DEPENDENT CAUSES OF DEATH

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Abstract. Many textbooks and research papers on the study of various causes of death are often based on the assumption of independence. Many researchers and practitioners, however, recognize that the assumption of independent causes of death is unrealistic. There is little or no evidence on the nature of the possible dependencies because there is extreme difficulty in establishing these dependencies. We use copulas as a tool for understanding these dependencies. Using 1992 Mortality Detail File, which consists of every death registered in the United States in 1992, we empirically show how to employ copula-based models to estimate dependencies of causes of death. In particular, we focus on the nature of dependencies of accidents with all other causes, and our estimation results show strong dependencies. This is not at all surprising as we provide possible explanation for these dependencies. We further illustrate how mispricing of accidental death benefit policies could occur if these dependencies are not taken into account.

1. Introduction. The analysis of the causes of human mortality is a well established area of research in medicine, public health, biostatistics (Chiang, 1968; Klein and Moeshberger, 1998), demography (Key…tz, 1985), and actuarial science (Bowers et al., 1997; Carriere, 1994). Analyzing causes of death is important because we wish to examine the overall health of the population, to evaluate the impact of public health programs, and to predict population growth more accurately, to name a few reasons. It is important in actuarial science because some contracts do not insure certain causes of death (e.g. suicide subject to a suicide clause) or in the case of accidental death bene…t coverages, the bene…t is a di¤erent amount from the basic sum insured.

The study of the various causes of death falls within a broad framework of the theory of competing causes. The term multiple decrement is generally preferred in actuarial science. See Bernoulli (1760) for an early paper on multiple decrements. In other disciplines such as biostatistics and engineering, the phrase competing risks is the preferred nomenclature. Another term used is multiple destination. Each of these terms describes a general theory that involves the assessment of a particular cause of failure under the complicating presence of other causes. In general, a system fails because of the occurrence of one of a set of competing causes. The system could be a person or a mechanical device with multiple components. A person dies because of one of several possible causes such as heart disease or cancer. The mechanical device or machine fails because one of its components malfunctions. Modelling the probability distribution of the lifetime of the system is the prime concern in the study of multiple decrements, to use the phrase preferred in actuarial science.

Despite the fact that it is a well established area of research, several researchers recognize the limitation that the theory is often based on the assumption that causes of death are independent. There is very little empirical evidence that causes are indeed not independent. However, there has been some work for analyzing dependent causes. Carriere (1994) introduced dependent decrement theory and in his paper showed that when heart and cerebrovascular diseases are correlated with all other causes of death, it could result in an erroneous analysis of removing heart and cerebrovascular diseases as a cause of death. Elandt-Johnson (1976) analytically developed the resulting distribution of failure times when some causes are removed, assuming possible dependent causes.

In the absence of any empirical evidence, we can justify why causes of death may be dependent. First is what we call the biological justification. The human body is a complex system that is made up of several related parts. Weakening of one part can lead to the weakening of another part. An analogy made in Keyfitz (1985) is that the human body is like a watch or a machine. The watch or machine may continue to function even with one part defective but puts strain on other parts leading to a breakdown. However, the cause of the eventual malfunction may be attributed to the piece of the machine that was subjected to the strain rather than to that piece of the machine that became weak in the first place. In terms of the human body, a disease could be attributed to a body part or organ. For instance, diseases of the heart are linked to that muscular organ, called the heart, that receives blood from the veins and pumps it through the arteries. Each body part or organ has its own survival function which, because of dependencies, alters the survival functions of the other body parts.

The second justification may well be called unobserved secondary causes. It is widely known that besides the underlying cause, there is a possible secondary cause that contributes to the death of a person. A better procedure of estimating dependencies among causes of death would be to examine secondary causes of death relative to the primary cause. However, although this type of information can be retrieved from death certificates, most mortality data sets, such as the 1992 Mortality Detail File used in our empirical investigation, do not contain the secondary cause information. In the absence of this information, we can only establish models, such as the copula structure used in our investigation, to estimate the dependencies.

Selection is another justification. It is possible that those who die from certain causes, such as accidents from mountain climbing or skiing, may be healthier on average than the general population. Hence, the effect of reducing deaths from these causes could substantially improve death rates.

The last justification is errors in variables. There is the possibility of errors in the values entered into the dataset. Some deaths may be misclassified. With several hundreds of ICD codes for classifying causes of death, it increases the likelihood of misclassification.

The main theme of this paper is to introduce copula models in empirically estimating the possible dependence of causes of death. To simplify the illustration, we confine ourselves to the case of only two competing causes of death, in particular, accidents and all other causes. The resulting empirical estimates are further used to illustrate how mispricing of accidental death benefit policies could occur if these dependencies are ignored in the pricing calculation. The rest of this paper is therefore arranged as follows. In Section 2, we provide preliminaries by reviewing fundamental elements of competing causes and copulas. Here, we introduce some of the notations and definitions used in later development. Section 3 describes the data used in our investigation, the estimation procedures, and the results of the estimation. In Section 4, we illustrate the usefulness of the estimation in assessing the price for an accidental death benefit insurance policy. Section 5 gives concluding remarks.

2. Preliminaries.

2.1. Theory of Competing Causes.

This section provides an introduction to the subject of competing causes. This introduction is brief and not comprehensive as it is meant to develop preliminaries, definitions, and notations useful in a later section. Many textbooks lay out foundations of the theory of competing causes. See Bowers et al. (1997), Birnbaum (1979), David and Moeschberger (1978), and Elandt-Johnson and Johnson (1980). The discussion is in the context of a population that is subject to dying from two causes of death.

A newborn individual has a lifetime random variable X and is exposed to two different causes of mortality. We shall denote the cause of death random variable by

J. Thus, $J = j$ indicates that death was due to cause $j = 1, 2$, which may denote deaths from accidents and all other causes. It is often convenient to introduce the theoretical "net" lifetime random variable X_j that corresponds to cause of death j. Its distribution, survival, and density functions will be denoted by F_j, S_j, and f_j, respectively.

The net lifetimes X_j will never be observed simultaneously. Instead, when an individual dies, we observe the cause of death J and the lifetime $X = \min(X_1, X_2)$. Let the net lifetime random vector be (X_1, X_2) whose joint density we assume exists and is denoted by $h(x_1, x_2)$. Its distribution and survival functions are to be denoted by $H(x_1, x_2)$ and $S(x_1, x_2)$, respectively. Thus, their marginal distribution functions can be derived using

$$F_1(x_1) = H(x_1, \infty) \text{ and } F_2(x_2) = H(\infty, x_2) \tag{1}$$

and survival functions using

$$S_1(x_1) = S(x_1, 0) \text{ and } S_2(x_2) = S(0, x_2). \tag{2}$$

For purposes of simplifying results, we additionally assume that a person dies from a single cause of death. In other words, the causes of death are mutually exclusive events. Thus, the value of J is uniquely determined with probability one. We can write this as follows $\text{Prob}(X_1 = X_2) = 0$. This assumption appears to be realistic because there is usually a single primary (called the underlying) cause of death recorded on the death certificate.

The joint distribution of (X, J) can be derived as follows. For $J = 1$, we have

$$\begin{aligned} F_{X,J}(x,1) &= \text{Prob}(X \le x, J = 1) = \text{Prob}(\min(X_1, X_2) \le t, J = 1) \\ &= \text{Prob}(X_1 \le x, X_1 < X_2) = \int_0^x \int_{z_1}^\infty h(z_1, z_2)\, dz_2 dz_1. \end{aligned} \tag{3}$$

Similarly, we have for $J = 2$,

$$F_{X,J}(x,1) = \int_0^x \int_{z_2}^\infty h(z_1, z_2)\, dz_1 dz_2. \tag{4}$$

The corresponding joint density is determined by

$$f_{X,J}(x, j) = \frac{\partial F_{X,J}(x, j)}{\partial x}. \quad (5)$$

To calculate the marginal distribution of X, we can sum the distribution functions in equations (3) and (4) as

$$F_X(x) = \sum_{j=1}^{2} F_{X,J}(x, j), \quad (6)$$

or directly compute it based on the survival function of (X_1, X_2) by noting that

$$F_X(tx) = 1 - S(x, x). \quad (7)$$

The survival functions of the net lifetimes X_j as calculated using (2) are often called the net survival functions. The survival function of the lifetime X, denoted by $S_X(t) = S(x, x)$, is called the total or overall survival function. Finally, the survival function that corresponds to (X, J) and defined by $S^{(j)}(x) = \text{Prob}(X > x, J = j)$, is called the crude survival function. For this reason, we shall call probabilities associated with (X, J) to be crude probabilities and they are useful for developing the likelihood function as discussed in our section on empirical analysis. Corresponding to the net, overall, and crude functions, we can define life table notations. We will not develop them here, but we suggest the reader to consult Bowers et al. (1997) and Carriere (1994).

