Measuring tail dependence for collateral losses using bivariate Lévy process

Jiwook Jang

Actuarial Studies, University of New South Wales, Sydney, NSW 2052, Australia, e-mail: j.jang@unsw.edu.au

Abstract In practice, insurance companies face collateral losses, for example, in worldwide, once a storm or earthquake arrives, it brings about damages in properties, motors and interruption of businesses. It occurred a couple of losses simultaneously from the World Trade Centre (WTC) catastrophe. However it has not been developed an applicable model for insurance companies to measure stochastic dependence for these collateral losses. The aim of this paper is to measure (upper) tail dependence for collateral losses, employing bivariate Lévy process, i.e. bivariate compound Poisson process with a copula, as insurance industry is more concerned with dependence between extreme values. In order to derive an explicit expression of joint Laplace transform of collateral losses, we use a member of Farlie-Gumbel-Morgenstern copula with exponential margins. Inversion of joint Fast Fourier transform obtained from the joint Laplace transform of collateral losses is used to calculate the coefficients of (upper) tail dependence numerically. We also provide the figures of the joint distribution of collateral losses and their contours at each value of the parameter in a Farlie-Gumbel-Morgenstern copula.

Keywords: Stochastic dependence for collateral losses; bivariate Lévy process; Farlie-Gumbel-Morgenstern copula; the coefficient of (upper) tail dependence; inversion of Fast Fourier transform.

1. Introduction

Practically insurance losses have dependence between their components, e.g. dependence between losses, dependence between loss arrivals and dependence between losses and their arrivals. So it requires for us to employ a multivariate model with dependence. Since copulae attracted academics and practitioners, multidimensional models have been developed to capture dependence between components of insurance risks (Lindskog and McNeil 2003; Pfeifer and Nešlehová 2004; Chavez-Demoulin et al. 2005 and Bäuerle and Grübel 2005). Most of these papers deal with covariance, linear correlation and simulation of dependent losses. In contrast, in this paper, we deal with measuring (upper) tail dependence of losses using bivariate Lévy process, i.e. bivariate compound Poisson process with a copula, as insurance industry is more concerned with dependence between extreme values. We are examining two losses that are occurring collaterally with dependence. In order to measure the dependence between losses, we may also consider using the Lévy copula, that can be found in Cont and Tankov (2004), rather than using a ordinary copula.

Ignoring the interest rate, let us assume that insurance company is experiencing dependent losses from one specific event such as flood, windstorm, hail, earthquake and terrorist attack. For example, WTC catastrophe brought about losses of properties, losses of vehicles, losses due to business interruptions, etc. out of one trigger. So for bivariate risk case, we can model

\[
L_{t}^{(1)} = \sum_{i=1}^{N_t} X_i, \\
L_{t}^{(2)} = \sum_{i=1}^{N_t} Y_i
\]

(1.1)

where \(L_{t}^{(1)}\) is the total losses arising from risk type 1, \(L_{t}^{(1)}\) is the total losses arising from risk type 2 and \(N_t\) is the total number of collateral losses up to time \(t\). \(X_i\) and \(Y_i\), \(i = 1, 2, \cdots\), are the loss amounts, which are assumed to dependent with a copula function \(C(H(x), H(y))\) \((x > 0, y > 0)\) where \(H(x)\) be the identically distribution function of \(X\) and \(H(y)\) be the identically distribution function of \(Y\). For details on copulae, we refer you Nelson (1998).

