# The hidden cost of delay in a credit loan portfolio

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**Abstract.** Using an actuarial model, we examine the delay in credit loan payments. It is assumed that the default arrival process follows the Poisson process and the loss sizes are assumed to be independent and identical exponential. We also assume that the delay between default occurrence and partially (or fully) recovered payment is an independent identical exponential random variable. For the recovery rate, we assume that it follows Beta distribution. Using the relationship between the shot noise process and accumulated/discounted aggregate losses process and applying the piecewise deterministic Markov processes theory, we obtain the explicit expressions of the expected value of losses occurred and the expected value of part (or whole) of loan paid with the delay. Based on these first moments, we define and predict the hidden cost of delay in a credit loan portfolio. Numerical examples are also provided based on these explicit expressions of the hidden cost of delay in a credit loan portfolio.

**Key words:** Delay in credit loan portfolio; accumulated/discounted aggregate losses; shot noise process; piecewise deterministic Markov process; the expectation of partial (or whole) payment of loans with the delay; the predictor of the hidden cost of delay.

## 1. Introduction

Let $X_i$, $i = 1, 2, \cdots$, be the loss amount i.e. the value of a loan that the bank suffers when the *i*-th borrower defaults, which are assumed to be independent and identically distributed with distribution function $H(x)$ $(x > 0)$. Ignoring the rate of interest, the total losses up to time *t* from the entire portfolio, $C(t)$ is given by

$$C(t) = \sum_{i=1}^{N(t)} X_i \tag{1.1}$$

where $N(t)$ is the number of losses occurred up to time *t*. If we consider the effect of the interest to the loss amounts, as most of risk models have been developed ignoring the rate of interest for mathematical simplicity (Bühlmann 1970; Gerber 1979 and

Beard et al. 1984), the accumulated value of aggregate losses up to time $t$, $L(t)$ is given by

$$L(t)=\sum_{i=1}^{N(t)} X_i e^{\delta(t-s_i)} \tag{1.2}$$

and the discounted value of aggregate losses occurred up to time $t$, denoted by $L^0(t)=e^{-\delta t}L(t)$, is given by

$$L^0(t)=\sum_{i=1}^{N(t)} X_i e^{-\delta s_i} \tag{1.3}$$

where $\delta$ is the instantaneous rate of interest and $s_i$'s are time points at which defaults occur $(s_i < t < \infty)$. Assuming that the borrowers pay the loss amounts immediately after default occurrences, the expectation of discounted value of aggregate losses up to time $t$ at present time 0 is given by

$$E\{L^0(t)\}\,. \tag{1.4}$$

However, in practice, the case that the loss amounts would be paid immediately after default occurrences is highly rare. So considering the delay between default occurrence and final settlement, denoted by $\tau$ $(0\le\tau<\infty)$, which is independent of $s$, the accumulated value of aggregate losses paid with the delay between default occurrence and final settlement, denoted by $L_\tau(t)$, is given by

$$L_\tau(t)=\sum_{i=1}^{N(t)} X_i e^{\delta(t-s_i-\tau_i)} \tag{1.5}$$

and its expectation is given by

$$E\left\{L_\tau(t)\right\}=E_\tau\left[E\{L(t-\tau)\}\right]. \tag{1.6}$$

Hence the expectation of discounted value of aggregate losses paid with delay is given by

$$E\{L_\tau^0(t)\}\,. \tag{1.7}$$

We have assumed that the loss amounts at default and at settlement are the same. However in reality, the eventual loss amounts paid at settlement can be different from the loss amounts at default. As the worst case, the final payment could be 0 but in most case, the bank can recover the part (or whole) of loans after the liquidation of borrowers' assets. So if we consider the recovery rate, $\kappa_i$ which is an independent, identical random variable, (1.5), (1.6) and (1.7) are given by

$$L_{\tau,\kappa}(t)=\sum_{i=1}^{N(t)} \kappa_i X_i e^{\delta(t-s_i-\tau_i)}\,, \tag{1.8}$$

$$E\left\{L_{\tau,\kappa}(t)\right\}=E_\tau\left[E\{L_\kappa(t-\tau)\}\right] \tag{1.9}$$

and

$$E\{L_{\tau,\kappa}^0(t)\}\,. \tag{1.10}$$

It is assumed that the number of default follows the Poisson process. In order to obtain the explicit expressions of the expected value of losses, we assume that the loss amounts are exponentially distributed, even though it has to be capped below $\infty$. We also assume that the delay between default occurrence and final settlement is an independent identical exponential random variable. For the recovery rate, $\kappa$ we assume that it follows Beta distribution.