Equation (3) is a useful probability statement. However, it can be particularly difficult to evaluate because of the number of times integration has to be performed. An alternative to equation (3) can be developed as follows. We note that we can write the overall survival function as

$$S(x_1, x_2) = \int_{x_1}^{\infty}\int_{x_2}^{\infty} h(z_1, z_2)\, dz_2 dz_1.$$

Now, take the partial derivative of both sides with respect to x_1 and evaluate both sides with $x_2 = x_1$, then we have

$$\left.\frac{\partial S(x_1, x_2)}{\partial x_1}\right|_{x_2 = x_1} = -\int_{x_1}^{\infty} h(x_1, z_2)\, dz_2.$$

Using this result, we can alternatively calculate the crude joint distribution in equation (3) for $j = 1$:

$$F_{X,J}(x,1) = \int_0^x \left[\left. -\frac{\partial S(x_1,x_2)}{\partial x_1} \right|_{x_2=x_1} \right] dx_1. \tag{8}$$

Following a similar argument for $j = 2$, we have an alternative formula to equation (4):

$$F_{X,J}(x,2) = \int_0^x \left[\left. -\frac{\partial S(x_1,x_2)}{\partial x_2} \right|_{x_1=x_2} \right] dx_2. \tag{9}$$

Equations (8) and (9) involve a single integration which is useful for numerically evaluating the joint distribution. A similar formula appears in Tsiatis (1975) to prove the identifiability in the case of independence and in Carriere (1994) as a simple representation of the crude survival function. Their formulas extend to several competing causes.

2.2. Copulas. This section briefly describes copulas. Used as a tool for understanding relationships among multivariate outcomes, a copula is a function that links (or couples) univeriate marginals to their full multivariate distribution. Copulas were intorduced by Sklar (1959) in the context of probabilistic metric spaces, a branch of mathematics that deals with measures. Today, there is a rapidly developing literature on the statistical properties and applications of copulas. As pointed out in Frees and Valdez (1998), there is a variety of applications of this tool in actuarial science. See also Genest and MacKay (1986a,1986b), Joe (1997), and Nelsen (1999) for further understanding of copulas.

An approach to examine dependence is to express the relationship in terms of a functional form. This approach is particularly commonplace. For instance, in regression analysis, a dependent variable is usually expressed as a linear relationship of several other independent variables. The statistical methods and estimation procedures in regression, as with other similar approaches of expressing relationships in functional form, are very well developed. A more general approach to model dependence between random variables is to specify the joint distribution of the variables using copulas. To define a copula more formally, we follow Schweizer and Sklar (1983). A two-dimensional copula, denoted by $C(u, v)$, is a two-dimensional probability distribution function defined on the unit square $[0, 1]^2$ and whose univariate marginals are uniform on $[0, 1]$. Thus, it follows that for all $u, v \in [0, 1]$, we have $C(u, 0) = C(0, v) = 0$, $C(u, 1) = u$, and $C(1, v) = v$. Furthermore, it is true that $C(u_1, v_1) - C(u_1, v_2) - C(u_2, v_1) + C(u_2, v_2) \geq 0$ whenever $u_1 \leq u_2, v_1 \leq v_2$ for all $u_1, v_1, u_2, v_2 \in [0, 1]$. The existence of a copula function for any multivariate distribution was established by Sklar (1959). In the case of two-dimensional copulas, he proved that for any random vector (X_1, X_2) with a bivariate distribution function $H(x_1, x_2) = \text{Prob}(X_1 \leq x_1, X_2 \leq x_2)$, there will always be a copula function C that will satisfy

$$H(x_1, x_2) = C(F_1(x_1), F_2(x_2)) \tag{10}$$

where $F_k(\cdot)$ for $k = 1, 2$ denotes the marginals. Because of (10), copulas are often referred to as functions that link or join or couple multivariate distribution functions to their marginal distribution functions.

For example, the Type B bivariate extreme value distribution function that appear in Johnson and Kotz (1972)

$$H(x_1, x_2) = \exp\left[-\left(e^{-\alpha x_1} + e^{-\alpha x_2}\right)^{1/\alpha}\right]$$

belongs to the following family of copulas

$$C(u, v) = \exp\left(-\left[(-\ln u)^{\alpha} + (-\ln v)^{\alpha}\right]^{1/\alpha}\right). \tag{11}$$

This family is known as the Gumbel-Hougaard family of copulas. For yet another example, consider the familiar bivariate normal distribution whose density can be expressed as

$$h(x_1, x_2) = \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left\{-\frac{x_1^2 - 2\rho x_1 x_2 + x_2^2}{2(1-\rho^2)}\right\}$$

and distribution function as

$$H(x_1, x_2) = \int_{-\infty}^{x_2}\int_{-\infty}^{x_1} h(z_1, z_2)\, dz_1 dz_2.$$

It follows that the corresponding copula can be expressed as

$$C(u,v) = H\left(\Phi^{-1}(u), \Phi^{-1}(v)\right) \tag{12}$$

where $\Phi(\cdot)$ is the distribution function of a standard normal random variable, i.e. $\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}w^2\right) dw$. Equation (12) is called the bivariate normal copula.

Note in both (11) and (12) that the copulas contain parameters that often describe the dependence. Schweizer and Wolff (1981) expressed familiar measures of correlation in terms of the copula functions. It is clear that the copula described by $C(u,v) = uv$ is the associated function for independent random variables. In the Gumbel-Hougaard family, we have the case of independence when $\alpha = 0$. Any departure from $\alpha = 0$ will imply dependence of the random variables. Similarly, in the normal copula family, we have $\rho = 0$ for the case of independence. For other familiar families of copulas, see Frees and Valdez (1998) and Nelsen (1999).

A copula is Archimedean if it can be written in the form

$$C(u,v) = \phi^{-1}(\phi(u) + \phi(v)), \tag{13}$$

for all $0 \le u,v \le 1$ and for some function ϕ that satisfies (i) $\phi(1) = 0$ and (ii) ϕ is decreasing and convex. The function ϕ is called the generator. It is straightforward to show that (13) satisfies the definition of a copula. One of the advantages of the Archimedean representation is that when searching for a copula suitable to describe random variables, we reduce the search for a single univariate function. It also allows us to construct copulas of higher than two dimensions.

The Gumbel-Hougaard copula in (11) belongs to the family of Archimedean copulas. Its generator is given by $\phi(t) = (-\ln t)^{\alpha}$. A family with generator $\phi(t) = \ln\left(\frac{e^{\alpha t} - 1}{e^{\alpha} - 1}\right)$ yields the familiar Frank's copula. In two dimension, the Frank's copula is expressed as

$$C(u,v) = \frac{1}{\alpha} \ln\left(1 + \frac{(e^{\alpha u} - 1)(e^{\alpha v} - 1)}{e^{\alpha} - 1}\right). \tag{14}$$

See Frank (1979) and Genest (1987) for details about the characteristics of this copula.

A source of generators for Archimedean copulas consists of the inverses of Laplace transforms of distribution functions, or correspondingly their moment generating functions. Suppose a non-negative random variable X has the distribution function $F(x)$. Then its Laplace transform is given by

$$\psi(t) = E\left(e^{-Xt}\right) = \int_0^\infty e^{-tx} dF(x).$$

It can be shown that the inverse of $\psi(t)$ satisfies the properties of a generator. Thus, $\phi(t) = \psi^{-1}(t)$ generates Archimedean copulas. See Nelsen (1999).

3. Data Characteristics and Estimation.

3.1. The Data.

The mortality data analyzed is derived from the 1992 Mortality Detail File[1] compiled by the National Center for Health Statistics (NCHS) of the Department of Health and Human Services. This collection consists of every death registered in the United States during 1992 but it is limited to deaths that occurred within the U.S. to both residents and non-residents. It does not include 1992 deaths of U.S. citizens that occurred outside of the United States.

