

It's a Utility! But not as we know it!
19th Banking and Finance Conference
UNSW



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Abstract

Utility, or happiness of consumption, provides the cornerstone of many models underpinning investor interactions in the market. More recently however, contemporary thinking is recognising that investor utility may be, at times, significantly different to many of the models. The very broad and new field of behavioural finance has started to offer a more individualised model of utility or value.

This paper expands the discussion of utility, originally presented as part of paper (Livanas (2006B)) on Investor Behaviour, presented at the 14th Colloquium of Superannuation Researchers, at the UNSW. In Livanas (2006B), as part of a wider study, superannuation investors were surveyed regarding their relative utilities of risk, returns and time horizon (of deferred consumption). The survey was constructed using partial profile choice tasks to assess utilities of investors over three attribute groups, with a sample size of 236 investors, drawn from Superannuation Investors.

This paper traces the development of utility and outlines some of the current thinking, before expanding empirical findings of investor utilities of risk, return and time horizon (of deferred consumption), presenting a quantitative expression of investor utilities developed though the partial choice task tradeoffs, that largely fit theory, except in one very important area: Investors' utility of time horizon seemed to be static, implying that they did not form a discriminating factor in investor choice. Finally the paper discusses the implications of the findings regarding time horizon utility and the equity risk premium.

Introduction

Utility forms the cornerstone of economic theory. Utility is often described as the happiness that consumers receive in the consumption of a good or a service. Happiness is of course a relative notion. Aristotle in his *Ethics* (Thomson (1976)) is imputed to contend that: ‘..views of happiness differ’. That of course has been the challenge of modern economic modelling: To impute a course of action that will result in the ‘greatest happiness to the greatest number’ as described by the enduring Mr Bentham.

Bernoulli, in 1738 investigated the utility of wealth, contemplating that utility is not absolute, but that it increases at an ever decreasing rate with increases in wealth. This provides the basis for the concept of *diminishing marginal utility*. Bernoulli’s postulation is written in the form of ‘Expected Utility Theory’ or the probability of an outcome occurring, and the value of the outcome.

More recently, Kahneman and Tversky (2003) developed Prospect Theory to argue that utility is better represented in terms of gains and losses or changes in wealth (Asset Integration), and that utility was not symmetric, but that decision makers took progressively less risk per unit gain, and relatively more risk to avoid a unit loss. “...losses loom larger than gains” (Risk Aversion). Individual utility, according to Kahneman and Tversky, is a mapped and edited version of absolute utility.

So where does this leave us in our understanding of utility in general? More particularly, what does it mean for Financial Markets; where investment decisions are presumed to follow standardised theories of the allocation of capital; where greater returns are preferable to lesser, where investors are risk averse, and where money has a time dependent value?

This paper is an extension to the paper entitled: ‘Are investors rational and does it matter?’ presented at the 14th Colloquium of Superannuation researchers, on the 21st July 2006 (Livanas (2006B)). In Livanas (2006B) investors were surveyed regarding their relative utilities of risk, returns and time horizon (of deferred consumption). The survey was constructed using partial profile choice tasks to assess utilities of investors over three attribute groups, with a sample size of 236 investors, drawn from Superannuation Investors.

A key finding from this research was that investors’ utility to different time horizons of investments, was indeterminate. Investors did not show any difference in utility between portfolio choices with difference time horizons. This paper outlines the research approach, the research findings and for the first time, provides some quantitative measures of utility. It goes on to discuss the implications for portfolio construction and initiates a discussion of the implications for the ‘Equity Risk Premium’.

The approach outlined in the paper may be interesting enough to be repeated with a much wider sample to provide more rigorous data as to utilities.

The theory of Utilities of Risk, Return and Time

Much has been written about the benefits of holding onto risky investments over longer horizons in order to achieve investment objectives. Modern literature often contains the argument that investors should hold onto risky investments for longer in order to gain the benefit of an increased return that results from holding a riskier asset. Furthermore, investors earlier on in their working career are often advised to invest in riskier assets than they would later on in their career, irrespective of their risk profiles.

This philosophy is echoed in many Superannuation portfolios, with lifecycle investing options that successively reduce the 'riskiness' of the portfolios as the investor nears retirement age. It is also reflected in the descriptions of many portfolios where three dimensions are highlighted for investors: the target or objective return of the portfolio; the risk of a negative return; and the time horizon of the portfolio. The dimension of time horizon, in determining portfolio characteristics implies that the portfolio risk and return characteristics have been determined over this time horizon.

