An Economic Model for Relative Importance*

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Abstract

The theory of competing causes involves assessing a cause of failure, such as death due to heart disease or cancer, in the presence of other competing causes. In this paper, we develop a fundamental economic model for examining the degree and order of importance in a competing cause framework. In particular, we demonstrate the usefulness of Von Neumann-Morgenstern utility theory in economics by evaluating the effect of a cause when its economic loss is eliminated. A quantity called the ratio of relative importance (RRI) is defined to measure the relative importance of a competing cause. We find that this quantity is a function of several factors that do indeed affect the "riskiness" of a competing cause relative to the other causes. These factors include the amounts of economic loss, the probabilities of failure associated with each cause, and the utility function that defines the risk aversion behavior of the decision maker. We are able to further demonstrate that using the ratio, we can develop intuitive explanations as to how one would assess the order of importance of one cause relative to another.

*Keywords: Relative importance, competing causes, Arrow-Pratt risk premiums.

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1 Introduction

A person dies from one of several possible causes of death: heart disease, cancer, motor vehicle accident, AIDS, to name but a few. The World Health Organization has over 282 codes (see World Health Organization, 1977) of classifying diseases, injuries and causes of death. Any of these causes of death is of course considered undesirable. However, some people attach special fear to certain causes which are therefore perceived to be relatively worse than others.

There is substantial evidence to support people's psychological perception that certain causes are relatively worse than others. In Savage (1993), a study shows that people fear the risk of cancer more than the risks of car accidents, home fires, and aviation. Other findings indicate that people would be willing to pay more to prevent cancer deaths than to prevent heart disease or motor vehicle accidents. See Sunstein (1997). Published in USA Today (February 1998), the results from a survey indicate that adults would be willing to pay larger taxes for research to find cures for cancer and AIDS.

How does one decide that one cause of death is relatively worse than another? Several surveys ranking various causes of death reveal that people have a sense of the magnitude of the risks of certain causes. See for example Consumer Reports (December 1996). It is not surprising to find that when people are asked to rank various causes, they can act more rationally when provided with additional information on the degree of seriousness of certain causes. Some measures that they find useful include (1) the rate of mortality for various causes, (2) the expected increase in life expectancy when a cause is eliminated, (3) the number of lives saved from eliminating certain causes, and (4) the decrease in the cost of insurance for eliminating certain causes. Table 1 provides a comparison of the 10 leading causes of death in the United States for selected periods between 1900 and 1992. These causes of death are ordered according to the percentage of total deaths.

Within the framework of expected utility theory, this paper develops a basic economic model that can be used to assess relative importance. The model may be applied in any system where failure is due to the occurrence of one of a set of competing causes. This sytem could be a person, an entity, or a mechanical device with multiple components. The development of the theory begins with the case of two competing causes and a single time period. A quantity called the ratio of relative importance (RRI) is defined to measure the relative importance of a competing cause. We find that this quantity is a function of several factors that do indeed affect the "riskiness" of a competing cause relative to other causes. These factors include the amounts of economic loss, the probabilities of failure for each competing cause, and the utility function describing the risk aversion behavior of the decision maker. In Section 2, we briefly outline the concept of relative importance drawn from various disciplines. Section 3 derives the fundamental economic model used to assess relative importance, defines the RRI, and derives them for various well-known utility function. This section also develops intuitive explanations about the order and importance of competing causes and provides asymptotic approximations when the form of the utility function may be unknown, but risk aversion measures are. The basic economic model is extended in Section 4 by assuming multiple competing causes and multiple time periods. Some simplifying results are derived when we restrict the class of decision makers to only those that are risk-neutral, or those with linear utility functions. We conclude in Section 5.

2 The Concept of Relative Importance

Suppose that the outcome of a random variable Y is influenced by several other independent variables, say $X_1, X_2, ..., X_m$, and that the influence can be expressed as a functional relationship $Y = g\left(X_1, X_2, ..., X_m\right)$ plus possibly some random error. Here, there is the natural question of how much influence each of the X's has on the outcome Y. Such is the concept of relative importance and its usefulness arises in several situations. A prime example is in the case of a linear function, i.e. g can be written as $\sum_{k=1}^{m} \beta_k X_k$, we have a regression equation.

To further illustrate the usefulness of relative importance, we examine some examples that have appeared in Kruskal (1984) and Kruskal and Majors (1989). Consider the situations of assessing the relative importance of factors such as: (a) occupation, geography, parental health, and smoking habits on the incidence of cancer; (b) education, training, employee morale, allocation of capital funds, and employee benefits on economic productivity; (c) duration and yield volatility on bond price volatility; and (d) monetary and fiscal policies in achieving economic goals. Frees (1998) examines relative importance of the various sources of risks in a general insurance framework. He assesses the order and degree of importance of the risk of claims, the risk of a disaster, and the risk of investments to the insurance company.

Table 1
Comparison of the Leading Causes of Death in the United States
for Selected Periods Between 1900 and 1992
(causes ordered as measured by percentage of total death)[1]

	1900	1918	1960	1992
1	Tuberculosis	Pneumonia & Influenza	Heart Disease	Heart Disease
2	Pneumonia	Heart Disease	Cancer	Cancer
3	Diarrhea & Enteritis	Tuberculosis	Cerebrovascular Diseases	Cerebrovascular Diseases
4	Heart Disease	Nephritis	Accidents	Obstructive Pulmonary Diseases
5	Nephritis	Cerebrovascular Diseases	Certain Dis. of Early Infancy	Pneumonia & Influenza
6	Accidents & Violence	Motor Vehicle & Other Accidents	Pneumonia & Influenza	Infectious Diseases
7	Cerebrovascular Diseases	Cancer	General Arteriosclerosis	Diabetes Mellitus
8	Cancer	Diarrhea & Enthritis	Diabetes Mellitus	Accidents
9	Bronchitis	Syphillis	Congenital Malformations	Suicide & Homicide
10	Diptheria	Whooping Cough	Cirrhosis of Liver	Chronic Liver Disease

[1]Sources: National Center for Health Statistics, *Vital Statistics of the United States*, 1944, 1960 and 1992.

As illustrated by our examples below, there is no single measure used to assess relative importance.