We assume that the collateral loss arrival process \(N_t\) follows a Poisson process with loss frequency rate \(\mu\). It is also assumed that is independent of \(X_i\) and \(Y_i\). We employ the Farlie-Gumbel-Morgenstern family copula, that is given by
\[ C(u, v) = uv + \theta uv(1 - u)(1 - v), \]  

where \( u \in [0, 1], \ v \in [0, 1] \) and \( \theta \in [-1, 1] \), to capture the dependence of collateral losses of \( X \) and \( Y \). In order to obtain the explicit expression of the function \( F(x, y) \), that is a two-dimensional distribution function with margins \( H(x) \) and \( H(y) \), we let \( X \) and \( Y \) be exponential random variables, i.e. \( H(x) = 1 - e^{-\alpha x} (\alpha > 0, \ x > 0) \) and \( H(y) = 1 - e^{-\beta y} (\beta > 0, \ y > 0) \), then the joint distribution function \( F(x, y) \) is given by

\[
F(x, y) = C(1 - e^{-\alpha x}, 1 - e^{-\beta y})
= 1 - e^{-\beta y} - e^{-\alpha x} - \beta y - \theta e^{-\alpha x - \beta y} - \theta e^{-\alpha x - 2\beta y} - \theta e^{-2\alpha x - \beta y} + \theta e^{-2\alpha x - 2\beta y}. \tag{1.3}
\]

and its derivative is given by

\[
dF(x, y) = dC(1 - e^{-\alpha x}, 1 - e^{-\beta y})
= (1 + \theta) \alpha \beta e^{-\alpha x - \beta y} - 2\theta \alpha \beta e^{-\alpha x - 2\beta y} - 2\theta \alpha \beta e^{-2\alpha x - \beta y} + 4\theta \alpha \beta e^{-2\alpha x - 2\beta y}. \tag{1.4}
\]

We examine (upper) tail dependence of collateral losses \( X \) and \( Y \) as insurance companies’ concerns are on extreme losses in practice. So we adopt the coefficient of (upper) tail dependence, \( \lambda_U \), used by Embrechts, Lindskog and McNeil (2003),

\[
\lim_{u \nearrow 1} \mathbb{P} \left\{ L_{i(1)} > G_{L_{i(1)}}^{-1}(u) \mid L_{i(2)} > G_{L_{i(2)}}^{-1}(u) \right\} = \lambda_U \tag{1.5}
\]

provided that the limit \( \lambda_U \in [0, 1] \) exists, where \( G_{L_{i(1)}} \) and \( G_{L_{i(2)}} \) are marginal distribution functions for \( L_{i(1)} \) and \( L_{i(2)} \).

In order to evaluate (1.5), we need to obtain the joint distribution of the aggregate losses \( L_{i(1)} \) and \( L_{i(2)} \), where their individual losses are occurring collaterally. Unfortunately, it is known that it is not possible for us to obtain the joint distribution the aggregate losses explicitly. So in section 2, we derive the general form of the joint Laplace transform of the distribution of the aggregate losses expressed with a copula function, applying the piecewise deterministic Markov processes theory. Based on this general form of the joint Laplace transform, we obtain the explicit expression of the joint Laplace transform of the distribution of the aggregate losses using (1.3). Section 3 provides the explicit expression of the covariance and linear correlation between \( L_{i(1)} \) and \( L_{i(2)} \) at time \( t \). In section 4, we illustrate the calculations of the coefficient of (upper) tail dependence using the joint Fast Fourier transform. We also provide the figures of the joint distribution of collateral losses and their contours at each value of \( \theta \). Section 5 concludes.

2. The joint Laplace transform of the distribution of aggregate losses

The piecewise deterministic Markov processes theory developed by Davis (1984) is a powerful mathematical tool for examining non-diffusion models. From now on, we present definitions and important properties of \( L_{i(1)} \) and \( L_{i(2)} \) with the aid of piecewise deterministic Markov processes theory (Dassios and Embrechts 1989; Rolski et al. 1998 and Dassios and Jang 2003). This theory is used to derive the general form of the joint Laplace transform of the distribution of aggregate losses \( L_{i(1)} \) and \( L_{i(2)} \).

