The Laplace transform of the distribution of the shot noise process with respect to the Esscher measure and its application to the accumulated aggregate insurance claims

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Abstract

We examine the time-dependent shot noise process by piecewise deterministic Markov process theory. The Laplace transform of the distribution of the time-dependent shot noise process is obtained using the martingale. Considering the decay rate of shot noise process as the rate of interest, the Laplace transform of the distribution of the accumulated aggregated claims, with respect to the Esscher measure, is obtained. Based on the Laplace transform under the Esscher measure, we derive the mean of the accumulated aggregated claims, assuming that the jump size follows the exponential distribution. The mean of the discounted aggregated claims, that is a fair (arbitrage-free) premium, is also derived.

Keywords: Time-dependent (reversed) shot noise process; Piecewise deterministic Markov process theory; The Laplace transform; Martingale; The Esscher transform; Accumulated aggregate claims.

1. Introduction

Let X_i be the claim amount, which are assumed to be independent and identically distributed with distribution function $H(x)$ $(x > 0)$. The accumulated value of aggregate claims up to time *t*, L_t is

$$L_t = X_1 e^{d(t-s_1)} + X_2 e^{d(t-s_2)} + \cdots + X_{N_t - 1} e^{d(t-s_{N_t - 1})} + X_{N_t} e^{d(t-s_{N_t})} \tag{1.1}$$

where d is the instantaneous rate of interest, s_i's are time points at which claims occur $(s_i < t < \infty)$ and N_t is the number of claims up to time *t*.

If we multiply e^{-dt} both sides in (1.1), it becomes the discounted value of aggregate claims up to time *t*, denoted by $L_0 = e^{-dt} L_t$,

$$L_0 = X_1 e^{-d s_1} + X_2 e^{-d s_2} + \cdots + X_{N_t - 1} e^{-d s_{N_t - 1}} + X_{N_t} e^{-d s_{N_t}} \ . \tag{1.2}$$

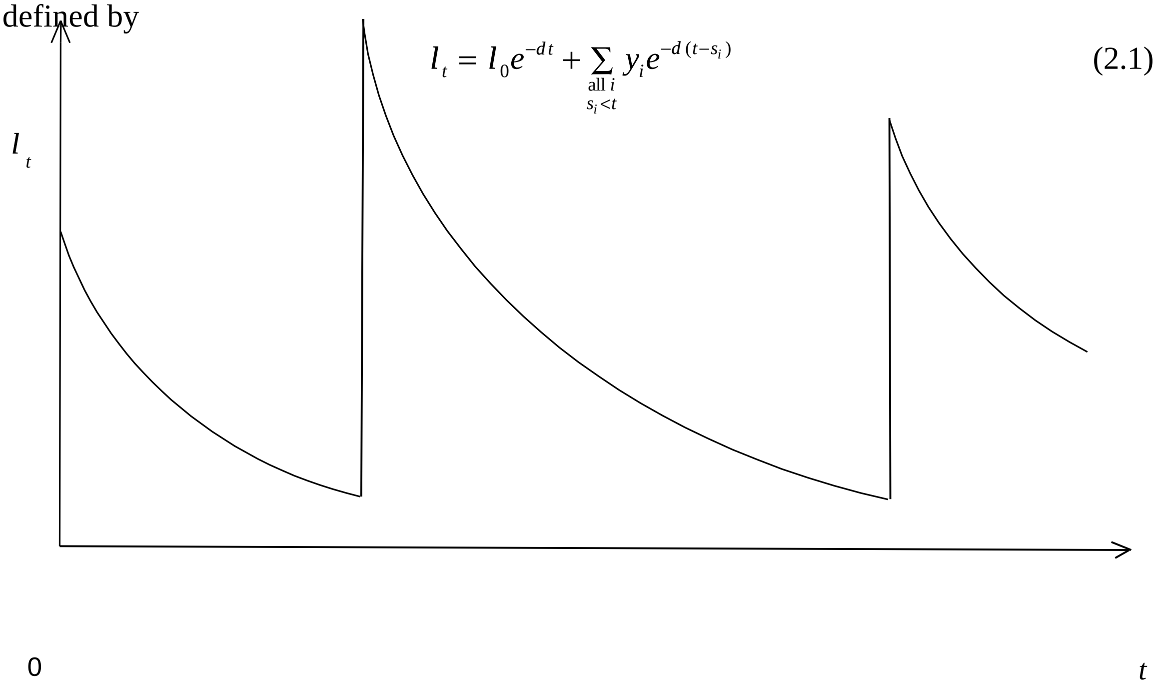
We are interested in obtaining the Laplace transform of the distribution of the accumulated value (or discounted value) of aggregated claims, with respect to the *Esscher measure* (i.e. risk-neutral measure) as we can easily derive the arbitrage-free moments of the accumulated value (or discounted value) of aggregated claims. In order to do so, we employ the time-dependent shot noise process and reverse one of its parameters.

