Solvency, Capital Allocation and Fair Rate of Return in Insurance*

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Abstract

In this paper we consider the links between solvency, capital allocation and fair rate of return in insurance. A method to allocate capital in insurance to lines of business is developed based on an economic definition of solvency and the market value of the insurer balance sheet. Solvency, and its financial impact, is determined by the value of the insolvency exchange option. The allocation of capital is determined using a comlete markets arbitrage-free model and, as a result, has desirable properties, such as the allocated capital "adds up" and is consistent with the economic value of the balance sheet assets and liabilities. A single period discrete state model example is used to illustrate the results. The impact of adding lines of business is briefly considered. The model is readily extended to a multi-period setting.

Keywords: capital allocation, solvency, economic value, insurance.

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1 Introduction

The determination of economic capital and the allocation of capital to lines of business is an important part of the financial and risk management of an insurance company. The Society of Actuaries Economic Capital Calculation and Allocation Subgroup has developed a Specialty Guide on Economic Capital [14] that provides a review of the concepts and literature in this area.

Solvency is assessed using regulatory capital which is often determined using prescribed rules. In practice insurance companies hold higher levels of capital and economic capital is assessed using risk based models. A number of alternative methods of determining regulatory and economic capital have been proposed when taking risk into account. The ruin probability is becoming a common risk measure used to determine regulatory capital in banking and insurance. For example, regulatory capital requirements under APRA Regulations in Australia allow insurers using internal models to determine capital at a level sufficient to meet a specified probability of ruin of 0.5% over a 1 year horizon. Probability of ruin is a similar concept to that of value-at-risk (VaR) used for market risk and economic capital in banking. In an early contribution, Borch (1962) [2] considers determining the safety loading in insurance premiums using a probability of ruin concept and game theory. The allocation of the safety loading is similar to the capital allocation problem since the aim is to allocate the safety loading to different classes of policyholders based on a company wide ruin probability.

Lowe and Stannard (1997) [5] refer to early contributions of Wittstein and Kanner in 1867 who define the "mean risk" and to Hattendorf who, in 1868, discusses this concept in terms of mortality risk in life insurance and the following translated abstract appears in Appendix B of [5]:

> Far more important is the mean risk. By this one means the sum of all possible operating losses, each multiplied by its probability. This definition is well defined, permits no uncertainty, and with it one can compute the mean risk for a given insurance portfolio.

This is an equivalent concept to that of expected policyholder deficit proposed in Butsic (1994) [3]. This is related to other risk measures that avoid some of the problems with VaR such as Tail Conditional Expectation, also referred to as TailVaR. Panjer [11] (2001) defines various risk measures including TailVaR.

Merton and Perold (1993) [7] propose a basis for allocating capital using the marginal contribution of a line of business and an options based definition for risk capital. This options based definition is not the same as the expected policyholder deficit. Allocation of capital is determined by considering the marginal impact on risk based capital from adding each line of business given the other lines of business.

Myers and Read (2001) [10] also propose a basis for allocation of capital in insurance companies based on the marginal contribution to the option based default value for each line of business. This marginal concept and the determination of capital differs from the approach of Merton and Perold (1993) [7].

The Myers and Read allocation "adds up", whereas the Merton and Perold allocation does not. The approach used by Myers and Read (2001) [10] to allocate capital to line of business is similar to that applied by Zeppetella (2002) [18] to the NAIC Risk Based Capital formula. This approach to allocating capital was also used in the 1960's and 1970's to allocate tax to line of business in life insurance in order to derive an additive allocation. Mildenhall (2002) [9] identifies a short-coming in the Myers and Read (2001) [10] approach related to the need for the loss distributions to be homogeneous. Meyers (2003) [8] discusses capital allocation including the homogeneity assumption.

Phillips, Cummins and Allen (1998) [12] claim that it is not appropriate to allocate capital by line of business since, in equilibrium, it is the overall default risk of the insurer that will reflect in the price of insurance. They state that:

> prices are predicted to vary across firms depending upon firm default risk, but prices of different lines of business within a given firm are not expected to vary after controlling for liability growth rates by line.

