Behavioural Finance implications in portfolio construction

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Abstract

The paper outlines some thoughts on portfolio construction where investors have asymmetric risk tolerance, with steeper downside risk intolerance. The paper presents the implications of this understanding on indifference curves by mapping Kahneman & Tversky's "Value' function as a set of 'Value indifference curves', and identifies an implication that portfolio optimisation may consider downside insurance. The paper goes on to discuss the appropriate time horizon of the portfolio given investor indifference to time, and given indeterminate cashflows.

Finally the paper outlines the process of risk budgeting, and again poses the question as to whether portfolio optimisation must consider asymmetric investor risk tolerance.

Introduction

Portfolio construction has developed significantly over the last decades, utilising the enormous advancements in computing power, and applying many of the lessons from portfolio theory.

Portfolio construction is ultimately based on the premise that risk can be diversified by adding in assets that are less than perfectly correlated, allowing the portfolio to achieve a better return per unit risk undertaken. From an investor's perspective, portfolios are to be constructed taking into account each investor's risk return preferences, with the optimal portfolio resting on the efficient frontier. With these two, sometimes competing objectives, and with the extensive assumptions in each, it's no wonder then that portfolio construction is sometimes seen as a black art.

More recently advances in behavioural finance have developed a better understanding of investor preferences and provided insight into the way investors make their decisions. For example, investor preferences on risk are asymmetric, with preferences dependant whether the risk results in a loss, or a gain. "Losses loom larger than gains" implying that volatility on the downside has a greater impact on investors, than volatility on the upside.

Furthermore, investors are seemingly unable to contemplate accurately differences in portfolio time horizons where they are greater than a year. So as an example, an investor finds it difficult to discriminate accurately between a portfolio that has a 5 year time horizon, and a portfolio that has a 7 year time horizon.

Finally, advances in understanding the risk characteristics of assets other than equities have started to allow portfolio managers to have more rigour in understanding portfolio risk characteristics. It is only recently that assets, other than equities, could even be contemplated as part of the principles behind modern portfolio theory (MPT).

These advances have significant implications for the way Superannuation Funds construct portfolios for their members, and it is likely that we will see a significant change in portfolio design in the coming years.

This paper builds on work completed on utilities (Livanas 2006 B and Livanas 2006 C) presents an argument for constructing portfolios to maximize utility for the investor rather than the fund manager.

What investors want…an asymmetric portfolio

Modern Portfolio Theory provided the environment that allowed the variance in asset prices to be used as a proxy for risk. This elegantly simple approach allowed a readily available set of tools to be applied to modelling portfolio characteristics and allowed a rigour to now be brought into the process. Using the statistical methods of determining correlation coefficients, equities could be analysed and the covariance of the relevant equities included in the portfolio would reduce the riskiness of the portfolio.

So, for example, if the distribution of the expected price of a security was expected to be normal, this security's expected mean and variance would completely describe its risk-return characteristic. Correspondingly, the expected mean, variance and covariance of two equities in a two-equity portfolio would determine the expected risk-return characteristics of the portfolio. Risk in this context is merely accorded the proxy of the expected distribution of returns, either side of the mean.

Interestingly, MPT does not explicitly talk to the variability of returns *over a period of time.* Rather, risk in this context is explicitly defined as the variation in the expected returns for a *single* period. Each period expected returns would need to be re-assessed.

The Optimal Portfolio

Now, assuming then that the risk-return characteristics of a portfolio are known, the theory goes further to derive an efficient frontier on which portfolios with risk-return criteria would have optimised returns. A portfolio can vary its mean and variance by varying the proportions of equities held in the portfolio. Optimising return per unit risk is called mean-variance optimisation, and the set of values for the optima of risk and return, forms the efficient frontier for the portfolio. Increases in returns diminish with increasing risk as shown in Figure 1 below.