Example 1: *Regression Methodology* In regression studies, one is naturally concerned with comparing the importance of various explanatory variables. Frees (1998), Bring (1996), and Kruskal (1987) summarize several measures used to assess importance of explanatory variables. Among these are: (a) measures of variable significance such as p-values and t-ratios; (b) the standardized regression coefficient expressed as $\beta_i \left[\sigma(X_i)/\sigma(Y)\right]$ where $\sigma(\cdot)$ denotes standard deviation; (c) change in the coefficient of determination; and (d) partial correlations. See Frees (1998) and Kruskal (1987) for other measures in regression methodologies.

Example 2: *Insurance Systems* Frees (1998) suggests the use of the random variable $E(Y|X)$ to assess the effect of X on Y. It naturally follows then to use the coefficient of determination

$$\rho^2_{XY} = \frac{Var(E(Y|X))}{Var(Y)} \tag{1}$$

as a measure of the risk of Y that is attributable to X. By comparing the approximate Arrow-Pratt risk premiums for the variables Y and $E(Y|X)$, that is

$$\pi(E(Y|X)) \approx \frac{1}{2} r_A Var(E(Y|X))$$

and

$$\pi(Y) \approx \frac{1}{2} r_A Var(Y)$$

where $r_A = -u''/u'$ defines the Arrow-Pratt measure of risk aversion, Frees provides an economic justification on the use of (1) as a measure of relative importance of risks. He extends this measure to a multivariate framework where more than two sources of risks are present and shows how one can use this measure to a portfolio of insurance policies that is subject to the risks of claims, a disaster, and investment.

Example 3: *Actuarial Ordering of Risks* In Kaas, van Heerwaarden, and Goovaerts (1994), a different approach to comparing the attractiveness of actuarial risks is considered. Actuarial risk X with distribution function $F_X(x)$ is defined to be the amount of claim. Risks are compared by using different types of ordering such as stochastic dominance, stop-loss oder, dangerousness, to name but a few. Properties of these orderings are studied. For example, Y is said to stochastically dominate X if X is preferred by all decision makers with increasing utility function, i.e. $E[u(-X)] \geq E[u(-Y)]$, for any increasing utility function u. Stochastic dominance implies larger distribution function, $F_X \geq F_Y$. Another type of ordering suggested is that of dangerousness which involves comparison of means. Here, risk Y is said to be more dangerous than risk X if (a) $E(Y) \geq E(X)$ and (b) there exists $c \geq 0$ such that $F_X(x) \leq F_Y(x)$ for all $x < c$ and $F_X(x) \geq F_Y(x)$ for all $x > c$. Other orderings are considered in Kaas, et al. (1994).

Example 4: *Health Research and Epidemiology* The question of relative importance also naturally arises in the study of diseases (epidemiology) and other branches of medical science. Here, one may be concerned about the outcome of a disease (such as cancer) in the presence of a risk factor (such as smoking). We may denote by D the presence (= 1) or absence (= 0) of the disease and F the presence (= 1) or absence (= 0) of the risk factor. If we use the notations

$$p_{ij} = P(D = i | F = j), \text{ for } i = 1, 2 \text{ and } j = 1, 2$$

with $\sum_{j=1}^{2} \sum_{i=1}^{2} p_{ij} = 1$, then some commonly used measures of relative importance are: (1) relative risk: $R = p_{11}/p_{10}$ which is the ratio of the risks of getting the disease when factor is present to that when factor is absent; (2) odds ratio: $O = \frac{p_{11}/p_{10}}{p_{10}/p_{00}}$ which is the ratio of the odds in the presence and absence of the factor; and (3) attributable risk: $A = \frac{(R-1)P(F=1)}{1+(R-1)P(F=1)}$ which is the proportion of those infected with the disease that could have been prevented had the risk factor been eliminated. See Walter (1976).

3 The Basic Economic Model

In this section, we use utility theory to develop a basic model for assessing relative importance of competing causes. The model assumes a single time period and two competing causes that we label as 1 and 2. In the next section, we extend this to multiple time periods and several competing causes.

The expected utility model has been a standard paradigm in the analysis of economic behavior of individuals who face risk and uncertainty. The theory, originally developed by Von Neumann and Morgenstern (1947), has had some success in deriving explnations to economic behavior phenomena and has been widely applied in the area of insurance decision making. According to the theory, a rational decision maker chooses from among a set of alternative consumption and investments by maximizing his expected utility function. A set of mathematical rationality axioms, e.g. weak order, independence, and continuity, is needed to assert existence of the utility function. We ignore the axiomatic development here, and if the reader is interested in the foundational aspects of the theory, see Fishburn (1970, 1982). In the ensuing paragraphs, we work with the Von Neumann Morgenstern expected utility model. We assume all axioms are satisfied so that the existence of a utility function is guaranteed. All decision makers under risk and uncertainty act according to these axioms and are therefore maximizers of expected utility.

3.1 Definitions and Notations

Consider that for a single time period, a decision maker faces the risk of losing

$$L = \begin{cases} \ell_1, & \text{if “cause 1” occurs, w.p. } p_1 \\ \ell_2, & \text{if “cause 2” occurs, w.p. } p_2 \\ 0, & \text{w.p. } 1 - p_1 - p_2 \end{cases},$$

where w.p. denotes “with probability.” In the case of life insurance, this decision maker may be an income earner who faces the risk of losing income for his family in the event of death. The two causes of death in this case could be, for instance, “cancer” and “other than cancer.” The amount of loss may be different for each cause of death because there may be large medical expenses incurred associated with “cancer” just prior to mortality. A similar development is applicable in the case of property insurance, where the decision maker may own a particular property that is subject to loss, for example, due to “fire” or “other than fire.”

For our purposes, an insurance policy will be denoted by a triplet (π, r_1, r_2) where π is the policy premium, and r_1, r_2 represent the proportions of insurance coverage for causes 1 and 2, respectively. An individual who chooses to purchase policy (π, r_1, r_2) will pay a premium π and will be reimbursed an amount of $r_i \ell_i$ when “cause i” occurs and nothing when neither cause occurs. We assume unlimited proportion of insurance coverage is available in the market, however, we do not treat insurance as gambling so that supply satisfies $0 \leq r_i \leq 1$ for $i = 1, 2$.