The generator of the process \( \left( L_{i(1)}, L_{i(2)} \right) \) acting on a function \( f \left( l^{(1)}, l^{(2)}, t \right) \) belonging to its domain is given by

\[
A f \left( l^{(1)}, l^{(2)}, t \right) = \frac{\partial f}{\partial t} + \mu \left[ \int_0^\infty \int_0^\infty f \left( l^{(1)} + x, l^{(2)} + y, t \right) dC(H(x), H(y)) - f \left( l^{(1)}, l^{(2)}, t \right) \right] \tag{2.1}
\]

where \( f : (0, \infty) \times (0, \infty) \times \mathbb{R}^+ \rightarrow (0, \infty) \). For \( f \left( l^{(1)}, l^{(2)}, t \right) \) to belong to the domain of the generator \( A \), it is sufficient that \( f \left( l^{(1)}, l^{(2)}, t \right) \) is differentiable w.r.t. \( l^{(1)}, l^{(2)}, t \) for all \( l^{(1)}, l^{(2)}, t \) and that
\[
\int_0^\infty \int_0^\infty f \left( l^{(1)} + x, l^{(2)} + y, t \right) dC(H(x), H(y)) - f \left( l^{(1)}, l^{(2)}, t \right) < \infty.
\]

Let us find a suitable martingale in order to derive the joint Laplace transform of the distribution of \( L_t^{(1)} \) and \( L_t^{(2)} \).

**Lemma 2.1** Considering constants \( \nu \geq 0 \) and \( \xi \geq 0 \),

\[
\exp \left( -\nu L_t^{(1)} \right) \exp \left( -\xi L_t^{(2)} \right) \exp \left[ \mu \int_0^t \{ 1 - \hat{c} (\nu, \xi) \} \, ds \right]
\]

is a martingale where \( \hat{c} (\nu, \xi) = \int_0^\infty \int_0^\infty e^{-\nu x} e^{-\xi y} dC(H(x), H(y)) \).

**Proof.** From (2.1), \( f \left( l^{(1)}, l^{(2)}, t \right) \) has to satisfy \( \mathcal{A} f = 0 \) for \( f \left( l^{(1)}, l^{(2)}, t \right) \) to be a martingale. Setting \( f \left( l^{(1)}, l^{(2)}, t \right) = \exp \left( -\nu l^{(1)} \right) \exp \left( -\xi l^{(2)} \right) e^{\beta(t)} \) we get the equation

\[
B'(t) + \mu \{ \hat{c} (\nu, \xi) - 1 \} = 0
\]

and the solution is

\[
B(t) = \mu \int_0^t \{ 1 - \hat{c} (\nu, \xi) \} \, ds
\]

by which the result follows. \( \blacksquare \)

Using the martingale obtained in Lemma 2.1, we can easily obtain the general form of the joint Laplace transform of the distribution of \( L_t^{(1)} \) and \( L_t^{(2)} \) at time \( t \),

\[
E \left\{ e^{-\nu L_t^{(1)}} e^{-\xi L_t^{(2)}} \mid L_0^{(1)}, L_0^{(2)} \right\} = \exp \left( -\nu L_0^{(1)} \right) \exp \left( -\xi L_0^{(2)} \right) \exp \left[ -\mu \int_0^t \{ 1 - \hat{c} (\nu, \xi) \} \, ds \right].
\]

(2.5)

For simplicity, we assume that \( L_0^{(1)} = 0 \) and \( L_0^{(2)} = 0 \), then it is given by

\[
E \left\{ e^{-\nu L_t^{(1)}} e^{-\xi L_t^{(2)}} \right\} = \exp \left[ -\mu \int_0^t \{ 1 - \hat{c} (\nu, \xi) \} \, ds \right].
\]

(2.6)

In order to obtain the explicit expression of the joint Laplace transform of the distribution of \( L_t^{(1)} \) and \( L_t^{(2)} \) at time \( t \), let us use the joint distribution function \( F(x, y) \) driven by (1.3), then it is given by

\[
E \left\{ e^{-\nu L_t^{(1)}} e^{-\xi L_t^{(2)}} \right\} = \exp \left[ -\mu \left\{ \frac{(\alpha\xi + \beta\nu + \nu\xi)}{(\alpha + \nu)(\beta + \xi)} \right\} t \right].
\]

(2.7)