The Laplace transform of the distribution of the accumulated value of aggregated claims up to time *t*, with respect to the original probability measure can be found in Dufresne (1990), Milevsky (1997), Goovaerts et al. (2000) and Jang (2001).

2. Time-dependent shot noise process and its generator

The shot noise process can be used in many diverse fields. In particular, it attracts us as it can be applied in financial and insurance field. Algebraically, the shot process can be defined by

$$l_t = l_0 e^{-dt} + \sum_{\substack{\text{all } i \\ s_i < t}} y_i e^{-d(t - s_i)} \tag{2.1}$$



where:

λ_0 initial value of λ
y_i jump size of primary event where $E(y_i) < \infty$
s_i time at which primary event i occurs where $s_i < t < \infty$
δ exponential decay
ρ the rate of primary event arrival.

The piecewise deterministic Markov processes theory developed by Davis (1984) is a powerful mathematical tool for examining non-diffusion models. The shot noise process is an example of piecewise deterministic Markov process. Therefore we can present definitions and important properties of the shot noise process with the aid of this theory (Dassios, 1987 and Dassios & Embrechts, 1989). It is used to derive the distribution of the shot noise process and its moments.

Some works of insurance application using shot noise process can be found in Dassios (1987), Klüppelberg & Mikosch (1995), Jang (1998, 2001) and Dassios & Jang (1998, 2001).

The three parameters of the shot noise process described are homogeneous in time. We are now going to generalise the shot noise process by allowing the parameters to depend on time. The rate of jump arrivals, $\rho(t)$, is bounded on all intervals [0, t) (no explosions). $\delta(t)$ is the rate of decay and the distribution function of jump sizes at any time t is $G(y;t)$ $(y > 0)$ with $E(y;t) = \mu_1(t) = \int_0^\infty y dG(y;t)$. We assume that $\delta(t)$, $\rho(t)$ and $G(y;t)$ are all Riemann integrable functions of t and are all positive.

The generator of the process (λ_t, t) acting on a function $f(\lambda, t)$ belonging to its domain is given by

$$\mathrm{A}\, f(\lambda,t) = \frac{\partial f}{\partial t} - \delta(t)\lambda \frac{\partial f}{\partial \lambda} + \rho(t)\{\int_0^\infty f(\lambda + y, t) dG(y;t) - f(\lambda,t)\}. \qquad (2.2)$$

For $f(\lambda,t)$ to belong to the domain of the generator A, it is sufficient that $f(\lambda,t)$ is differentiable w.r.t. λ, t for all λ, t and that $\left|\int_0^\infty f(\lambda + y, t) dG(y;t) - f(\lambda,t)\right| < \infty$.

Now let us find a martingale in order to derive the Laplace transform of the distribution of l_t at time *t*. To do so, we begin with a related theorem that is used by Jang (1998).

Theorem 2.1 *Considering a constant such that* $n \geq 0$*, then*

$$\exp\left\{-n e^{\Delta(t)} l_t\right\} \exp\left[\int_0^t r(s)[1-\hat{g}\{n e^{\Delta(s)}; s\}]ds\right] \qquad (2.3)$$

is a martingale where $\Delta(t) = \int_0^t d(s)ds$ *and* $\hat{g}(k; s) = \int_0^\infty e^{-ky} dG(y; s)$.

