Economic Valuation:
Something Old, Something New

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Abstract

Economic valuation involves the application of economic assumptions and an economic model to derive the value of an asset or set of cashflows or the price of a commodity or service. This paper discusses the theory underlying economic valuation models of interest to actuaries and reviews their application to insurance and energy markets. The insurance CAPM is an example of an economic valuation framework that is used in practice and worthy of discussion, particularly in respect of recent developments in frictional cost approaches to insurance pricing. Electricity markets and economic valuation of electricity assets has been an area where actuaries have been involved in recent years. A new and recently developed economic valuation model using the theory of storage, and equilibrium analysis from microeconomics, is reviewed.

1 Introduction

Economic valuation models of most interest to actuaries have been popularised in the finance discipline with an emphasis on asset and contingent claim valuation. International accounting standards propose the use of market or fair valuations of asset and liabilities for reporting profits and, in particular, the use of economic valuation models is becoming increasingly important as a basis for determining fair values of both assets and liabilities. In this paper, economic valuation models are reviewed and the application of these to insurance and electricity markets illustrated. There is something old - the economic theory underlying the models, and something new - the applications, particularly to electricity markets.

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A recent paper by Fitzherbert (2001) [1] discusses the Capital Asset Pricing Model (CAPM) and its place in actuarial education. CAPM is an often used model in practice for assessing risk margins for rates of return on capital and is an example of an economic valuation model. The economic valuation models reviewed in this paper contain CAPM as a special case. Errors and misconceptions in Fitzherbert (2001) [1] will be clarified in another paper (Sherris and Wong, 2002 [16]).

The CAPM will be developed in the context of more general economic valuation models. Financial pricing of insurance is based on the application of economic valuation models to insurance company cash flows of which the insurance CAPM is a particular case. A comprehensive survey of financial pricing models in insurance is found in Cummins and Phillips (2000) [6]. See also Briys and Varenne (2001) [4].

Integrated risk management provides a synthesis of corporate finance and financial risk management with insurance and is leading to a deeper understanding of economic valuation of insurance and insurance pricing. Doherty (2000) [7] provides a coverage of integrated risk management.

There is an extensive literature in the areas of economic valuation. It is not the aim of this paper to provide a complete literature review and references. Instead a number of key textbooks will be used as major references. In the case of economic valuation models used in finance theory a recommended reference is Cochrane (2001) [3]. An introductory coverage of the basic ideas in financial economic valuation can be found in Luenberger, Chapters 9 to 16 (1998) [13].

New and interesting problems in risk management have arisen in the energy markets and in the electricity market in particular. There is a need for economic valuation models in these new and challenging areas in order to price and manage risk. The economic valuation models for commodities and also for energy, such as electricity, require consideration of issues of the optimal conversion of fuels to produce energy and optimal storage models for commodities and fuels that can be stored. The basic economic issues required in these new areas are introduced in this paper.

It is important to understand that the assumptions or requirements of an economic valuation model do not necessarily need to hold in the real world. The usefulness or validity of an economic valuation model will depend on its assumptions and how realistic they are to the problem at hand. Even though some of the underlying assumptions of a valuation model do not hold in the real world, an economic valuation model can be very valuable in assisting with financial decision-making and risk management. It is generally acknowledged that all models are wrong, but we are interested in those that are most useful in practice. Training, expertise, and experience all play a part in applying economic valuation models for decision making in practice.

Economic valuation models are also used as an agreed basis for determining fair values between parties. In regulated markets, prices are often set by regulatory authorities using a competitive economic model as the basis for establishing prices.

Australian actuaries will benefit from a sounder understanding of the theory and shortcomings of economic models, including the CAPM, as well as broader issues in economic modelling in insurance. Although this paper contains formulae and mathematics, the key ideas are fundamental to financial economic valuation and to many problems that actuaries face in valuing cash flows. The purpose of this paper is educational and hopefully informative.

2 Economic Valuation

2.1 Fixed and known cash flows

The most basic assumption that can be used to develop an economic valuation model is that the model is arbitrage-free. This assumption actually underlies the most basic model familiar to actuaries, the discounted cash flow model. The arbitrage-free assumption is a basic requirement for an economic model to be consistent and provide sensible results.

To illustrate the arbitrage-free requirement of an economic valuation model, consider the formula used to value a set of fixed and known cash flows with amounts $x(t)$ occurring at times $t = 0, 1, \ldots, t, t+1, \ldots, T$.

Assume that entitlement to the individual cash flows and an entitlement to the whole portfolio of cash flows can be bought or sold. We must have

$$V_P(0) = \sum_{t=0}^{T} V_{x(t)}(0),$$

where $V_P(0)$ is the current value of the portfolio of cash flows and $V_{x(t)}(0)$ is the current, time 0, value of cash flow of amount $x(t)$ payable at time t. The value is additive in the cash flows, something most actuaries take for granted. Under what conditions must the value be additive in the cash flows?

The additive requirement must hold otherwise the value of the portfolio of cash flows will not equal the sum of the values of the component cash flows in the portfolio. If $V_P(0) > \sum_{t=0}^{T} V_{x(t)}(0)$, then we would purchase each of the individual cash flows with cost $\sum_{t=0}^{T} V_{x(t)}(0)$, package them together in a portfolio, and sell the portfolio at $V_P(0)$, for an instant risk free profit of $V_P(0) - \sum_{t=0}^{T} V_{x(t)}(0)$. We would do the reverse of this process if $V_P(0) < \sum_{t=0}^{T} V_{x(t)}(0)$.

The only value relationship that can hold is $V_P(0) = \sum_{t=0}^{T} V_{x(t)}(0)$ if such arbitrage profits are to be avoided.

In order to determine $V_{x(t)}(0)$ for any given cash flow consider the arbitrage-free valuation of a single cash flow of 1 at time t. Assume that we can agree today to exchange a dollar at time t for $1 + f(t)$ dollars at time $t+1$. The $f(t)$ is determined in economic theory from marginal rates of substitution for consumption at time t with consumption at time $t+1$ and marginal rates of transformation for productive investment. Normally we take the value of $f(t)$ as an exogenous rate and do not use an economic model to determine the rate. With traded financial markets, this forward interest rate can be determined from market interest rates. If there are no markets that allow us to determine this forward interest rate then, at least in theory, we can use an economic model to determine it. We would need to estimate marginal rates of transformation for productive investments and marginal rates of substitution for individuals. This is covered in standard micro-economic texts and many finance texts. See for example Eichberger and Harper (1997) [8].

For economic reasons we require that $f(t) > 0$ for all t.

Construct a portfolio at time 0 of an entitlement to 1 of the cash flow at time $t-1$ and δ of the cash flow at time t and select δ so that this portfolio has a zero net cost

$$V_{t-1}(0) + \delta V_t(0) = 0$$

where $V_{t-1}(0)$ is the value at time 0 of 1 due at time $t-1$. We have that

$$\delta = -\frac{V_{t-1}(0)}{V_t(0)}$$

and this portfolio will have a pay-off of 1 at time $t-1$ and $-\frac{V_{t-1}(0)}{V_t(0)}$ at time t. Since we can agree today to exchange 1 at time $t-1$ for an amount of $1+f(t-1)$ at time t we require that

$$\frac{V_{t-1}(0)}{V_t(0)} = 1 + f(t-1)$$

or

$$\frac{V_t(0)}{V_{t-1}(0)} = \frac{1}{1+f(t-1)}.$$

Otherwise there will be arbitrage opportunities between the prices for the cash flows and the forward exchange agreements.

Define

$$m(t-1) = \frac{1}{1+f(t-1)}$$

so that $m(t)$ is the single period discount factor from time $t-1$ to t.