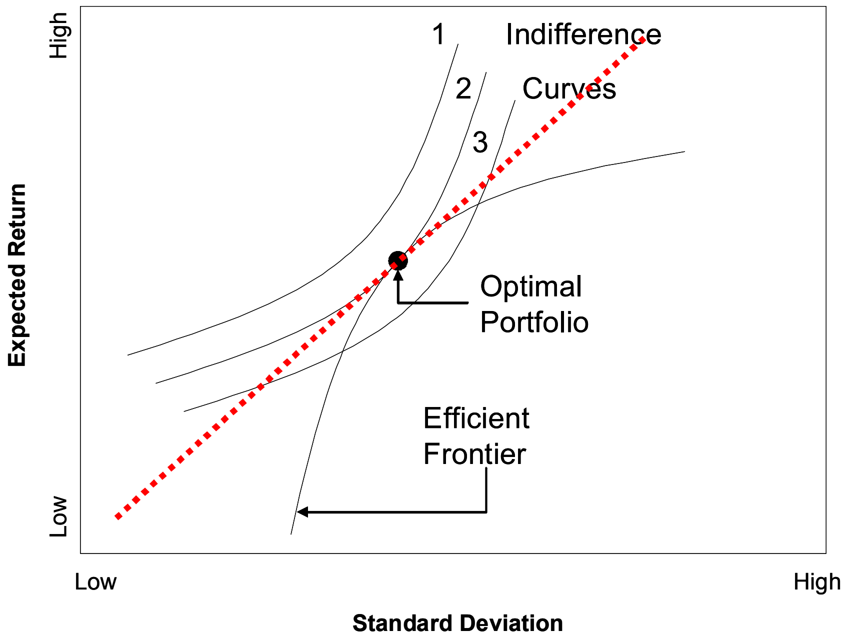

Figure 1: Efficient Frontier and Investor Indifference Curves

Investors, in turn, can be imagined to require a higher return per additional unit risk that they take. This is the ‘law of diminishing utility’ whereby any additional unit of risk taken requires a proportionately larger unit of return. Based on this argument, a set of indifference curves define the investors’ willingness to take additional risk per unit return. The intersection of each investor’s indifference curve and the efficient frontier results defines the ‘optimal portfolio’ along the tangent marking the market line.

However, ask investors what they think of losing money, and they’ll tell you that they characterise the effect of a loss as significantly different to the effect of a gain. “Losses loom larger than gains.” Determining investor’s asymmetry to gains versus losses was one of the key components of ‘Prospect Theory’ developed by psychologists Kahneman and Tversky, for which they won the Nobel Prize for Economics. This theory derived a value function that demonstrated this asymmetry as shown in Figure 2 below. (The value curve has been rotated about the axis to correspond with the more usual axis naming convention for risk-return plots).

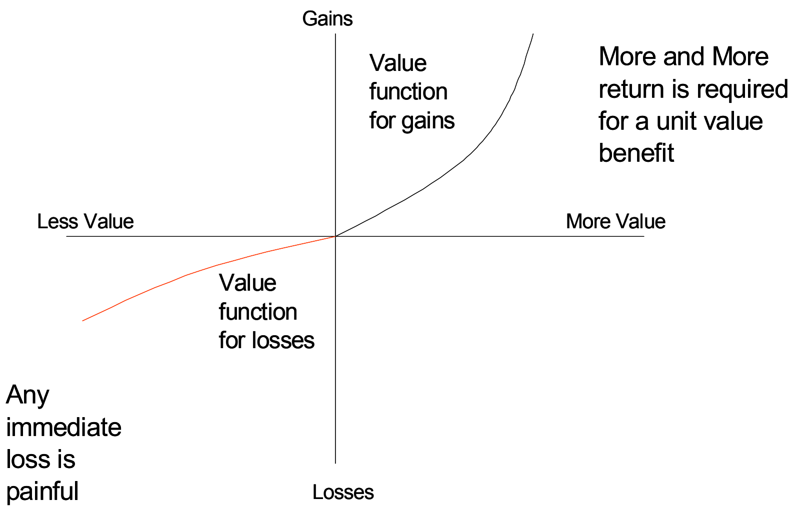

Figure 2: Investor Values (or mapped utilities) of expected returns as described by Kahneman and Tversky in ‘Prospect Theory’.

So, if investors are asymmetric in their response to gains versus loss taking, what are the implications for portfolio construction? Surely then investors’ indifference curves are expected to vary according to Expected Gains (E(G)) in contrast to Expected Losses (E(L)), with implications for the location of the optimal portfolio.