Sherris (2003) [13] develops a model that reflects the overall default risk in equilibrium pricing using the CAPM mean-variance framework and derives a basis for capital allocation proportionate to the actuarial fair value of liabilities by line of business. There is an implicit assumption made in Sherris (2003) [13] for the allocation of the default value to lines of business similar to the implicit assumption also made in Phillips, Cummins and Allen (1998) [12]. This assumption is discussed and clarified in this paper.

In Myers and Read (2001) [10], a single period model is used and results are given for normal and log-normal distributions. Phillips, Cummins and Allen (1998) [12] use a continuous time model but applied to a single period. Risk capital in these models allows for the default or insolvency exchange option value where the insurer is assumed to exchange the liabilities due for the assets whenever the asset value is lower than the liability obligations of the insurer.

Cummins (2000) [4] reviews different methods of allocating capital to lines of business and concludes that

> more research is needed to determine which model is more consistent with value maximization.

This paper considers insurer solvency based on the economic value of the balance sheet of the insurer and the allocation of capital to lines of business. In our paper, capital is the option based default value of the insurer, which we will also refer to as the insolvency exchange option, plus the insurer surplus. This is the same as the economic value of the balance sheet equity of the insurer. The option based default value, or insolvency exchange option, is the value of the option that the shareholders have to exchange the liability obligations for the assets whenever the insurer becomes insolvent. We consider property and casualty (non-life or general) insurance although similar considerations apply to life insurance. The model used is similar to that in Myers and Read (2001) [10] except that we use a general economic valuation framework with a stochastic discount factor. Results are also derived using a discrete state model. We show that the ratio of the default value to liability value for a line of business does not change with different allocations of capital to lines of business, as implied by the allocation formula derived in Myers and Read (2001) [10]. For any given line of business the default value to liability ratio depends on the distribution of the liability value for the line of business and its correlation with the assets, and not the line of business surplus ratio. We show how capital allocation to lines of business can be carried out so that the allocation will always "add up" and can be made positive for each line of business.

The paper begins by considering the economic value of the insurer balance sheet. The value of the insolvency exchange option and its effect on the market value of the liabilities and equity of the insurer is covered. The payoffs to each line of business allowing for the insurer insolvency and the value of the insolvency exchange option by line of business is determined. The allocation of the capital of the insurer to line of business is then considered, including both the insurer surplus and the insolvency exchange option components of the insurer equity. This allocation is shown to "add up" and is shown to require an allocation of assets, liabilities and the insolvency exchange option by line of business. The allocation of assets is not unique and a number of methods for doing this are considered. A discrete state model is then used to illustrate the results. Finally, proposed methods of allocating capital in insurance based on covariances are related to the method of this paper.

2 Economic Valuation of the Insurer Balance Sheet

In an insurance company, surplus is defined as the difference between the value of the assets and the value of the liabilities. Realistic measures of surplus require that assets be valued at market value and liabilities be valued using actuarial techniques that place a fair value on the expected future claim payments. Actuarial techniques ignore the default, or credit, risk of the insurer and value the projected expected future claim amounts.

The value of the liabilities ignoring default on payment of claims will not capture the option value that the shareholders have arising from limited liability. If the assets are insufficient to meet the claim payments then an insurer is insolvent and shareholders are only liable for the capital they have invested into the company and the retained earnings that have not been previously distributed as dividends.

The model we use for the insurance company is similar to that in Merton and

Perold (1993) [7], Myers and Read (2001) [10], Butsic (1994) [3], and Phillips, Cummins and Allen (1998) [12]. The model is a single period model. The model assumes that the cashflows for assets and liabilities are valued so that the model is arbitrage-free. We assume that there exists a bank account accumulating at the risk free interest rate and a risk neutral Q-measure. The model is assumed to be complete so that the Q-measure uniquely values all cashflows in the model. We could have selected a different numeraire for valuation purposes, such as the value of the market portfolio of assets, and an equivalent martingale measure corresponding to this numeraire, and all the results in the paper would hold.

In the model, the fair price charged for insurance will reflect the impact of insolvency on the payoff on the claims under the insurance contracts at the end of period. This will depend on the amount of capital subscribed and also the investment policy of the company through the volatility of the assets. Thus in order to determine a fair price for insurance in the model it is necessary to specify in advance the solvency ratio and investment policy of the company. In the model the company writes insurance business with a known distribution of losses and subscribes capital to meet a fixed and known solvency ratio. The investment policy as given by the proportion of total assets invested in different assets is known at the start of the period. The return distribution for the assets is also known. We assume in the model that the company does not purchase reinsurance, or equivalently, that the reinsurance strategy is fixed, known and fairly priced and all results are considered net of reinsurance.