In general, we have, for any t,

$$\frac{V_t(0)}{V_0(0)} = \frac{V_t(0)}{V_{t-1}(0)} \frac{V_{t-1}(0)}{V_{t-2}(0)} \cdots \frac{V_2(0)}{V_1(0)} \frac{V_1(0)}{V_0(0)}$$

or

$$\frac{V_t(0)}{V_0(0)} = \frac{1}{1+f(t-1)} \frac{1}{1+f(t-2)} \cdots \frac{1}{1+f(1)} \frac{1}{1+f(0)}$$

and since $V_0(0) = 1$, the value of 1 due at time t becomes

$$\begin{aligned} V_t(0) &= \frac{1}{1+f(t-1)} \frac{1}{1+f(t-2)} \cdots \frac{1}{1+f(1)} \frac{1}{1+f(0)} \\ &= m(t-1)\, m(t-2) \ldots m(1)\, m(0) \end{aligned}$$

If we treat an entitlement to $x(t)$ at time t as a portfolio of $x(t)$ entitlements each to an amount of 1 at time t then

$$V_{x(t)}(0) = V_t(0)\, x(t)$$

Assuming that we can buy or sell multiples of cash flows, then

$$\begin{aligned} V_P(0) &= \sum_{t=0}^{T} V_{x(t)}(0) \\ &= \sum_{t=0}^{T} V_t(0)\, x(t) \\ &= \sum_{t=0}^{T} v(t)\, x(t), \end{aligned}$$

where $x(t)$ is the cash flow at time t, and

$$v(t) = m(t-1)\, m(t-2) \ldots m(1)\, m(0)$$

is the valuation factor for cash flow at time t and $V_P(0)$ is the current value.

Some of the properties of the discount factor $v(t)$ are $v(0) = 1$, $\frac{v(t+1)-v(t)}{v(t)} = \frac{v(t+1)}{v(t)} - 1 = \frac{1}{1+f(t)} - 1 = \frac{-f(t)}{1+f(t)} < 0$ and $v(t) \to 0$ as $t \to \infty$.

The valuation model can also be written in recursive form

$$\begin{aligned} V_P(t) &= \frac{1}{1+f(t)} \left[V_P(t+1) + x(t+1) \right] \\ &= m(t) \left[V_P(t+1) + x(t+1) \right] \quad \text{for } t = 0, 1, 2, \ldots, T-1, \end{aligned}$$

with $V_P(T) = x(T)$.

We have illustrated how the discounted cash flow formula for fixed and known cash flows, that is taken for granted by actuaries, requires economic assumptions to justify its use. These include an arbitrage-free assumption and either traded financial markets for instruments or economic model assumptions related to investment opportunities.

2.2 Random cash flows

Random cash flows introduce risk into the valuation model. In this paper, risk will refer to the situation where the future cash flow value is random but we know the probability distribution of the cash flow. Uncertainty will refer to the case where we do not know the probability distribution. In reality future cash flows are uncertain and historical data is used to estimate the future probability distribution. We will assume that we know the probability distribution so that we are in a situation of risk only. In practice model and parameter estimation uncertainty must be taken into account.

Utility functions are used in insurance and finance to quantify risk and uncertainty. There are models based on non-expected utility but these are not explored here. See for example van der Hoek and Sherris (2001) [20] and Landsman and Sherris (2001) [11]. The standard economic valuation models are based on individual preferences in an economy where individuals must decide how much of their wealth to consume, how much to invest in risky cash flows and how much of their assets to insure in order to optimise their discounted expected utility of future consumption. Expected utility is derived from a set of axioms on preferences.

Risk is usually allowed for by considering future states of the economy. We might assume s discrete possible states of nature at time $t+1$ each with known probability of occurrence. These probabilities are often referred to as real world probabilities. If we are estimating the probabilities from data then they are referred to as empirical probabilities.

At time t individuals are usually assumed to have a utility function over consumption given by

$$U\left(c_t, c_{t+1}\right) = u\left(c_t\right) + \beta \mathbb{E}_t\left[u\left(c_{t+1}\right)\right],$$

where c_t is consumption at time t, $u(\cdot)$ is an increasing concave utility function, β is a subjective discount factor and $\mathbb{E}_t$ is the conditional expectation operator over future states at time $t+1$. Individuals are assumed to maximise this utility function by selecting their optimal amount of consumption, asset holdings and insurance. Asset holdings may be direct investments in productive assets or financial assets giving a share in future cash flows from productive investments.

Value in equilibrium Following the approach in Cochrane (2001) [3], we derive a fundamental economic valuation result. Assume that the optimal level of consumption has been determined as C_t^* and $C_{t+1}^*(s)$ in state s at time $t+1$. Consider the valuation of a security that has pay-off at time $t+1$ of $X(t+1,s)$ in state s with an equilibrium price, its economic value, of $P(t)$ at time t. Denote by $\xi > 0$ an additional amount of this security purchased at price $P(t)$ over and above the equilibrium holding. By definition of equilibrium, the excess amount demanded at the equilibrium price must be zero.

An individual will select ξ for any given security price by solving the following economic valuation model

$$\begin{aligned} J\left(c_t\right) &= \max_{\{\xi\}}\left[u\left(c_t\right) + \beta \mathbb{E}_t\left[u\left(c_{t+1}\right)\right]\right] \\ &\text{such that} \\ c_t &= C_t^* - P(t)\,\xi \\ c_{t+1}(s) &= C_{t+1}^*(s) + X(t+1,s)\xi \quad \text{for all } s. \end{aligned}$$

Substituting the constraints into the objective gives

$$J\left(c_t\right) = \max_{\{\xi\}}\left[u\left(C_t^* - P(t)\,\xi\right) + \beta \mathbb{E}_t\left[u\left(C_{t+1}^* + X(t+1)\xi\right)\right]\right].$$

Differentiating with respect to ξ and setting ξ to zero gives the condition for optimal holding of the security and optimal consumption at equilibrium as follows

$$\frac{\partial}{\partial \xi} J(c_t)\,|_{\xi=0} = -P(t)\,u'(C_t^*) + \beta \mathbb{E}_t\left[X(t+1)u'\left(C_{t+1}^*\right)\right] = 0$$

so that

$$P(t) = \mathbb{E}_t\left[\beta \frac{u'\left(C_{t+1}^*\right)}{u'\left(C_t^*\right)} X(t+1)\right].$$

If we let

$$m(t+1) = \beta \frac{u'\left(C_{t+1}^*\right)}{u'\left(C_t^*\right)},$$

then

$$P(t) = \mathbb{E}_t\left[m(t+1)\,X(t+1)\right],$$

where expectation is taken with respect to the real world probabilities. In this case $m(t+1)$ is a stochastic discount factor. Since the optimal consumption differs by state, the discount factor differs by state. The stochastic discount factor is a marginal rate of substitution of consumption at time $t+1$ in state s for consumption at time t. It is also referred to as a pricing kernel, a state price density and, in recent actuarial literature, as a deflator.

For a risk free security, an investment of 1 at time t is assumed to pay-off $R_t = 1 + r_f$ in one time period, where r_f is the risk-free interest rate, r_f, so that

$$1 = \mathbb{E}_t\left[m(t+1)\,R_t\right]$$

or

$$\frac{1}{R_t} = \mathbb{E}_t\left[m(t+1)\right].$$

The approach of Cochrane (2001) [3] is that asset pricing can be summarised by two equations

$$p_t = \mathbb{E}\left(m_{t+1} x_{t+1}\right)$$

and

$$m_{t+1} = f\left(\text{data, parameters}\right),$$

where p_t is the price, x_{t+1} is the payoff, and m_{t+1} is the stochastic discount factor. The derivation of this result does not require any particular assumptions about return distributions nor about the form of the utility function other than the usual assumptions such as individuals prefer more to less and risk aversion. The pricing model has been derived over a single time period, but this could be a single time period in a multi-period model since conditional expected values are used. It includes single time period models.