Risk and Return Indifference Curves

Utility for investors is commonly represented based on the conventional axiom that investors prefer more return to less; that investors are risk averse; and that the investor's marginal utility decreases with increasing wealth. Figure 1 below shows the conventional indifference curve diagram, with the utility received by an investor from trading off a unit of risk (standard deviation) for incremental E(U) or expected return, lying along an

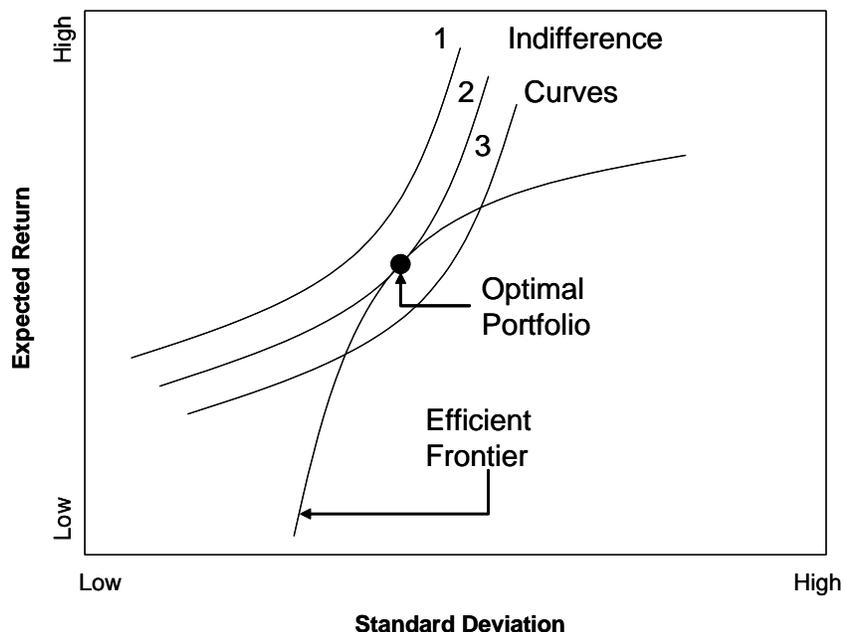


Figure 1: Conventional Indifference Curves and Optimal Portfolio Selection

indifference curve 1,2, or 3, with each curve representing an investor with a specific coefficient of risk aversion.

An increasingly higher return is demanded for taking on an increase in risk, as the investor moves along the indifference curve. Notably, the indifference curves operate on the basis of the marginal utility received by the investor for additional return, given additional risk – the first derivative of the utility curve. This concept of marginal utility is one of the underlying assumptions used in designing the partial choice tasks in the tradeoff tests

In Figure 1 then, the optimal portfolio chosen by an investor with a given risk aversion, is one that has a risk-return characteristic located at the tangent of the indifference curve for that particular investor.

Figure 1 demonstrates the characteristics of two of the dimensions of portfolio choice. A third dimension, time horizon, can be ascribed as the z-axis above the risk-return plane, with the indifference curves forming surface contours.

In order to plot the surface, the investors marginal utility curve with respect to time, must be determined. This is then the partial derivative of $E(U)$ with respect to time: $\frac{dE(U)}{dt}$

What are the implications of time horizon in a portfolio?

Time diversification is the idea of reducing risk by investing in a risky asset, over a longer period. Kritzman (2003) provides an excellent summary of the arguments for and against time diversification.

The argument for time diversification is based on the fact that, in a serially independent series of returns, the standard deviation of annualised returns diminishes over time. In constructing a portfolio p , assuming non serially correlated, normally distributed returns, with mean μ_p and standard deviation, σ_p the probability of a negative return from portfolio p , for a single period, can be calculated as follows:

$$\frac{T - \mu_p}{\sigma_p} = z \tag{1}$$

Where T is the target return (equal to zero where the investor is concerned to measure the chance of a negative return), and z returns the standard deviations below the mean. The probability of the average annualised return being less than T over period h (the time horizon) is represented as follows:

$$\frac{1}{\sqrt{h}} \frac{T - \mu_h}{\sigma_h} = z_a \tag{2}$$

Clearly, the probability of a negative annualised return diminishes as the time horizon lengthens, and approaches the mean. Equation 2 can readily be rewritten as the Sharpe ratio, by substituting T with the Risk Free rate. This neatly ties in with the discussion of

the 'Equity Risk Premium, with the notion that investors are rewarded for holding on to a more volatile security with a greater expected return (In exchange, to increase their chances of achieving this greater return, they would need to hold onto the security for longer). Without going into the debate as to the Equity Risk premium, on which countless papers have been written, including excellent ones by Thaler, there is a strong counter argument for the principle of time diversification, summarised brilliantly by Kritzman:

Kritzman (2003) raises some interesting debates regarding the principle of time diversification, including Samuelson's argument that although the chance of a negative return reduces, the magnitude increases, to the extent that the utility of investing in a risky asset is static for any time horizon. Kritzman also highlights Zvi Bodie's argument against time diversification, whereby using option pricing theory, Bodie contends that the price for insurance (a put option) will increase with time due to the dispersion of outcomes. This implies that an increased time horizon results in higher risk costs, as the price of the insurance (a 'put' option) increases.