Using the relationship between the shot noise process and accumulated/discounted aggregate losses process and applying the piecewise deterministic Markov processes theory, the explicit expressions of the expected value of losses and of the expected value of partial (or whole) payment of loan with the delay are obtained. Based on these first moments, we define and predict the hidden cost of delay in a credit loan portfolio. Numerical examples are also provided based on the predictor of the hidden cost of delay.

In this paper, the migration risk and market risk are not examined as we employ an actuarial approach to model default risk. A similar work can be found in Micocci (2000). If you are interested in for credit risk modeling relating to the migration risk and market risk we refer you Merton (1974), Jarrow and Turnbull (1995), Jarrow et al. (1997), Lando (1997, 1998), Duffie and Singleton (1999), Bielecki and Rutkowski (2000), and Elliot et al. (2000).

## 2. Shot noise process and its generator

The shot noise process can be used in many diverse fields. In particular, it attracts us as it can be applied in financial and insurance field. The shot noise process is particularly useful as it measures the frequency, magnitude and time period needed to determine the effect of primary events. As time passes, the shot noise process decreases until another event occurs which will result in a positive jump in the shot noise process. We will adopt the shot noise process used by Cox & Isham (1980):

$$\lambda_t = \lambda_0 e^{-\delta t} + \sum_{\substack{\text{all } i \\ s_i < t}} y_i e^{-\delta(t-s_i)} \tag{2.1}$$

where:

$\lambda_0$ initial value of $\lambda$

$y_i$ jump size of primary event , where $E(y_i) < \infty$

$s_i$ time at which primary event *i* occurs, where $s_i < t < \infty$

$\delta$ exponential decay
$\rho$ the rate of primary event arrival.

The shot noise process is a piecewise deterministic Markov process (also a generalised Lévy process). The piecewise deterministic Markov processes theory developed by Davis (1984) is a powerful mathematical tool for examining non-diffusion models. Therefore we present definitions and important properties of the shot noise process with the aid of piecewise deterministic Markov processes theory (Dassios and Embrechts 1989 and Rolski et al. 1998). It is used to derive the mean of the shot noise process. Some works of insurance application using shot noise process can be found in Klüppelberg and Mikosch (1995a, 1995b), and Dassios and Jang (2003).

The three parameters of the shot noise process described are homogeneous in time. The decay is exponential $\delta$, which is a constant, can never reach 0 and the number of primary event arrivals follows a Poisson distribution with the frequency rate $\rho$. We will have generally distributed jump sizes with density function $g(y)$ and distribution function $G(y)$ $(y > 0)$. We can also generalise the shot noise process by allowing three parameters to depend on time (Dassios and Jang 2003). However, throughout the paper, for simplicity, we assume the parameters to be homogeneous in time.

The generator of the process $(\lambda_t,\ t)$ acting on a function $f(\lambda, t)$ belonging to its domain is given by

$$\mathrm{A}\, f(\lambda,t) = \frac{\partial f}{\partial t} - \delta\lambda \frac{\partial f}{\partial \lambda} + \rho\{\int_0^\infty f(\lambda + y, t)dG(y) - f(\lambda,t)\} \tag{2.2}$$

where $f : (0,\infty)\times\mathbb{R}^+ \to (0,\infty)$. It is sufficient that $f(\lambda,t)$ is differentiable w.r.t. $\lambda$, $t$ for all $\lambda$, $t$ and that $\left|\int_0^\infty f(\lambda + y, t)dG(y) - f(\lambda,t)\right| < \infty$ for $f(\lambda,t)$ to belong to the domain of the generator A. For details on the piecewise deterministic Markov processes theory and the generator, we refer you the appendix.