From the definition of covariance

$$cov\left(x,y\right)=\mathbb{E}\left(xy\right)-\mathbb{E}\left(x\right)\mathbb{E}\left(y\right),$$

so the price can be re-written as

$$\begin{aligned} p_t &= \mathbb{E}\left(m_{t+1}\right)\mathbb{E}\left(x_{t+1}\right)+cov\left(m_{t+1},x_{t+1}\right) \\ &= \frac{\mathbb{E}\left(x_{t+1}\right)}{R_t}+cov\left(m_{t+1},x_{t+1}\right). \end{aligned}$$

Then the prices reflect expected values of cash flows discounted at the risk free rate plus a covariance or risk adjustment term. The risk adjustment term reflects the covariance of the cash flow with the stochastic discount factor, which will be model dependent. However, note that this is just a result of the definition of covariance and is not something unique to a particular model such as CAPM. A covariance term will appear in all economic valuation models based on equilibrium to allow for risk.

The model can be expressed in terms of returns. In this case we have

$$1=\mathbb{E}\left(m_{t+1}r_{t+1}\right),$$

where

$$r_{t+1}=\frac{x_{t+1}}{p_t}$$

is the rate of return per period and can be re-written as

$$1=\frac{\mathbb{E}\left(r_{t+1}\right)}{R_t}+cov\left(m_{t+1},r_{t+1}\right)$$

or

$$\mathbb{E}\left(r_{t+1}\right)-R_t=-cov\left(\frac{m_{t+1}}{\mathbb{E}\left(m_{t+1}\right)},r_{t+1}\right).$$

Once again no explicit assumptions about the form of the utility function or distribution of returns were made to derive this result.

Note that, if you assume a stationary returns process, so that the expected return is constant in each time period, then the estimator of $\mathbb{E}\left(r_{t+1}-1\right)$ would be the sample arithmetic average of discrete per period returns. Issues in estimation of returns and the use of arithmetic and geometric averages in determining expected returns is covered in a forthcoming paper (Sherris and Wong, 2002 [16]).

The returns could have been expressed as continuous compounding returns with $\delta_{t+1} = \ln(r_{t+1}) = \ln\left(\frac{x_{t+1}}{p_t}\right)$ and we would then have the pricing equation in return form given by

$$1 = \mathbb{E}\left(m_{t+1}e^{\delta_{t+1}}\right).$$

This section of the paper has reviewed and derived the basic economic valuation model using economic assumptions of equilibrium and the quantification or risk through a utility function. The economic valuation model includes many models as special cases including CAPM, multi-period CAPM, and option pricing models.

2.3 Fundamental Theorem of Asset Pricing

Applying the basic requirement for an economic valuation model to be arbitrage-free will ensure that the stochastic discount factor is strictly positive, $m(t+1) > 0$ in all states. We saw earlier no-arbitrage in financial markets was required to justify a deterministic discounted cash flow valuation model. The importance of the no-arbitrage assumption in an economic valuation model is highlighted by a version of the Fundamental Theorem of Asset Pricing.

This very powerful theorem states that, a strictly positive stochastic discount factor m_{t+1} will exist such that $p_t = \mathbb{E}(m_{t+1}x_{t+1})$, if and only if, there are no arbitrage opportunities in the valuation model.

We are able to derive a very powerful economic valuation framework from a very basic requirement of an economic valuation model.

If a money market account exists, where cash can be invested at the risk free rate of interest, then the Fundamental Theorem of Asset Pricing states that it is possible to find a probability measure, Q, which is equivalent, in the probability measure theory sense of the word, to the real world or historical probability measure, such that

$$p_t = \frac{\mathbb{E}^Q(x_{t+1})}{R_t}.$$

The Q probability measure is referred to as the risk-neutral measure and the probabilities used to determine the expected value are called risk-neutral probabilities. This is not the probability distribution that is used to project or estimate future cash flows for assessing solvency, or for risk management purposes.

Under this risk neutral probability measure, the discounted asset price at the risk free rate will be a martingale. Martingales are a probability model for a fair game. They are stochastic processes with zero expected growth so that the expected value in the future is just the current value. The change to the Q probability measure represents a risk adjustment to the expected value and the discounting at the risk free rate converts the expected value to present value. The risk adjusted present value determined using this approach can be regarded as a fair game providing a fair risk-adjusted return.

In this approach the risk adjustment is made in the determination of the expected value and not in the discount rate used to present value the expected cash flow. The standard approach to applying CAPM is to adjust for risk in the discount rate.

In a multi-period model, for a random cash flow x_T payable at time T, the time $t < T$ value p_t, will be given by

$$p_t = \frac{\mathbb{E}^Q\left(x_T\right)}{\left(1 + r_f\right)^{T-t}}.$$

The risk free rate r_f is assumed to be constant in each time period. This interest rate is also referred to as the short rate in asset pricing models. If interest rates are assumed to be random, then the price will be given by

$$p_t = \mathbb{E}^Q\left(\frac{x_T}{\prod_{i=t}^{T-1}\left(1 + r_{fi}\right)}\right)$$

where $\prod_{i=t}^{T-1}\left(1 + r_{fi}\right)$ is the accumulated value of \$1 invested in a bank account earning the short rate in each future time period. The Q probabilities are such that the value of the cash flow denominated in units of the bank account accumulated value will be a martingale. This is the reason that the short interest rate is often used in economic valuation models.

2.4 CAPM - Linear Discount Factor

Many assumptions can be used to derive CAPM. These include normally distributed returns and exponential utility as well as quadratic utility. Full details are given in Cochrane (2001) Chapter 9 [3].

The main feature of the CAPM is that the stochastic discount factor is linear so that

$$m_{t+1} = a + bR_{t+1}^W,$$

where W is economy-wide wealth.

Using the result

$$\mathbb{E}\left(r_{t+1}\right) - R_t = -cov\left(\frac{m_{t+1}}{\mathbb{E}\left(m_{t+1}\right)}, r_{t+1}\right)$$

and a linear discount factor, gives

$$\begin{aligned}\mathbb{E}\left(r_{t+1}\right) - R_t &= -cov\left(\frac{a + bR_{t+1}^W}{\mathbb{E}\left(a + bR_{t+1}^W\right)}, r_{t+1}\right)\\ &= -\frac{1}{\frac{a}{b} + \mathbb{E}\left(R_{t+1}^W\right)}cov\left(R_{t+1}^W, r_{t+1}\right).\end{aligned}$$

Since this must hold for the portfolio of market wealth we have

$$\mathbb{E}\left(R_{t+1}^{W}\right)-R_t=-\frac{1}{\frac{a}{b}+\mathbb{E}\left(R_{t+1}^{W}\right)}\sigma^2\left(R_{t+1}^{W}\right)$$

and solving for $\frac{a}{b}$ gives

$$\frac{a}{b}=-\frac{\sigma^2\left(R_{t+1}^{W}\right)}{\left[\mathbb{E}\left(R_{t+1}^{W}\right)-R_t\right]}-\mathbb{E}\left(R_{t+1}^{W}\right).$$

Substituting back gives

$$\mathbb{E}\left(r_{t+1}\right)-R_t=\beta_W\left[\mathbb{E}\left(R_{t+1}^{W}\right)-R_t\right],$$

where

$$\beta_W=\frac{cov\left(R_{t+1}^{W},r_{t+1}\right)}{\sigma^2\left(R_{t+1}^{W}\right)}$$

and we have derived CAPM in expected return form.

