)},\ e_4^{(T-3,\omega_k,T,\omega_j)} \},$$

with each surrender behavior is as follows

$$e_1^{(T-3,\omega_k,T,\omega_j)} = (s, s, s),$$

$$e_2^{(T-3,\omega_k,T,\omega_j)} = (s, k, s),$$

$$e_3^{(T-3,\omega_k,T,\omega_j)} = (s, s, k),$$

and

$$e_4^{(T-3,\omega_k,T,\omega_j)} = (s, k, k),$$

where s denotes surrendering and k denotes keeping.

– $SV\,(e_l^{(t,\omega_k,T,\omega_j)})$ = the accumulated value from surrender at time T and on a path $\omega_j \in H^{(t)}(\omega_k)$ under the surrender behavior $e_l^{(t,\omega_k,T,\omega_j)}$, l = 1,2, ……, 2^{T-1-t}.

– Example : If the surrender behavior $e^{(t,\omega_k,T,\omega_j)}$ = (s,s,k,k,k. ……, k), then we have

$$SV\,(e^{(t,\omega_k,T,\omega_j)})$$

$$= AV\,(\mathrm{t},\omega_k)\ (1\text{-Sc(t)})\ (1+i_m(\mathrm{t},\omega_j))\ (1\text{-Sc(1)})\ (1+i_m(\mathrm{t}+1,\omega_j))$$
$$*\prod_{u=t+2}^{T-1}(1+i_c(u,\omega_j,u^*)),$$

where

u^* = 0, if there is no surrender before time u

= the latest surrender time before time u,

and $i_c(\mathrm{u},\omega_j,\ u^*)$ is the crediting rate at time u, on path $\omega_j \in H^{(t)}(\omega_k)$. Since we assume that surrender occurs at the end of period we apply the surrender charge Sc(1) in the above formula. We also assume that there is no surrender charge at time T.

– $\overline{SV}(\mathrm{t},\omega_k,\mathrm{T},\omega_j)$ = the average of the accumulated value from surrender, given $\omega_j \in H^{(t)}(\omega_k)$At time T,

$$\overline{SV}(\mathrm{t},\omega_k,\mathrm{T},\omega_j) = E^Q\,[SV(e^{(t,\omega_k,T,\omega_j)})\,|\,H^{(t)}(\omega_k)]$$

$$= \sum_{e^{(t,\omega_k,T,\omega_j)} \in C(t,\omega_k,T,\omega_j)} \Pr(e^{(t,\omega_k,T,\omega_j)}) SV(e^{(t,\omega_k,T,\omega_j)}) ,$$

where $\Pr(e^{(t,\omega_k,T,\omega_j)})$ is a risk neutral probability of the surrender behavior $e^{(t,\omega_k,T,\omega_j)} \in$ C(t, ω_k ,T, ω_j).

– SV (t, ω_k) = the value from surrender at current time t,

$$SV(t,\omega_k) = E^Q\left[\frac{\overline{SV}(t,\omega_k,T,\omega)}{\prod_{u=t}^{T-1}(1+i(u,\omega))} \mid H^{(t)}(\omega_k)\right]$$

$$= \sum_{\omega \in H^{(t)}(\omega_k)} \Pr(\omega) \frac{\overline{SV}(t,\omega_k,T,\omega)}{\prod_{u=t}^{T-1}(1+i(u,\omega))},$$

where Pr(ω) is a risk neutral probability.

– F(t, ω_k ,T, ω_j) = the set of surrender behaviors of the policy holder from time t to time T-1, <u>given keeping at time t</u>, on a path $\omega_j \in H^{(t)}(\omega_k)$,

$$F(t,\omega_k,T,\omega_j) = \{ f_1^{(t,\omega_k,T,\omega_j)},\ f_2^{(t,\omega_k,T,\omega_j)},\ f_3^{(t,\omega_k,T,\omega_j)},\ \ldots\ldots,\ f_{2^{T-1-t}}^{(t,\omega_k,T,\omega_j)} \},$$

where each $f_l^{(t,\omega_k,T,\omega_j)}$, l = 1,2, ……, 2^{T-1-t}, is a surrender behavior of the policy holder from time t to time T-1, given keeping at time t, on a path $\omega_j \in H^{(t)}(\omega_k)$.

– KV ($f_l^{(t,\omega_k,T,\omega_j)}$) = the accumulated value at time T and on a path $\omega_j \in H^{(t)}(\omega_k)$ if the policy holder invests the amount of money, $AV(\mathrm{t},\omega_k)$, until the option maturity T under the surrender behavior $f_l^{(t,\omega_k,T,\omega_j)}$, l = 1,2, ……, 2^{T-1-t}.