The interesting outcome of this debate, is that time diversification, for a population of investors, will mean that the return experienced on average by each investor, for the risky investment, will always be more than that of the risk free asset. 'The greatest good for the greatest number' premise by Bentham. However for individual investors, there is a strong argument that time diversification doesn't result in increased utility.

Inferring Risk

The purpose of the survey, simply, was to develop an empirical model of risk, return and time horizon utilities, with which to revisit the theory, and with which to provide guidance into communicating with investors, developing appropriate investment and product portfolios and developing appropriate investment choices.

Investors have a fairly good understanding of the notion of Expected Return and of Time Horizon over which an investment would be evaluated. However, it is unrealistic to expect investors to have an understanding of risk in terms of variance or standard deviation. Rather, investors have an understanding of the *effects of risk in terms of the likelihood of a negative return, or of a return below a specific target.*

Consequently, the survey investigated investors' risk preferences in terms of 'chance of a negative return' with specific parameters. The transform of 'chance of negative returns' for specific return and time horizon dimensions using (2), solves for standard deviation as follows:

$$\sigma_h = \frac{T - \mu_h}{z_a \sqrt{h}} \quad (3)$$

Research Design and Methodology

Survey Design

Much of the research into investor behaviour relies on either historical data of investor activities, or on experimental observations of investors in a simulation. The results in this paper were compiled in a different manner. The study comprised investors who had recently executed an investment choice. Furthermore, this study did not rely on investors responding to a website, or direct mail. Rather, investors in the sample were telephoned directly by Woolcott Pty Ltd (an Australian market research organisation), and requested to participate as they would in normal market research questions.

A sample of over 4,000 investors formed the primary basis of the survey. These were investors who had made a decision in the preceding four years, regarding choice of investment. Investors had had the option to choose between five investment portfolios with standardised risk return characteristics. From this group, a group of 186 investors was ultimately surveyed. Additional data was collected regarding their investment activities and this data was used to compare actual investment activities to beliefs. Furthermore, an additional 50 investors were surveyed who had not made any investment choice in the last four years. This group acted as the control sample in the group.

The survey comprised several parts, with open ended questions, and multiple alternative questions used in developing investor preferences.

Partial Choice Tasks Tradeoffs

In order to derive a systematic approach to determining the relative utilities of risk, return and time horizon, choice-modelling was used to ensure a robust and quantitative method of research. A series of alternative investment options, with varying risk, return, and minimum time to achieve the objective of the investment, were presented to the survey participants as binary options. These investment options reflected the usual investments that the investor would have had to decide upon, in making their investment decision.

The attributes were chosen from groups as shown in Table 1 below:

Table 1 Groups of Attributes used in the Choice Task tradeoffs

Return	Risk of a Negative Return	Time Horizon / Maturity of Investment
3.9%	no chance	1 year
6.0 - 6.3%	13% chance	3 year
6.5 - 7.2%	20% chance	5 year
7.2 - 8.1%	25% chance	10 year
8.0 - 9.0%	33% chance	

As can be seen, there are 14 individual attributes that could be used in pair-wise modelling. In order to limit the imposition on the participants, the choice tasks were limited to partial sets of combinations, rotated throughout the survey, with investors asked to choose between two combinations each time.

The risk of a negative return was explained as the risk of an average annualised return over the time horizon, allowing the use of equation (3). The values of z used in transforming the values of risk of a negative return, and the imputed standard deviation, for the various time horizons, are as shown in Table 2 below, for a time horizon of 1. This can be readily repeated for $h=2$; 5 and 10:

Table 2 Expected Values of s for $h=1$

Risk of a Negative Return	Values of $E(R)$		$\sigma_h = \frac{T - \mu_h}{z_a \sqrt{h}}$			
	Values of Z	3.90%	6.15%	6.85%	7.65%	8.50%
no chance	-8.50	0%	1%	1%	1%	1%
13% chance	-1.13	3%	5%	6%	7%	8%
20% chance	-0.84	5%	7%	8%	9%	10%
25% chance	-0.67	6%	9%	10%	11%	13%
33% chance	-0.43	9%	14%	16%	18%	20%

Investor Utilities as derived from the Survey

The results of the partial choice task tradeoffs were aggregated to create a set of relative utilities for each attribute.