Deriving a set of indifference curves based on the ‘value’ function, requires mapping the ‘value’ function onto the risk-return plane. As discussed, Gains and Losses may be substituted with Expected Returns (positive or negative) as a transform.

$$E(R) = f\,\{E(G);\, E(L)\}$$

While it can be argued that Gains and Losses may not necessarily be linearly related to Expected Returns, it can be contemplated that the mapping of the utility function onto a value curve may be able to take into account any non linear relationships.

Risk (or variance as a proxy) is more difficult to map from the value function. However assuming we are only interested in the efficient frontier as presented in Figure 1, each Expected Return can derive an Expected Risk value for any given 'Value' point.

Consequently Kahneman and Tversky's 'Value' function maps (in a stylised manner) onto the risk-return plane with 'Value' indifference curves as shown in Figure 3 below:

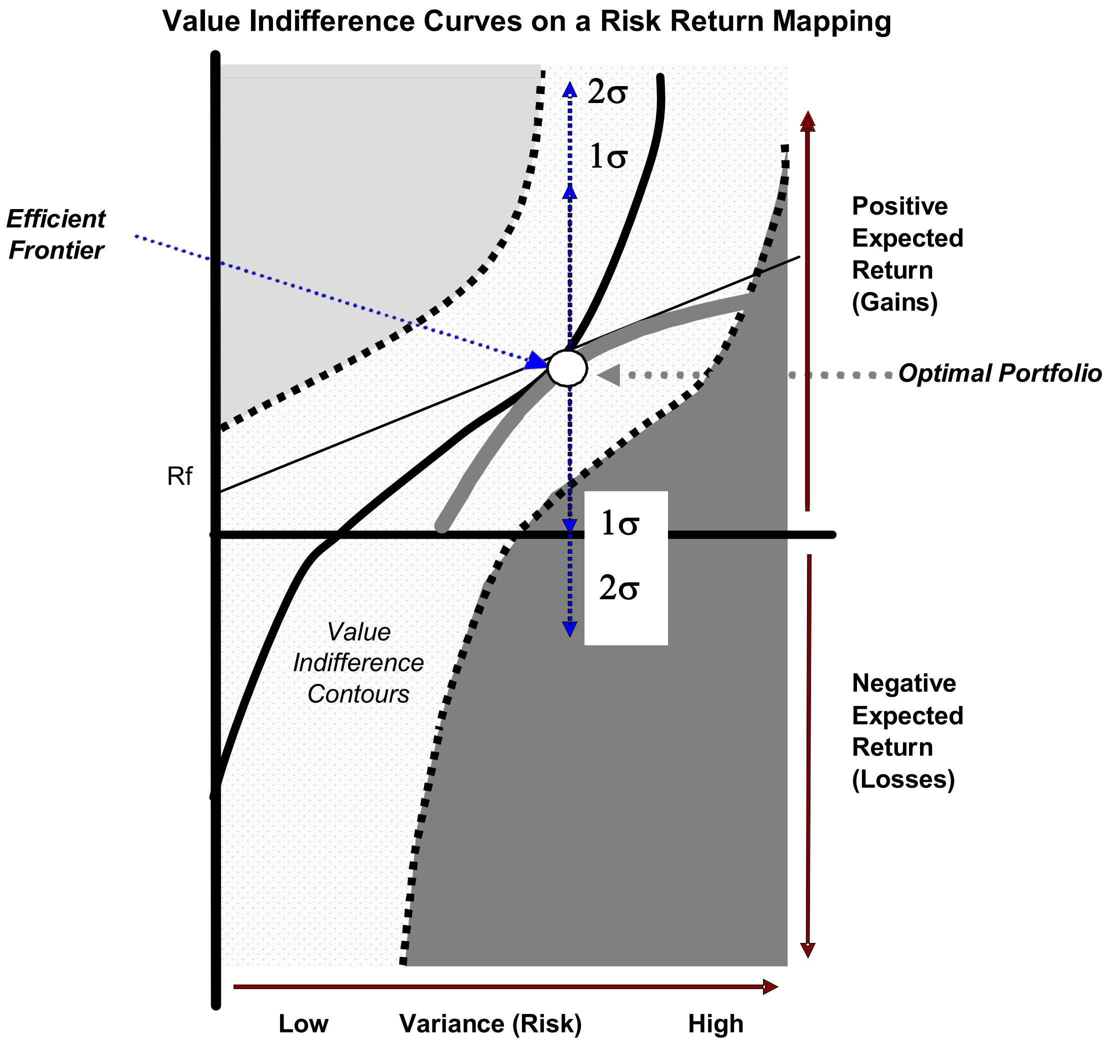

Figure 3: Investor indifference curves modified for losses and gains

This figure shows that the optimal portfolio can only exist above a minimum return line, given that one will not knowingly create a portfolio with a return less than the risk free rate. The intersection between the efficient frontier and the indifference curve is the optimal portfolio.