Proof

From (2.2), $f(l, t)$ has to satisfy $A f = 0$ for it to be a martingale. Setting $e^{-A(t)l} e^{B(t)}$ we get the equation

$$-l A'(t) + B'(t) + d(t) l A(t) + r(t)[\hat{g}\{A(t); t\} - 1] = 0 \qquad (2.4)$$

and solving (2.4) we obtain

$$A(t) = n \exp\left\{\int_0^t d(s)ds\right\} \text{ and } B(t) = \int_0^t r(s)[1-\hat{g}\{n e^{\int_0^s d(u)du}; s\}]ds$$

and the result follows.

□

3. The Laplace transform of the distribution of the time-dependent shot noise process and accumulated aggregated claims

We can easily obtain the Laplace transform of the distribution of l_t at time *t* using the martingale found in theorem 2.1.

Theorem 3.1 *The Laplace transform of the distribution of* l_t *at time t is given by*

$$E\left(e^{-n l_t} \mid l_0\right) = \exp\left\{-n l_0 e^{-\Delta(t)}\right\} \exp\left[-\int_0^t r(s)[1-\hat{g}\{n e^{-\Delta(t)} e^{\Delta(s)}; s\}]ds\right]. \qquad (3.1)$$

Proof

From (2.3), for a fixed time t^* and a fixed constant $n^* \geq 0$, we have

$$E\left\{\exp\left(-n^* e^{\Delta(t^*)} l_{t^*}\right) \mid l_0\right\} = \exp\left(-n^* l_0\right) \exp\left[-\int_0^{t^*} r(s)[1-\hat{g}\{n^* e^{\Delta(s)}; s\}]ds\right]$$

and setting $n^* = n e^{-\Delta(t^*)}$ we have

$$E\left(e^{-n l_{t^*}} \mid l_0\right) = \exp\left\{-n l_0 e^{-\Delta(t^*)}\right\} \exp\left[-\int_0^{t^*} r(s)[1-\hat{g}\{n e^{-\Delta(t^*)} e^{\Delta(s)}; s\}]ds\right]. \qquad (3.2)$$

Since (3.2) holds for an arbitrary fixed t^*, it holds for all $t \ge 0$ and the theorem is proved.

□

It will be interesting to find the Laplace transform of the distribution of l_t using a specific jump size distribution of $G(y;t)$ $(y > 0)$. We use an exponential jump size distribution, i.e. its density is $g(y;t) = (a + ge^{dt})\exp\{-(a + ge^{dt})y\}$, $y > 0$, $a + ge^{dt} > 0$. Let us also assume that $r(t) = r\dfrac{a}{a + ge^{dt}}$. The reason for this particular assumption will become apparent later when we change the probability measure.

Let us assume that $d(t) = d$ throughout the rest of this paper.

Corollary 3.2 *Let the jump size distribution be exponential i.e.* $g(y;t) = (a + ge^{dt})\exp\{-(a + ge^{dt})y\}$, $y > 0$, $-a e^{-dt} < g \le 0$. *Assuming that* $r(t) = r\dfrac{a}{a + ge^{dt}}$ *then*

$$E\left\{e^{-n l_{t_1}} \mid l_{t_0}\right\} = \exp\left\{-n l_0 e^{-d(t_1 - t_0)}\right\} \left(\frac{g e^{d t_0} + a e^{-d(t_1 - t_0)}}{g e^{d t_0} + a}\right)^{\frac{r}{d}} \left(\frac{g e^{d t_0} + n e^{-d(t_1 - t_0)} + a}{g e^{d t_0} + (n + a) e^{-d(t_1 - t_0)}}\right)^{\frac{r}{d}}. \qquad (3.3)$$

<u>Proof</u>

If we set $r(t) = r\dfrac{a}{a + ge^{dt}}$ and $g(y;t) = (a + ge^{dt})\exp\{-(a + ge^{dt})y\}$, $y > 0$, $-a e^{-dt} < g \le 0$ in (3.1), the result follows immediately.

□

Let us examine how the results we have obtained above can be applied in deriving the Laplace transform of distribution of accumulated aggregated claims.