The analysis was able to establish how much more 'utility' is received by the sampled investors, for each of the attributes. For example, Utility of an expected annualised return of 15% could be compared to the utility of a 10% return. Utilities are arbitrarily zero centred and while a greater score in a specific attribute is better than a lesser score, one cannot compare utilities across categories. i.e. a level of utility for risk cannot be compared to a level of utility for return. Nevertheless, an investor's experience of a portfolio with a combination of attributes can be considered as an aggregate of the utilities generated by each of the attributed, each scaled according to some function.

The utilities thus derived are marginal utilities. Marginal utilities are derived from ordinal preference of the choice tasks, and hence have no units. The nomenclature to describe these marginal utilities will be as follows:

- $mE(R)$ – This is the marginal utility of a unit of Expected Annualised Return expressed as percentage per annum.
- $mE(Z)$ – This is the marginal utility of a unit of Expected Loss expressed as the Expected Probability of a negative annualised return over the time horizon
- $mE(H)$ – This is the marginal utility of the time horizon, expressed as the Expectation of the number of years an investor would need maintain his or her investment to be assured of the target return.

The marginal utilities describe expectations of each of the attributes. The survey respondents were chosen on the basis that most had recently made a decision to switch portfolios with varying attributes, and those were encouraged to recall their experiences in the survey. The control group had not made any investment decision.

The total marginal Expected Utility $E(U_p)$ of a specific portfolio p can be considered as:

$$mE(U_p) = f_1 mE(R_p) + f_2 mE(Z_p) + f_3 m(H_p) \quad (4)$$

The implications of the survey are significant for portfolio design, investment selection and choice communication. Furthermore, in total, the findings also allow an aggregate representation of investor risk, return and time horizon trade-offs. This will be explored later in the paper.

The outcomes of utilities for each of the Expected Return, Risk and Time Horizon are discussed below:

Marginal Utility of Expected Returns

The first outcome of the choice modelling was assessing the marginal utility of returns. Figure 2 below shows the utility curve for investors' preferences plotted against expected annualised investment returns.

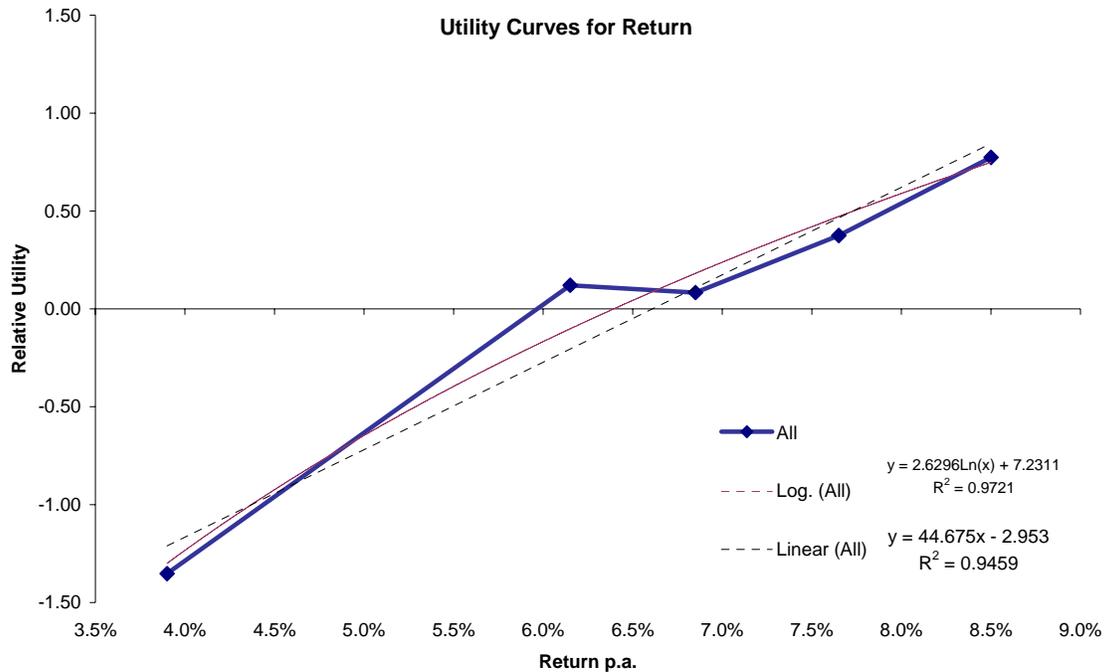


Figure 2 Utility curve for Return

As anticipated, the utility for return has a positively sloping utility function. A trendline fit using a log function yielded a slightly higher correlation than a linear function, but the difference in the correlations for the range of values studied do not allow sufficient confidence to draw any conclusions at this stage, other than the comfort that the representation from this sample was consistent with theory.