Let $u(\cdot)$ be the individual's utility function. Assuming an initial wealth of w_0 and no insurance purchased, the individual has a random wealth of $w_0 - L$ and an expected utility of $E[u(w_0 - L)]$ during the single period. Assuming insurance policy (π, r_1, r_2) is purchased, the expected utility will be a function of the demand r_1 and r_2. Denoting this expectation by $z(r_1, r_2)$, we would have

$$\begin{aligned} z(r_1, r_2) &= p_1 u(w_0 - \pi - (1 - r_1)\ell_1) + p_2 u(w_0 - \pi - (1 - r_2)\ell_2) \\ &\quad + (1 - p_1 - p_2) u(w_0 - \pi). \end{aligned}$$

Now suppose the individual has nondecreasing utility function and is risk averse, i.e. $u' \geq 0$ and $u'' \leq 0$. Because of risk aversion, the individual will always prefer a sure loss to any gamble. Thus, if $\pi = p_1 r_1 \ell_1 + p_2 r_2 \ell_2$, he will purchase full insurance, that is, $r_1 = r_2 = 1$. In Mossin (1968), a similar result of full insurance as the optimal coverage is also derived.

Given that full insurance coverage is to be purchased, the maximum premium π that the individual is willing to pay will be the unique solution to

$$u(w_0 - \pi) = p_1 u(w_0 - \ell_1) + p_2 u(w_0 - \ell_2) + (1 - p_1 - p_2) u(w_0)$$

or after some re-arrangement, we have

$$\begin{aligned} & u(w_0) - u(w_0 - \pi) \\ = \; & p_1 [u(w_0) - u(w_0 - \ell_1)] + p_2 [u(w_0) - u(w_0 - \ell_2)]. \end{aligned} \tag{2}$$

The premium π in this case is called the Arrow-Pratt risk premium. See Arrow (1965) and Pratt (1964). Equation (2) has a nice interpretation. The left-hand side gives the reduction in the utility for paying the premium for full protection from all causes of loss. This is equal to the sum of the reductions in utility for losing amounts ℓ_i, with each reduction multiplied by its corresponding probability p_i, $i = 1, 2$.

Now, consider the situation where the individual is able to take action to eliminate losses arising from "cause 1" and hence $\ell_i = 0$. In this case, it is straightforward to show that policy $\left(\pi_{(-1)}, 0, 1\right)$ will be purchased. Here, $\pi_{(-1)}$ denotes the Arrow-Pratt risk premium the individual is willing to pay when "cause 1" is eliminated. Note that we assume elimination of the amount of loss, but not the probability of loss. However, it is not difficult to show that the same solution will result if the probability of loss is eliminated. Thus, $\pi_{(-1)}$ is the unique solution to

$$u(w_0) - u\left(w_0 - \pi_{(-1)}\right) = p_2\left[u(w_0) - u(w_0 - \ell_2)\right]. \tag{3}$$

Equation (3) is intepreted to be that the reduction in utility for paying the premium to cover loss due to "cause 2" is the same as the reduction in utility for losing amount of loss ℓ_2, due to "cause 2," but multiplied by its corresponding probability. Similarly, when "cause 2" is eliminated, we have

$$u(w_0) - u\left(w_0 - \pi_{(-2)}\right) = p_1\left[u(w_0) - u(w_0 - \ell_1)\right]. \tag{4}$$

Equation (4) can be similarly interpreted as equation (3). In this case, the loss is due to "cause 1." We call the premium $\pi_{(-i)}$ the cause i-eliminated risk premium, $i = 1, 2$ and is used as the basis for assessing relative importance.

Definition 1 *The measure*

$$RRI_i = \frac{\pi - \pi_{(-i)}}{\pi}, \text{ for } i = 1, 2 \tag{5}$$

is called the ratio of relative importance of competing "cause i."

We can interpret the numerator of this ratio as the risk premium attributable to the presence of the i-th competing cause. The ratio is therefore a proportion to the overall risk premium. A competing cause with a higher ratio of relative importance is considered "more risky" than one with a lower ratio. Furthermore, if we deduct equation (2) from (3), we get

$$u\left(w_0 - \pi_{(-1)}\right) - u(w_0 - \pi) = p_1\left[u(w_0 - \ell_1) - u(w_0)\right].$$

Because we assume that utility function is non-decreasing, the right-hand side of the above equation has to be non-negative; so must therefore be the left-hand side. Thus, we must have $\pi_{(-1)} \leq \pi$, or equivalently $RRI_1 \leq 1$. In a similar fashion, we can prove that $RRI_2 \leq 1$. Our measure of relative importance is obviously non-negative and never exceeds 1. As pointed out by Frees (1998), a measure of relative importance with these properties is desirable.

It is clear that the ratios of relative importance are functions of a person's initial wealth, the economic loss of each cause, and their associated probabilities. Intuitively, we observe that a cause with higher probability of occurrence and larger economic loss will be less preferred, and therefore more risky, than another cause with lower probability and smaller economic loss. In this case, the cause will have a higher ratio of relative importance. Thus, for instance, in an insurance contract that pays twice the amount of benefit if death were due to an accident, then accidents will have higher ratios of relative importance if probability of death due to an accident is higher than due to all other causes. This may be true for very young ages when accidents are relatively worse than many other causes. Similarly, in the other direction, when a cause has lower probability and smaller economic loss, it will be more preferred. We prove these intuitively appealing results as a proposition.

We also state, as corollary to the proposition, that when the economic losses are equal, a comparison based on RRI leads to a comparison of probabilities. This result is again intuitively appealing because it is natural to determine the riskiness of competing causes by simply comparing probabilities of occurrence. A risk with lower probability of occurrence is often perceived as "less risky"; a risk with higher probability is "more risky."

Similarly, when two competing causes have equal probabilities, we determine the riskiness by comparing economic losses. Intuitively, the one with larger economic loss will be perceived as "more risky." This is also stated as a corollary.

Proposition 2 *For the class of decision makers with non-decreasing utility functions, the following holds:*

(i) If $\ell_1 \geq \ell_2$ and $p_1 \geq p_2$, then $RRI_1 \geq RRI_2$.

(ii) If $\ell_1 \leq \ell_2$ and $p_1 \leq p_2$, then $RRI_1 \leq RRI_2$.

(iii) If $\ell_1 \geq \ell_2$ and $RRI_1 \leq RRI_2$, then $p_1 \leq p_2$.

(iv) If $\ell_1 \leq \ell_2$ and $RRI_1 \geq RRI_2$, then $p_1 \geq p_2$.