If we set $-d$ to d in (2.1), it becomes

$$x_t = x_0 e^{dt} + \sum_{\substack{\text{all } i \\ s_i < t}} y_i e^{d(t - s_i)}. \qquad (3.4)$$

Interestingly, we can see that it is equivalent to (1.1) if we substitute 'x' with 'L' and 'y' with 'X' in (3.4), assuming that x_0, that can be considered as the total claims up to present time 0, is 0. The decay rate d in (2.1) is the rate of interest in (4.1).

Similar to the previous section, based on

$$\mathrm{A}\, f(x,t) = \frac{\P f}{\P t} + d x \frac{\P f}{\P l} + r(t)\{\int_0^{\infty} f(x+x,t)dH(x;t) - f(x,t)\},$$

we can easily derive the Laplace transform of the distribution of x_t at time *t*, i.e.

$$E\left\{e^{-nx_t} \middle| x_0\right\} = \exp\left\{-n x_0 e^{dt)}\right\} \exp\left[-\int_0^t r(s)\{1-\hat{h}(n e^{ds};s)\}ds\right] \tag{3.5}$$

and if the jump size distribution is exponential, i.e. $h(x;t) = (a + g e^{-dt})\,\mathrm{e}^{-(a+g e^{-dt})x} \quad x > 0$, $-a e^{dt} < g \le 0$, we have

$$\exp\left\{-n x_0 e^{dt}\right\}\left(\frac{g + a e^{dt}}{g + a}\right)^{-\frac{r}{d}} \left(\frac{g + n e^{dt} + a}{g + (n + a)e^{dt}}\right)^{-\frac{r}{d}}. \tag{3.6}$$

Assuming that $x_0 = 0$, the Laplace transform of the distribution of the accumulated aggregated claims is

$$\exp\left[-\int_0^t r(s)\{1-\hat{h}(n e^{ds};s)\}ds\right] \tag{3.7}$$

and if the jump size distribution is exponential, it becomes

$$\left(\frac{g + a e^{dt}}{g + a}\right)^{-\frac{r}{d}} \left(\frac{g + n e^{dt} + a}{g + (n + a)e^{dt}}\right)^{-\frac{r}{d}}. \tag{3.8}$$

If we differentiate (3.8) with respect to n and put $n = 0$, we can obtain the mean of the accumulated aggregated claims, i.e.

$$E(L_t) = \frac{a r}{(a + g e^{-dt})(a + g)}\left\{\frac{e^{dt} - 1}{d}\right\} = \frac{a r}{(a + g e^{-dt})(a + g)} \bar{s}_{\overline{t|}} \tag{3.9}$$

and the higher moments can be obtained by differentiating it further.

4. The Esscher transform and change of probability measure

In general, the Esscher transform is defined as a change of probability measure for certain stochastic processes. An Esscher transform of such a process induces an equivalent probability measure on the process. The parameters involved for the Esscher transform are determined so that the process is a martingale under the new probability measure. We will examine an equivalent martingale probability measure obtained via the Esscher transform (Gerber & Shiu, 1996 and Dassios & Jang, 2002).

A probability measure P^* is called an equivalent martingale probability measure if:

(i) $P^*[A] = 0$ iff $P[A] = 0$, for any $A \in \Im_t$;

(ii) The Radon-Nikodym derivative $\dfrac{dP^*}{dP}$ belongs to $L^2(\Omega, \Im_t, P)$;

(iii) A stochastic process X_t is a martingale under P^*, i.e.

$$E^*\left[X_t \middle| \Im_s\right] = X_s \ , \quad P^* - \text{a.s.}$$

for any $0 \le s \le t \le T$, where E^* denotes the expectation with respect to P^* (Harrison & Kreps, 1979 and Sondermann, 1991).

We here offer the definition of the Esscher transform that is adopted from Gerber & Shiu (1996).

Definition 4.1 *Let X_t be a stochastic process and h^* a real number. For a measurable function f, the expectation of the random variable $f(X_t)$ with respect to the equivalent martingale probability measure is*

$$E^*[f(X_t)] = E\left[f(X_t)\frac{e^{h^* X_t}}{E(e^{h^* X_t})}\right] = \frac{E\left[f(X_t)e^{h^* X_t}\right]}{E[e^{h^* X_t}]} \tag{4.1}$$

where the process $e^{h^* X_t}$ is a martingale and $E(e^{h^* X_t}) < \infty$.