– $\overline{KV}(\mathrm{t},\omega_k,\mathrm{T},\omega_j)$ = the average of the accumulated value at time T, given $\omega_j \in H^{(t)}(\omega_k)$,

$$\overline{KV}(\mathrm{t},\omega_k,\mathrm{T},\omega_j) = E^Q[KV(f^{(t,\omega_k,T,\omega_j)}) \mid H^{(t)}(\omega_k)]$$

$$= \sum_{f^{(t,\omega_k,T,\omega_j)} \in F(t,\omega_k,T,\omega_j)} \Pr(f^{(t,\omega_k,T,\omega_j)}) KV(f^{(t,\omega_k,T,\omega_j)}),$$

where Pr($f^{(t,\omega_k,T,\omega_j)}$) is a risk neutral probability of the surrender behavior $f^{(t,\omega_k,T,\omega_j)} \in \mathrm{F}(\mathrm{t},\omega_k,\mathrm{T},\omega_j)$.

– $KV(\mathrm{t},\omega_k)$ = the keeping value at current time t,

$$KV(\mathrm{t},\omega_k) = E^Q\left[\frac{\overline{KV}(t,\omega_k,T,\omega)}{\prod_{u=t}^{T-1}(1+i(u,\omega))} \mid H^{(t)}(\omega_k)\right]$$

$$= \sum_{\omega \in H^{(t)}(\omega_k)} \Pr(\omega) \frac{\overline{KV}(t,\omega_k,T,\omega)}{\prod_{u=t}^{T-1}(1+i(u,\omega))},$$

where Pr(ω) is a risk neutral probability.

– $EV(\mathrm{t}, \omega_k)$ = exercise value at time t on a path ω_k,

$$EV(\mathrm{t}, \omega_k) = p(t, \omega_k) * q_s(t, \omega_k) * \{ SV(\mathrm{t}, \omega_k) - KV(\mathrm{t}, \omega_k) \}.$$

– Value of surrender option(VSO) for the total exercise values at time 0,

$$VSO = \sum_{k=1}^{K} \Pr(\omega_k) \sum_{t=1}^{T-1} \frac{EV(t, \omega_k)}{\prod_{u=0}^{t-1} (1 + i(u, \omega_k))},$$

where K denotes the number of paths at time 0.

<u>Value of Surrender Option (VSO)</u>

	Surrender Charge	VSO(BDT)	VSO(HL)
Case 1	7% - 0%	-158.07	-156.63
Case 2	0%	4.77	4.81

Here the initial single premium is 10,000.

- Surrender charges really have an effect on the value of the surrender option(VSO) of IIA.

- The two values of the surrender option(VSO) of IIA with BDT model and HL model are almost the same but not exactly the same. So the values may be dependent on the particular choice of interest rate model.

- The values of the surrender option (VSO) with surrender charges are <u>negative numbers</u>.

  These negative values may be some profits to the insurance companies not to the policy holders who have the option (or the right!).
  The surrender option is a right given to the policy holders and we may expect that the value of the surrender options be positive.
  It may not be really surprising for someone who notice that some insurance companies get positive gains from surrender.

5. FAIR SURENDER CHARGES

- The values of the surrender option with surrender charges are negative numbers; some profits to the insurance companies, not to the policy holders who have the option

- We may find fair surrender charges not only for the company but also for the policy holders

- The choice of interest rate model is a consideration in valuing interest rate contingent cash flows.

Table 3. Finding Fair Surrender Charges

Surrender Charges	VSO (BDT)	VSO (HL)
7% - 0%	-158.07	-156.63
5% - 0%	-94.79	-94.12
1%	-25.60	-25.27
0%	4.77	4.81

6. CONCLUSION

Conclusions

- We consider surrender rate models for IIA using Logit functions with variables

  (a) the difference between reference rates(new money rates) and product crediting rates with surrender charges,

  (b) the policy age since the contract was issued,

  (c) unemployment rates,

  (d) economy growth rates,

  (e) seasonal effects and so on.

- It is interesting to note that the values of the surrender option (VSO) with surrender charges are negative, which may be some profits to the insurance companies not to the policy holders who have the option (or the right!).

- Finding fair surrender charges should be considered.

Future Research Topics

- We can investigate rational and irrational surrender behaviors of the policy holders.

- We may also try to calculate the value of the optimal surrender options.

- There are a few theories on the pricing of American options with Markov properties. But it is still difficult to price American options with path dependent cash flows which do not have Markov properties. We may consider this problem for the future research topic.

THANK YOU VERY MUCH !!!