The 'Value' indifference curves have a point of inflexion at the zero (or minimum acceptable) return line. Investor's (dis) utility behaves differently in the domain of an Expected Negative Return (Loss). The implications are quite interesting. While the optimal portfolio can exist on the efficient frontier, assuming that the expected returns of this portfolio are normally distributed, the effect of a 1 standard deviation dispersion of outcomes will have very different impact in 'Value' to an investor when this outcome results in an increased gain instead of a loss.

Practically an optimal portfolio would be optimised only when a one standard deviation(1σ) results in an equivalent gain (or loss) of value, leading the argument that hedging of returns or portfolio insurance may derive more value than thought.

The aggregate value of the optimal portfolio is:

$$V= \prod^{N} \{V_g \, E(G) - V_l \, (E(L)\}$$

Where the Value of gains is V_g and the Value of losses is V_l. The loss function is steeper than the function for gains, and at a 1σ dispersion, the value of gains has a smaller value than the equivalent value of a loss.

Portfolio Insurance

Portfolio insurance has the effect of shifting the optimal portfolio vertically as shown in Figure 4. This will have the effect of truncating the potential dispersion of losses, but also shifting the dispersion of possible gains and the mean:

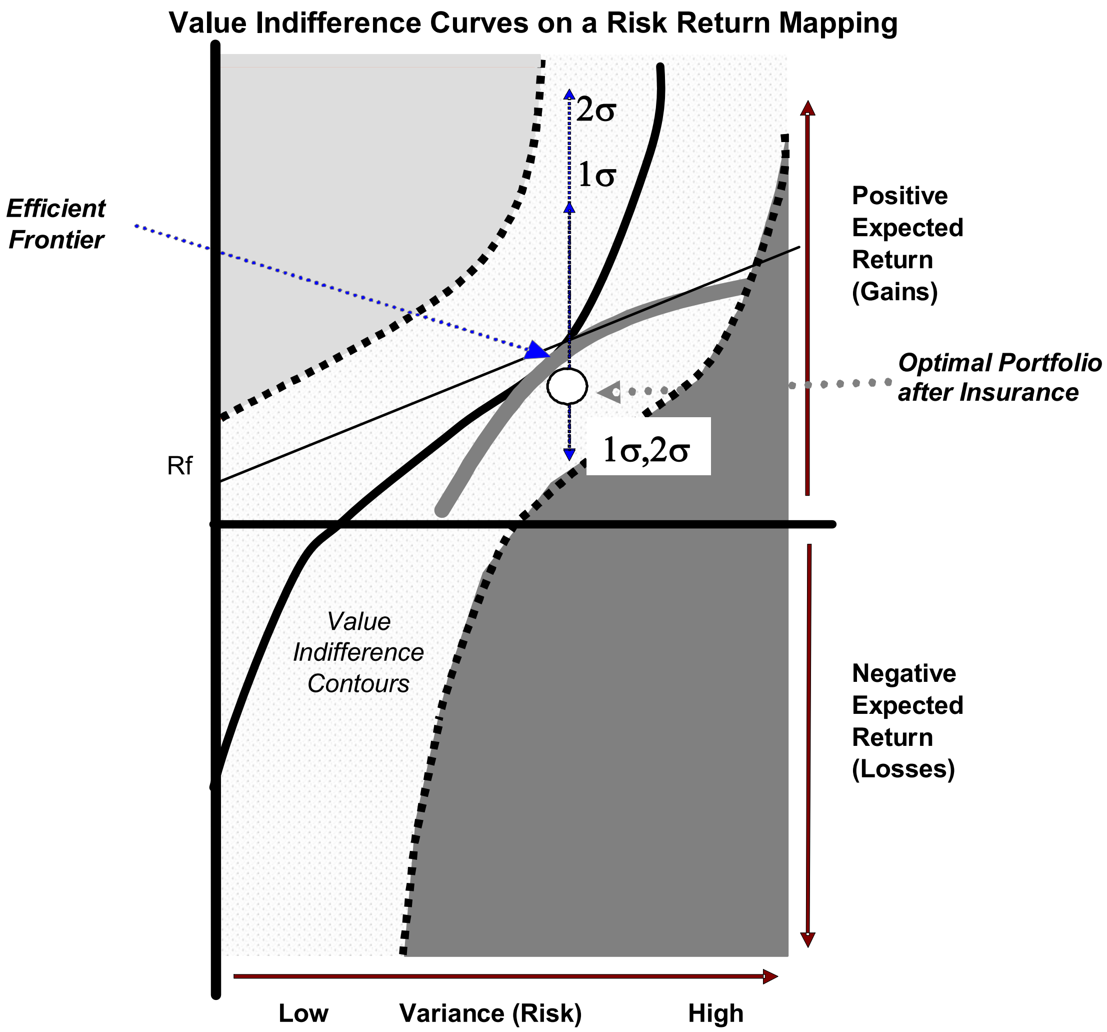

Figure 4: The dispersion of returns compresses.

The dispersion of potential returns is shifted for gains, but the effect of the truncation of losses, and the associated greater (dis)utility avoided means that the overall utility of this portfolio to the investor is actually increased.