If \( \theta = 0 \), then we have

\[
E \left\{ e^{-\nu L_t^{(1)}} e^{-\xi L_t^{(2)}} \right\} = \exp \left[ -\mu \left\{ \frac{(\alpha\xi + \beta\nu + \nu\xi)}{(\alpha + \nu)(\beta + \xi)} \right\} t \right],
\]

(2.8)

which is the case that two losses \( X \) and \( Y \) occur same time from a sharing loss frequency rate \( \mu \), but their sizes are independent each other. It will be of interest to find more explicit expressions of the joint Laplace transform of the distribution of \( L_t^{(1)} \) and \( L_t^{(2)} \) at time \( t \), using other copulae and other margins \( H(x) \) and \( H(y) \). However it is obvious that we will not be able to derive explicit forms of the joint Laplace transform of the distribution of \( L_t^{(1)} \) and \( L_t^{(2)} \) if we apply heavy-tailed distributions for margin \( H(x) \) and \( H(y) \).
If we set $\xi = 0$, then the Laplace transform of the distribution of $L_t^{(1)}$ is given by

$$E \left\{ e^{-\nu L_t^{(1)}} \right\} = \exp \left\{ -\mu \left( \frac{\nu}{\alpha + \nu} \right) t \right\}$$

(2.9)

and if we set $\nu = 0$, then the Laplace transform of the distribution of $L_t^{(2)}$ is given by

$$E \left\{ e^{-\xi L_t^{(2)}} \right\} = \exp \left\{ -\mu \left( \frac{\xi}{\beta + \xi} \right) t \right\}$$

(2.10)

which are the Laplace transform of the distribution of the compound Poisson process with exponential loss sizes. Due to the dependence of collateral losses of $X$ and $Y$ with sharing loss frequency rate $\mu$, it is obvious that

$$E \left\{ e^{-\nu L_t^{(1)}} e^{-\xi L_t^{(2)}} \right\} \neq E \left\{ e^{-\nu L_t^{(1)}} \right\} E \left\{ e^{-\xi L_t^{(2)}} \right\}.$$  

(2.11)

If loss $X$ occurs with its frequency rate $\mu^{(x)}$ and loss $Y$ occurs with its frequency rate $\mu^{(y)}$ respectively and everything is independent each other, we can easily derive the explicit expression of the joint Laplace transform of the distribution of $L_t^{(1)}$ and $L_t^{(2)}$ at time $t$, i.e.

$$E \left\{ e^{-\nu L_t^{(1)}} e^{-\xi L_t^{(2)}} \right\} = E \left\{ e^{-\nu L_t^{(1)}} \right\} E \left\{ e^{-\xi L_t^{(2)}} \right\}$$

$$= \exp \left\{ -\mu^{(x)} \left( \frac{\nu}{\alpha + \nu} \right) t \right\} \exp \left\{ -\mu^{(y)} \left( \frac{\xi}{\beta + \xi} \right) t \right\}.$$  

(3.2)

If we set $\mu = \mu^{(x)} = \mu^{(y)}$, i.e. frequency rate for loss $X$ and $Y$ are just the same, then (2.12) becomes

$$E \left\{ e^{-\nu L_t^{(1)}} e^{-\xi L_t^{(2)}} \right\} = E \left\{ e^{-\nu L_t^{(1)}} \right\} E \left\{ e^{-\xi L_t^{(2)}} \right\}$$

$$= \exp \left\{ -\mu \left( \frac{\nu}{\alpha + \nu} \right) t \right\} \exp \left\{ -\mu \left( \frac{\xi}{\beta + \xi} \right) t \right\}$$

$$= \exp \left\{ -\mu \left( \frac{(\alpha \xi + \beta \nu + 2\nu \xi)}{(\alpha + \nu)(\beta + \xi)} \right) t \right\}.$$  

(2.13)

Equation (2.13) looks similar to (2.8) as loss size $X$ and $Y$ are independent and their frequency rates are the same. However the joint Laplace transform of the distribution of $L_t^{(1)}$ and $L_t^{(2)}$ at time $t$ expressed by (2.13) are the case that they are occurring independently, not collaterally like (2.8).