From definition 4.1, we need to obtain a martingale that can be used to define a change of probability measure, i.e. it can be used to define the Radon-Nikodym derivative $\dfrac{dP^*}{dP}$ where P is the original probability measure and P^* is the equivalent martingale probability measure with parameters involved.

Let M_t be the total number of claim jumps up to time *t*. Then the generator of the process (l_t, M_t, t) acting on a function $f(l, m, t)$ belonging to its domain is given by

$$\text{A}\, f(l, m, t) = \frac{\P f}{\P t} - d\, l\, \frac{\P f}{\P l} + r[\int_0^\infty f(l + y, m+1, t)dG(y) - f(l, m, t)]. \tag{4.2}$$

Now let us find a suitable martingale in order to change measure applying the Esscher transform.

Theorem 4.2 *Considering constants* y^* *and* g^* *such that* $y^* \geq 1$ *and* $g^* \leq 0$,

$$y^{*M_t} \exp\left(-g^* l_t e^{dt}\right) \exp\left[r \int_0^t \{1 - y^* \hat{g}(g^* e^{ds})\} ds \right] \tag{4.3}$$

is a martingale where $\hat{g}(u) = \int_0^\infty e^{-uy} dG(y)$.

Proof

From (4.2), $f(l, m, t)$ has to satisfy $\mathcal{A} f = 0$ for $f(l_t, M_t, t)$ to be a martingale. Trying $y^{*m} \exp\left(-g^* l e^{dt}\right) e^{R(t)}$ we get the equation

$$-l d g^* e^{dt} + R'(t) + d l g^* e^{dt} + r\{y^* \hat{g}(g^* e^{dt}) - 1\} = 0 \tag{4.4}$$

and solve (4.4) we have

$$R(t) = r \int_0^t \{1 - y^* \hat{g}(g^* e^{ds})\} ds$$

and the result follows.

□

Let us look at the how the process l_t changes after changing probability measure. To do so we start with a technical lemma.

Lemma 4.3 *Assume that* $f(l, m, t) = f(l, t)$ *for all* m *and that* $e^{-n^* l_t}$ *is a martingale. Consider a constant* n^* *such that* $n^* \geq 0$. *Then*

$$\mathcal{A}^* f(l, 0) = \frac{\mathcal{A}\{f(l, 0) e^{-n^* l}\}}{e^{-n^* l}}. \tag{4.5}$$

Proof

The generator of the process (l_t, t) acting on a function $f(l, t)$ with respect to the equivalent martingale probability measure is

$$\mathcal{A}^* f(l, 0) = \lim_{t \downarrow 0} \frac{E^*[f(l_t, t) | l_0 = l] - f(l, 0)}{t}. \tag{4.6}$$

We will use $\dfrac{e^{-n^* l_t}}{E(e^{-n^* l_t})}$ as the Radon-Nikodym derivative to define equivalent martingale probability measure where $E(e^{-n^* l_t}) < \infty$. Hence, the expected value of $f(l_t, t)$ given l with respect to the equivalent martingale probability measure is

$$E^*\left\{f(l_t, t) \middle| l_0 = l\right\} = \frac{E[f(l_t, t) \cdot e^{-n^* l_t} \,|\, l_0 = l]}{E(e^{-n^* l_t} \,|\, l_0 = l)}. \tag{4.7}$$

Since the denominator in (3.7) is a martingale, it becomes

$$E^*\left\{f(l_t, t) \middle| l_0 = l\right\} = \frac{f(l, 0) \cdot e^{-n^* l} + \int_0^t E[\mathrm{A}\ f(l_s, s) \cdot e^{-n^* l_s} \,|\, l_0 = l] ds}{e^{-n^* l}}. \tag{4.8}$$

Set (4.8) in (4.6) then

$$\mathrm{A}^*\ f(l, 0) = \frac{1}{e^{-n^* l}} \lim_{t \downarrow 0} \int_0^t \frac{E[\mathrm{A}\ f(l_s, s) \cdot e^{-n^* l_s} \,|\, l_0 = l] ds}{t}. \tag{4.9}$$

Therefore, from Dynkin's formula (Øksendal, 1992) (4.5) follows immediately.