The aggregate value of the optimal portfolio with insurance is:

$$V= \prod^{N-i} \{V_g\, E(G) - V_l\, [(E(L)-E(L)^{N-j})]\}^{j}$$

Where the extent of the dispersion is limited to N-j, and the insurance truncates the losses at j.

So, perhaps conventional wisdom is correct after all. The optimal portfolio when aggregated for the dispersion of returns in a standard deviation sense, actually provides greater potential value when insurance truncates losses.

Time, hyperbolic discounting and utilities

Livanas 2006 B shows that investor utility does not vary as a function of the time horizon of portfolios that have time horizons between 1 year and 10 years. This flies in the face of the dispersion of variance over time, with the conventional wisdom being that variance compresses as a function to $1/\sqrt{h}$, where h is the time horizon of the portfolio.(Of course the assumptions of this are that the distribution of returns over time is consistent, implying a static portfolio at the least!)

However, the evidence presented by Livanas is entirely consistent with other work concluded on investor perceptions of the time value of money. Utkus (2006) highlights that investor discounting follows a hyperbolic function with a limit at some specific value, in contrast to the power function of the normal time value of money formulas. Figure 5: below highlights this:

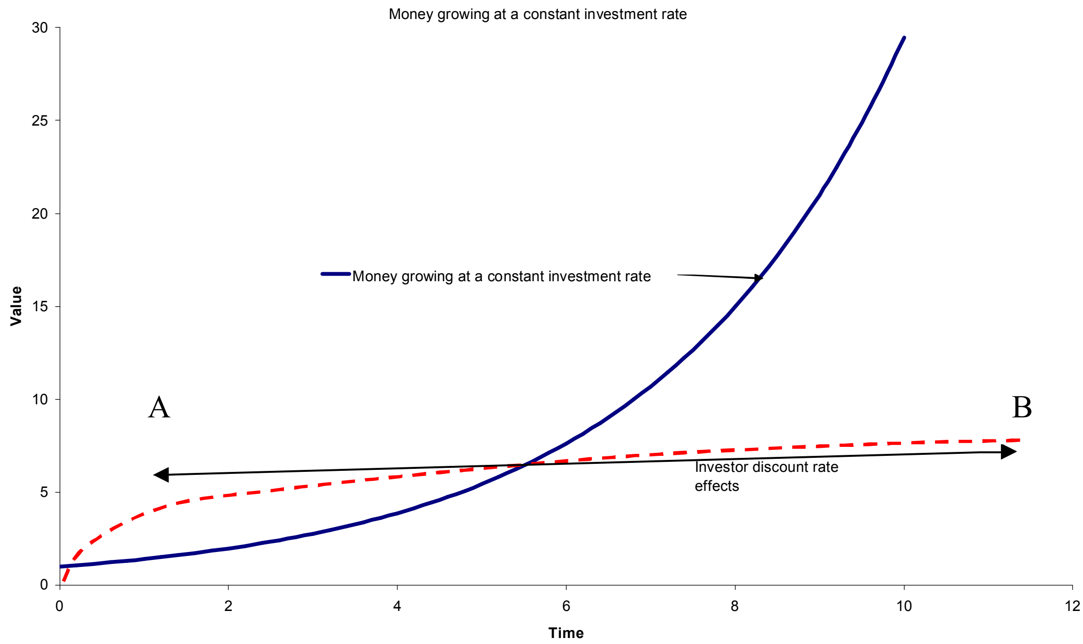

Figure 5: Time value of money and investor perceptions

Between periods A and B, it can be argued that investor inability to discriminate between the differing utilities of portfolios with different time horizons is consistent with hyperbolic discounting.