The death statistics are collected from all 50 states, the District of Columbia, and New York city (which is apparently independent of the state of New York for the purpose of death registration) and tabulated by the NCHS. Information from the mortality detail file includes the month of death, day of the week of death, sex, race, age, education, work industry, place of residence, place of death, and whether an autopsy was performed. There were a total of 2,179,187 deaths recorded in the data file. Of these, 3,574 are foreign residents; another 4,048 have unknown age at death and are therefore removed from our analysis.

When we speak of a cause of death, we refer to the so-called underlying cause of death. This is important to emphasize because in the death certificate, the certifier may provide immediate, underlying, as well as contributing causes. Using the example

[1] The author thanks Laura Guy of the Data Program Library Services, University of Wisconsin - Madison for assisting in retrieving this dataset.

in Zopf (1992), "the immediate cause of death of a very elderly person may be pneumonia, but the underlying cause may be congestive heart failure, and a contributing cause of the heart problem may be atherosclerosis, or the long-term accumulation of deposits on arterial walls. Thus, the victim died of an illness contracted perhaps only a few days before death, partly because of weakness and susceptibility from the diminishing capacity of the heart to pump blood." To ensure uniformity in reporting different types of causes of death, the World Health Organization (WHO) sets standards documented in its Manual of the International Statistical Classification of Diseases, Injuries, and Causes of Death (ICD). See WHO (1997). It is standard practice to report statistics of causes of death using underlying causes. This paper focuses on accident as a cause of death, and thereafter, our analysis divides the causes of death into accidents and all others besides accidents.

For estimation purposes, data about living population in 1992 is required and is retrieved from the 1993 U.S. Statistical Abstract. There were an approximate 255 million people living in 1992, and this translates to an overall death rate of 8.53 for every thousand of population. The death rate per thousand for female is 8.06 and is lower than that for male which is 9.02. Figure 1 provides a further comparison of the death rate in 1992 by sex for some causes of death. The accident rate per thousand was 0.20, accounting for 1.9% of all deaths in 1992.

[Insert Figure 1 here]

To prevent inadvertent disclosure of individuals, NCHS removed all distinguishing characteristics and statistics on each death that may possibly lead to identification of the data subject. As such, the data file does not contain the actual date of death nor the actual date of birth of each deceased. Unfortunately, for our purposes, we lose some very useful information for our analysis and that is, the knowledge of the exact age at death. The information on age consists of categorizing each decedent into various age groups which are of one-year intervals in the first five years and five-year intervals thereafter.

Our analysis begins therefore by building a non-parametric two-decrement table based on accidents and all other causes of death. Because we wish to apply the results of our analysis by analyzing accidental death benefit insurance coverages, we dropped all deaths prior to age 19 and past age 79. This age range is most useful for insurance companies. To produce rates for one-year age intervals, we performed linear interpolation for five-year age intervals where needed. We shall define

$\hat{q}_x^{(1)} =$ the nonparametric estimate of the one-year probability of dying from accident for a person age x,

$\hat{q}_x^{(2)} =$ the nonparametric estimate of the one-year probability of dying from causes other than accidents for a person age x, and

$\hat{q}_x^{(T)} =$ the nonparametric estimate of the one-year probability of dying from any cause for a person age x.

Clearly, $\hat{q}_x^{(T)} = \hat{q}_x^{(1)} + \hat{q}_x^{(2)}$. Figure 2 provides a comparison of the one-year crude probabilities by cause of death: accidents and all other causes.

[Insert Figure 2 here]

3.2. Development of the Likelihood Function.

Each observation in our dataset consists of a person, labeled i and is observed within one year time period $(x_i, x_i + 1)$. At the end of the observation period, either he or she dies from a specified cause of death or lives to attain age $x_i + 1$. We shall use the symbol $\delta_{i,j}$ to indicate death was due to cause j and is thus defined as

$$\delta_{i,j} = \begin{cases} 1 & \text{if the } i\text{th person died from cause } j \\ 0 & \text{if the } i\text{th person stayed alive or died from cause other than } j \end{cases}$$

Our observed data therefore consists of $(x_i, \delta_{i,j})$ for $i = 1, 2, \ldots, N$ where N denotes the total number of observed persons between ages 20 and 64 at the beginning of the one year period. In this case, we have a total of $N = 149,761,000$. We assume no migration during the one year observation period. We use maximum likelihood procedures to estimate our paramters. To develop the likelihood, we need to distinguish those who were observed to be dead and those who remain alive.

If an observation fails to reach age $x_i + 1$, then his or her contribution to the likelihood function is given by

$$\prod_{j=1}^{2} \left[\mathrm{Prob}\left(X \le x_i + 1, J = j \mid X > x_i\right)\right]^{\delta_{i,j}}, \tag{15}$$

where

$$\begin{aligned} \mathrm{Prob}\left(X \le x_i + 1, J = j \mid X > x_i\right) &= \frac{\mathrm{Prob}\left(x_i < X \le x_i + 1, J = j\right)}{\mathrm{Prob}\left(X > x_i\right)} \\ &= \frac{\mathrm{Prob}\left(x_i < X \le x_i + 1, J = j\right)}{S\left(x_i, x_i\right)}. \end{aligned} \tag{16}$$

Equations (8) and (9) can be used to evaluate the "crude" probability in the numerator of equation (16).

On the other hand, for an observation who survived to attain $x_i + 1$, his or her contribution to the likelihood function is given by

$$\left[\mathrm{Prob}\left(X > x_i + 1 \mid X > x_i\right)\right]^{1 - \sum_{j=1}^{2} \delta_{i,j}}, \tag{17}$$

where

$$\mathrm{Prob}\left(X > x_i + 1 \mid X > x_i\right) = \frac{\mathrm{Prob}\left(X > x_i + 1\right)}{\mathrm{Prob}\left(X > x_i\right)} = \frac{S\left(x_i + 1, x_i + 1\right)}{S\left(x_i, x_i\right)}. \tag{18}$$

By expressing the joint distribution function in terms of the copula,i.e.

$$H\left(x_1, x_2\right) = C\left(F_1\left(x_1\right), F_2\left(x_2\right)\right)$$

we can write the joint survival functions in equation (18) also in terms of the copula as

$$\begin{aligned} S(z, z) &= 1 - F_1(z) - F_2(z) + H(z, z) \\ &= 1 - C\left(F_1(z), 1\right) - C\left(1, F_2(z)\right) + C\left(F_1(z), F_2(z)\right). \end{aligned} \tag{19}$$

We further simplify the likelihood by aggregating the contributions according to one-year intervals for the range 20 through 64. The aggregate log-likelihood function to maximize simplifies to

$$\log L\left(x_i;\delta_{i,j}\right) = \sum_{j=1}^{2}\sum_{g=1}^{G} N_{j,g}\log\left[\mathrm{Prob}\left(X \le x_i+1; J=j \mid X > x_i\right)\right]^{\delta_{i,j}} \tag{20}$$
$$+\sum_{g=1}^{G}\left(M_g - \sum_{j=1}^{2} N_{j,g}\right)\log S\left(x_g+1; x_g+1\right) - \sum_{g=1}^{G} M_g \log S\left(x_g; x_g\right);$$

where N = overall exposure (that is, the total population under consideration at the beginning of 1992), g = the gth age group (for instance, $g = 1$ refers to the age interval $[20; 21)$, G = the total number of age group intervals, $N_{j;g}$ = the number of persons who died from cause j and who belong to age group g, and M_g = total beginning population who belong to age group g (thus, $N = \sum_{g=1}^{G} M_g$). A more detailed derivation and discussion of the likelihood in (20) can be found in Valdez (1998).

3.3. Choice of Marginals and Copula Function.

The choice of our model is parametric, that is, we specify the parametric distribution of the marginals and so with the copula function that defines the joint distribution of the lifetimes.