3. Covariance and linear correlation of collateral losses

Let us examine the covariance and linear correlation between $L_t^{(1)}$ and $L_t^{(2)}$ at time $t$. Differentiating (2.7) w.r.t. $\nu$ and $\xi$ and set $\nu = 0$ and $\xi = 0$, then we can easily derive the joint expectation of $L_t^{(1)}$ and $L_t^{(2)}$ at time $t$, i.e.

$$E \left\{ L_t^{(1)} L_t^{(2)} \right\} = \frac{\mu^2}{\alpha \beta} \nu^2 + \frac{\mu}{\alpha \beta} \left( 1 + \frac{\theta}{4} \right) t.$$  

(3.1)

Also from (2.9) and (2.10) we can easily derive the expectation of $L_t^{(1)}$ and $L_t^{(2)}$ at time $t$, i.e.

$$E \left\{ L_t^{(1)} \right\} = \frac{\mu}{\alpha} t$$  

(3.2)

and

$$E \left\{ L_t^{(2)} \right\} = \frac{\mu}{\beta} t.$$  

(3.3)

The higher moments of $L_t^{(1)}$ and $L_t^{(2)}$ at time $t$ can be obtained by differentiating it further, i.e.
\[ Var \{ L_t^{(1)} \} = \frac{2\mu}{\alpha^2} t. \] (3.4)

and

\[ Var \{ L_t^{(2)} \} = \frac{2\mu}{\beta^2} t. \] (3.5)

The covariance between \( L_t^{(1)} \) and \( L_t^{(2)} \) at time \( t \) is given by

\[ Cov(L_t^{(1)}, L_t^{(2)}) = E \{ L_t^{(1)} L_t^{(2)} \} - E \{ L_t^{(1)} \} E \{ L_t^{(2)} \} = \frac{\mu}{\alpha \beta} \left( 1 + \frac{\theta}{4} \right) t. \] (3.6)

As linear correlation (or Pearson’s correlation) has been most popularly used in practice as a measure of dependence, we present the expression of the linear correlation coefficient for \( L_t^{(1)} \) and \( L_t^{(2)} \) at time \( t \), denoted by \( \rho \left( L_t^{(1)}, L_t^{(2)} \right) \).

\[ \rho \left( L_t^{(1)}, L_t^{(2)} \right) = \frac{Cov(L_t^{(1)}, L_t^{(2)})}{\sqrt{Var \{ L_t^{(1)} \} \sqrt{Var \{ L_t^{(2)} \} }} = \frac{\frac{\mu}{\alpha \beta} \left( 1 + \frac{\theta}{4} \right) t}{\sqrt{\frac{2\mu}{\alpha^2} t} \sqrt{\frac{2\mu}{\beta^2} t}} = \frac{4 + \theta}{8}. \] (3.7)

Let us now illustrate the calculations of the covariance and linear correlation between \( L_t^{(1)} \) and \( L_t^{(2)} \) at time \( t \).

Example 3.1

The parameter values used to calculate the covariance and linear correlation using (3.6) and (3.7) are

\[ \mu = 4, \ \alpha = 1, \ \beta = 0.5, \ t = 1. \]

From (3.6) and (3.7), the calculations of covariance and linear correlation between \( L_t^{(1)} \) and \( L_t^{(2)} \) at time \( t \) are shown in Table 3.1 and Table 3.2 respectively.

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>( Cov(L_t^{(1)}, L_t^{(2)}) )</th>
<th>( \theta )</th>
<th>( \rho \left( L_t^{(1)}, L_t^{(2)} \right) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>6</td>
<td>-1</td>
<td>0.375</td>
</tr>
<tr>
<td>-0.5</td>
<td>7</td>
<td>-0.5</td>
<td>0.4375</td>
</tr>
<tr>
<td>0</td>
<td>8</td>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>0.5</td>
<td>9</td>
<td>0.5</td>
<td>0.5625</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>1</td>
<td>0.625</td>
</tr>
</tbody>
</table>

4. Tail dependence of collateral losses

Let us look at how the joint Laplace transform derived in Section 2 can be used to evaluate the coefficient of (upper) tail dependence, \( \lambda_U \). As it is not possible for us to obtain the joint distribution of \( L_t^{(1)} \) and \( L_t^{(2)} \) explicitly, we invert the joint Fast Fourier transform obtained from the joint Laplace transform of collateral losses to approximate the coefficient of (upper) tail dependence (Castleman 1996; Gonzalez and Woods 2002 and Gonzalez et al. 2004). The below figures are the joint distribution of collateral losses and their contours at each value of \( \theta \).