The pattern of utilities did not vary significantly by age group.

Marginal Utility of Expected Risk

The second outcome of the choice modelling was in assessing the marginal utility of expected risk to investors.

Risk was identified to respondents as the expectation of an average annualised negative return over the investment horizon. This risk was expressed as a probability over the time horizon.

Figure 3 below shows the marginal utility curve for investor preferences given expectations of risk, expressed as a percentage probability.

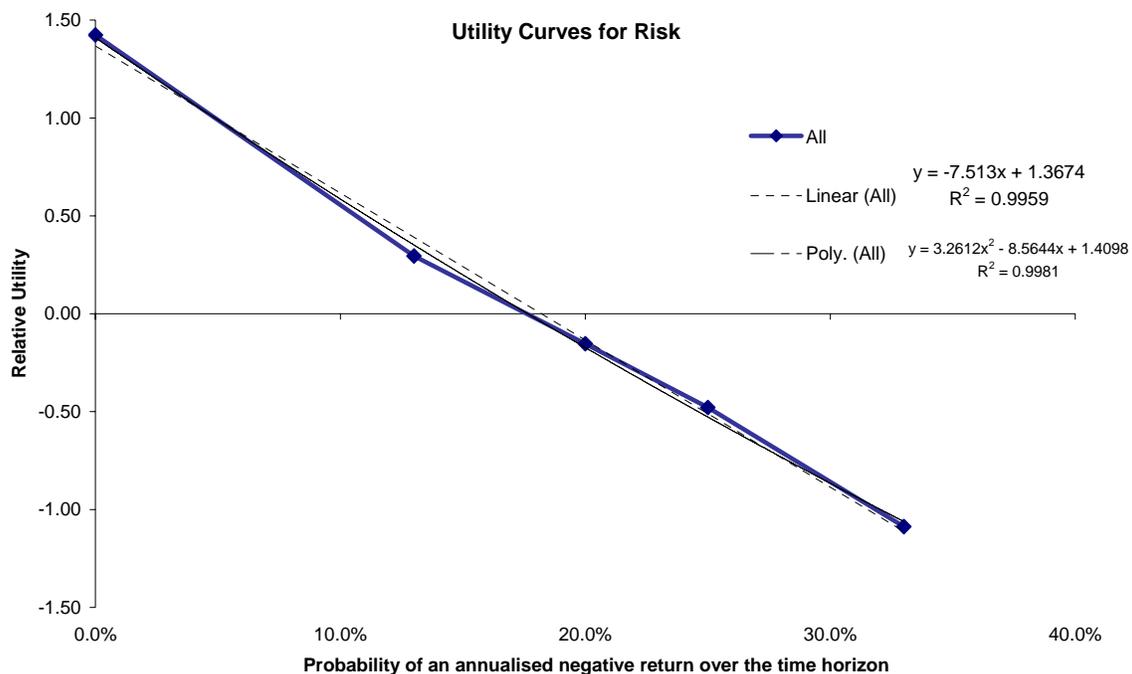


Figure 3 Utility curve for Risk

As expected, the marginal utility curve for risk is negative. Investors prefer less risk to more risk. The data is not sufficiently comprehensive to determine a best fit with a log trendline, although a quadratic function does provide a good correlation. The important finding is that there is a clearly negative slope as anticipated.

It must be noted again, that the measure of risk in this question is not standard deviation or variance. It would be highly unusual for investors to be able to accurately estimate expected standard deviation, and the marginal utility derived from this. Rather, asking investors to consider, say a 33% chance of an annualised negative return over the time horizon, provided a more consistent response. Table 3 shows the mapping of these probabilities, with the more usual measure of standard deviation, given an expected return and expected time horizon.

Marginal Utility of Expected Time Horizon

The third outcome of choice modelling was to assess marginal utility of different maturity profiles of investment options. This is the expected time horizon that the investor would need to maintain his or her investment, in order to achieve an expected return, given an expected risk profile. Portfolios will generally have a time horizon over which the risk-return parameters are constructed. Holding onto investments for longer, should increase the likelihood of a particular outcome being achieved, given an expected variance of returns.