Under these assumptions, in equilibrium, the fair rate of return for a liability of the insurer taking into account the insolvency risk, can be determined since the payoffs at the end of the period have known distributions. Capital also earns a fair rate of expected return based on the balance sheet solvency ratio of the insurer and the expected return and risk of the assets and liabilities.

2.1 Assets

The investment strategy of the company is known and given by w_j the weight of asset $j = 1, \dots, J$ in the insurer's portfolio. The end of period payoff distribution for all assets $j = 1, \dots, J$ is also known. The initial value of the assets, V_A, consists of the fair market premiums of the policyholders and the capital of the shareholders. The insurer invests the premiums and capital in a portfolio of assets determined according to its known investment policy.

The random end of period payoff for asset j is denoted by A_j, and this incorporates any loss arising from default of the issuer of the asset through its equilibrium market price.

The initial market value of the assets will be

$$\begin{aligned} V_A &= \sum_{j=1}^{J} E^Q\left[\frac{A_j}{1+r}\right] \\ &= E^Q\left[\frac{A}{1+r}\right] \end{aligned}$$

where $A_j = w_j A$, and where Q is a risk-neutral equivalent probability measure equivalent to the real-world probability measure P and r is the no default risk free interest rate on an investment/borrowing that always pays a fixed amount in all states of the world, usually referred to as a bank account.

Denote by R_{A_j} the return on asset A_j, which implicitly allows for credit risk, so that it is the actual realised return on the asset, and by R_A the return on the asset portfolio. We then have

$$A = V_A (1 + R_A) = V_A \left(1 + \sum_{j=1}^{J} w_j R_{A_j} \right)$$

Insolvency occurs when the assets of the insurer are insufficient to meet the outstanding claims. The equity holders in the insurer effectively exercise an option to exchange the liability to pay the insurance outstanding claim amounts for the assets of the company whenever the value of the assets is below the total outstanding claim liability. This could be described as an asset-liability mismatch risk since if the assets were to exactly match the liability payoffs then there would be no risk of insolvency.

When the insurer defaults on its claim payments this happens for all lines of business at the same time. When default occurs it is assumed that the losses arising from default are shared amongst the policyholders in proportion to their claims payable at the date of default. In other words, policyholders who are owed money arising from an insurance claim rank proportionately according to the amount of their claim in the event of default. This means that the default value for a line of business depends on the distribution of the claims for the line of business as well as on the covariance of the claims for the line of business with the assets and the other lines of business.

2.2 Liabilities

The insurer writes multiple (K) lines of business denoted by $k = 1, \ldots, K$. These could be considered as individual policies. Line of business k incurs the random claim amount L_k at the end of the period, assuming unlimited liability. L_k is not affected by the amount of capital, dividend policy, investment policy, reinsurance strategy and any other actions of the insurer that may impact on its ability to pay the liabilities under the insurance contracts.

The end-of-period total claim payments for the insurance company is

$$L = \sum_{k=1}^{K} L_k.$$

The value of the liability, assuming full payment, can be written as

$$V_L = E^Q \left[\frac{L}{1+r} \right] = \sum_{k=1}^{K} E^Q \left[\frac{L_k}{1+r} \right]$$

Even though these are assumed to be paid in full, the liability claim payments are still risky since the future pay-off is a random variable. We can also write the value of the liability in terms of real world or historical probabilities as

$$\begin{aligned} V_L &= E^P[mL] \\ &= E^P[m]\, E^P[L] + cov^P(m, L) \\ &= \frac{E^P[L]}{1+r} + cov^P(m, L) \end{aligned}$$

where m is a stochastic discount factor. This value of the liabilities allows for relevant economic risk factors but does not take into account the insolvency of the insurance company since it assumes that the policyholder claims are always paid. In general L will depend on a variety of risk factors and m will price these risk factors, along with all risk factors that cashflows in the model are dependent on.