For the CAPM, the stochastic discount factor is linear in the return on total wealth in the economy. This is not the return on the equity market index, although for empirical testing of the model, the return on an equity market index is usually used as a proxy for the return on economy wide wealth.

There are a range of assumptions about investor preferences and distributions of asset returns, and hence wealth, that are consistent with CAPM. There are also extensive empirical studies that test CAPM for security returns that are not discussed here. The linear discount factor can be treated as an approximation to other stochastic discount factor models.

There are many economic valuation models that extend CAPM, such as the intertemporal CAPM and factor models such as Arbitrage Pricing Theory. These are all subject to econometric issues when testing how well they represent reality, including time-varying discount factors and a high degree of parameter uncertainty. Empirical evidence suggests that betas are time varying, implying that discount rates should also be time varying, and there is uncertainty with respect to the estimation of the market risk premium. Cochrane (2001) [3] and Campbell, Lo and MacKinlay (1997) [2] cover issues in and results from empirical tests of asset pricing models.

3 Insurance Pricing

In recent years, CAPM and other financial models have been applied to insurance pricing particularly for the purposes of fair rate of return rating in regulated insurance markets. In the insurance context, the CAPM is usually used to determine the expected rate of return in a discounted cash flow model. A detailed coverage of the application of CAPM and related insurance pricing models is given in Taylor (1994) [18]. Insurance pricing models based on equilibrium models of insurance markets have been developed that are applications of the CAPM to a leveraged insurance company, where the leverage is provided from the reserves and provisions for policyholder liabilities. This leverage is often called the "float".

The Myers-Cohn model is often used for premium determination in a fair rate of return approach and is effectively an extension of the "principle of equivalence" taught in actuarial mathematics to risky cash flows allowing for taxation. Taylor (1994) [18] provides an extensive coverage of the Myers-Cohn model and its relationship to other models used in fair insurance pricing including allowances for taxation and expenses. Internal rate of return (IRR) models using equity cash flows are also used in general insurance pricing. The relationship between the IRR approach and the Myers-Cohn approach is also covered in Taylor (1994) [18].

Consider the equilibrium price derived in the previous section

$$p_t = \frac{\mathbb{E}\left(x_{t+1}\right)}{R_t} + cov\left(m_{t+1}, x_{t+1}\right),$$

where the cash flow $x_{t+1} < 0$ is a loss assumed to occur at the end of a time period. If we regard an insurance contract as a security paying $x_{t+1} > 0$, then the equilibrium pricing of such a contract will take into account the covariance of the loss with the stochastic discount factor. The first term in the pricing formula is the expected value of the loss, present valued at the risk free interest rate. So this would be consistent in practice with using government bond yields from a government bond yield curve to present value expected future claims payments in pricing an insurance contract before allowing for risk adjustment.

The covariance term is the risk adjustment and represents the covariance between the insurance loss and the stochastic discount factor. If the stochastic discount factor is above average whenever the loss is below average, and below average whenever the loss is above average, then the covariance will be negative. If individuals regard situations where the loss is larger than expected as a "bad" outcome and situations where the loss is smaller than expected as a "good" outcome, then it would be expected that the price for a cash flow in "bad" outcome states will be higher, because individuals should value cash flows in such states more than in "good" states. This means that the stochastic discount factor will be lower for the "bad" states. Similarly if the price for cash flows in "good" outcome states is less than for "bad" outcome states, the stochastic discount factor should be higher for these states. In these circumstances the risk load for insurance risk in an insurance contract is expected to be negative if the insurance price is to be a fair price. However, a negative risk load in an insurance price does not result in insurance company equity returns being negative. They will be fair in terms of the return from the risk borne by the company as will be discussed in the following sections.

3.1 Life insurance, Profit Testing and Relationships to Myers-Cohn and IRR Approaches

It is interesting to note the similarities between insurance pricing and valuation of liabilities in both general insurance and in life insurance. The equivalent model to the Myers-Cohn approach is the direct method in the fair valuation of life insurance and the internal rate of return approach is known as the indirect method. Girard (2002) [9] provides a coverage of the direct and indirect approaches in life insurance.

Life insurance actuaries developed net premium methods of valuing traditional with-profits business which included a loading in the premium for a reversionary bonus and used a reduced interest rate to discount the expected sum insured and premium payments when valuing the liabilities. This approach ensured that the emergence of surplus over time was of an amount to support the reversionary bonus. In the life insurance case, the premium loading and the method of valuing the liabilities was designed to provide a planned surplus to the policyholders in the form of a reversionary bonus. As the actual profit emerged, the policyholders were paid a reversionary bonus that not only reflected the loading in the premium but also the actual experience with respect to investment earnings, claims and expenses. In life insurance the loading was returned to the policyholders. In general insurance, the equivalent to these premium loadings for with-profits life insurance are the premium loadings for underwriting profit other than risk loadings. However, in contrast to with-profit life insurance, where these bonus loadings were planned to be returned to the policyholders equitably, the profit loadings in general insurance go to the shareholders after allowing for the actual investment, claims and expense experience.

The profit testing technique for determining a premium developed by life insurance actuaries is similar to the IRR approach in general insurance. In life insurance profit testing, the net cash flow to equity from an insurance contract, referred to as a profit signature, is determined from a cash flow projection model. A profit target is selected, such as a target return to equity or net present value of profits as a percentage of the premium, and the premium is then determined that will meet the profit target based on the projected cash flows.

When a target return to equity is specified, the premium is determined so that the target return to equity is the IRR on the net cash flows to equity. This is equivalent to the IRR approach.

3.2 Discounted Cash Flow - Myers-Cohn

Consider an insurance contract with a premium as a single payment at the commencement of a policy for an amount P and claims paid at the end of future time periods. Expenses are not included. No allowance for taxation on investment and underwriting income is included although tax at a rate of τ on profits is included in Taylor (1994) [18]. Claim payments related to this policy are paid over time with an amount L_t assumed paid at the end of time period t, at time $t+1$. This is a random variable and would be typically determined by estimating the claims run-off from historical data for this class of business.

Assume that all claim payments on the policy are paid by time T. The insurance company balance sheet will consist of assets, with market value at time t denoted by A_t, liabilities for policy claims with market value R_t, and equity E_t which by definition equals $A_t - R_t$. We assume that values are market values for assets and that markets exist to determine these and that economic values, or at least fair values as determined using an appropriate and agreed fair valuation based on discounted cash flows, are used for liabilities.

In Taylor (1994) [18] a deterministic loading factor v_t is assumed to be applied to the claim payments at time t in determining provisions. This is not a loading for risk under CAPM since the risk load in the insurance CAPM is determined by the interest rate used to discount the cash flows and not from a risk adjustment to the cash flows. The discounted expected value of the liabilities at time t will be

$$R_t = \sum_{\tau=t}^{T} PV_t \left[(1 + v_t) L_\tau \right],$$

where the PV of the liability cash flows needs to adjust for risk and for time value.

The insurance CAPM involves an application of the CAPM to the liability cash flows. In doing this we are applying an inherently single period model (the CAPM) in a multiperiod valuation model and using a market portfolio as a proxy for economy wide wealth. It is also assumed that expected returns on assets, expected discount rates for liabilities and expected returns to equity are constant.

From the definition of the liability discount rate, r_L,

$$PV_t (L_\tau) = \frac{\mathbb{E}[L_\tau]}{[1 + \mathbb{E}(r_L)]^{\tau+1-t}}$$

with

$$\mathbb{E}(r_L) = r_f + \beta_L (\mathbb{E}(r_M) - r_f),$$

where r_M is the return on the market index, $(\mathbb{E}(r_M) - r_f)$ is the equity market risk premium, β_L is the underwriting beta which is defined as

$$\beta_L = \frac{cov(r_M, r_L)}{\sigma_{r_M}^2},$$

and $\sigma_{r_M}^2$ is the variance of the return on the market index.