Now let us derive the mean of $\lambda_t$ assuming that $\lambda_0$ is given. If we set $f(\lambda) = \lambda$ in (2.2), then

$$\mathrm{A}\ \lambda = -\delta\lambda + \mu_1\rho$$

where $\mu_1 = \int_0^\infty y dG(y)$. From $E(\lambda_t \,|\lambda_0) - \lambda_0 = E[\int_0^t \{\mathrm{A}\ f(\lambda_s)\,|\lambda_0\} ds]$,

$$E\left(\lambda_t \,|\, \lambda_0\right) = \lambda_0 - \delta \int_0^t E\left(\lambda_s \,|\, \lambda_0\right) + \mu_1 \rho t \, .$$

Differentiate w.r.t $t$

$$\frac{dE\left(\lambda_t \,|\, \lambda_0\right)}{dt} = -\delta E\left(\lambda_t \,|\, \lambda_0\right) + \mu_1 \rho$$

and solve the differential equation we have

$$E\left(\lambda_t \,|\, \lambda_0\right) = \lambda_0 e^{-\delta t} + \frac{\mu_1 \rho}{\delta}\left(1 - e^{-\delta t}\right). \tag{2.3}$$

It will be interesting to find the mean of $\lambda_t$ using a specific jump size distribution of $G(y)$ $(y > 0)$. We use an exponential jump size distribution, i.e. $g(y) = \alpha e^{-\alpha y}$, $y > 0$, $\alpha > 0$ and the expectation of $\lambda_t$ is given by

$$E\left(\lambda_t \,|\, \lambda_0\right) = \lambda_0 e^{-\delta t} + \frac{\rho}{\alpha \delta}\left(1 - e^{-\delta t}\right). \tag{2.4}$$

## 3. Relationship between shot noise process and accumulated/discounted aggregate losses

Let us examine how the results we have obtained in section 2 can be applied in deriving the expectation of accumulated aggregate losses. If we set $-\delta$ to $\delta$ in (2.1), it becomes

$$\xi(t) = \xi_0 e^{\delta t} + \sum_{\substack{\text{all } i \\ s_i < t}} y_i e^{\delta(t - s_i)} \, . \tag{3.1}$$

Interestingly, we can easily find that it is equivalent to (1.2) if we substitute '$\xi$' with '$L$' and '$y$' with '$X$' in (3.1), assuming that $\xi_0 = 0$. It is also equivalent to (1.3) if we multiply $e^{-\delta t}$ both sides in (3.1). Therefore similar to the previous section, based on

$$\mathrm{A} \, f(l,t) = \frac{\partial f}{\partial t} + \delta l \frac{\partial f}{\partial l} + \rho\left\{\int_0^\infty f(l + x, t) dH(x) - f(l,t)\right\}$$

and assuming that the loss size distribution is exponential, i.e. $h(x) = \alpha e^{-\alpha x}$, $x > 0$, $\alpha > 0$, we can easily obtain that

$$E\left\{L(t)\right\} = \frac{\rho}{\alpha \delta}\left(e^{\delta t} - 1\right) = \frac{\rho}{\alpha} \bar{s}_{\overline{t}|} \tag{3.2}$$

where $\bar{s}_{\overline{t}|} = \dfrac{e^{\delta t} - 1}{\delta}$. By multiplying $e^{-\delta t}$ both sides in (3.2) we obtain

$$E\left\{L^0(t)\right\} = \frac{\rho}{\alpha \delta}\left(1 - e^{-\delta t}\right) = \frac{\rho}{\alpha} \bar{a}_{\overline{t}|} \tag{3.3}$$

where $\bar{a}_{\overline{t}|} = \dfrac{1 - e^{-\delta t}}{\delta}$.

In practice, other thick-tail distributions such as log-normal, gamma and Pareto, etc. can also be applied for loss size distribution of $H(x)\ \ (x>0)$.

## 4. The expected value of loans paid with delays and prediction of hidden cost of delay

We can easily obtain the expected value of loans paid with delays using (3.2), (1.5) and (1.6). Assuming that the claim size distribution is exponential, i.e. $h(x)=\alpha e^{-\alpha x},\ \ x>0,\ \ \alpha>0$, we have

$$E\{L_\tau(t)\}=E_\tau\left[E\{L(t-\tau)\}\right]=E_\tau\left[\frac{\rho}{\alpha\delta}\left\{e^{\delta(t-\tau)}-1\right\}\right]. \tag{4.1}$$

If we assume that the delay between default occurrence and final settlement follows exponential distribution with its density function $j(\tau)=\beta e^{-\beta\tau},\ \ \tau>0,\ \ \beta>0$, (4.1) is given by

$$E\{L_\tau(t)\}=\frac{\rho}{\alpha\delta}\left\{\left(\frac{\beta}{\beta+\delta}\right)e^{\delta t}-1\right\}. \tag{4.2}$$