Figure 3 below shows the utility curve for time horizon:

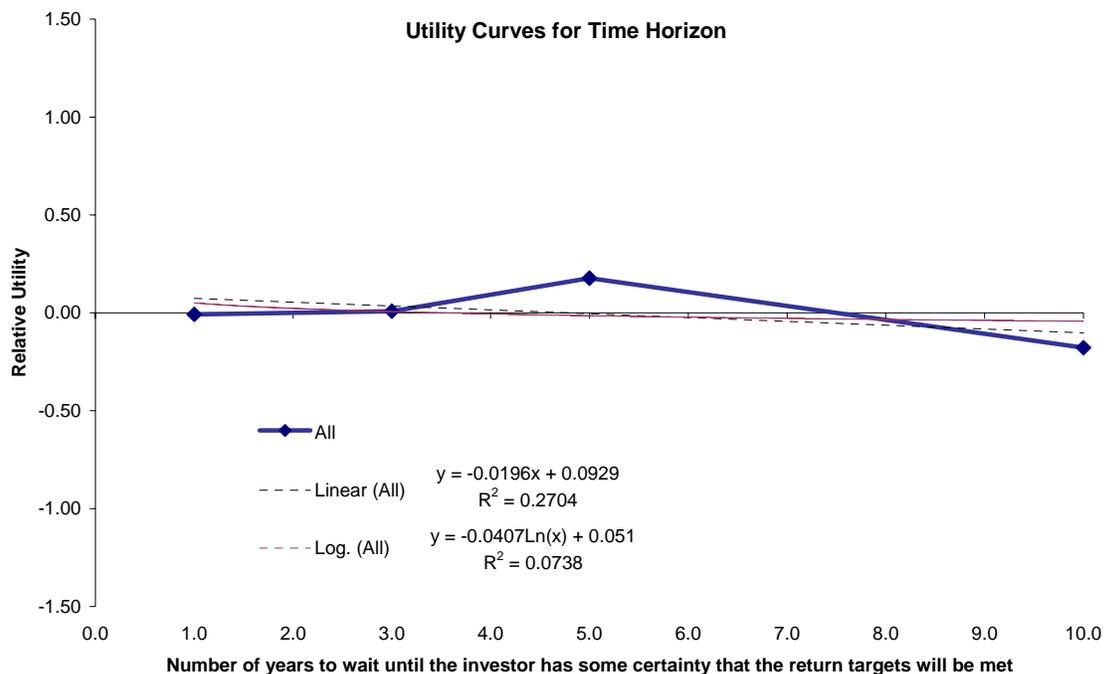


Figure 4 Utility curve for Time Horizon

Astonishingly there was no clear discernible relative utility for time horizon! Neither a linear or logarithmic line provided any meaningful correlation with the trend. Fundamentally, there was no clear correlation between the time horizon and utility, just as Samuelson would have you believe. A best fit for this utility could just as readily be a constant.

Investors were seemingly indifferent whether the maturity profile or objective time horizon of the portfolio was 1 year of 5 years or even 10 years!

This inability of investors to appreciate temporal aspects of investments has been discussed before, but this evidence provides unambiguous clues as to investor perceptions. It perhaps also supports the debate arguing that time diversification is illusory on an individual basis.

Error Terms

Table 3 below shows the error terms for Risk and Return using the quadratic and log functions as best fit.

Table 3 Error terms for Return and Risk

Marginal Utility of Expected Return				Marginal Utility of Expected Risk			
$y = 2.6296\ln(x) + 7.2311$				$y = 3.2612x^2 - 8.5644x + 1.4098$			
R2 = 0.9721				R2 = 0.9981			
Expected Return	Expected Value	Actual Value	Arithmetic Error	Expected Risk	Expected Value	Actual Value	Arithmetic Error
3.9%	-1.300	-1.352	-0.052	0.00%	1.410	1.425	0.015
6.0% - 6.3%	-0.102	0.121	0.223	13.00%	0.352	0.295	-0.057
6.5% - 7.2%	0.181	0.083	-0.098	20.00%	-0.173	-0.153	0.019
7.3% - 8.0%	0.472	0.375	-0.097	25.00%	-0.527	-0.479	0.049
8.0% - 9.0%	0.749	0.774	0.025	33.00%	-1.061	-1.087	-0.026
Mean			0.000	Mean			0.000
Sdev			0.1343	Sdev			0.0414

Table 4 shows only the error terms for the aggregated utilities. The error terms for the partial choice tasks have not been calculated at this stage.