(v) If $p_1 \geq p_2$ and $RRI_1 \leq RRI_2$, then $\ell_1 \leq \ell_2$.

(vi) If $p_1 \leq p_2$ and $RRI_1 \geq RRI_2$, then $\ell_1 \geq \ell_2$.

Proof. Deducting equation (3) from (4), we have

$$u\left(w_0 - \pi_{(-2)}\right) - u\left(w_0 - \pi_{(-1)}\right) = (p_2 - p_1)\, u(w_0) + p_1 u(w_0 - \ell_1) - p_2 u(w_0 - \ell_2).$$

Adding and deducting the term $(p_2 - p_1)\, u(w_0 - \ell_2)$ to the right-hand side above, we get

$$\begin{aligned} u\left(w_0 - \pi_{(-2)}\right) - u\left(w_0 - \pi_{(-1)}\right) &= (p_2 - p_1)\left[u(w_0) - u(w_0 - \ell_2)\right] \\ &\quad + p_1\left[u(w_0 - \ell_1) - u(w_0 - \ell_2)\right]. \quad (6) \end{aligned}$$

To prove (i) in the proposition, assuming $\ell_1 \geq \ell_2$ and $p_1 \geq p_2$, then both terms on the right-hand side of (6) must be negative since utility function is non-decreasing. Thus, the left-hand side must also be negative. Again, because of non-decreasing utility, we must have $\pi_{(-1)} \leq \pi_{(-2)}$, which proves the result. We can prove (ii) by making similar observations from equation (6). After some re-arrangement, note that (6) is equivalent to the following:

$$\begin{aligned} (p_2 - p_1)\left[u(w_0) - u(w_0 - \ell_2)\right] &= p_1\left[u(w_0 - \ell_2) - u(w_0 - \ell_1)\right] \\ &\quad + u\left(w_0 - \pi_{(-2)}\right) - u\left(w_0 - \pi_{(-1)}\right) \quad (7) \end{aligned}$$

and

$$\begin{aligned} p_1\left[u(w_0 - \ell_1) - u(w_0 - \ell_2)\right] &= (p_1 - p_2)\left[u(w_0) - u(w_0 - \ell_2)\right] \\ &\quad + u\left(w_0 - \pi_{(-2)}\right) - u\left(w_0 - \pi_{(-1)}\right). \quad (8) \end{aligned}$$

Equation (7) is used to prove (iii) and (iv). Equation (8) is used to prove (v) and (vi).■

Corollary 3 *Assume $\ell_1 = \ell_2$. For the class of decision makers with non-decreasing utility function, $RRI_1 \leq RRI_2$ if and only if $p_1 \leq p_2$.*

Proof. Use (i) and (iv) of the proposition.■

Corollary 4 *Assume $p_1 = p_2$. For the class of decision makers with non-decreasing utility function, $RRI_1 \leq RRI_2$ if and only if $\ell_1 \leq \ell_2$.*

Proof. Use (ii) and (iii) of the proposition.■

3.2 RRI for Some Utility Functions

This section derives ratios of relative importance for some familiar utility function. All utility functions considered are non-decreasing and therefore will clearly satisfy the proposition proved in the previous section.

Example 1: *Linear Utility Function* Here, we assume utility has the form $u(x) = a + bx$, with $b > 0$. From equation (2), we have $\pi = p_1\ell_1 + p_2\ell_2$. From equations (3) and (4), we have $\pi_{(-1)} = p_2\ell_2$ and $\pi_{(-2)} = p_1\ell_1$. Thus, the ratios of relative importance are

$$RRI_1 = \frac{\pi - \pi_{(-1)}}{\pi} = \frac{p_1\ell_1}{p_1\ell_1 + p_2\ell_2}$$

and

$$RRI_2 = \frac{\pi - \pi_{(-2)}}{\pi} = \frac{p_2\ell_2}{p_1\ell_1 + p_2\ell_2}.$$

Full coverage premium is additively decomposed into the sum of the cause-i eliminated risk premiums. That is, we have $\pi = \pi_{(-1)} + \pi_{(-2)}$. Consequently, the sum of the ratios of relative importance will be unity, i.e. $RRI_1 + RRI_2 = 1$. When $\ell_1 = \ell_2$, the ratios of relative importance reduce to the ratios of probabilities as follows:

$$RRI_1 = \frac{p_1}{p_1 + p_2} \text{ and } RRI_2 = \frac{p_2}{p_1 + p_2}.$$

Example 2: *Logarithmic Utility Function* We assume utility has the form $u(x) = \log(x)$. From equation (2), we have

$$\pi = w_0 \left[1 - (1 - \ell_1/w_0)^{p_1} (1 - \ell_2/w_0)^{p_2}\right].$$

From equations (3) and (4), we have

$$\pi_{(-1)} = w_0 \left[1 - (1 - \ell_2/w_0)^{p_2}\right]$$

and

$$\pi_{(-2)} = w_0 \left[1 - (1 - \ell_1/w_0)^{p_1}\right].$$

Thus, the ratios of relative importance are, for "cause 1,"

$$RRI_1 = \frac{(1 - \ell_2/w_0)^{p_2} \left[1 - (1 - \ell_1/w_0)^{p_1}\right]}{1 - (1 - \ell_1/w_0)^{p_1} (1 - \ell_2/w_0)^{p_2}}$$

and for "cause 2,"

$$RRI_2 = \frac{(1 - \ell_1/w_0)^{p_1} \left[1 - (1 - \ell_2/w_0)^{p_2}\right]}{1 - (1 - \ell_1/w_0)^{p_1} (1 - \ell_2/w_0)^{p_2}}.$$

Example 3: *Exponential Utility Function* We assume utility has the form $u(x) = \frac{1}{a}\left(1 - e^{-ax}\right)$, where $a > 0$ so that clearly, we have

$$u(w_0) - u(w_0 - \pi) = \frac{1}{a} e^{-aw_0} \left(1 - e^{-a\pi}\right).$$

From equation (2), we have

$$\pi = a \log \left\{1 - \left[p_1 \left(1 - e^{a\ell_1}\right) + p_2 \left(1 - e^{a\ell_2}\right)\right]\right\}.$$