In deriving the utility curves for investors, Livanas (2006C) excluded time horizon as a factor in determining the risk-return dimensions. To include time horizon, the derivation would need to assume that risk was constant for all time horizons. That is, that the utility curves derived from investors for portfolios of different time horizons can only be consistent if risk doesn't alter with time horizon.

Samuelson (1963) argued strongly that the law of large numbers operates at a point in time, and is in fact temporally indifferent. Markowitz also started his Modern Portfolio Theory derivation by avoiding time as a factor, preferring to look at 'flow of returns', much like this paper looks at dispersion of returns at any one point.For the purposes of this paper however, it is sufficient to accept the very well documented principles of hyperbolic discounting, and consider these for the purposes of portfolio construction.

So, if investor utility is not affected by portfolios of longer or shorter time horizons, how should portfolios be constructed?

Matching time horizon of the portfolio to the time horizon of the investor

The fundamental principle of a portfolio for an investor is to allow the investor to diversify across a range of less than perfectly correlated assets. Practically, investors will invest while capital is surplus, and withdraw funds when capital is required. Investors risk profile, or propensity to risk is relevant only for the period between those events. Investors' risk profile may change over time, but that requires a re-assessment of the riskiness of the portfolio at that time. To presume that to alter the risk profile of the portfolio within the period of investment, without the assessment of the investor risk profile, is a position that cannot be supported. Therefore lifecycle style investment can only be supported if there is evidence that the risk profile of the investor alters over the lifecycle, and cannot be seen as a way of optimising portfolios.

So, if the time horizon is only relevant to the extent of the investment cycle of the investor, the only valuable calculation is to derive the behaviour of the portfolio over this period alone. Sharpe (2001) concludes similarly, that the relevant time horizon of a portfolio for a pension fund is likely to be measured in "….years if not decades." Whereas financial institutions concerned with short-term values, may have a "…(time) horizon over which risk is calculated…typically measured in days rather than weeks, months, or years."

Churn, and implications for portfolio design

Two complexities arise however when portfolio design is expanded to include multiple investors in a pooled portfolio.

Firstly, the individual time horizon for individual investors will in itself be dispersed. Investors capacity and timing to invest, and requirements to be paid out, will vary significantly amongst investors, dependant on each investor's individual circumstance. In addition, the investor's investment into a pooled portfolio is often one of a number of investments with differing time horizons.

Furthermore, Livanas (2006B) indicated that investors often do not discriminate based on the time horizon of the pooled portfolio in selecting portfolio options.

Secondly, the mean time horizon of the portfolio itself, will also vary, dependant on money flows from investors, timing of investment income, payment of tax, fees and other costs, etc.

As a result, selecting the appropriate time horizon for a pooled portfolio is often complex. (Paradoxically, the time horizon of a defined benefit fund is often more predictable, with constraints on timing of the cashflows often imposed by the trust deed). Consequently any portfolio design is a compromise, and to talk about optimising the time horizon for a pool of investors is not defendable.

Techniques to determine the appropriate time horizon of a pooled portfolio could be borrowed from Bond Theory, and it might be useful to use the Macaulay Duration calculation to derive the time horizon of the portfolio.

Using the calculation for the Macaulay duration, discounting the expected cashflows with the risk free rate, the Expected time horizon of the portfolio can be modelled as follows:

$$\hat{E}(H_p) = \text{MEAN} \left\{ \sum_{t=1}^{N} [1/(1+R_f)^t * (E(FI_t) - E(FO_t)) * t] \right\}$$

Where $\hat{E}(H_p)$ is the expected mean time horizon of the expected net flows of the portfolio, $E(FI_t)$ and $E(FO_t)$ are the times of expected funds inflows and outflows at time t, over period N, and R_f is the risk free rate.