□

Let us examine the generator A^* of the process (l_t, M_t, t) acting on a function $f(l, m, t)$ with respect to the equivalent martingale probability measure.

Theorem 4.4 *Consider constants y^* and g^* such that $y^* \geq 1$ and $g^* \leq 0$. Suppose that $\hat{g}(g^* e^{dt}) < \infty$. Then*

$$\mathrm{A}^*\ f(l, m, t) = \frac{¶ f}{¶ t} - d l \frac{¶ f}{¶ l} + r^*(t)[\int_0^\infty f(l + y, m + 1, t) dG^*(y; t) - f(l, m, t)] \tag{4.10}$$

where $r^*(t) = y^* r\, \hat{g}(g^* e^{dt})$ *and* $dG^*(y; t) = \dfrac{\exp(-g^* e^{dt} y) dG(y)}{\hat{g}(g^* e^{dt})}$.

<u>Proof</u>

From theorem 4.2, we can use

$$\frac{y^{*M_t}\exp\left(-g^{*} l_t e^{dt}\right)\exp[r\int_0^t\{1-y^{*}\hat{g}(g^{*}e^{ds})\}ds]}{E[y^{*M_t}\exp\left(-g^{*} l_t e^{dt}\right)\exp[r\int_0^t\{1-y^{*}\hat{g}(g^{*}e^{ds})\}ds]]} \qquad (4.11)$$

as the Radon-Nikodym derivative to define an equivalent martingale probability measure. Therefore from lemma 4.3,

$$\mathrm{A}^{*} f(l_t, M_t, t) = \frac{\mathrm{A}\, f(l_t, M_t, t) y^{*M_t}\exp\left(-g^{*} l_t e^{dt}\right)\exp[r\int_0^t\{1-y^{*}\hat{g}(g^{*}e^{ds})\}ds]}{E[y^{*M_t}\exp\left(-g^{*} l_t e^{dt}\right)\exp[r\int_0^t\{1-y^{*}\hat{g}(g^{*}e^{ds})\}ds]]}.$$

From (4.2), using the generator with respect to the original probability measure,

$$\mathrm{A}\, f(l_t, M_t, t) y^{*M_t}\exp\left(-g^{*} l_t e^{dt}\right)\exp[r\int_0^t\{1-y^{*}\hat{g}(g^{*}e^{ds})\}ds]$$

$$=\frac{\P f}{\P t} - d l\frac{\P f}{\P l} + r[y^{*}\int_0^{\infty} f(l+y, m+1, t)dG^{*}(y;t)\exp(-g^{*}e^{dt}y)dG(y) - y^{*}\hat{g}(g^{*}e^{ds}) f(l, m, t)]$$

$$\cdot\; y^{*M_t}\exp\left(-g^{*} l_t e^{dt}\right)\exp[r\int_0^t\{1-y^{*}\hat{g}(g^{*}e^{ds})\}ds].$$

Therefore

$$\mathrm{A}^{*} f(l, m, t) = \frac{\P f}{\P t} - d l\frac{\P f}{\P l} + r^{*}(t)[\int_0^{\infty} f(l+y, m+1, t)dG^{*}(y;t) - f(l, m, t)] \qquad (4.12)$$

where $r^{*}(t) = y^{*} r\,\hat{g}(g^{*}e^{dt})$ and $dG^{*}(y;t) = \dfrac{\exp(-g^{*}e^{dt}y)dG(y)}{\hat{g}(g^{*}e^{dt})}$.

□

Theorem 4.4 yields the following:

(i) The rate of jump arrival r has changed to $r^{*}(t) = y^{*} r\,\hat{g}(g^{*}e^{dt})$ (it now depends on time);

(ii) The jump size measure $dG(y)$ has changed to $dG^{*}(y;t) = \dfrac{\exp(-g^{*}e^{dt}y)dG(y)}{\hat{g}(g^{*}e^{dt})}$ (it now depends on time).