If we let one plus the liability growth rate be denoted by $1 + R_L = \frac{L}{V_L}$ then

$$\begin{aligned} 1 + E[R_L] &= \frac{E^P[L]}{V_L} \\ &= (1+r)\left[1 - cov^P\left(m, \frac{L}{V_L}\right)\right] \\ &= (1+r)\left[1 - cov^P(m, 1 + R_L)\right] \end{aligned}$$

We also have for line of business k

$$1 + E[R_{L_k}] = (1+r)\left[1 - cov^P(m, 1 + R_{L_k})\right]$$

Thus the expected growth rate of a liability will be greater than or less than the risk free rate depending on whether $cov^P(m, 1 + R_{L_k})$ is less than or greater than zero. It is the covariation of the liability growth rate with the stochastic discount factor that determines the risk adjustment to the liability payoff in determining values. A negative covariance with the stochastic discount factor results in a positive risk premium over the risk free return.

The economic value of the insurer liabilities will be based on the pay-off to the policyholders taking into account the impact of insolvency on the claim payments and will depend on the investment policy and the amount of capital of the particular insurer issuing the policy. This economic value is the price paid for the insurance policy in a competitive market and represents a fair rate of return to policyholders taking into account the insurer limited liability for insurance claims. It will reflect both the underlying economic risk factors of the insurance liability as well as the level of solvency of the insurance company. Insurers with higher levels of economic capital will, in equilibrium, receive higher prices for their insurance contracts under the assumptions of the model.

The competitive premium that policyholders will pay in total will be $V_L - D$

where D is the value of the insolvency exchange option for the insurer given by

$$\begin{aligned} D &= \frac{E^Q\left[\max\left(L-A,0\right)\right]}{1+r} \\ &= \frac{E^Q\left[L-A|\, L-A>0\right]\Pr^Q\left[L-A>0\right]}{1+r} \\ &= \frac{E^Q\left[L-A|\,\frac{A}{L}<1\right]\Pr^Q\left[\frac{A}{L}<1\right]}{1+r} \end{aligned}$$

The insolvency exchange option value reflects both the probability of insolvency and the expected severity of the insolvency based on the risk neutral probabilities. For extreme events, assuming risk aversion, the risk neutral probabilities will usually exceed the actual historical or real world probabilities. Using real world ruin probabilities, such as in VaR or probability or ruin approaches to setting risk based capital, will not be an adequate measure of insolvency risk.

We will consider the competitive market pricing of individual lines of business allowing for the insurer insolvency exchange option later. This will require a basis for allocating the value of the insolvency exchange option to lines of business. For the time being we consider only the total balance sheet liabilities.

2.3 Insurer Capital and Equity

In the model we assume that the total amount of initial assets is determined so that the solvency ratio using the market value balance sheet will be a fixed and known proportion of the value of the liabilities ignoring the default option. This can be determined in many different ways, as it is in practice. For instance, it might be based on regulatory capital requirements plus a margin or be based on an economic capital risk measure such as Value-at-Risk or TailVaR.

Since the insurer is solvent at the start of the period, the amount of initial capital will always be such that the initial value of the assets exceeds the initial value of the liabilities. We denote the solvency ratio by s so that $V_A = (1+s)\,V_L$. This insurer solvency ratio is reflected in the market value and price for of the insurance contracts written by the insurer. A solvent insurer will have $s > 0$.

The market value of the initial actuarial surplus is given by

$$S = V_A - V_L > 0$$

where the asset values allow for the issuer default but the actuarial liability values of the insurer do not allow for the insurer default risk. The market value of the equity will be the actuarial surplus plus the value of the insolvency exchange option. Since the market value of the equity is the market value of the assets less the market value of the liabilities we have

$$\begin{aligned} V_X &\equiv V_A - (V_L - D) \\ &= sV_L - D > 0 \end{aligned}$$

This is the same balance sheet value as in Myers and Read (2001) [10].

We can write

$$
\begin{aligned}
V_X &= \left[\frac{E^P[A]}{1+r} + cov^P(m,A)\right] - \left[\frac{E^P[L]}{1+r} + cov^P(m,L)\right] + D \\
&= \left[\frac{E^P\left[\sum_{j=1}^{J} A_j\right]}{1+r} + cov^P\left(m, \sum_{j=1}^{J} A_j\right)\right] \\
&\quad - \left[\frac{E^P\left[\sum_{k=1}^{K} L_k\right]}{1+r} + cov^P\left(m, \sum_{k=1}^{K} L_k\right)\right] + D \\
&= \sum_{j=1}^{J}\left[\frac{E^P[A_j]}{1+r} + cov^P(m,A_j)\right] - \sum_{k=1}^{K}\left[\frac{E^P[L_k]}{1+r} + cov^P(m,L_k)\right] + D
\end{aligned}
$$