Example 4.1

The parameter values used to approximate the coefficient of (upper) tail dependence are

\[ \mu = 4, \ \alpha = 1, \ \beta = 0.5, \ t = 1. \]

From (2.9) and (2.10), the mean of the aggregate losses arising from risk type 1 is given by
Figure 1: The joint distribution of collateral losses with $\theta = 1$

Figure 2: The contour of the joint distribution of collateral losses with $\theta = 1$
Figure 3: The joint distribution of collateral losses with $\theta = 0.5$

Figure 4: The contour of the joint distribution of collateral losses with $\theta = 0.5$
Figure 5: The joint distribution of collateral losses with $\theta = 0$

Figure 6: The contour of the joint distribution of collateral losses with $\theta = 0$
Figure 7: The joint distribution of collateral losses with $\theta = -0.5$

Figure 8: The contour of the joint distribution of collateral losses with $\theta = -0.5$
Figure 9: The joint distribution of collateral losses with $\theta = -1$

Figure 10: The contour of the joint distribution of collateral losses with $\theta = -1$
\[ E \left\{ L_i^{(1)} \right\} = \frac{\mu}{\alpha} t = 4 \]

and the mean of the aggregate losses arising from risk type 2 is given by

\[ E \left\{ L_i^{(2)} \right\} = \frac{\mu}{\beta} t = 8. \]

Using Matlab, the calculations of mean of aggregate losses arising from risk type 1 and 2 respectively are shown in Table 4.1.

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>( E \left{ L_i^{(1)} \right} )</th>
<th>( E \left{ L_i^{(2)} \right} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.0000</td>
<td>7.9998</td>
</tr>
<tr>
<td>0.5</td>
<td>4.0000</td>
<td>7.9997</td>
</tr>
<tr>
<td>0</td>
<td>4.0000</td>
<td>7.9996</td>
</tr>
<tr>
<td>−0.5</td>
<td>4.0000</td>
<td>7.9996</td>
</tr>
<tr>
<td>−1</td>
<td>4.0000</td>
<td>7.9999</td>
</tr>
</tbody>
</table>

Using Matlab, the calculations of the coefficients of (upper) tail dependence for collateral losses are shown in Table 4.2, Table 4.3, Table 4.4 and Table 4.5 using the different VaR at 90%, 95%, 99% and 99.9%.

Table 4.2

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>( \mathbb{P} \left{ L_i^{(1)} &gt; 7.84, L_i^{(2)} &gt; 15.68 \right} )</th>
<th>( \mathbb{P} \left{ L_i^{(1)} &gt; 7.84 \mid L_i^{(2)} &gt; 15.68 \right} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.038425</td>
<td>0.38425</td>
</tr>
<tr>
<td>0.5</td>
<td>0.034221</td>
<td>0.34221</td>
</tr>
<tr>
<td>0</td>
<td>0.030239</td>
<td>0.30239</td>
</tr>
<tr>
<td>−0.5</td>
<td>0.026450</td>
<td>0.26450</td>
</tr>
<tr>
<td>−1</td>
<td>0.022831</td>
<td>0.22831</td>
</tr>
</tbody>
</table>

where \( \mathbb{P} \left\{ L_i^{(1)} > 7.84 \right\} = \mathbb{P} \left\{ L_i^{(2)} > 15.68 \right\} = 0.1 \)