Therefore, the only defendable method of developing a perspective of time horizon for a portfolio is to model the flows of assets into and out of the portfolio. In superannuation with significantly positive flows, the time horizon of the portfolio can in fact be decades.

This has very significant implications for the asset classes selected and the modelling of the portfolio characteristics.

Risk Budgeting

The preceding sections discussed the implications of including portfolio risk where the investor 'Value' function was considered, and the consideration of the appropriate time horizon in constructing portfolios.

The remaining section will deal with portfolio construction given the risk characteristics of different asset classes. Modern Portfolio Theory (MPT) defines the Market Portfolio, as the portfolio of all assets. This portfolio contains the broadest diversification of risk possible.

Sidestepping the relevant merits of the Market portfolio, any portfolio created from a population of investible assets is an optimisation under constraint, and risk budgeting is a technique to allocate these assets, given the dearth of tools to derive reliable covariances between every possible asset in a portfolio set.

Building a portfolio

Most large pension funds build a portfolio based on a two step approach, with the first step being the allocation of assets. Given that 'asset allocation accounts for well over 90% of the risk.." (Sharpe (2001)), this element of building a portfolio is by far the most important.

Given the many possible investment options, and the complexity in estimating risks and correlations for these options, the standard premise of Risk Budgeting is to utilise some form of a factor model to model the performance of the option, including the shape of the possible probability distributions.

An efficient portfolio minimises variance for a given level of expected return. This creates the efficient frontier.

Generally, standard one period mean-variance optimisation techniques are used to maximise the Expected Utility of the portfolio (EU) with the task to maximise the Excess Returns (EER_p) of each of the asset classes, while minimising the portfolio variance V_p of the excess returns.

"Maximize: $EU = EER_p - V_p / rt$

Subject to: $\sum_i X_i = 1$

where EER_p and V_p are respectively the expected value and variance of portfolio excess return." (Sharpe 2001). X_i is the proportion of the ith asset in the portfolio, and rt is the 'Risk Tolerance'

Investor's Asymmetric Risk Tolerance

Based on the above problem, the marginal EU (MEU) of the portfolio, can be shown to be a function purely of the Risk Tolerance of the investor : MEU = f(rt)[1]

However the investor's risk tolerance is asymmetric, therefore the more accurate representation is to define risk tolerance merely as the risk tolerance towards a return below a determinable minimum. In practice this is likely to be the measure used, however in recognising that only the Risk Tolerance with respect to a negative outcome is relevant, and that this risk tolerance is otherwise asymmetric, the language used in discussing the process is a more accurate reflection of the process.

Summary

The preceding paper outlines some thoughts on portfolio construction where investors have asymmetric risk tolerance, with steeper downside risk intolerance.

The paper presents the implications of this understanding on indifference curves by mapping Kahneman & Tversky's "Value' function as a set of 'Value indifference curves', and identifies an implication that portfolio optimisation may consider downside insurance. The paper goes on to discuss the appropriate time horizon of the portfolio given investor indifference to time, and given indeterminate cashflows.

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End Notes

[1] The proof of this can be seen in Sharpe (2001) as follows:
"Consider the *marginal expected utility* (MEU) of a position in a portfolio, defined as the rate of change of expected utility (EU) per unit change in the amount invested in that position. This will equal:

$$MEU_i = \delta EU/\delta X_i = \delta EER_p/\delta X_i - \delta V_p/\delta X_i) / rt$$

But under the assumption that the expected return of an asset is the same regardless of the amount invested in it, the derivative of EER_p with respect to X_i will equal EER_i Moreover, the derivative of V_p with respect to X_i is the value that we have defined as the manager's marginal risk, MR_i. So we can write:

$$MEU_i = EER_i - MR_i / rt$$

Imagine a portfolio in which the marginal expected utilities of two managers differ. Clearly the portfolio is not optimal. Why? Because one could take some money away from the manager with the lower MEU and give it to the manager with the higher MEU, thereby increasing the expected utility of the portfolio. It thus follows that a condition for portfolio optimality in the absence of binding constraints is that:

$$MEU_i = k, \quad \text{for all } i$$

where k, is a constant"

Following from this, where Vp and EERp are variables of the asset, the constant relates entirely to the Risk Tolerance (rt) of the individual, therefore MEU = f(rt)