Mortality in Australia: Marking the 150th Anniversary of the First Australian Life Table

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Life Tables and Insurance Applications

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Early Life Table Insurance Applications

- In the early third century, Roman Praetorian prefect Aemilius Macer and praetorian prefect Domitius Ulpianus (Ulpian) constructed life tables used as an annuities table for determining tax based on age and the annuity value.

- John Graunt (1620-1674) was the first to produce a life table based on the Bills of Mortality of London. The Bills of Mortality included details of the deaths each week and the cause of death.

- Johann de Witt (1625-1672), the Prime Minister of the Netherlands, used the chances of death to value Life Annuities issued by the government.
Early Life Table Insurance Applications

• Edmond Halley (1656-1742), of Halley's Comet fame, developed a mortality table from the bills of mortality of the town of Breslau and used his table of mortality to calculate values of life annuities.

• Abraham de Moivre (1667-1754) developed the first treatment of probability in English, the Doctrine of Chances, and applied the theory of probability to problems related to annuities on human lives in his Annuities upon Lives.

James Dodson (1710–1757) was a British mathematician, actuary and innovator in the insurance industry. He used mortality rates based on the Bills of Mortality for the City of London for the period 1728-50 to compute long term insurance premiums used by The Equitable.
Life Tables and Insurance Applications

• Morris Birkbeck Pell (1827–1879) was a mathematician, professor, lawyer and actuary.

• In 1854 he had become actuarial consultant to the Australian Mutual Provident Society.

• In 1870 he was a director and consulting actuary of the Mutual Life Association of Australasia.

• He published papers 'On the Rates of Mortality and Expectation of Life in New South Wales' (1867) and 'On the Constitution of Matter' (1871).

• He also published in the Journal of the Institute of Actuaries, London, 'On the Distribution of Profits in Mutual Insurance Societies' (1869) and 'On the Institute of Actuaries' life tables' (1879) among other papers.

Source: Australian Dictionary of Biography
Data for Life Tables

Individual data
- Risk factors: Age, sex, smoking status, education, occupation, ethnicity, income, geographical location, marital status
- Cause of death
- Survey data (HRS)
- Government administrative data

Population by country
- Aggregate deaths by age, gender, period, cohort
- Aggregate cross sectional health data
- Prevalence of health conditions
- Mortality rates by Causes of death
- Human Mortality Database

Life insurance, pension fund and annuity pools
- Aggregation of deaths and exposures by underwriting risk factors
- Effect of selection
- Insured lives
- Annuitants
- Pensioners
- Role of Actuarial associations (CMIB, SoA)
Frailty and Markov Physiological Ageing Models

Life insurers “select” lives using underwriting allowing for heterogeneity in individual mortality

Frailty Model – 1945 Australian Male Cohort with varying levels of frailty

![Distribution of Physiological Age at Age 65: Male](chart1)

Markov Physiological Age Model – Distribution of Physiological Ages for a 65 year old Australian male

Source: Shu and Sherris (2010)

Life Tables for Insurance allowing for Risk Factors

\[ q_{it} = E(Y_{it} | X_{it}) = 1 - \exp[-\exp(X_{it}\beta' + b_i)] \]

\[ q_{it}(\alpha) = 1 - \exp[-\exp(X_{it}\hat{\beta}' + z_{it}\hat{\sigma}_b)] \]

GLMM can allow for both underwriting risk factors and residual variability (frailty)

Heterogeneity still significant after underwriting

Insurer can adjust annuity prices by pricing using adjusted mortality reflecting risk profiles and a frailty factor

Table 3: Risk profiles for underwritten annuity purchases

<table>
<thead>
<tr>
<th></th>
<th>Low risk</th>
<th>Avg risk</th>
<th>High risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>Education</td>
<td>College or above</td>
<td>HS or some college</td>
<td>Not completed HS</td>
</tr>
<tr>
<td>Partnered</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>BMI</td>
<td>Normal</td>
<td>Overweight</td>
<td>Underweight</td>
</tr>
<tr>
<td>Smoker</td>
<td>Never</td>
<td>Before, not currently</td>
<td>Before and currently</td>
</tr>
<tr>
<td>Medical conditions</td>
<td>None</td>
<td>High BP</td>
<td>High BP and Diabetes</td>
</tr>
</tbody>
</table>