Therefore the expectation of discounted value of aggregate losses paid fully with delay between default occurrence and final settlement is given by

$$E\{L_\tau^0(t)\}=\frac{\rho}{\alpha\delta}\left\{\left(\frac{\beta}{\beta+\delta}\right)-e^{-\delta t}\right\}=\frac{\rho}{\alpha}\left(\frac{\beta}{\beta+\delta}\right)\bar{a}_{\overline{t}|}-\frac{\rho}{\alpha(\beta+\delta)}e^{-\delta t}. \tag{4.3}$$

In reality, we can also use other thick-tail distributions such as mixture of exponential, gamma, log-normal and Pareto, etc. for the delay distribution of $j(\tau)$.

Considering the recovery rate, $\kappa_i$, which is an independent, identical Beta random variable, i.e. $u(\kappa)=\frac{\Gamma(\gamma+\xi)}{\Gamma(\gamma)\Gamma(\xi)}\kappa^{\gamma-1}(1-\kappa)^{\xi-1},\ \ 0<\kappa<1,\ \gamma>0,\ \ \xi>0$ then from (4.3) and (1.10), the expectation of discounted value of aggregate losses paid partially with delay is given by

$$E\{L_{\tau,\kappa}^0(t)\}=\frac{\rho}{\alpha}\left(\frac{\gamma}{\gamma+\xi}\right)\left(\frac{\beta}{\beta+\delta}\right)\bar{a}_{\overline{t}|}-\left(\frac{\gamma}{\gamma+\xi}\right)\frac{\rho}{\alpha(\beta+\delta)}e^{-\delta t}. \tag{4.4}$$

Now, using (3.3), (4.3) and (4.4), let us define the predictor of the hidden cost of delay as

$$E\{L^0(t)\}-E\{L_\tau^0(t)\}=\frac{\rho}{\alpha}\left[\left\{1-\left(\frac{\beta}{\beta+\delta}\right)\right\}\bar{a}_{\overline{t}|}+\frac{1}{(\beta+\delta)}e^{-\delta t}\right]\ \text{with full recovery} \tag{4.5}$$

and

$$E\{L^0(t)\} - E\{L^0_{\tau,\kappa}(t)\} = \frac{\rho}{\alpha}\left[\left\{1 - \left(\frac{\gamma}{\gamma+\xi}\right)\left(\frac{\beta}{\beta+\delta}\right)\right\}\bar{a}_{\overline{t}|} + \left(\frac{\gamma}{\gamma+\xi}\right)\frac{1}{(\beta+\delta)}e^{-\delta t}\right]$$

with partial recovery. (4.6)

Based on the predictors of the hidden cost of delay in a credit loan portfolio, let us illustrate their calculations.

**Example 4.1**

The parameter values used to calculate the predictor of the hidden cost of delay are $\alpha = 0.01$, $\beta = 3$, $\gamma = 10$, $\xi = 10$, $\delta = 0.05$, $\rho = 5$ and $t = 1$. The calculations of the expectation of discounted value of aggregate losses occurred, the expectation of discounted value of aggregate losses paid fully/partially with the delay between claim occurrence and final settlement are shown in Table 4.1. The calculations of the predictors of the hidden dost of delay are shown in Table 4.2.

**Table 4.1**

| | |
|---|---|
| $E\left\{L^0(t)\right\} = \frac{\rho}{\alpha}\bar{a}_{\overline{t}\vert}$ | 4,877.1 |
| $E\left\{L^0_{\tau}(t)\right\} = \frac{\rho}{\alpha}\left(\frac{\beta}{\beta+\delta}\right)\bar{a}_{\overline{t}\vert} - \frac{\rho}{\alpha(\beta+\delta)}e^{-\delta t}$ | 3,237.7 |
| $E\left\{L^0_{\tau,\kappa}(t)\right\} = \frac{\rho}{\alpha}\left(\frac{\gamma}{\gamma+\xi}\right)\left(\frac{\beta}{\beta+\delta}\right)\bar{a}_{\overline{t}\vert} - \left(\frac{\gamma}{\gamma+\xi}\right)\frac{\rho}{\alpha(\beta+\delta)}e^{-\delta t}$ | 1,618.9 |