Exponential and Gompertz Marginals. Figure 2 provides strong support of constant force of mortality across age to model the marginal for accident causes and a hazard function that exponentially increases with age to model the marginal for all other causes. Thus, we choose the exponential distribution for the marginal of accident causes whose density function is expressed as

$$f_1(x_1) = \frac{1}{1}\exp\left(-\frac{1}{1}x_1\right) \tag{21}$$

and distribution function is given by

$$F_1(x_1) = 1 - \exp\left(-\frac{1}{1}x_1\right). \tag{22}$$

For causes other than accident, we choose the Gompertz distribution (Gompertz, 1825) for its marginal whose density function can be expressed as

$$f_2(x_2) = \frac{1}{\sigma}\exp\left[\frac{x_2 - m}{\sigma} + e^{-m/\sigma}\left(1 - e^{x_2/\sigma}\right)\right] \tag{23}$$

and distribution function is given by

$$F_2(x_2) = 1 - \exp\left[e^{-m/\sigma} \left(1 - e^{x_2/\sigma} \right) \right]. \qquad (24)$$

The parameterization of the Gompertz distribution has m as the mode of the distribution and σ as the dispersion about this mode. See Carriere (1992). Both the exponential and Gompertz distributions are familiar ones to actuaries and are often used to describe the pattern of the overall lifetime, particularly that of the Gompertz. See Bowers, et al. (1997). These marginals are to be used to describe the lifetimes attributable to accidents and all others. That is, we assume that a newborn is endowed with different patterns of survivorship; each pattern describes a survivorship attributable to a cause of death. Unfortunately, these patterns are never observed separately. Because of this, it becomes difficult to assess the quality of our marginal distributions. It is also interesting to note that the distribution of the minimum of the lifetimes attributable to all the causes of death is a weighted average of the marginal distributions. See appendix.

The Frank's Copula. To examine evidence of dependencies of accidents with all other causes, we express the joint distribution of the lifetimes from the two causes in terms of a copula. As described in Section 2, copulas are helpful in describing the dependence because they do contain a parameter that defines the possible dependence. For our purposes, we find the Frank copula function as expressed in equation (14) the appropriate copula for our models as justified below. See also Frank (1979) and Genest (1987).

Choosing a copula is not an easy procedure. We implemented a trial and error procedure for choosing a copula. First, we considered three familiar copulas: the Gumbel-Hougaard, Frank, and Cook-Johnson. The first two families are described in Section 2 and for the Cook-Johnson, see Frees and Valdez (1998). We considered these copulas because they are mathematically tractable in the sense that equation (20) can be readily evaluated. Note that computing these parametric probabilities involve partial derivatives of the copula. Partial derivatives of the Frank and Gumbel-Hougaard copulas are well documented in Frees, et al. (1996) and Frees and Valdez (1998). Minimization routines for the log-likelihood function were programmed in SAS, using the procedure called IML (Interactive Matrix Language). We find that optimization procedures converge well when the Frank's copula is used.

The Frank's copula has been used to model mortality dependence of paired individuals as in Frees, et al. (1996). It belongs to the family of Archimedean copulas which has several interesting properties. Other theoretically appealing properties of this family of copulas are studied in Genest (1987).

3.4. Estimation Results and Discussion.

There were four parameters to estimate: μ, m, σ, and α for our model of dependence. Table 1 presents the numerical estimates for these parameters and compared the values against the case where we assume that accident as a cause of death is independent of all other causes. Note that there is no α parameter for the case of independence.

Table 1[2]

Numerical Estimates of Parameters

Parameter	Frank's copula	Independence
μ	709.90 (1.6508)	1,264.98 (1.7298)
m	111.45 (0.0685)	118.90 (0.0844)
σ	32.30 (0.0641)	31.97 (0.0593)
α	-13.27 (0.0563)	not applicable
Log-Likelihood	-16,861,311	-16,881,212

In the dependence model, the parameter estimate of $\hat{\alpha} = -13.27$ translates to a 91% Spearman's correlation coefficient, a very strong dependence. When other pairs of causes of death were considered, there is a similarly strong evidence of dependencies (Valdez, 1998). To interpret this correlation, it is best to consult the result of Carriere (1994). In his paper, he showed that when heart/cerebrovascular diseases is

[2]Standard errors are the values in parenthesis.

removed as a cause of death, the median age at death of a newborn increases with decreasing correlations and decreases with increasing correlations. In e¤ect, mortality improvement with higher correlation is expected to be much worse than that indicated by assuming no correlation. In our "accident" illustration above, because the Spearman's correlation is 91% and is therefore higher than a zero correlation, we expect that if mortality were estimated assuming "accidents" were removed, the mortality improvement would have been worse than that indicated if the causes were assumed to be independent.

4. Pricing for Accidental Death Bene…t Coverages. This section illustrates how one can use the results above to assess the price of an accidental death bene…t insurance coverage. For illustration purposes, we will assume that the death bene…t is $1,000, is doubled to $2,000 if death is due to accident, and coverage is a term to age 65. Thus, bene…t ceases at age 65. We value the cost of insurance using our model with dependence together with the parameter estimates derived in the previous section. We compare this cost against the cost derived assuming independence, which is the standard procedure used in practice. We caution the pricing actuary that our model was estimated using data from a general population whose mortality pattern may be di¤erent from a group of insureds who may have been carefully selected by the insurance company. However, it should become straightforward for the actuary to derive his or her own mortality estimates using all the procedures described in this paper.

Assuming a constant force of interest δ, the present value random variable for an accidental death bene…t issued to a person age x is given by

$$Z_x = \begin{cases} 2000e^{-\delta T(x)}; & T(x) < 65 - x; J = 1 \\ 1000e^{-\delta T(x)}; & T(x) < 65 - x; J = 2 \\ 0; & T(x) \geq 65 - x. \end{cases} \tag{25}$$

The cost of the insurance is measured by the expected value of this random variable.

To derive the distribution function of the future lifetime for a person age x, we use

$$F_{T(x)}(t) = \frac{F_X(x+t) - F_X(x)}{1 - F_X(x)}. \tag{26}$$

Thus, we see that the expected value of (25) can be expressed as follows:

$$\begin{aligned} E(Z_x) &= \frac{2000}{1 - F_X(x)} \int_0^{\omega - x} e^{-\delta t} dF_{X;J}(t;1) + \frac{1000}{1 - F_X(x)} \int_0^{\omega - x} e^{-\delta t} dF_{X;J}(t;2) \quad (27) \\ &= \frac{2000}{S(x;x)} \int_0^{\omega - x} e^{-\delta t} f_{X;J}(t;1) dt + \frac{1000}{S(x;x)} \int_0^{\omega - x} e^{-\delta t} f_{X;J}(t;2) dt; \end{aligned}$$

where $f_{X;J}(\cdot;\cdot)$ can be evaluated by taking the derivatives in the expressions in (8) and (9). Thus, it is straightforward to show that

$$f_{X;J}(x;1) = (1 - C_1) f_1(x) \quad \text{and} \quad f_{X;J}(x;2) = (1 - C_2) f_2(x)$$

where $C_1 = \partial C / \partial u_1$ and $C_2 = \partial C / \partial u_2$, both evaluated at $u_1 = F_1(x)$ and $u_2 = F_2(x)$. In the case of independence, we have $C_1 = F_2(x)$ and $C_2 = F_1(x)$. Assuming the Frank's copula to account for dependence, these partial derivatives have been derived by Frees, et al. (1996). The appendix also derives the distribution of the overall lifetime without the need for the assumption of independence.

Table 2 provides estimates for the cost of accidental death benefit insurance for selected issue ages. The table decomposes the total costs attributable to accident and all other causes. The table also provides a comparison of these cost estimates between the assumption of dependence and no dependence of causes. As especially noted in this paper, the assumption of independent causes is typical among many pricing actuaries. When the total cost is considered, there appears to be no clear pattern of whether the independent assumption underestimates or overestimates the true cost. It appears that there is underestimation for younger ages and overestimation for later ages.

Table 2

Expected Insurance Cost of Accidental Death Bene…t

Dependent versus Independent Causes

	Dependent Causes			Independent Causes		
Age	Accidents	All Other	Total	Accidents	All Other	Total
20	187.32	184.39	371.71	152.42	190.04	342.47
30	156.10	238.04	394.14	148.91	247.22	396.14
40	121.27	300.41	421.67	140.66	305.26	445.92
50	81.29	337.11	418.41	119.43	329.54	448.97
60	32.16	221.29	253.45	61.36	207.26	268.62

In Figure 3, we display how these costs decompose by cause of death across several issue ages, assuming the causes of death are dependent therefore using the parameter estimates derived assuming dependence. As the graph indicates, accidents account for a larger proportion of the cost for younger issue ages. This appears intuitively interesting because as several mortality studies indicate, accidents is a relatively major cause of death for younger persons. The cost of insurance declines at much later issue ages because the bene…ts in the policy pays only until age 65. Figure 4 is the independent counterpart of Figure 4. Here, we use the parameter estimates derived assuming the causes are independent.