Similarly, we can solve for cause-eliminated risk premiums

$$\pi_{(-1)} = a \log \left[1 - p_2 \left(1 - e^{a\ell_2}\right)\right]$$

and

$$\pi_{(-2)} = a \log \left[1 - p_1 \left(1 - e^{a\ell_1}\right)\right].$$

Thus, the ratios of relative importance are

$$RRI_1 = \frac{\log\left(1 - \frac{p_{1,\exp}}{1 - p_{2,\exp}}\right)}{\log\left[1 - \left(p_{1,\exp} + p_{2,\exp}\right)\right]}$$

and

$$RRI_2 = \frac{\log\left(1 - \frac{p_{2,\exp}}{1 - p_{1,\exp}}\right)}{\log\left[1 - \left(p_{1,\exp} + p_{2,\exp}\right)\right]}$$

where

$$p_{1,\exp} = p_1 \left(1 - e^{a\ell_1}\right) \text{ and } p_{2,\exp} = p_2 \left(1 - e^{a\ell_2}\right).$$

Example 4: *Power Utility Function* Here we assume utility has the form $u(x) = x^a$, where $0 < a < 1$. From equation (2), we have

$$\pi = w_0 \left\{1 - \left\{1 - \left[p_1 \left[1 - \left(1 - \ell_1/w_0\right)^a\right] + p_2 \left[1 - \left(1 - \ell_2/w_0\right)^a\right]\right]\right\}^{1/a}\right\}.$$

Similarly, we can solve for cause-eliminated risk premiums

$$\pi_{(-1)} = w_0 \left\{1 - \left\{1 - p_2 \left[1 - \left(1 - \ell_2/w_0\right)^a\right]\right\}^{1/a}\right\}$$

and

$$\pi_{(-2)} = w_0 \left\{ 1 - \{1 - p_1 [1 - (1 - \ell_1/w_0)^a]\}^{1/a} \right\}.$$

Thus, the ratios of relative importance are

$$RRI_1 = \frac{(1 - p_{1,power})^{1/a} - [1 - (p_{1,power} + p_{2,power})]^{1/a}}{1 - [1 - (p_{1,power} + p_{2,power})]^{1/a}}$$

and

$$RRI_2 = \frac{(1 - p_{2,power})^{1/a} - [1 - (p_{1,power} + p_{2,power})]^{1/a}}{1 - [1 - (p_{1,power} + p_{2,power})]^{1/a}}$$

where

$$p_{1,power} = p_1 [1 - (1 - \ell_1/w_0)^a] \text{ and } p_{2,power} = p_2 [1 - (1 - \ell_2/w_0)^a].$$

3.3 Asymptotic Approximations

In the previous subsection, we developed the definition of the ratio of relative importance as a useful device for comparing the riskiness of the competing causes. This section develops a Taylor-series procedure to approximate the ratio of relative importance. The idea is to generalize the result to a class of utility functions so that the result will be free of the form of the utility function. But, first, we have the following useful proposition.

Proposition 5 *For the class of decision markers with non-decreasing utility function, the following decomposition of risk premiums* $\pi = \pi_{(-1)} + \pi_{(-2)}$ *holds true if and only if the utility function is linear.*

Proof. Sufficiency has been proven as example in the previous subsection. For the necessity part, assume risk premium decomposes into $\pi = \pi_{(-1)} + \pi_{(-2)}$. By combining equations (2), (3) and (4), we have

$$u(w_0) - u(w_0 - \pi) = \left[u(w_0) - u\left(w_0 - \pi_{(-1)}\right)\right] + \left[u(w_0) - u\left(w_0 - \pi_{(-2)}\right)\right]$$

or after some re-arrangement,

$$\left[u(w_0) - u\left(w_0 - \pi_{(-1)}\right)\right] - \left[u\left(w_0 - \pi_{(-1)}\right) - u(w_0 - \pi)\right] = 0.$$

This expression is equivalent to

$$\int_0^{\pi_{(-1)}} u'(w_0 - z)\, dz - \int_0^{\pi - \pi_{(-2)}} u'\left(w_0 - \pi_{(-2)} - z\right) dz = 0.$$

Since $\pi_{(-1)} = \pi - \pi_{(-2)}$, then we must have

$$\int_0^{\pi_{(-1)}} \left[u'\left(w_0 - z\right) - u'\left(w_0 - \pi_{(-2)} - z\right)\right] dz = 0.$$

By the assumption of non-decreasing utility function, then it must be that u' is constant which implies linear utility function.■

The proposition above is particularly useful because the risk premiums in a multiple decrement framework are impossible to decompose additively into components for which each component is attributable to each competing cause. Such decomposition is possible only for the class of risk-neutral decision makers.

We now derive approximations to the measure of relative importance defined earlier. We note that the resulting formula to compute RRI's depends on the form of the utility function. Our goal in the following paragraphs is to derive approximation formula that can be free of the utility function. We employ Taylor's approximation to do this. First, we note that

$$u\left(w_0 - \pi_{(-1)}\right) = u\left(w_0\right) - \pi_{(-1)} u'\left(w_0\right) + O\left(\pi_{(-1)}^2\right),$$

where the remainder term $O\left(\pi_{(-1)}^2\right)$ means terms of order two and higher. We assume $\pi_{(-1)}$ is small enough so that $1/\pi_{(-1)} \rightarrow \infty$ such as for causes that have a high probability of occurrence or of high economic loss, or for wealty decision makers. From equation (3), we have

$$\pi_{(-1)} \approx \frac{p_2\left[u\left(w_0\right) - u\left(w_0 - \ell_2\right)\right]}{u'\left(w_0\right)},$$

assuming that the remainder term $O\left(\pi_{(-1)}^2\right)$ is small enough to be ignored.