Table 4: Expected value of $1 for life for a male aged 65, at 4%

<table>
<thead>
<tr>
<th></th>
<th>Baseline Standard</th>
<th>Underwritten Low</th>
<th>Underwritten Average</th>
<th>Underwritten High</th>
</tr>
</thead>
<tbody>
<tr>
<td>GLM</td>
<td>12.33</td>
<td>18.57</td>
<td>17.29</td>
<td>4.20</td>
</tr>
<tr>
<td>Frailty - 50%</td>
<td>12.28</td>
<td>18.66</td>
<td>17.03</td>
<td>3.57</td>
</tr>
<tr>
<td>Frailty - 75%</td>
<td>15.95</td>
<td>19.92</td>
<td>19.15</td>
<td>6.75</td>
</tr>
<tr>
<td>Frailty - 85%</td>
<td>17.54</td>
<td>20.25</td>
<td>19.78</td>
<td>8.85</td>
</tr>
<tr>
<td>Frailty - 95%</td>
<td>19.36</td>
<td>20.54</td>
<td>20.34</td>
<td>12.55</td>
</tr>
<tr>
<td>Frailty - 99.5%</td>
<td>20.44</td>
<td>20.68</td>
<td>20.63</td>
<td>17.33</td>
</tr>
</tbody>
</table>

Multi-State Life Tables for Functional Disability

Disability & recovery transition
Intensities – Males on left, Females on right

U.S. HRS (Health and Retirement Study) data:
- Rates of becoming LTC disabled are significantly higher for women than men.
- Force of disability > mortality hazard for females of all ages.
- Distinct age patterns of recovery.
- Used to produce life tables by health state

Multi-State Life Tables for Functional Disability

Mortality transition Intensities –
Males on left and Females on right

(a) Nondisabled to Dead, \( \mu \)

(b) Disabled to Dead, \( \nu \)

US Mortality rates from HRS data
Higher male mortality
Higher disabled mortality

Multi-State Life Tables for Functional Disability

Male transition rates by age – HRS data

Female transition rates by age – HRS data

### Multiple State Functionally Disabled Life Table

<table>
<thead>
<tr>
<th>Age</th>
<th>Survivors</th>
<th>Healthy</th>
<th>Mildly Disabled</th>
<th>Severely Disabled</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Males</td>
<td>Females</td>
</tr>
<tr>
<td>65</td>
<td>40,000</td>
<td>100.00%</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>70</td>
<td>35,834</td>
<td>90.30%</td>
<td>7.45%</td>
<td>2.25%</td>
</tr>
<tr>
<td>75</td>
<td>29,735</td>
<td>83.96%</td>
<td>11.68%</td>
<td>4.36%</td>
</tr>
<tr>
<td>80</td>
<td>22,129</td>
<td>78.44%</td>
<td>14.54%</td>
<td>7.02%</td>
</tr>
<tr>
<td>85</td>
<td>13,912</td>
<td>71.37%</td>
<td>17.91%</td>
<td>10.72%</td>
</tr>
<tr>
<td>90</td>
<td>6,612</td>
<td>61.04%</td>
<td>21.46%</td>
<td>17.50%</td>
</tr>
<tr>
<td>95</td>
<td>2,064</td>
<td>47.77%</td>
<td>24.90%</td>
<td>27.33%</td>
</tr>
<tr>
<td>100</td>
<td>340</td>
<td>27.35%</td>
<td>22.94%</td>
<td>49.71%</td>
</tr>
</tbody>
</table>

HRS data – combining transition rates to produce a life table for non-disabled lives

Functional Disability Life Table and LTC Insurance

- Disability (functional) free life expectancy
- Estimated from HRS data

<table>
<thead>
<tr>
<th>Demographic Characteristics</th>
<th>Males</th>
<th>Females</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean years of life after age 65</td>
<td>16.33</td>
<td>19.43</td>
</tr>
<tr>
<td>Mean years with mild disability</td>
<td>1.78</td>
<td>2.80</td>
</tr>
<tr>
<td>Mean years with severe disability</td>
<td>0.89</td>
<td>1.68</td>
</tr>
<tr>
<td>Share with disability</td>
<td>56.43%</td>
<td>72.70%</td>
</tr>
<tr>
<td>Share with mild disability</td>
<td>47.89%</td>
<td>63.37%</td>
</tr>
<tr>
<td>Share with severe disability</td>
<td>26.82%</td>
<td>42.39%</td>
</tr>
<tr>
<td>Average age of first disability, conditional on</td>
<td>76.23</td>
<td>76.52</td>
</tr>
<tr>
<td>becoming disabled</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average age of first mild disability, conditional</td>
<td>75.83</td>
<td>76.38</td>
</tr>
<tr>
<td>on becoming mildly disabled</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average age of first severe disability, conditional</td>
<td>80.51</td>
<td>81.70</td>
</tr>
<tr>
<td>on becoming severely disabled</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Static Life Tables
Society of Actuaries Mortality Tables

Mortality and Other Rate Tables (MORT)