[Insert Figures 3 and 4 here]

Figure 5 further provides a graphical display of a comparison of the expected cost between the independence and dependence assumption across more several ages. The …gure displays the ratio of the expected cost assuming independent causes to that assuming dependent causes. A ratio above 1.0 indicates an overestimation of expected cost. A ratio below 1.0 is an underestimation. We observe a possible underestimation of cost for issue ages below 30 because the pricing actuary fails to account for the dependence that exists between accidents and all other causes. On the other hand, there is possible overestimation for ages 30 and above. The range of misestimation is approximately between -8% to +7%.

[Insert Figure 5 here]

As yet another illustration, we now consider the case where the coverage is only for one year. Such is usually the case for instance in a Group Accidental Death and Dismemberment (Group AD&D, for short) policy. Several group life insurance policies provide this type of a coverage as supplemental benefits. See Black and Skipper, Jr. (1988). Besides paying twice the death benefit if employee dies due to an accident, the supplemental coverage pays for loss of sight, hand, or foot resulting from an accident. Ignoring this in the benefit calculation, the expected cost formula will be exactly that in equation (27) except that the integrals are evaluated from 0 to 1, in other words, only for a period of one year as opposed to until reaching age 65 as in the previous illustration. Figure 6, 7, and 8 give the complete analogue of Figures 3, 4, and 5 in the case where we have term to age 65. In Figures 6 and 7, the expected cost of one year insurance increases with age. However, as Figure 6 illustrates for the dependent case, the proportion of the cost attributable to accidents declines with age unlike that of independence where it is constant over the age range. In Figure 7, the shape of the ratio of misestimation is similar in pattern to that in Figure 5. However, it is worth noting two major differences: (1) there is underestimation up until the age around 40, and (2) the range of misestimation is -17% to +8%.

[Insert Figures 6, 7 and 8 here]

5. Concluding Remarks. While many researchers, academicians, and practitioners (Makeham, 1874; Promislow, 1991; Seal, 1986) recognize the limitation of models assuming independent causes of death, no study has ever measured the extent of possible dependence of causes of death. This paper suggests the use of copula-based models to understand the possible dependencies that exist between causes of mortality. The suggested procedures retain all the inherent characteristics and qualities of competing risk models used to examine causes of death. Assuming only two causes of death such as accidents and all other causes, we assume that a newborn individual is endowed with two lifetime random variables where each lifetime is associated with each competing cause of death. The lifetime random variables possess marginal distributions which can be specified parametrically. The joint distribution of the lifetime random variables is then expressed as a copula function which exists because of a proposition by Sklar made in 1959.

In effect, assuming only two causes of death, the model of dependence can be established as follows:

1. A newborn is endowed with a set of lifetime random variables to be denoted by X_1 and X_2, each lifetime variable is associated with each cause of death.

2. The lifetime random variables possess marginal distributions F_1 and F_2 to be specified parameterically.

3. The joint distribution of $(X_1; X_2)$ is expressed in terms of a copula function as $H(x_1; x_2; \alpha) = C(F_1(x_1); F_2(x_2))$ for which α provides a measure of dependence.

Using 1992 U.S. mortality data, this paper further develops procedures for empirically estimating dependencies for various causes of death. Examining accidents and all other causes, our empirical estimates support evidence of dependence between the two causes of death. We provided interpretations as to the meaning of these dependencies, and we justified them with: (a) biological justification, (b) unobserved secondary causes, (c) selection, and (d) errors in variables.

While there is limitation for using the results to price insurance products because the empirical data used were based on the experience of the general population, we illustrated the range of cost misestimation that could possibly result for failing to account for the dependence. We did this illustration both in the case where the policy coverage is until age 65 where such is common for individual life insurance contracts, and in the case where the policy coverage is only for a period of one year where such is more common for group life insurance contracts. The important point is that the procedure outlined in this paper to measure the effect of dependencies of causes of death can be applied in other similar situations such as in pricing contracts with a different policy term and even contracts where the benefits vary for causes other than accidents.

6. Appendix: The Distribution of the Overall Lifetime.

This appendix derives the form of the distribution of the overall lifetime $X = \min(X_1; X_2)$ without having to assume independence. We begin by noting that

$$F_X(x) = \text{Prob}(X \leq x) = 1 - S(x; x)$$

and then using equation (19), we have

$$F_X(x) = F_1(x) + F_2(x) - C(F_1(x); F_2(x)):$$

The density function is derived by differentiating the distribution function. Hence,

$$f_X(x) = \left(1 - \frac{\partial C}{\partial u}\right) f_1(x) + \left(1 - \frac{\partial C}{\partial v}\right) f_2(x)$$

where the partial derivatives are evaluated at $u = F_1(x)$ and $v = F_2(x)$.

In the special case where we assume independence, we have

$$f_X(x) = (1 - F_2(x)) f_1(x) + (1 - F_1(x)) f_2(x)$$

and in the case of Frank's copula as expressed in (14), we have

$$f_X(x) = \frac{\left(e^{\alpha} - e^{\alpha F_2(x)}\right) f_1(x) + \left(e^{\alpha} - e^{\alpha F_1(x)}\right) f_2(x)}{(e^{\alpha} - 1) + (e^{\alpha F_1(x)} - 1)(e^{\alpha F_2(x)} - 1)}:$$

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Figure 1: Comparison of death rates for some causes of death in 1992

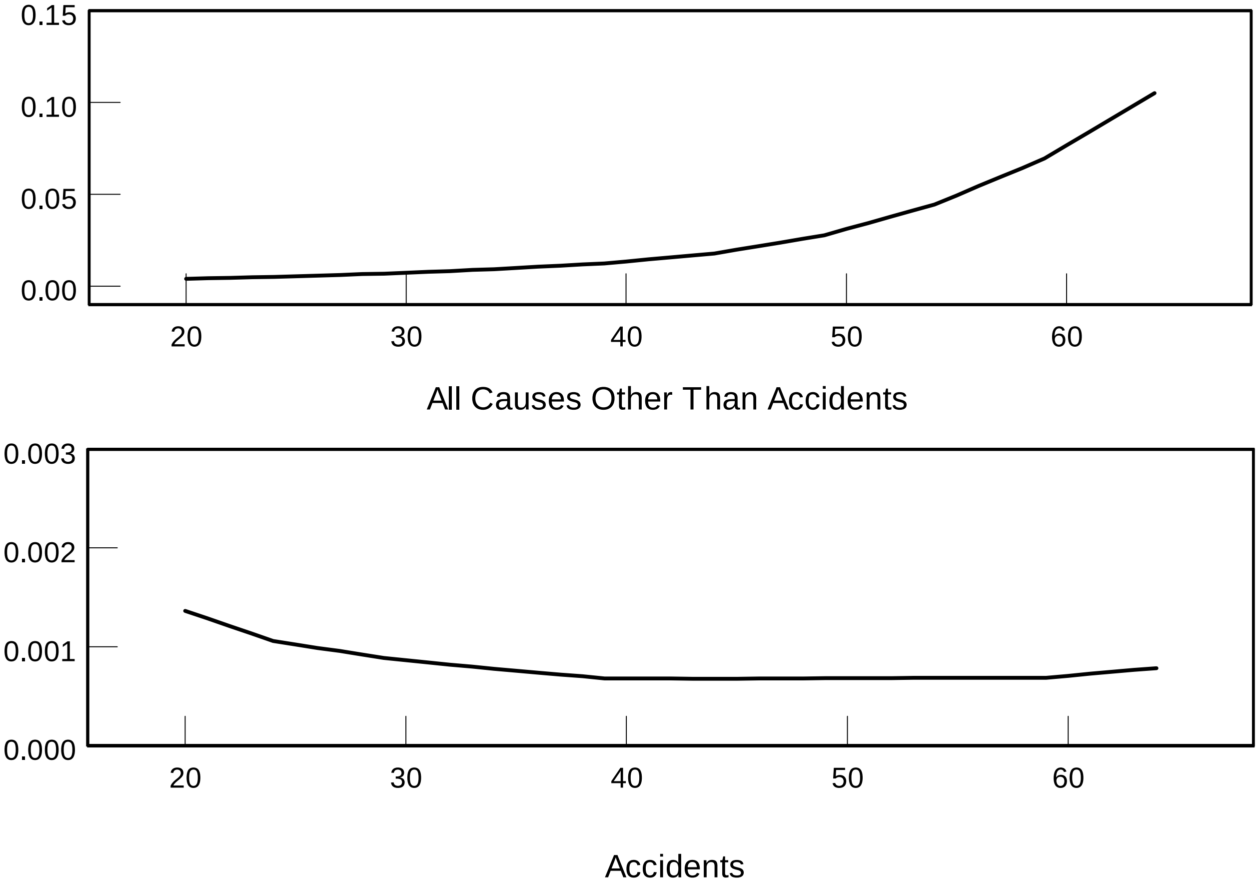

Figure 2: Comparison of one-year crude probabilities $\tilde{q}_x^{(1)}$ (accidents) and $\tilde{q}_x^{(2)}$ (all other causes). The figures show that the crude probabilities for accidents across the age range 20 – 64 are fairly constant and small relative to those for all other causes.