Table 4.3

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>( \mathbb{P} \left{ L_i^{(1)} &gt; 9.37, L_i^{(2)} &gt; 18.74 \right} )</th>
<th>( \mathbb{P} \left{ L_i^{(1)} &gt; 9.37 \mid L_i^{(2)} &gt; 18.74 \right} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.014593</td>
<td>0.29187</td>
</tr>
<tr>
<td>0.5</td>
<td>0.012565</td>
<td>0.25129</td>
</tr>
<tr>
<td>0</td>
<td>0.010681</td>
<td>0.21363</td>
</tr>
<tr>
<td>−0.5</td>
<td>0.0089309</td>
<td>0.17862</td>
</tr>
<tr>
<td>−1</td>
<td>0.0073023</td>
<td>0.14605</td>
</tr>
</tbody>
</table>

where \( \mathbb{P} \left\{ L_i^{(1)} > 9.37 \right\} = \mathbb{P} \left\{ L_i^{(2)} > 18.74 \right\} = 0.05 \)
Table 4.4

<table>
<thead>
<tr>
<th>θ</th>
<th>( \mathbb{P}{L_i^{(1)} &gt; 12.61, L_i^{(2)} &gt; 25.22} )</th>
<th>( \mathbb{P}{L_i^{(1)} &gt; 12.61 \mid L_i^{(2)} &gt; 25.22} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0015290</td>
<td>0.15290</td>
</tr>
<tr>
<td>0.5</td>
<td>0.0012168</td>
<td>0.12168</td>
</tr>
<tr>
<td>0</td>
<td>0.00094405</td>
<td>0.094405</td>
</tr>
<tr>
<td>-0.5</td>
<td>0.00070781</td>
<td>0.070781</td>
</tr>
<tr>
<td>-1</td>
<td>0.00050554</td>
<td>0.050554</td>
</tr>
</tbody>
</table>

where \( \mathbb{P}\{L_i^{(1)} > 12.61\} = \mathbb{P}\{L_i^{(2)} > 25.22\} = 0.01 \)

Table 4.5

<table>
<thead>
<tr>
<th>θ</th>
<th>( \mathbb{P}{L_i^{(1)} &gt; 16.81, L_i^{(2)} &gt; 33.62} )</th>
<th>( \mathbb{P}{L_i^{(1)} &gt; 16.81 \mid L_i^{(2)} &gt; 33.62} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.000059365</td>
<td>0.059365</td>
</tr>
<tr>
<td>0.5</td>
<td>0.000042203</td>
<td>0.042203</td>
</tr>
<tr>
<td>0</td>
<td>0.000028667</td>
<td>0.028667</td>
</tr>
<tr>
<td>-0.5</td>
<td>0.000018274</td>
<td>0.018274</td>
</tr>
<tr>
<td>-1</td>
<td>0.000010590</td>
<td>0.010590</td>
</tr>
</tbody>
</table>

where \( \mathbb{P}\{L_i^{(1)} > 16.81\} = \mathbb{P}\{L_i^{(2)} > 33.62\} = 0.001 \)

5. Conclusion

In this paper, we examined (upper) tail dependence for collateral losses using bivariate compound Poisson process as insurance industry is more concerned with dependence between extreme values. The covariance and linear correlation of collateral losses were also studied. In order to measure dependence between losses, we used a member of Farlie-Gumbel-Morgenstern copula with exponential margins. Employing piecewise deterministic Markov processes theory and martingale approach, we derived the explicit expression of the joint Laplace transform of the distribution of collateral losses. Inversion of joint Fast Fourier transform obtained from the joint Laplace transform was used to calculate the coefficients of (upper) tail dependence numerically. We also provided the figures of the joint distribution of collateral losses and their contours at each value of the parameter in a Farlie-Gumbel-Morgenstern copula. For further research, firstly we can examine inverting analytical forms of the joint Laplace transform of the distribution of losses, using other copulae and other margins, in particular heavy-tailed distributions for loss sizes. Secondly, other dependence structures, i.e. dependence between loss arrivals or dependence between losses and their arrivals can be included in loss processes. Lastly, but challenging enough, we can consider extending to multidimensional risk from bivariate risk.

References