On the other hand, since we have

$$u\left(w_0 - \pi\right) \approx u\left(w_0 - \pi_{(-1)}\right) - \left(\pi - \pi_{(-1)}\right) u'\left(w_0 - \pi_{(-1)}\right)$$

and equation (2), we get

$$\pi - \pi_{(-1)} \approx \frac{p_1\left[u\left(w_0\right) - u\left(w_0 - \ell_1\right)\right]}{u'\left(w_0 - \pi_{(-1)}\right)}. \tag{9}$$

Thus,

$$\pi \approx \frac{p_1\left[u\left(w_0\right) - u\left(w_0 - \ell_1\right)\right]}{u'\left(w_0 - \pi_{(-1)}\right)} + \frac{p_2\left[u\left(w_0\right) - u\left(w_0 - \ell_2\right)\right]}{u'\left(w_0\right)}. \tag{10}$$

The ratio of relative importance for competing "cause 1" is then the ratio of (9) to (10). To further the approximation, we assume equal losses for both competing causes, that is to say, $\ell_1 = \ell_2 = \ell$. The ratio is then simplified to:

$$RRI_1 \approx \frac{p_1}{p_1 + p_2 \left[u'\left(w_0 - \pi_{(-1)}\right)/u'\left(w_0\right)\right]}.$$

Similarly, one can show that

$$RRI_2 \approx \frac{p_2}{p_1 \left[u'\left(w_0 - \pi_{(-2)}\right)/u'\left(w_0\right)\right] + p_2}.$$

We can re-write these ratios further by using the measure of risk aversion introduced by Arrow (1965) and Pratt (1964) to further simplify them. Recall that $r_A(w_0) = -u''(w_0)/u'(w_0)$ is the measure of local risk aversion. From equations (3) and (4), if we differentiate both sides of each equation with respect to initial wealth w_0, we get

$$u'\left(w_0 - \pi_{(-1)}\right) = u'(w_0) - p_2\left[u'(w_0) - u'(w_0 - \ell)\right]$$

and

$$u'\left(w_0 - \pi_{(-2)}\right) = u'(w_0) - p_1\left[u'(w_0) - u'(w_0 - \ell)\right].$$

Therefore, we have

$$\begin{aligned} RRI_1 &\approx \frac{p_1}{p_1 + p_2\left[1 - p_2 \frac{u'(w_0) - u'(w_0 - \ell)}{u'(w_0)}\right]} \\ &\approx \frac{p_1}{p_1 + p_2\left[1 - p_2 \frac{\ell u''(w_0)}{u'(w_0)}\right]} \\ &= \frac{p_1}{p_1 + p_2\left[1 + \ell p_2 r_A(w_0)\right]}. \end{aligned} \tag{11}$$

Similarly, we can show that

$$RRI_2 \approx \frac{p_1}{p_1\left[1 + \ell p_2 r_A(w_0)\right] + p_2}. \tag{12}$$

Because the approximations above involve the measure of local risk aversion measure, these may still implicitly be functions of the utility. However, as Table 2 shows, measures for some utility function are not at all complicated. Because of the non-additivity of the risk premiums except in the case of risk-neutral decision makers, the ratios of relative importance do not add up to one. The approximations in (11) and (12) also provide the components which contribute to the non-additivity of the ratios of relative importance. There is non-additivity because of three components: (a) the economic loss associated with each cause, (b) the associated probabilities; and (c) risk aversion which depends on the individua's initial wealth and shape of the utility function. In effect, we observe that small losses, low probabilities, or low risk aversion increases the chance of additivity.

Table 2
Local Risk Aversion Measures and Approximate Ratios
of Relative Importance for Various Utility Functions

Utility	Local Risk Aversion	Equation (11) $\approx RRI_1$	Equation (12) $\approx RRI_2$
Linear	0	$\dfrac{p_1}{p_1+p_2}$	$\dfrac{p_2}{p_1+p_2}$
Logarithmic	$\dfrac{1}{x}$	$\dfrac{p_1}{p_1+p_2\left(1+\ell\frac{p_2}{w_0}\right)}$	$\dfrac{p_1}{p_1\left(1+\ell\frac{p_1}{w_0}\right)+p_2}$
Exponential	a	$\dfrac{p_1}{p_1+p_2\left(1+a\ell p_2\right)}$	$\dfrac{p_1}{p_1\left(1+a\ell p_1\right)+p_2}$
Power	$\dfrac{(1-a)}{x}$	$\dfrac{p_1}{p_1+p_2\left(1+\ell p_2\frac{(1-a)}{w_0}\right)}$	$\dfrac{p_1}{p_1\left(1+\ell p_1\frac{(1-a)}{w_0}\right)+p_2}$

4 Model Extensions

In the previous section, we develop measures of relative importance in the case where we only have two competing causes and a single time period. The development was made by sequentially removing each competing cause and calculating the Arrow-Pratt risk premium the individual would be willing to pay after removal of the cause. The removal of risk is equated with making the correponding loss from that cause to be zero. This procedure is acceptable provided the decision maker is able to realistically eliminate the risk of losing from a particular competing cause. However, in reality, the decision maker is often faced with more than two competing causes and/or for a longer time period. This section provides extensions to the basic economic model. In section 4.1, we extend the development with more than two competing causes. In section 4.2, we extend it to multiple time periods.

4.1 Multiple Competing Causes

The extension to more than two competing causes is rather straightforward. Here, for the case where we have k competing causes, we re-define the loss random variable as follows:

$$L = \begin{cases} \ell_j, & \text{if "cause } j\text{" occurs, w.p. } p_j, \text{ for } j = 1, 2, ..., k \\ 0, & \text{w.p. } 1 - \sum_{j=1}^{k} p_j \end{cases} .$$

Thus, the expected utility is

$$E\left[u\left(w_0 - L\right)\right] = \sum_{j=1}^{k} p_j u\left(w_0 - \ell_j\right) + \left(1 - \sum_{j=1}^{k} p_j\right) u\left(w_0\right). \qquad (13)$$

The Arrow-Pratt risk premium π can be determined from

$$u\left(w_0 - \pi\right) = E\left[u\left(w_0 - L\right)\right].$$

Now consider the case where the decision maker is able to take action to eliminate losses arising from say "cause i," and hence $\ell_i = 0$ where $i = 1, 2, ..., k$ is any one of the competing causes. In this situation, the maximum premium $\pi_{(-i)}$ he would then be willing to pay satisfies

$$u\left(w_0 - \pi_{(-i)}\right) = \sum_{j \neq i} p_j u\left(w_0 - \ell_j\right) + \left(1 - \sum_{j \neq i} p_j\right) u\left(w_0\right). \qquad (14)$$

Subtracting (13) from (14), we clearly see that

$$u\left(w_0 - \pi_{(-i)}\right) - u\left(w_0 - \pi\right) = p_i\left[u\left(w_0 - \ell_i\right) - u\left(w_0\right)\right]. \qquad (15)$$

To simplify, suppose the individual is risk neutral and hence has a linear utility function. It is well-known that the Arrow-Pratt risk premium will be the expected value of the loss so that

$$\pi = \sum_{j=1}^{k} p_j \ell_j.$$

From (15), we should have

$$\pi - \pi_{(-i)} = p_i \ell_i.$$

The ratio of relative importance of competing "cause i" in this case yields

$$RRI_i = \frac{\pi - \pi_{(-i)}}{\pi} = \frac{p_i \ell_i}{\sum_{j=1}^{k} p_j \ell_j}.$$

The numerator here is the contribution of competing cause i to the total expected loss.