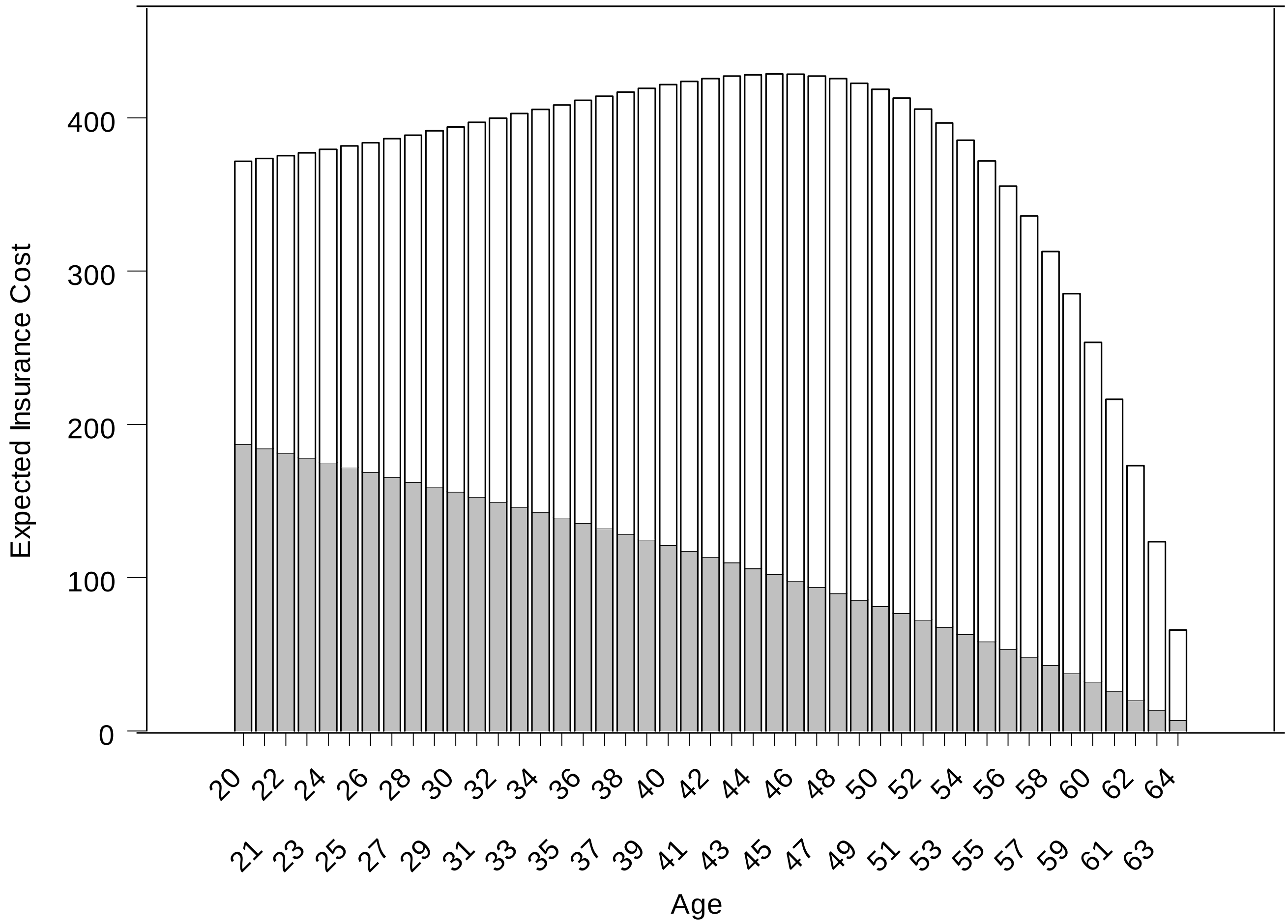

Figure 3: **The Dependent Case (Term to Age 65).** Decomposition of the expected insurance cost by cause of death across different ages at issue. The height of the bar gives the expected insurance cost, with the lightly shaded region indicating the portion attributable to *accidents* and the not-shaded region to *all other causes*.

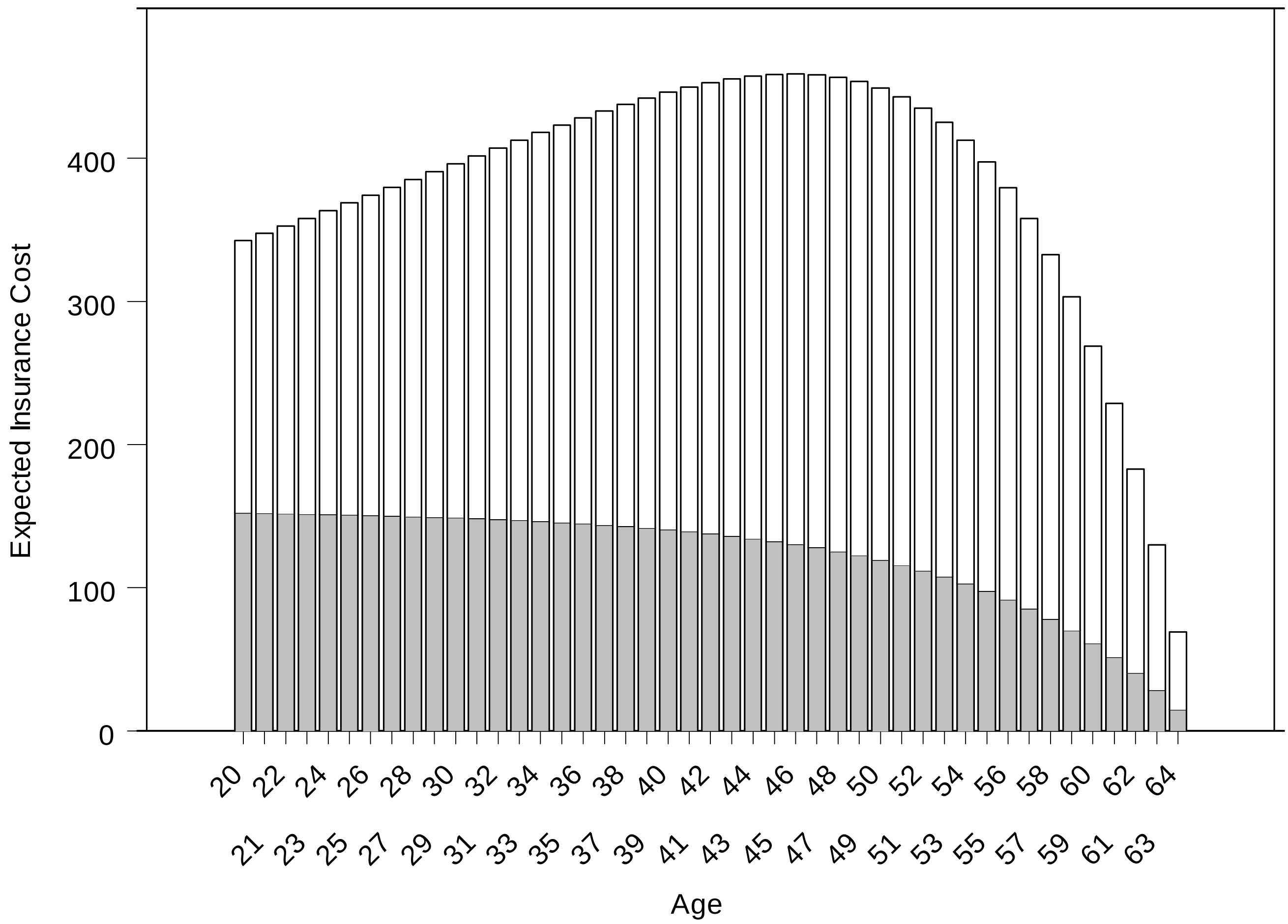

Figure 4: **The Independent Case (Term to Age 65).** Decomposition of the expected insurance cost by cause of death across different ages at issue. The height of the bar gives the expected insurance cost, with the lightly shaded region indicating the portion attributable to *accidents* and the not-shaded region to *all other causes*.