This allows us to write

$$\begin{aligned} R_t &= \sum_{\tau=t}^{T} \frac{(1 + v_\tau) \mathbb{E}[L_\tau]}{[1 + \mathbb{E}(r_L)]^{\tau+1-t}} \\ &= (1 + \mathbb{E}[r_L]) R_{t-1} - (1 + v_t) \mathbb{E}[L_t]. \end{aligned}$$

Since the claim payments are made from the assets we also have

$$A_t = (1 + \mathbb{E}[r_A]) A_{t-1} - \mathbb{E}[L_t]$$

The relationship between the underwriting beta and the insurance company equity beta is derived from the identity

$$\mathbb{E}[r_E] E_t = \mathbb{E}[r_A] A_t - \mathbb{E}[r_L] R_t.$$

Rearranging gives

$$\mathbb{E}[r_E] = \mathbb{E}[r_A] \frac{A_t}{E_t} - \mathbb{E}[r_L] \frac{R_t}{E_t}$$

which will only be constant if the expected returns on assets and liabilities as well as the capital structure of the company are constant.

Assuming CAPM holds for the expected asset and liability returns allows the expected return on equity to be written as

$$\mathbb{E}[r_E] = r_f + \beta_E (\mathbb{E}(r_M) - r_f)$$

with

$$\beta_E = \left(\beta_A \frac{A_t}{E_t} - \beta_L \frac{R_t}{E_t}\right) = \left(\beta_A \frac{A_t}{E_t} + \beta_L \left[1 - \frac{A_t}{E_t}\right]\right),$$

where $\frac{A_t}{E_t}$ is the leverage in the insurance company balance sheet from writing insurance.

To illustrate this result, assume that the risk free rate is 4% ($r_f = 0.04$), the equity market risk premium is 6% ($[\mathbb{E}(r_M) - r_f] = 0.06$), and an insurance company has an equity beta for its equity of $\beta_E = 1.4$. These can be estimated or determined from market data, although with a quantifiable degree of uncertainty. If the company invests in assets with a beta of $\beta_A = 0.5$ and underwrites insurance business with an underwriting beta of $\beta_L = 0$ then this implies that

$$\frac{A_t}{E_t} = \frac{\beta_E - \beta_L}{\beta_A - \beta_L} = 2.8$$

with a solvency ratio of

$$\frac{E_t}{R_t} = \frac{1}{\frac{A_t}{E_t} - 1} = 0.56.$$

Of course if the market value of the assets and the market value of the equity of the insurance company were known, then this leverage ratio could be determined from the balance sheet. Regulations may also prescribe a minimum capital requirement. In this case leverage would be influenced by regulatory requirements. In this case, the expected return on equity could be estimated based on assumptions about leverage, underwriting beta and asset beta. For example, for an insurance company with $\beta_A = 1.2$, $\beta_L = -0.2$, $\frac{A_t}{E_t} = 2$, the insurance CAPM would imply that, using the same values for the risk free rate and the equity risk premium as above,

$$\mathbb{E}[r_E] = 0.04 + (1.2 \times 2 - (-0.2)(-1))\, 0.06 = 0.172$$

or 17.2%. In this case, even though the risk-adjusted interest rate used to present value claims, $\mathbb{E}[r_L]$, is less than the risk free rate, shareholders earn a fair rate that reflects the return expected from taking a leveraged position in the investment markets financed from the insurance "float". Insurance is both an underwriting and a spread business.

The Myers-Cohn model determines the premium as the discounted expected value of claim payments including the loading and an allowance for tax on profits from investment and underwriting income. Ignoring tax and assuming a single premium gives

$$P = \sum_{t=1}^{T} \frac{(1+v_t)\,\mathbb{E}[L_t]}{[1+\mathbb{E}(r_L)]^t}$$

If the loading factor is assumed to be constant $v_t = v$ for all t, then

$$P = (1+v) \sum_{t=1}^{T} \frac{\mathbb{E}[L_t]}{[1+\mathbb{E}(r_L)]^t}$$

The discounted expected value of the claims payments is

$$\sum_{t=1}^{T} \frac{\mathbb{E}[L_t]}{[1+\mathbb{E}(r_L)]^t}$$

and this is "risk-adjusted" since the discount rate is a CAPM based risk adjusted rate.

If the loading factor is set to zero, so that $v_t = 0$ for all t, then there is no explicit profit loading added to the premium and this is what an economic valuation model based purely on CAPM with an adjustment for risk only in the discount rate requires. There is still a risk loading using CAPM and this is in the risk adjustment to the discount rate. In order for the CAPM risk load to be positive, we require

$$P \geq \sum_{t=1}^{T} \frac{\mathbb{E}[L_t]}{[1+r_f]^t}$$

or

$$\mathbb{E}(r_L) < r_f$$

which implies that

$$\beta_L < 0.$$

The risk loading arising from the CAPM depends on the sign of β_L. Most actuaries assume that a risk adjustment to the present value of a liability should be in the form of a positive risk load. For liabilities with $\beta_L > 0$, this will not be the case.

We can also consider the expected return on equity $\mathbb{E}[r_E]$ implied by the Myers-Cohn approach at time $t-1$. There is no reason for this to be constant for each time period. For example, consider a time period t, then conditional on values at time $t-1$,

$$\begin{aligned}
\mathbb{E}[r_E] &= \frac{\mathbb{E}[E_t] - E_{t-1}}{E_{t-1}} \\
&= \frac{(\mathbb{E}[A_t] - A_{t-1}) - (R_t - R_{t-1})}{E_{t-1}} \\
&= \frac{\mathbb{E}[r_A]A_{t-1} - [\mathbb{E}[r_L]R_{t-1} - v_t\mathbb{E}[L_t]]}{E_{t-1}} \\
&= \mathbb{E}[r_A]E_{t-1} + [\mathbb{E}[r_A] - \mathbb{E}[r_L]]\frac{R_{t-1}}{E_{t-1}} + \frac{v_t\mathbb{E}[L_t]}{E_{t-1}}.
\end{aligned}$$

The first term is the return expected by equity from investing their own funds in the investment markets through the insurance company. The next term is the expected spread from the "float", which is in fact a leveraged position. The final term is the expected loading factor included in the premium.

The other approach for determining premiums, the IRR approach, uses the expected discounted value of net cash flow to equity present valued at the expected equity return. The expected profit in any time period from an insurance product is

- expected investment income, including realised and unrealised gains, given by $\mathbb{E}[r_A]A_{t-1}$;
- premium income, which for a single premium will be an amount of P at time 0 and zero otherwise, less;
- expected claims, $\mathbb{E}[L_t]$, less;
- the increase in the liability (claims provision), $[R_t - R_{t-1}]$.

The profit is an equity cash flow and is present valued at the expected return on equity. The IRR approach sets the premium so that the expected return to equity, $\mathbb{E}(r_E)$, is the internal rate of return on the net cash flow to equity. Taylor (1994) [18] derives the conditions under which the IRR approach produces the same premium as the Myers-Cohn approach.

The expected discounted value approach covered above does not explicitly allow for the effect of insolvency on the value of the claim payments. If the probability of insolvency was allowed for then the value of the liability would be the present value of the liabilities assuming no insolvency less the value of a put option on the assets of the insurance company with strike price equal to the value of the liabilities. The policyholders have written a put option to the equity holders. If the value of the liabilities exceeds those of the assets, then the insurance company is in default and the equity holders put the assets to the policyholders. The value of this insolvency put needs to be included in the premium in order to value the expected claim payments. The less capital an insurance company has the higher the value of the insolvency put option and the lower the premium.