In other words, the risk-neutral Esscher measure is the measure with respect to which l_t becomes the shot noise process with three parameters of d, $r^*(t) = y^* r \hat{g}(g^* e^{dt})$, $dG^*(y;t) = \frac{e^{-g^* e^{dt} y} dG(y)}{\hat{g}(g^* e^{dt})}$.

5. The Laplace transform of the distribution of accumulated aggregated claims with respect to the Esscher measure

Now let us examine the Laplace transform of the distribution of accumulated aggregated claims, with respect to the Esscher measure. This can be achieved by using an equivalent martingale probability measure, P^*, within the Laplace transform obtained in section 3. Therefore, from (3.7), the Laplace transform of the distribution of accumulated aggregated claims, with respect to the Esscher measure is

$$\exp\left[-\int_0^t r^*(s)\{1-\hat{h}(n e^{ds};s)\}ds\right]. \tag{5.1}$$

We will assume that the jump size distribution is exponential, i.e. $h(x) = a e^{-ax}$, $x > 0$, $a > 0$, i.e. we can obtain that $h^*(x;t) = (a + g^* e^{-dt}) e^{-(a+g^* e^{-dt})x}$, $x > 0$, $-a e^{dt} < g^* \leq 0$ since $dH^*(x;t) = \frac{\exp(-g^* e^{-dt} x) dH(x)}{\hat{h}(g^* e^{-dt})}$. Therefore, from (3.9) and theorem 4.4, assuming that the jump size distribution is exponential, the Laplace transform of the distribution of accumulated aggregated claims, with respect to the Esscher measure, becomes

$$\left(\frac{g^* + a e^{dt}}{g^* + a}\right)^{-\frac{y^* r}{d}} \left(\frac{g^* + n e^{dt} + a}{g^* + (n+a) e^{dt}}\right)^{-\frac{y^* r}{d}} \tag{5.2}$$

where all symbols have previously been defined.

If we differentiate (5.2) with respect to n and put $n = 0$, we can obtain the mean of the accumulated aggregated claims, with respect to the Esscher measure, i.e.

$$E^*(L_t) = y^* \frac{a r}{(a + g^* e^{-dt})(a + g^*)} \bar{s}_{\overline{t|}} \tag{5.3}$$

and the higher moments can be obtained by differentiating it further. If we multiply e^{-dt} both sides in (5.3), we can also obtain

$$E^{*}\left(L_{0}\right)=y^{*}\frac{a\,r}{(a+g^{*}e^{-d t})(a+g^{*})}\;\bar{a}_{\overline{t|}} \qquad (5.4)$$

which is the mean of the discounted aggregated claims, with respect to the Esscher measure.

Harrison & Kreps (1979) and Harrison & Pliska (1981) launched the approach for the pricing and analysis of movements of the financial derivatives whose prices are determined by the price of the underlying assets. Their mathematical framework originates from the idea of risk-neutral, or non-arbitrage, valuation of Cox & Ross (1976). Sondermann (1991) introduced the non-arbitrage approach for the pricing of reinsurance contracts. He proved that if there is no arbitrage opportunities in the market, reinsurance premiums are calculated by the expectation of their value at maturity with respect to a new probability measure and not with respect to the original probability measure. This new probability measure is called the equivalent martingale probability measure. Dassios & Jang (1998, 2002) and Jang (1998, 2000) also employed this non-arbitrage pricing technique for catastrophe reinsurance & derivatives.

The existence of an equivalent martingale probability measure is equivalent to the assumption of no arbitrage opportunities in the market. Therefore if we assume the market, i.e. by defining the insurance strategy, the equation (5.4) can be easily justified as an arbitrage-free insurance premium. However, as we can calculate a family of arbitrage-free premiums by changing the value of y^{*} and g^{*}, we have not achieved the complete market. The insurance companies' attitude towards risk determines which equivalent martingale probability measure should be used. The attractive thing about the Esscher transform is that it provides us with at least one equivalent martingale probability measure in incomplete market situations. For details, we refer you Dassios & Jang (1998, 2002) and Jang (1998, 2000).

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