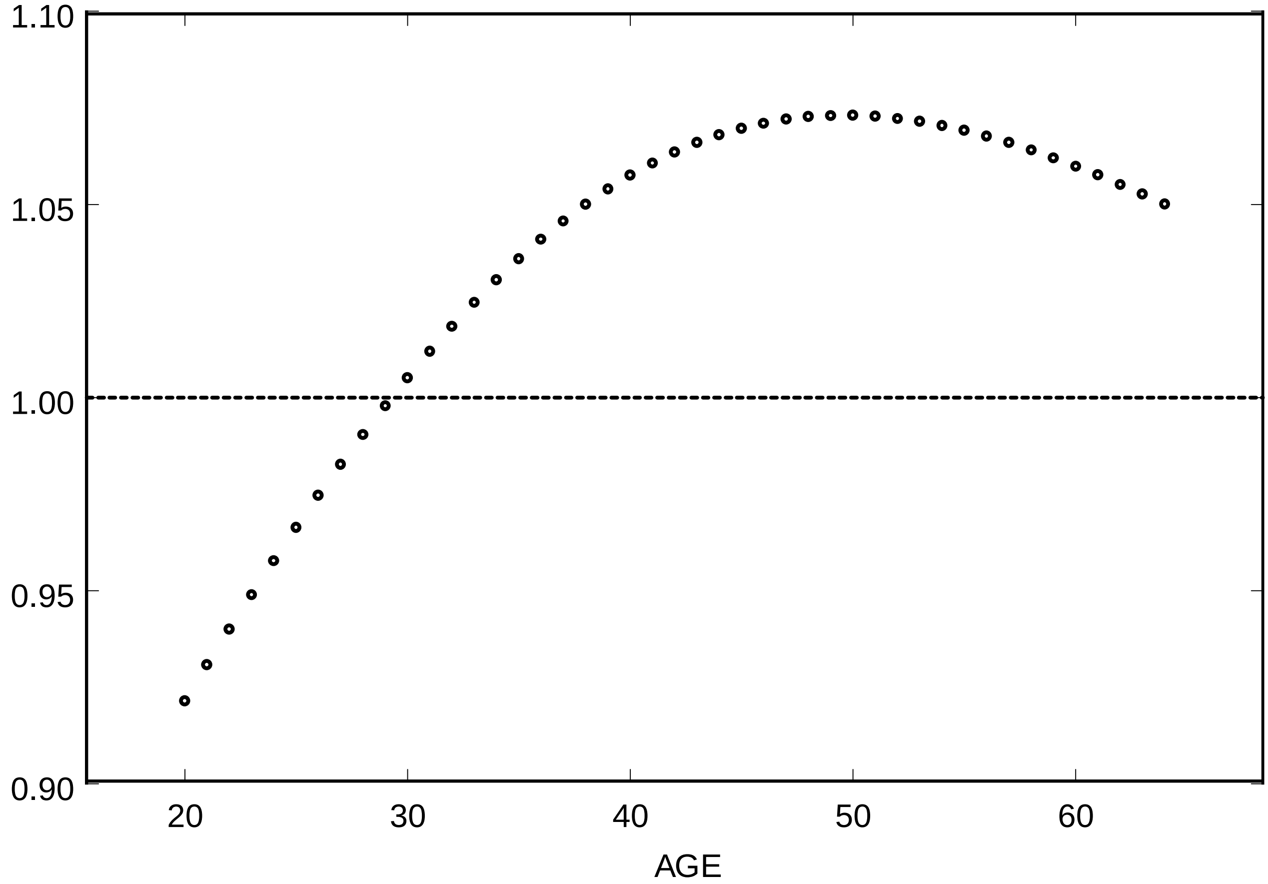

Figure 5: **Term to Age 65.** Expected cost of accidental death benefit: ratio of independent to dependent causes. A ratio above 1.0 indicates an overestimation of cost and a ratio below 1.0 indicates an underestimation.

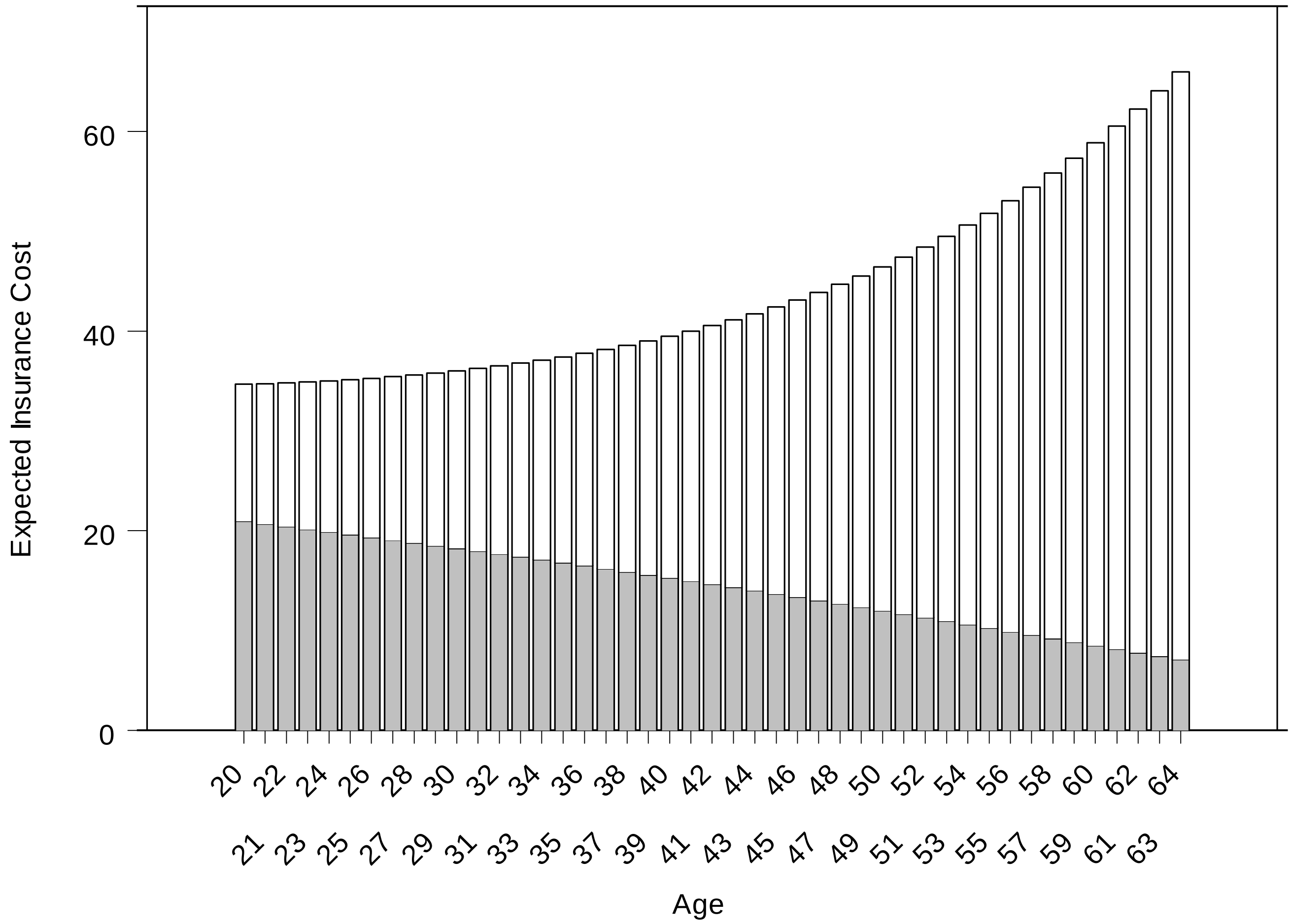

Figure 6: **The Dependent Case (One Year Term only).** Decomposition of the expected insurance cost by cause of death across different ages at issue. The height of the bar gives the expected insurance cost, with the lightly shaded region indicating the portion attributable to *accidents* and the not-shaded region to *all other causes*.