The more general valuation framework developed earlier in this paper can allow for the effect of insolvency. The insurance CAPM does not include an allowance for insolvency since CAPM effectively assumes unlimited liability, at least in the case of the assumption of normally distributed returns. To apply CAPM to the valuation of cash flows that include an allowance for insolvency requires the use of time-varying discount rates but it is more appropriate to use option pricing techniques in this case.

The Myers-Cohn approach does not determine an optimal amount of capital for an insurance company to hold. Under this pricing framework, the amount of capital held by the insurance company is exogenous. The amount of capital that the company holds does not affect the premium charged unless an explicit allowance for the cost of the insolvency put is included. Even then, there is no optimal level of capital in the model. This is equivalent to the capital structure puzzle in corporate finance, where there is no optimal capital structure unless taxes and other frictions such as agency costs and bankruptcy costs are included.

Taylor (1995) [19] develops a model that includes insolvency and derives an equilibrium level of capital for insurance companies that is endogenous to the economic model considered. He formally derives the insurance CAPM in an economic model with insurance companies, in a similar manner to the derivation of CAPM allowing only for financial markets. Allowing for capital structure is an important factor in insurance pricing. Readers are referred to Taylor (1995) [19] for the formal development of an insurance pricing model including optimal capital structure.

3.3 Frictional Cost Approach

Shareholder capital in insurance companies serves many purposes. Insurance companies provide a range of services to policyholders. These services are charged at their marginal expected costs in the premium so that it is standard in determining premiums to include an expense load to recover expenses. In fact insurance companies, will maximise profit by including loadings for expenses equal to the marginal cost.

Premiums also include an allowance for the additional taxes that the insurance company incurs and this is an expense shareholders should be compensated for, otherwise they would not provide the capital to run the insurance business. The additional tax that is incurred by the insurance company, over and above that incurred by investors in competing investments, is loaded for in the premium at its expected value discounted at the risk free rate. Full details of the impact of taxation on premiums is covered in Taylor (1994) [18].

There are also other costs involved with running an insurance company that should be allowed for in the premiums. These costs are referred to as frictional costs and they have been studied extensively in corporate finance particularly with respect to debt pricing and capital structure. An insurance company is basically a leveraged company financed by the policyholder "float" which is the provisions for outstanding claims. As a result, it is possible to draw an analogy between risky debt pricing and insurance pricing which means that research in pricing of credit risk will also provide insights into insurance pricing.

Frictional costs do provide the potential to determine an optimal capital structure for an insurance company. This is an area that has attracted recent research interest in insurance. Culp (2002) [5] provides a detailed coverage of the theory of optimal capital structure including the "trade-off theory" and the "pecking order theory".

Insurance companies hold capital for a range of reasons including regulatory requirements for risk based capital. Many insurance companies hold capital in excess of the amount required by regulation. This capital reduces the probability of insolvency and reduces bankruptcy costs that would be incurred if the company were to become insolvent.

Apart from costs of financial distress, there are agency costs of equity and debt that also need to be considered. Since management of an insurance company are the agents of the shareholders, any misalignment of the interests of management and shareholders and policyholders will result in less than optimal financial performance because of agency costs. Agency costs basically arise because the management of the insurance company does not act in the interests of shareholder, or may even act against the interests of policyholders, leading to costs that would not otherwise have occurred if incentives had been aligned.

Including frictional costs provides a model for determining an optimal level of capital for an insurance company. The interaction of capital, risk and pricing in insurance is an area that is the subject of current research by both academics and practitioners. These costs would need to be included as part of the expected costs for the insurance contract. They are company specific and reflect the leverage of the company balance sheet. They provide an additional loading in the determination of the premium rate.

3.4 Actuarial Pricing and Valuation

Actuaries have developed a range of premium principles that include a load to the expected value of losses. Chapters 10 and 11 of Lane (2002) [12] discuss actuarial approaches to pricing insurance risks as applied to reinsurance. These include the standard deviation principle which takes the expected loss and loads this with a factor times the standard deviation of the loss. There are many other principles suggested in the literature including that of Wang (Chapter 11 of Lane (2002) [12]). Further discussion of premium principles and use of non-expected utility is covered in Landsman and Sherris (2002) [11].

A concern of regulators, and increasingly actuaries in setting premiums for insurance, is the probability of insolvency of the insurance company. Dynamic

Financial Analysis is increasingly used as a modelling technique to assess this probability and to design strategies to manage this risk. Value-at-risk is a quantile based risk measure for a specified probability of loss, used for capital and risk management in banking and finance. Its drawbacks are well known, but the concept is similar to that of probability of ruin in insurance. Capital requirements and even premiums could be set that will maintain a specified probability of insolvency for an insurance company.

In recent research, Gründl and Schmeiser (2002) [10] compare the pricing of a reinsurance contract called a double-trigger contract using an actuarial premium principle with that using a financial economic model. The pricing models considered in the paper include a pricing model based on CAPM with

$$\begin{aligned} P &= \frac{1}{1+r_f}\left[\mathbb{E}\left[x\right]-\lambda cov\left(x,R_M\right)\right] \\ \text{and } \lambda &= \frac{\mathbb{E}\left[R_M\right]-r_f}{\sigma^2_{r_M}}, \end{aligned}$$

where x is the payment on the contract assumed to be at the end of the year. Gründl and Schmeiser (2002) [10] also consider the policyholder put option in the pricing.

The actuarial pricing premium principle used is based on a risk load that ensures the probability of ruin of the insurance company issuing the contract remains at a pre-specified level. Although this is not commonly used as a basis for setting premiums, this is an approach used by regulators and actuaries when setting provisions for outstanding claims on general insurance business. The prudential margin is based on a pre-specified probability of adequacy. This could be used as a justification for the loading factor in the premiums. However, Gründl and Schmeiser (2002) suggest that this approach charges policyholders a capital charge that should be met by the shareholders not the policyholders.

In banking, it is standard to provision for expected losses and to hold capital for unexpected losses. Actuaries, when reserving for insurance liabilities, provision for expected losses plus a prudential margin. Capital is then held to provide additional resources to cover unexpected losses beyond that provided in the prudential margin. This could also be used to justify a loading factor in the premiums. The two approaches differ in that, by charging a prudential margin in the form of a loading factor, policyholders are providing capital to finance the insurance company with no basis for a return for this being provided to policyholders.

Using reinsurance rates, Major and Kreps (in Chapter 10 of Lane (2002) [12] estimate a best fitting loading as a function of expected losses of the form

$$\alpha\mathbb{E}\left[L\right]^{\beta}.$$

The reinsurance market, and in particular the insurance securitisation market, provides an insurance market where prices are observable to a certain extent and where there is a primary and a limited secondary financial market for insurance risk. This allows the analysis of actual price data and an empirical basis for testing and developing insurance risk pricing models. Because the risks in the reinsurance and insurance securitisation market are often low frequency and high severity events, the loading can be very significant compared with the expected loss. The size of the loading and the availability of price data means that this is an ideal market to develop and understand insurance risk and pricing based on both economic valuation and actuarial models.

4 Electricity Pricing

With the deregulation of wholesale electricity markets and an increased interest in risk management in energy markets, the application of financial economic models to electricity risk management and valuation of electricity generation assets has become an area of increased interest to a variety of professionals. The electricity market has features not found in other financial markets such as high volatility of prices (1000% p.a. common), seasonal variation in prices (varies by time of day, week, year), price volatility correlated with price level, and both positive and negative price spikes.