When we have multiple causes, we can also make pairwise comparison similar to the case of only two competing causes. We state a proposition similar to Proposition (2). We omit the proof as it is very similar to the one in that proposition.

Proposition 6 *For the class of decision makers with non-decreasing utility functions, the following holds for any pairs of $i, j = 1, 2, ..., k$:*

(i) If $\ell_i \geq \ell_j$ and $p_i \geq p_j$, then $RRI_i \geq RRI_j$.

(ii) If $\ell_i \leq \ell_j$ and $p_i \leq p_j$, then $RRI_i \leq RRI_j$.

(iii) If $\ell_i \geq \ell_j$ and $RRI_i \leq RRI_j$, then $p_i \leq p_j$.

(iv) If $\ell_i \leq \ell_j$ and $RRI_i \geq RRI_j$, then $p_i \geq p_j$.

(v) If $p_i \geq p_j$ and $RRI_i \leq RRI_j$, then $\ell_i \leq \ell_j$.

(vi) If $p_i \leq p_j$ and $RRI_i \geq RRI_j$, then $\ell_i \geq \ell_j$.

4.2 Model with Several Periods

In the case of multiple time periods, there may be some difficulties of extending the basic model. One way to handle multiple time periods is to define individual preferences using a utility function that would be additive over time, but with an appropriate weighting factor, say β. See Varian (1992). This factor accounts for items like the time value of money and possible differences in the quality of satisfaction over time. To simplify, suppose we only have two competing causes and define the loss random variable for period t as follows:

$$L_t = \begin{cases} \ell_{1,t}, & \text{if "cause 1" occurs, w.p. } p_{1,t} \\ \ell_{2,t}, & \text{if "cause 2" occurs, w.p. } p_{2,t} \\ 0, & \text{w.p. } 1 - p_{1,t} - p_{2,t} \end{cases}.$$

Note that, to emphasize time-dependence,we have subscripted both the probabilities and economic losses associated with each cause. Assuming infinite time periods, then the expected utility will be

$$E\left[\sum_{t=1}^{\infty} \beta^t u\left(w_0 - L_t\right)\right] = \sum_{t=1}^{\infty} \beta^t \ {}_{t-1}p \times \left[\begin{array}{c} p_{1,t} u\left(w_0 - \ell_{1,t}\right) \\ +p_{2,t} u\left(w_0 - \ell_{2,t}\right) \\ +\left(1 - p_{1,t} - p_{2,t}\right) u\left(w_0\right) \end{array}\right]$$

with the maximum premium satisfying

$$\sum_{t=1}^{\infty} \beta^t \ {}_{t-1}p u\left(w_0 - \pi\right) = E\left[\sum_{t=1}^{\infty} \beta^t u\left(w_0 - L_t\right)\right], \tag{16}$$

where ${}_{t-1}p = \prod_{m=0}^{t-1}\left(1 - p_{1,m} - p_{2,m}\right)$ is the cumulative probability of "no loss" from time 0 until the beginning of period t.

In (16), we assume amount of risk premium is payable once at contract inception. Furthermore, following a similar argument as in the previous section, with an insurance coverage that excludes "cause 1," the resulting maximum premium $\pi_{(-1)}$ the individual is willing to pay will satisfy

$$\sum_{t=1}^{\infty} \beta^t \ {}_{t-1}p u\left(w_0 - \pi_{(-1)}\right) = \sum_{t=1}^{\infty} \beta^t \ {}_{t-1}p \left\{u\left(w_0\right) + p_{2,t}\left[u\left(w_0\right) - u\left(w_0 - \ell_{2,t}\right)\right]\right\}. \tag{17}$$

Similarly, we can derive such equation for $\pi_{(-2)}$:

$$\sum_{t=1}^{\infty} \beta^t \ {}_{t-1}p u\left(w_0 - \pi_{(-2)}\right) = \sum_{t=1}^{\infty} \beta^t \ {}_{t-1}p \left\{u\left(w_0\right) + p_{1,t}\left[u\left(w_0\right) - u\left(w_0 - \ell_{1,t}\right)\right]\right\}. \tag{18}$$

In the case where we have a risk-neutral decision maker, it is easy to verify using equations (16) and (17) above that

$$\pi = \frac{\sum_{t=1}^{\infty} \beta^t \ {}_{t-1}p E\left(L_t\right)}{\sum_{t=1}^{\infty} \beta^t \ {}_{t-1}p} = \frac{\sum_{t=1}^{\infty} \beta^t \ {}_{t-1}p \left(p_{1,t}\ell_{1,t} + p_{2,t}\ell_{2,t}\right)}{\sum_{t=1}^{\infty} \beta^t \ {}_{t-1}p}$$

and that

$$\pi_{(-1)} = \frac{\sum_{t=1}^{\infty} \beta^t \ {}_{t-1}p \cdot p_{2,t}\ell_{2,t}}{\sum_{t=1}^{\infty} \beta^t \ {}_{t-1}p}.$$

Similarly, one can derive

$$\pi_{(-2)} = \frac{\sum_{t=1}^{\infty} \beta^t \ {}_{t-1}p \cdot p_{1,t}\ell_{1,t}}{\sum_{t=1}^{\infty} \beta^t \ {}_{t-1}p}.$$

Thus, our measures of relative importance when the utility function is linear are given by

$$RRI_1 = \frac{\sum_{t=1}^{\infty} \beta^t \ {}_{t-1}p \cdot p_{1,t}\ell_{1,t}}{\sum_{t=1}^{\infty} \beta^t \ {}_{t-1}p \left(p_{1,t}\ell_{1,t} + p_{2,t}\ell_{2,t}\right)}$$

and

$$RRI_2 = \frac{\sum_{t=1}^{\infty} \beta^t \ {}_{t-1}p \cdot p_{2,t}\ell_{2,t}}{\sum_{t=1}^{\infty} \beta^t \ {}_{t-1}p \left(p_{1,t}\ell_{1,t} + p_{2,t}\ell_{2,t}\right)}.$$