The Mortality and Other Rate Tables (MORT) application was developed by the Society of Actuaries (SOA) to provide easy electronic access to a variety of rate tables of interest to actuaries and others. As of 2014, the inventory of rate tables available via this database numbers over 2,500 and encompasses SOA experience mortality and lapse tables, regulatory valuation tables, population tables and various international tables.
Dynamic Life Tables

Factor model for mortality (age x, current time t survival to time T)

Risk factors reflect level, slope and curvature of the mortality curve

\[
\tilde{\mu}(t, T) = \frac{1}{T - t} \left[ (T - t)Z_{1,t} + \frac{1 - e^{-\delta(T-t)}}{\delta} Z_{2,t} + \left( \frac{1 - e^{-\delta(T-t)}}{\delta} - (T - t)e^{-\delta(T-t)} \right) Z_{3,t} - C(t, T) \right]
\]

Mortality rate dynamics

\[
\begin{pmatrix}
\frac{dZ_{1,t}}{dt} \\
\frac{dZ_{2,t}}{dt} \\
\frac{dZ_{3,t}}{dt}
\end{pmatrix} = -\begin{pmatrix}
\delta_1 & 0 & 0 \\
0 & \delta_2 & 0 \\
0 & 0 & \delta_3
\end{pmatrix} \begin{pmatrix}
Z_{1,t} \\
Z_{2,t} \\
Z_{3,t}
\end{pmatrix} dt + \begin{pmatrix}
\sigma_1 & 0 & 0 \\
0 & \sigma_2 & 0 \\
0 & 0 & \sigma_3
\end{pmatrix} \begin{pmatrix}
\frac{dW_{1,t}^Q}{dt} \\
\frac{dW_{2,t}^Q}{dt} \\
\frac{dW_{3,t}^Q}{dt}
\end{pmatrix}
\]

\[
d\mu(t, T) = \sum_{i=1}^{n} \left[ \frac{\sigma_i^2 e^{-\delta_i(T-t)}}{\delta_i} \left( e^{-\delta_i(T-t)} - 1 \right) dt - \sigma_i e^{-\delta_i(T-t)} dW_i^Q \right]
\]

Dynamic Factor Model for Survival Curve

- Survivor curve
- Swedish mortality 1910 to 2007 ages 50 to 100

Dynamic Life Tables – Mortality Survival Curve with Confidence Intervals

Cohort survival curves
Uncertainty calibrated to historical data
Parameter uncertainty included

Welcome to the Actuaries Longevity Illustrator

Planning for retirement can be complicated, and there are many factors that must be taken into account. One of the most important, and sometimes misunderstood, is your longevity — that is, how long you might actually live. This is different from your life expectancy, which is how long an individual of your age, gender, and health would be anticipated to live on average. There is still a significant chance that you will live for many years beyond that, and you should consider this possibility when planning your retirement. This Actuaries Longevity Illustrator helps you do that by letting you see how long you might live with different degrees of certainty based on the expectations for an average individual with your characteristics. Take a look. You might be surprised by the results!

If your retirement plans involve two people, the considerations become even more complex. The Longevity Illustrator addresses two crucial concerns, “How long can we expect to live as a couple, and how long can we expect a survivor to live after one of us has died?” The Longevity Illustrator helps you to consider the likelihood of these possible outcomes.

Developed by the American Academy of Actuaries and the Society of Actuaries, the Longevity Illustrator is designed to provide you with perspectives on your longevity risk—the uncertainty of how long you and your spouse/partner might live. It does not address your finances, your investments, your earning potential or your anticipated expenses; consult with a financial professional about those aspects of your retirement planning. We invite you to use the Longevity Illustrator to enhance your understanding of the potential risk for outliving your financial resources.

How it works

You will answer a few questions about your health and demographic characteristics. The Longevity Illustrator will then produce charts that allow you to see the probabilities associated with how long you (and your spouse/partner, if applicable) may live, which will help you understand the likelihood that you may live for a much longer time than your life expectancy would suggest. This will allow you to consider the risks of outliving your financial resources, i.e., the chance of running out of money during your lifetime(s). You can view the results as either charts or as tables of values. You can also print out a summary sheet of the information provided by the Longevity Illustrator.
Individual Life Tables using Facial Recognition

- Facial recognition can be used to estimate physiological age, BMI, gender, smoker status - application of data science and machine learning. Example – Lapetus Solutions [https://www.lapetussolutions.com/about/](https://www.lapetussolutions.com/about/)

From just a simple selfie, our Facial Analytics technology examines 100’s of points and 1000’s of regions on the face to instantly provide: BMI calculation, Estimated Age, Gender, Smoking Indication.
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References


Questions and Discussion