There are two approaches to modelling the economics of electricity markets found in the literature. The first takes existing financial economic models and applies these to electricity by modifying the assumed price dynamics to capture observed features of electricity prices. The second approach, developed primarily by Routledge, Seppi, and Spatt (1999) [14], models the underlying micro-economics that drive electricity prices. Stavrou (2001) [17] surveys both of these approaches and places the Routledge, Seppi, and Spatt model in an equilibrium economic framework with production and consumption. The first approach treats the price process as exogenous, whereas the second approach derives the price process from the underlying economic model and the resulting price process is endogenous to the model.

More details of the topics covered in this section of the paper, including references are found in Stavrou (2001) [17].

4.1 Exogenous Price Models

To model electricity prices it is necessary to allow for a seasonal component. Empirical studies of electricity prices suggest that a log-normal, mean-reversion model with "jumps" capture the main features of these prices. As an example, assume that prices are given by

$$\begin{aligned} \ln P_t &= f(t) + Y_t, \quad \text{where} \\ dY_t &= -\kappa Y_t dt + \sigma dZ. \end{aligned}$$

Y_t is a state variable that follows an Ornstein-Uhlenbeck (mean reverting) process. Thus

$$P_t = e^{f(t)+Y_t},$$

where $f(t)$ captures the seasonal deterministic variation in electricity prices. For example

$$f(t) = \alpha + \beta D_t + \gamma \cos\left[(t+\tau)\frac{2\pi}{365}\right],$$

where D_t is a day-of-the-week dummy variable and a 365 day cycle is also included.

The price at time t is log-normally distributed with

$$E_0(\ln P_t) = f(t) + [\ln P_0 - f(0)]\, e^{-\kappa t}$$

and

$$Var_0(\ln P_t) = \frac{\sigma^2}{2\kappa}\left(1 - e^{-2\kappa t}\right).$$

Applying the Itô formula gives

$$\begin{aligned}
\frac{dP_t}{P_t} &= \mu_P dt + \sigma_P dZ, \quad \text{where} \\
\mu_P &= \kappa\left[\frac{1}{\kappa}\left(\frac{\sigma^2}{2} + \frac{df(t)}{dt}\right) + f(t) - \ln P_t\right] dt \quad \text{and} \\
\sigma_P &= \sigma.
\end{aligned}$$

The approach used to value securities or assets such as electricity generators, whose value depends on the electricity price, assumes that there is a market, such as a futures or an options market, for securities whose pay-offs depend on the electricity price. An economic assumption that the market is arbitrage free is used in order to determine a price of risk for electricity, which is denoted by $\lambda(P_t, t)$.

This is a similar approach which was originally used to develop term structure models. They are known as relative valuation models, or partial equilibrium models, since they only value one security relative to another.

To illustrate the development of an arbitrage-free economic model for contracts that depend on the electricity price, assume that two securities trade with market values at time t denoted by $G(P,t)$ and $H(P,t)$, depending on P. The security values are assumed to be perfectly correlated with the factor that drives electricity prices. The Itô formula allows us to write the dynamics of the market values as

$$dG = \mu_G dt + \sigma_G dZ_t$$

and

$$dH = \mu_H dt + \sigma_H dZ_t.$$

Prices can be determined for these securities that will not allow arbitrage profits by constructing a portfolio of σ_G units of security H, at a price of $H(P,t)$

per security, and $-\sigma_H$ units of security G, at a price per security of $G(P,t)$. The portfolio is required to be self-financing through the use of borrowing and lending at a continuous rate r, assumed to be risk free. The volatility terms in this self-financing portfolio will offset, making it no longer risky and we obtain

$$(\sigma_G\mu_H - \sigma_H\mu_G)\,dt = r\,(\sigma_G H - \sigma_H G)\,dt$$

or

$$\frac{\mu_H - rH}{\sigma_H} = \frac{\mu_G - rG}{\sigma_G}.$$

This means that, in order for there to be no arbitrage between the two securities, the excess return on each security per unit of volatility, called the "market price of risk", must be the same for each and will only depend on t and P. The same result applies for any traded security that depends only the price P.

This gives the fundamental arbitrage-free requirement for any traded security S depending on P

$$\frac{\mu_S - rS}{\sigma_S} = \lambda\,(P,t)\,,$$

where

$$\begin{aligned}\mu_S &= \frac{\partial S}{\partial t} + \mu_P P\frac{\partial S}{\partial P} + \frac{1}{2}\sigma_P^2 P^2\frac{\partial^2 S}{\partial P^2} \text{ and}\\ \sigma_S &= \sigma_P P\frac{\partial S}{\partial P}\end{aligned}$$

Substituting into the arbitrage-free requirement gives the following valuation partial differential equation

$$\frac{\partial S}{\partial t} + (\mu_P - \lambda\sigma)\,P\frac{\partial S}{\partial P} + \frac{1}{2}\sigma^2P^2\frac{\partial^2 S}{\partial P^2} - rS = 0.$$

Actuaries familiar with derivative valuation will recognise this as a fundamental partial differential equation used in the financial economic valuation of options, bonds and other contingent claims. The only difference is the form of the price process driving the electricity price and the boundary conditions. There are many techniques available to solve this PDE that have been developed in financial mathematics.

The underlying price process under the equivalent "risk neutral" measure Q is given by

$$\ln P_t = f\,(t) + Y_0e^{-\kappa t} - \frac{\lambda\sigma}{\kappa}\left(1 - e^{-\kappa t}\right) + \sigma\int_0^t e^{\kappa(s-t)}dZ^*\,(s)\,,$$

where dZ^* is an increment in a Wiener process under the risk neutral measure Q. This is the probability distribution for electricity prices which is used to determine the expected value of the pay-offs on securities that depend on the electricity price for the purposes of risk neutral valuation. The expected values under this risk neutral probability distribution are present valued at the risk free rate - typically a government bond spot rate for the maturity of the pay-off. This is "risk-neutral" valuation, although a better term for this is arbitrage-free relative valuation using a money-market account as numeraire.

Risk neutral valuation is a particular case of a more general result. In the risk neutral case, a money market account accumulating at the risk free interest rate is used as a numeraire and the Fundamental Theorem of Asset Pricing states that a probability measure Q exists such that the value of a traded asset expressed in units of the numeraire will be a martingale. Other numeraires can be used for valuation and for some valuation problems the choice of numeraire can be important.

4.2 Forward Prices

For commodities, including energy, forward prices are assumed to depend on the spot price of the commodity and a convenience yield. Convenience yields are best considered using the theory of storage where they are seen to represent an embedded timing option of the holder of the commodity if they can store the commodity for later consumption. This assumes that the commodity can be stored. Electricity is not economically storable so caution must be used when applying standard convenience yield assumptions in electricity markets.

For a storable commodity, where the convenience yield δ is assumed constant, the forward price is determined by arbitrage as

$$F_{t,T} = P_t e^{(r-\delta)(T-t)},$$

where $F_{t,T}$ is the forward price at time t for delivery at time T, P_t is the spot price at time t, and r is the continuous compounding risk free rate of interest.

It is possible to assume that the convenience yield is stochastic which leads to a 2 factor model with an exogenous convenience yield. This is the basis for many electricity pricing models. They are not covered further since in the electricity market the assumption of storage is not appropriate and convenience yields are best modelled using the theory of storage for commodities that can be stored, as in Routledge, Seppi and Spatt (2000) [15].

4.3 Endogenous Price Models

Commodity price models or forward commodity price models, as discussed in the previous section, assume complete and arbitrage-free markets. They assume exogenous dynamics for the spot price and convenience yield. These are not determined by the underlying economics of supply and demand for the commodity nor the underlying technology required to produce the commodity. They ignore optimal storage and the embedded option that storage provides. Electricity is different to many commodities and other forms of energy in that is non-storable, at least in an economical way. This means that intertemporal arbitrage is not possible. Inventories can not smooth price fluctuations.