**Table 4.2**

| | |
|---|---|
| $E\{L^0(t)\} - E\{L^0_{\tau}(t)\} = \frac{\rho}{\alpha}\left[\left\{1 - \left(\frac{\beta}{\beta+\delta}\right)\right\}\bar{a}_{\overline{t}\vert} + \frac{1}{(\beta+\delta)}e^{-\delta t}\right]$ | 1,639.4 |
| $E\{L^0(t)\} - E\{L^0_{\tau,\kappa}(t)\} = \frac{\rho}{\alpha}\left[\left\{1 - \left(\frac{\gamma}{\gamma+\xi}\right)\left(\frac{\beta}{\beta+\delta}\right)\right\}\bar{a}_{\overline{t}\vert} + \left(\frac{\gamma}{\gamma+\xi}\right)\frac{1}{(\beta+\delta)}e^{-\delta t}\right]$ | 3,258.2 |

**Example 4.2**

We will now examine the effect on the predictor of the hidden cost of delay caused by changes in the value of the instantaneous rate of interest $\delta$ and exponential delay parameter $\beta$. The calculations of the predictor of the hidden cost of delay at each value of $\delta$ and $\beta$ with partial recovery are shown in Table 4.3.

**Table 4.3**

| | $\delta = 0.03$ | $\delta = 0.05$ | $\delta = 0.07$ |
|---|---|---|---|
| $\beta = 10$ | 2,712.1 | 2,687.3 | 2,662.8 |
| $\beta = 3$ | 3,288.0 | 3,258.2 | 3,228.8 |
| $\beta = 1$ | 4,890.1 | 4,819.5 | 4,751.0 |

**Example 4.3**

Let us examine the effect on the predictor of the hidden cost of delay caused by changes in the recovery rate (i.e. the values of the parameters of Beta distribution $\gamma$ and $\xi$). The calculations of the predictor of the hidden cost of delay at each value of $\gamma$ and $\xi$ with partial recovery are shown in Table 4.4.

**Table 4.4**

| | |
|---|---|
| $\gamma = 1.5$ and $\xi = 28.5$ | 4,715.2 |
| $\gamma = 10$ and $\xi = 10$ | 3,258.2 |
| $\gamma = 14$ and $\xi = 6$ | 2,610.7 |

## 5. Conclusion

We examined the delay in credit loan portfolio using an actuarial model. In order to derive the explicit formulae for the predictors of the hidden cost of delay, we assumed the Poisson process for the default arrival process, an independent and identical exponential distribution for the loss sizes and another independent and identical exponential distribution for the delay between default occurrence and partially (or fully) recovered payment. Beta distribution was employed for the recovery rate.

The relationship between the shot noise process and accumulated/discounted aggregate claims process was used to consider the effect of the interest to the loss amounts and applying the piecewise deterministic Markov processes theory, we obtained the expected value of losses occurred and the expected value of part (or whole) of loan paid with the delay.

We provided numerical examples of the calculations of the predictors of the hidden cost of delay in a credit loan portfolio. For further research, using data other heavy-tail distributions such as gamma, log-normal and Pareto, etc. can be examined for delay between default occurrence and payment. Also other distributions can be considered for loss amount distribution rather than an exponential as it allows loss amount to be $\infty$. However as an approximation of the hidden cost delay we can employ a theoretical distribution, as the expected value of losses is only required in the predictors of the hidden cost of delay. We can also consider stochastic interest rate to make the model more realistic.

**Appendix**

This appendix explains the basic definition of a piecewise deterministic Markov process (PDMP) that is adopted from Dassios and Embrechts (1989). A detailed discussion can also be found in Davis (1984) and Rolski et al. (1999).