Portfolio Indifference Curves - Isoutilities

Based on the utilities for each combination of portfolio, and assuming a constant utility for time horizon, figure 5 shows a plot of the marginal utilities shown as isoutilities. Any point on an isoutility will correspond to the equivalent investor utility for those combinations of attributes of a portfolio that appear on the isoutility.

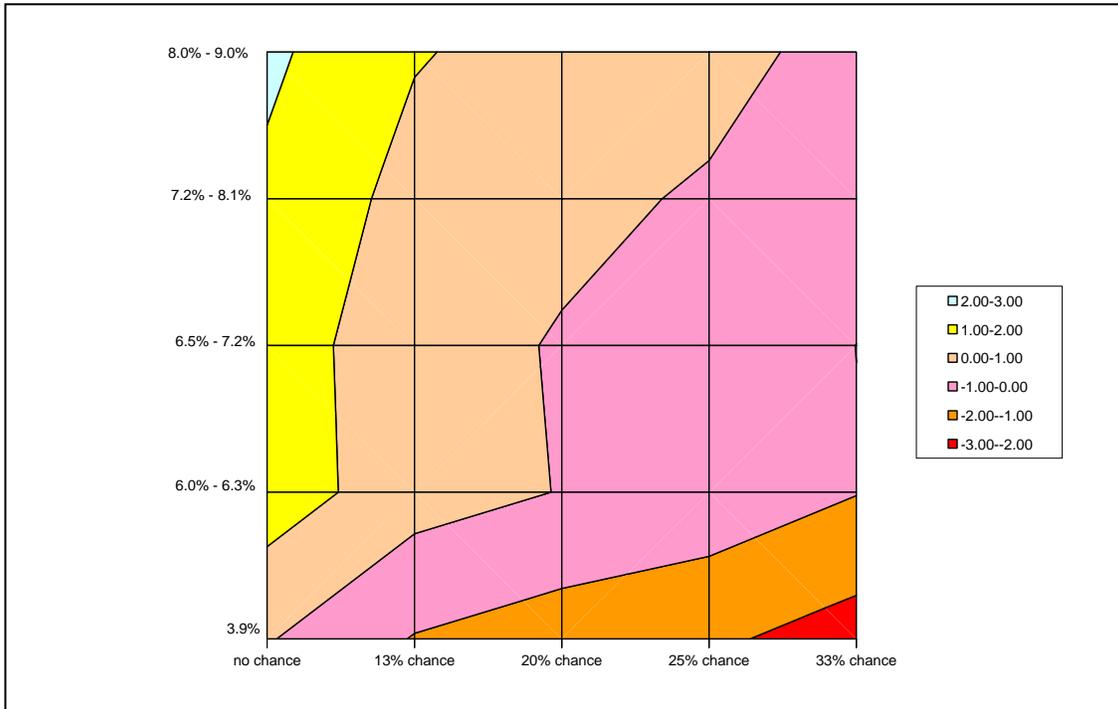


Figure 5 Investor Indifference Isoutilities

Implications from the findings of the study

The survey data allowed an aggregation of investor utilities through the partial choice task trade-offs for each of marginal utilities of Expected Return, Expected Risk and Expected Time Horizon.

(4) developed a relationship between the aggregated marginal utilities as follows:

$$mE(U_p) = f_1 mE(R_p) + f_2 mE(Z_p) + f_3 m(H_p) \quad (4)$$

Best fit estimates to the data provide an estimate of the underlying functions f_1, f_2 and f_3 of each of the marginal utilities, allowing analysis and optimisation. Importantly, it is not necessary to transform the Expected Risk utility to reference standard deviation instead of the probability of a negative return, for two reasons. Firstly, there is a linear scalar between the one property and the other, and secondly, we are interested specifically in utilities, not necessarily statistical risk measures.

Using (4) in analysis, it is worthwhile to revisit the principles of portfolio optimisation (Sharpe 2006B).