Following the development in the previous subsection on multiple causes, it becomes straightforward to evaluate ratios of relative importance when you have more than two competing causes. It can be shown that, in the case where we have k competing causes and multiple time periods, we have the following result for risk-neutral decision makers:

$$RRI_i = \frac{\sum_{t=1}^{\infty} \beta^t \ {}_{t-1}p \cdot p_{i,t}\ell_{i,t}}{\sum_{t=1}^{\infty} \beta^t \ {}_{t-1}p \left(\sum_{i=1}^{k} p_{i,t}\ell_{i,t}\right)} \tag{19}$$

for $i = 1, 2, ..., k$. The difficulty with a multiple time period framework lies from the obvious fact that we will need to estimate probabilities $p_{j,t}$ for each competing cause and at each time period. In (19), note that, if $p_{j,t}$ does not depend on t for all $j = 1, 2, ..., k$, then the expression becomes independent of the discount factor. This explains the usefulness of single-period models for property insurance where in some cases, the probabilities are independent of time.

Moreover, in the case of several causes and multiple time periods, we can derive results similar to Propositions (2) and (6). We state these results without proof. The proof is very similar again to that provided in Proposition (2) using the following relationship that is easily derived using equations (16), (17), and (18):

$$
\begin{aligned}
& \sum_{t=1}^{\infty} \beta^t \ {}_{t-1}p \left[u\left(w_0 - \pi_{(-2)}\right) - u\left(w_0 - \pi_{(-2)}\right) \right] \\
= & \sum_{t=1}^{\infty} \beta^t \ {}_{t-1}p \left(p_{2,t} - p_{1,t}\right) \left[u\left(w_0\right) - u\left(w_0 - \ell_{2,t}\right) \right] \\
& - \sum_{t=1}^{\infty} \beta^t \ {}_{t-1}p \cdot p_{1,t} \left[u\left(w_0 - \ell_{1,t}\right) - u\left(w_0 - \ell_{2,t}\right) \right].
\end{aligned}
$$

Proposition 7 *For the class of decision makers with non-decreasing utility functions, the following holds for any pairs of* $i, j = 1, 2, ..., k$*:*

(i) If $\ell_{i,t} \geq \ell_{j,t}$ *and* $p_{i,t} \geq p_{j,t}$ *for all period* t*, then* $RRI_i \geq RRI_j$*.*

(ii) If $\ell_{i,t} \leq \ell_{j,t}$ *and* $p_{i,t} \leq p_{j,t}$ *for all period* t*, then* $RRI_i \leq RRI_j$*.*

Note that this proposition does not have similar versions of (iii) through (vi) of Propositions 2 and 5. It becomes difficult to draw similar conclusions because of the multiplicity of the time period. However, we can draw similar results when we make the simplifying assumptions that the economic loss for each cause is constant over time. We drop the subscript t on the conomic losses. We do not make such assumption for the probabilities. Such assumptions realistically hold for instance, in life insurance, where the benefit amount is constant at each time period, but it is well-known that probabilities of death increase with age, and hence, the probabilities are not constant over time. There is no strict comparison of probabilities at each period. Instead, when probabilities are compared, they are compared based on their "present values." We state, again without proof, the results.

Proposition 8 *For the class of decision makers with non-decreasing utility functions, the following holds for any pairs of* $i, j = 1, 2, ..., k$*:*

(i) *If* $\ell_i \geq \ell_j$ *and* $\sum_{t=1}^{\infty} \beta^t \ {}_{t-1}p \cdot p_{i,t} \geq \sum_{t=1}^{\infty} \beta^t \ {}_{t-1}p \cdot p_{j,t}$*, then* $RRI_i \geq RRI_j$.

(ii) *If* $\ell_i \leq \ell_j$ *and* $\sum_{t=1}^{\infty} \beta^t \ {}_{t-1}p \cdot p_{i,t} \leq \sum_{t=1}^{\infty} \beta^t \ {}_{t-1}p \cdot p_{j,t}$*, then* $RRI_i \leq RRI_j$.

(iii) *If* $\ell_i \geq \ell_j$ *and* $RRI_i \leq RRI_j$*, then* $\sum_{t=1}^{\infty} \beta^t \ {}_{t-1}p \cdot p_{i,t} \leq \sum_{t=1}^{\infty} \beta^t \ {}_{t-1}p \cdot p_{j,t}$.

(iv) *If* $\ell_i \leq \ell_j$ *and* $RRI_i \geq RRI_j$*, then* $\sum_{t=1}^{\infty} \beta^t \ {}_{t-1}p \cdot p_{i,t} \geq \sum_{t=1}^{\infty} \beta^t \ {}_{t-1}p \cdot p_{j,t}$.

(v) *If* $\sum_{t=1}^{\infty} \beta^t \ {}_{t-1}p \cdot p_{i,t} \geq \sum_{t=1}^{\infty} \beta^t \ {}_{t-1}p \cdot p_{j,t}$ *and* $RRI_i \leq RRI_j$*, then* $\ell_i \leq \ell_j$.

(vi) *If* $\sum_{t=1}^{\infty} \beta^t \ {}_{t-1}p \cdot p_{i,t} \leq \sum_{t=1}^{\infty} \beta^t \ {}_{t-1}p \cdot p_{j,t}$ *and* $RRI_i \geq RRI_j$*, then* $\ell_i \geq \ell_j$.

5 Concluding Remarks

This paper uses expected utility theory to derive formulas for assessing the relative importance of competing causes. We define the relative importance to be the ratio of two risk premiums; the numerator is the risk premium an individual is willing to pay when he eliminates the cause, and the denominator is that when no cause is eliminated. Using this definition, we further derive intuitively appealing results. Generally, we find that the relative importance of competing cause depends on a variety of factors: (a) the economic loss associated with the cause relative to the other economic losses, (b) the probability that failure due to the cause relative to the other probabilities, (c) the shape of the utility function of the decision marker which describes his risk aversion behavior, and (d) in the case of multiple time periods, a type of discount that accounts for the time value of money and possible differences in quality of satisfaction over time.

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