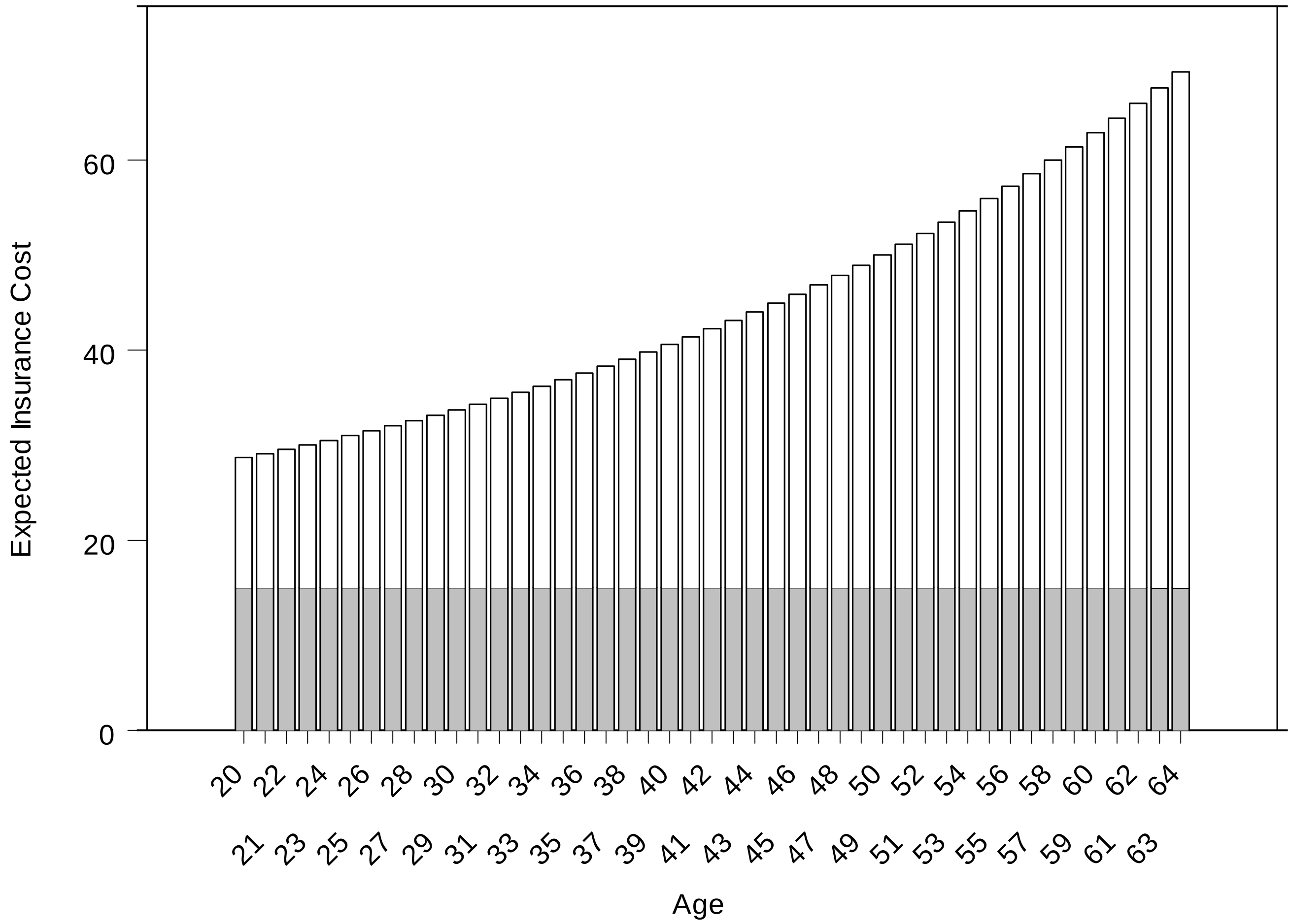

Figure 7: **The Independent Case (One Year Term only).** Decomposition of the expected insurance cost by cause of death across different ages at issue. The height of the bar gives the expected insurance cost, with the lightly shaded region indicating the portion attributable to *accidents* and the not-shaded region to *all other causes*.

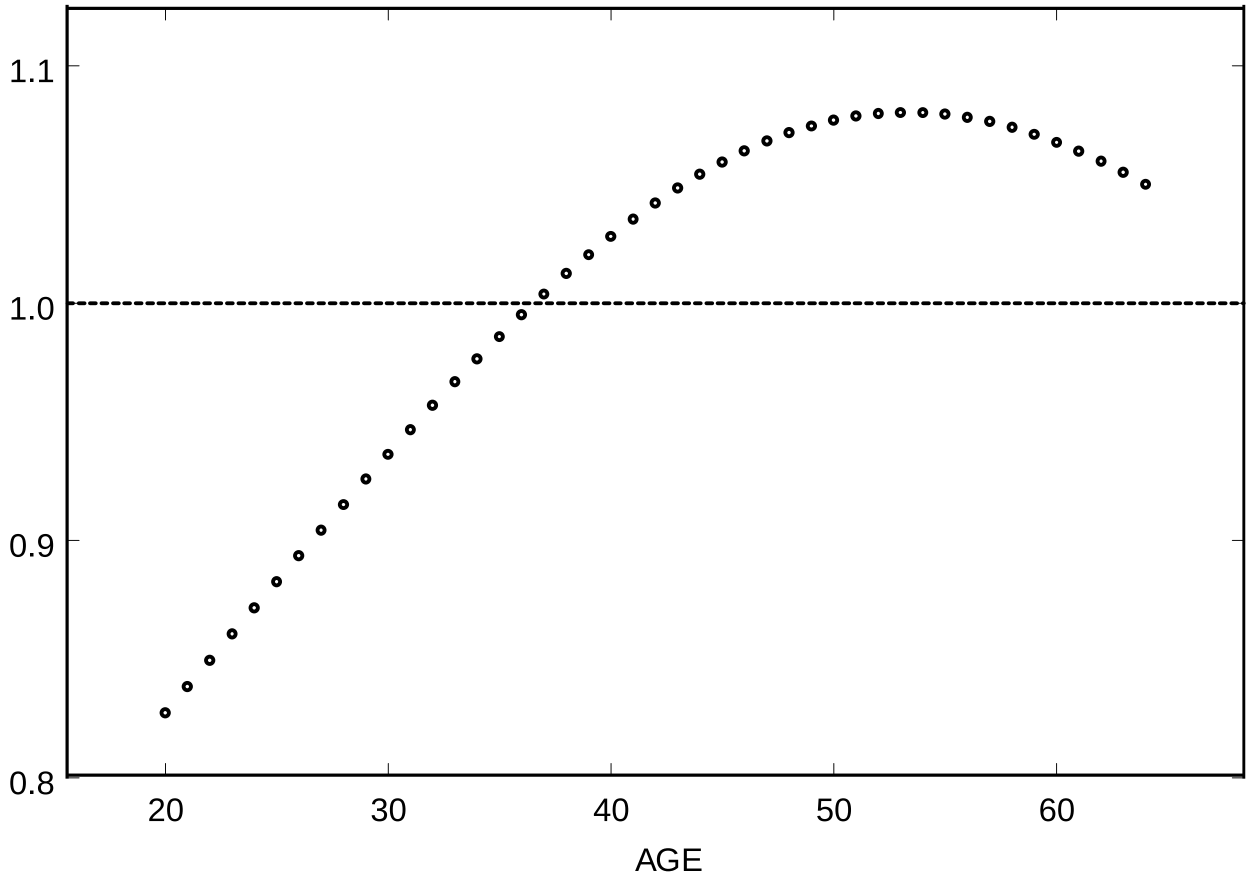

Figure 8: **One Year Term only.** Expected cost of accidental death benefit: ratio of independent to dependent causes. A ratio above 1.0 indicates an overestimation of cost and a ratio below 1.0 indicates an underestimation.