In portfolio optimisation, *expected utility* provides the theoretical framework for developing the optimal portfolio, given an efficient frontier. Expected utility is the expected return (or expected probability of outcomes) to be derived by the investor, at some point in the future, in exchange for deferred consumption (investment) today, given a certain value of risk and the risk aversion of the investor. One way to look at this is to assess the likely 'states' of the portfolio in the future, viewing outcomes as discrete possibilities, with investor preferences between each. Sharpe (2006)

In general terms, a portfolio's expected utility $E(U)$ is the sum of the probability of outcomes π of each of the portfolio returns R in each of the states s :

$$E(U) = \sum_s \pi_s u(R_{ps}) \quad (6)$$

As such, (4) can be adapted to show the utility of portfolio p , during a state 's' as follows:

$$mE(U_{ps}) = C_R mE(R_{ps}) + C_Z mE(Z_{ps}) + C_H m(H_{ps}) \quad (7)$$

where each of the 'states' represent a different set of attributes as shown in Table 2 and a factor C , provides scaling for each of the utilities $mE(R)$; $mE(Z)$ and $mE(P)$

For example, the Expected Utility of portfolio p , at state '1', where the Expected Return is 6.5%-7.2%, and there is a 33% chance of a negative annualised return over a 1 year time horizon, would be expressed as:

$$mE(U_{p1}) = C_R mE(R_{p6.5\%-7.2\%}) + C_Z mE(Z_{p33\%}) + C_H m(H_{p1yr}) \quad (7a)$$

The expected utilities for each state can be inferred from the function of each of the marginal utilities, or from combining partial utilities for each of the attributes at each of the states. For the purposes of an example, the utilities as shown in Table 4 can be referred to as follows:

Table 4 Survey Utilities for each Expected Return:

E(R)	mE(R)	E(Z)	mE(Z)	E(H)	mE(H)
3.9%	-1.352	no chance	1.425	1 year	-0.008
6.0 - 6.3%	0.121	13% chance	0.295	3 year	0.009
6.5 - 7.2%	0.083	20% chance	-0.153	5 year	0.177
7.2 - 8.1%	0.375	25% chance	-0.479	10 year	-0.178
8.0 - 9.0%	0.774	33% chance	-1.087		

Consequently, for the state s=1, for all C's =1, (5) solves as: 0.375-1.087-0.008 = -1.013

There is of course the real issue as to whether utilities are additive merely with a linear factor. It is worth bearing in mind that the data has been extracted from investors in choice task tradeoffs. In each pair of tradeoffs, the two portfolios presented contained drawings from each of the three attribute groups were presented. The initial implication is that utilities were determined in the first instances, by comparing each of the states of the portfolios, and then disaggregated to determine the marginal utilities for each of the attributes. This implies that the relative utilities of Return scale in the same manner as those of Risk or Time Horizon, implying that the additive properties imputed, allow (7) to provide a reasonable assumption.

Utilities and the Equity Risk premium

A surprising extension of the approach developed in this paper, is its potential use in developing an aggregate view of investor risk premium, which can further be disaggregated, knowing the portfolio composition, to an equity risk premium calculation. This development is beyond the scope of this paper, but the concept will be introduced for further study.

Sharpe proposed an algorithm for portfolio optimisation based on the relationship between expected utility, expected portfolio returns μ_p , investor risk aversion stated as λ_{ra} , the coefficient of risk r for investor a, i and portfolio return variance σ_p^2 , stated as follows:

$$E(U) = \mu_p - \lambda_{ra} \sigma_p^2 \tag{8}$$

Building on Sharpe, Cochrane (1997) proposed representing utility in the form of the relationship as follows:

$$\frac{E(r) - r_f}{\sigma(r)} = \gamma \sigma_{\Delta C} \text{corr}(\Delta C, r) \quad (9)$$

The left term is the Sharpe measure. Cochrane proposes that the investor utility is a function of the investor consumption factor C , the investor risk aversion γ , the standard deviation at the change in consumption, and the correlation between the investor consumption factor C , and the portfolio return. Assuming that the correlation between the consumption factor C and $r = 1$, and that instantaneous consumption occurs at state s , the equation reduces to (3). On this basis, the Sharpe measure, $E(r) - r_f / \sigma_r^2$ is a function of utility of the investor at state s , as follows:

$$\frac{E(r) - r_f}{\sigma(r) \sigma_{\Delta C}} = \frac{E_s(U)}{\sigma_s^2}$$

That is the risk premium demanded by investors as a function of the investor risk aversion. Assuming that the coefficient for risk aversion by the investor stays constant and allowing σ_H to be transformed from $E(Z)$ and scaled for time horizon H , the Expected Utility as derived from the survey can be solved for any combination of portfolios and the Risk Premium determined individually and in aggregate. Furthermore, knowing the portfolio asset allocation, and assuming a large enough sample size to allow such disaggregation, the risks can be disaggregated to determine the equity risk premium.

Summary

This paper proposes a novel approach to determining utilities using partial choice task tradeoffs through a survey. The paper demonstrated consistency in developing utilities and highlighted some important implications of the results for portfolio optimisation.

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