Electricity modelling needs to take into account that it is produced by converting storable commodities - such as hydro, coal, gas - and that it is necessary to allow for cross commodity prices in an optimal conversion-storage economic model to derive the price dynamics. Such an approach leads naturally to a way of providing a link between forward markets for electricity and spot markets for electricity and commodities that are used in the production of electricity.

The main ideas underlying such an economic valuation model will be illustrated with a simple case with 2 commodities - coal (or gas) and electricity with a competitive market. The model is an economic model with both producers and consumers.

Producers have to determine an amount of coal (or gas) to store, Q^c, and also an amount to convert to electricity, B^c. Consumers determine an amount to consume of coal C^c and of electricity C^e. Consumers own shares in the electricity producers as part of their financial assets. The economic model is a representative consumer model with production (and financial markets), as well as consumers. There is an endowment of coal.

In financial economic models there is a very important result that takes the form of a separation theorem. This theorem states that managers of firms should independently maximise profits in order to maximise consumer (shareholder) expected utility. When applied in this case, electricity producers should maximise their profit from electricity production through the conversion and storage of coal in order to maximise shareholder expected utility.

Producers are assumed to convert $B^c \geq 0$ units of coal to produce $g\left(B^c\right)$ units of electricity. The technology of producing electricity is assumed to be determined by the function $g\left(\bullet\right)$. The net demand functions for electricity and coal are assumed given by $D^e\left(P_t^e, z_t\right), D^c\left(P_t^c, z_t\right)$, where z_t reflects the impact of supply and demand shocks (such as weather and breakdowns in generators) and P_t^e, P_t^c are the spot prices for electricity and coal. These demand functions are determined based on market analysis. Technical assumptions are required to ensure the market clearing prices exist.

Storage is costly so that for $Q_t^c \geq 0$ of coal stored at time t yields $\left(1-\delta_c\right) Q_t^c$ units at time $t+1$. Since electricity is non-storable we have $\delta_e = 1$. It is assumed that coal input for conversion is limited to a maximum level such that $0 \leq B^c \leq B_{\max}^c$, and it is assumed that $g\left(B^c\right)$ is twice differentiable, increasing and concave.

The producer has to maximise profit by determining the optimal amount of coal converted and stored in each period given the amount of coal stored from the last period and allowing for the randomness arising from the demand and supply shocks. The producer solves the following profit maximisation problem

$$\max_{Q_t^c, B_t^c} E\left[\left.\frac{P_{t+1}^e g_{t+1}\left(B_{t+1}^c\right)+\left(\left(1-\delta_c\right) Q_t^c - B_{t+1}^c\right) P_{t+1}^c}{1+r} - Q_t^c P_t^c \right| Q_{t-1}^c, z_t\right]$$

$$\text{such that } Q_t^c \geq 0,\ 0 \leq B_t^c \leq B_{\max}^c.$$

Consider first the optimal storage decision. The first order condition for maximising profit is given by differentiating with respect to Q_t^c, and allowing for $Q_t^c \geq 0$, to get

$$\begin{aligned} P_t^c &= E\left[\frac{(1-\delta_c) P_{t+1}^c}{1+r}\middle| Q_{t-1}^c, z_t\right] \quad \text{if } Q_t^c > 0 \\ P_t^c &\geq E\left[\frac{(1-\delta_c) P_{t+1}^c}{1+r}\middle| Q_{t-1}^c, z_t\right] \quad \text{if } Q_t^c = 0. \end{aligned}$$

Thus the coal price will be determined by the optimal storage amount of coal.

The optimal conversion decision is determined by taking the first order condition $\frac{\partial}{\partial B_t^c}$ allowing for $B_t^c \geq 0$ to get $\left(\text{where } g_t'(B_t^c) = \frac{\partial}{\partial B_t^c} g_t(B_t^c)\right)$

$$\begin{aligned} E\left[P_t^e g_t'(B_t^c) - P_t^c | Q_{t-1}^c, z_t\right] &= 0; \quad 0 < B_t^c < B_{\max}^c \\ E\left[P_t^e g_t'(B_t^c) - P_t^c | Q_{t-1}^c, z_t\right] &\leq 0; \quad B_t^c = 0 \\ E\left[P_t^e g_t'(B_t^c) - P_t^c | Q_{t-1}^c, z_t\right] &\geq 0; \quad B_t^c = B_{\max}^c. \end{aligned}$$

These first order conditions can be rearranged to get

$$\begin{aligned} \frac{P_t^c}{P_t^e} &= g_t'(B_t^c); \quad 0 < B^c < B_{\max}^c \\ \frac{P_t^c}{P_t^e} &\geq g_t'(B_t^c); \quad B^c = 0 \\ \frac{P_t^c}{P_t^e} &\leq g_t'(B_t^c); \quad B^c = B_{\max}^c \end{aligned}$$

and solving for the optimal conversion demand for coal gives a relationship between demand and the relative prices of coal and electricity that takes the form

$$B_t^c = g_t'^{-1}\left(\frac{P_t^c}{P_t^e}\right).$$

This model and the first order conditions are solved to determine the economic equilibrium price for electricity. The equilibrium electricity and coal prices are determined so that they clear the spot markets and satisfy the optimal storage and conversion requirements for coal. The net demand is $D^e(P_t^e, z_t)$ for electricity, and $D^c(P_t^c, z_t) + Q_t^c - (1-\delta^c) Q_{t-1}^c + B_t^c$ for coal, which includes demand for both storage and conversion of coal. Market clearing occurs when excess demand is zero. The conditions for equilibrium and optimal storage and conversion are solved numerically.

Clearly to implement the model it is necessary to calibrate the model including estimating net demand, conversion, and the shock process. However there is an abundant literature on these aspects of the electricity market so that it is possible to develop a realistic model. The model has been implemented in a simple example in Routledge, Seppi, and Spatt (1999) [14] for a model with gas and electricity. The results of the model produce an electricity price process that fits the features of actual data quite well. Electricity price distributions are skewed with the electricity skewness induced by the storage and conversion of gas. Price distributions derived from the model are heteroscedastic and the correlation between electricity and input fuels is not constant because of the linkage through storage and conversion of the input. These are features of empirical price data not captured in the exogenous price models.

The model provides an economic equilibrium price process that can be applied for valuation and risk management purposes. It does not require the assumptions of storage of electricity that other approaches often assume.

Computable general equilibrium models are being applied to a wide variety of areas. Electricity markets is an area where such models can provide new insights.

5 Conclusions

This paper has aimed to cover some key ideas in economic valuation from simple no-arbitrage to equilibrium pricing. The no-arbitrage assumption is very powerful and is the basis for the most commonly used actuarial technique, discounted cash flow.

In models with risk, arising from stochastic cash flows, the existence of positive stochastic discount factors is a result of the arbitrage-free requirement. An economic valuation framework using discounted expected values was developed from a simple economic equilibrium. Special cases of the framework include CAPM and option pricing models. The stochastic discount factor that results depends on the form of the utility function. Stochastic discount factors, referred to as deflators in the actuarial literature, are being used increasingly in actuarial valuations.

Since insurance has been an area where CAPM has been applied, the insurance economic valuation model has been reviewed and provides an opportunity for actuaries to discuss this topic and clarify issues about this model in the actuarial profession. In particular, recent developments in frictional costs are reviewed since these have implications for loadings in premium rates for insurance contracts.

Finally, an economic valuation model using optimal storage and conversion of fuel to generate electricity was set out in order to highlight the more complex models that can be developed and applied for risk management and valuation where the simple no-arbitrage assumptions are not sufficient.

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