PDMP is a Markov process $X_t$ with two components $(\eta_t,\ \xi_t)$ where $\eta_t$ takes values in a discrete set $\mathrm{K}$ and given $\eta_t = n \in \mathrm{K}$, $\xi_t$ takes values in an open set $\mathrm{M}_n \subset \Re^{d(n)}$ for some function $d : \mathrm{K} \to \mathbb{N}$. The state space of $X_t$ is equal to $\mathrm{E} = \{(n, z) :\ n \in \mathrm{K},\ z \in \mathrm{M}_n\}$. We further assume that for every point $x = (n, z) \in \mathrm{E}$, there is a unique, deterministic integral curve $\phi_n(t, z) \subset \mathrm{M}_n$, determined by a differential operator $\chi_n$ on $\Re^{d(n)}$, such that $z \in \phi_n(t, z)$. If for some $t_0 \in \Re^+$, $X_{t_0} = (n_0, z_0) \in \mathrm{E}$, then $\xi_t$, where $t \ge t_0$ follows $\phi_{n_0}(t, z_0)$ until either $t = T_0$, some random time with hazard rate of function $\rho$ or until $\xi_t = \partial \mathrm{M}_{n_0}$, the boundary of $\mathrm{M}_{n_0}$. In both cases, the process $X_t$ jumps, according to a Markov transition measure $Q$ on $\mathrm{E}$, to a point $(n_1, z_1) \in \mathrm{E}$. $\xi_t$ again follows the deterministic path $\phi_{n_1}$ till a random time $T_1$ (independent of $T_0$) or till $\xi_t = \partial \mathrm{M}_{n_1}$, etc…. The jump times $T_i$ are assumed to satisfy the following condition:

$$\forall t > 0,\ E\left( \sum_i I(T_i \le t) \right) < \infty. \tag{A.1}$$

The stochastic calculus that will enable us to analyse various models rests on the notion of (extended) generator $\mathrm{A}$ of $X_t$. Let $\Gamma$ denotes the set of boundary points of $\mathrm{E}$, $\Gamma = \{(n, z) :\ n \in K,\ z \in \partial \mathrm{M}_n\}$, and let $\mathrm{A}$ be an operator acting on measurable functions $f : \mathrm{E} \bigcup \Gamma \to \Re$ satisfying

(i) The function $t \to f\left(n,\ \phi_n(t, z)\right)$ is absolutely continuous for $t \in \left[0,\ t(n, z)\right]$, for all $(n, z) \in \mathrm{E}$.

(ii) For all $x \in \Gamma$, $f(x) = \int_{\mathrm{E}} f(y) Q(x; dy)$ (Boundary condition).

(iii) For all $t \ge 0$, $E\left\{ \sum_{T_i \le t} \left| f(X_{T_i}) - f(X_{T_i^-}) \right| \right\} < \infty$.

Hence the set of measurable functions satisfying (i), (ii) and (iii) form a subset of the domain of the extended generator $\mathrm{A}$, denoted by $D(\mathrm{A})$. Now for piecewise deterministic Markov processes, we can explicitly calculate $\mathrm{A}$ by (Davis 1984, Theorem 5.5)

$$\forall f \in D(\mathrm{A}): \; \mathrm{A}\, f(x) = \chi f(x) + \rho(x) \int_{\mathrm{E}} \{ f(y) - f(x) \} Q(x; dy). \qquad \text{(A.2)}$$

In some cases, it is important to have time $t$ as an explicit component of the PDMP. In those cases $\mathrm{A}$ can be decomposed as $\frac{\partial}{\partial t} + \mathrm{A}_t$, where $\mathrm{A}_t$ is given by (A.2) with possibly time-dependent coefficients.

An application of Dynkin's formula provides us with the following important result (Martingales will always be with respect to the natural filtration $\sigma\{X_s : \; s \le t\}$):

(a) If for all $t$, $f(\cdot,\; t)$ belongs to the domain of $\mathrm{A}_t$ and $\frac{\partial}{\partial t} f(x,t) + \mathrm{A}_t\, f(x,t) = 0$, then process $f(X_t, t)$ is a martingale.

(b) If $f$ belongs to the domain of $\mathrm{A}$ and $\mathrm{A}\, f(x) = 0$, then $f(X_t)$ is a martingale.

The generator of the process $X_t$ acting on a function $f(X_t)$ belonging to its domain as described above is also given by

$$\mathrm{A}\, f(X_t) = \lim_{h \downarrow 0} \frac{E\{ f(X_{t+h}) \,|\, X_t = x\} - f(X_t)}{h}.$$

In other words, $\mathrm{A}\, f(X_t)$ is the expected increment of the process $X_t$ between $t$ and $t+h$, given the history of $X_t$ at time $t$. From this interpretation the following inversion formula is plausible, i.e.

$$E\left[ f(X_{t+h}) \,|\, X_t = x \right] - f(X_t) = \int_0^h E\left\{ \mathrm{A}\, f(X_s) \right\} ds$$

which is Dynkin's formula.

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