The terms $cov^P\left(m, \sum_{j=1}^{J} A_j\right)$ and $cov^P\left(m, \sum_{k=1}^{K} L_k\right)$ are usually regarded as risk loadings by actuaries. The risk loadings for the total insurer comprise risk premiums for each asset and for each liability and also a contribution from the insolvency exchange option. The risk loadings reflect the covariance of the payoffs on the assets and liabilities with the stochastic discount factor. They are additive and can be negative if the covariance of the market stochastic discount factor with the payoff is negative. Under the economic valuation model used here, based on the assumption of a complete market, the risk loadings are additive. Allocating the liability risk loadings is not the same as allocating capital to lines of business. To allocate capital it is also necessary to allocate assets and the insolvency exchange option value D.

2.4 The Economic Balance Sheet

At the end of the period the payoffs on the balance sheet of the insurer will be:

Balance Sheet	**Initial Value**	**End of Period Payoff**
Assets	V_A	A
Liabilities	$V_L - D$	$\min(L,A)$ $= L + \min(A-L,0)$ $= L - \max(L-A,0)$
Equity	$S + D$	$\max(A-L,0)$ $= A - L - \min(A-L,0)$ $= A - L + \max(L-A,0)$

At the start of the period the following is assumed to happen. The insurer sets its investment policy (w_j for all j), determines the liability risks that the company will underwrite and its solvency ratio, s. This information is assumed known and reflected in the valuation of cashflows. The distribution of liability risks, L, is known and the value of these liabilities ignoring the insurer default option, V_L, is given by the risk neutral Q-probabilities, or equivalently the stochastic discount factor, since we assume a complete market. The total value of the assets is determined from the liability value and the solvency ratio, since this is just $(1+s)\, V_L$.

The amount of capital subscribed and the market premium charged to the policyholders must then be determined taking into account the insolvency exchange option. The solvency ratio, s, determines the amount of insurer capital. Given the distribution of both A and L, the value of the insolvency exchange option is

$$D = \frac{E^Q \left[\max\left(L - A, 0\right)\right]}{1 + r}$$

The total market premiums for the policyholders is $V_L - D$, and the capital subscribed is $V_A - (V_L - D)$.

The premiums charged are fair, allowing for the insolvency of the insurer, and the balance sheet structure is determined by the liabilities underwritten and the target solvency ratio. Given V_L and the distribution of L, once s is fixed then V_A is also known. The distribution of the payoff from the assets, A, is determined by both V_A and the known investment policy of the insurer. The distributions of both A and L are used to determine the insolvency exchange option value D. Given V_A, V_L, and D the value of V_X is determined and the economic value of the insurer balance sheet is fully derived.

The capital earns a fair rate of expected return since all assets and liabilities, including the insolvency exchange option, are fairly priced under the risk neutral Q-measure. The fair rate of return reflects the leverage of the insurer balance sheet.

3 Allocation to Line of Business

The allocation of the capital to lines of business is often used in practice to measure the financial performance of a line of business in terms of its expected return on allocated capital. In this case the aim is to allocate the total equity V_X to line of business to determine an expected return on equity by line of business. The return to a line of business will include a component for investment income so it is also necessary to allocate assets to lines of business as well as allocating the insolvency exchange option.

To begin with consider the allocation of the insolvency exchange option. We assume that all lines of business rank equally in the event of default so that policyholders who have claims due and payable in line of business k will be entitled to a share $\frac{L_k}{L}$ of the assets of the company where the total outstanding claim amount is $L = \sum_{k=1}^{K} L_k$. This is the standard situation for policyholders of insurers. They rank equally for outstanding claim payments in the event of default of the insurer. No priority is assumed for any line of business.

The end-of-period payoff to line of business k is therefore well defined based on this equal priority as

$$\begin{array}{ll} \frac{L_k}{L}A & \text{if } L > A \text{ (or } \frac{L}{A} > 1) \\ L_k & \text{if } L \leq A \text{ (or } \frac{L}{A} \leq 1) \end{array}$$

In either case the payoff on the assets will be A.

If we let the value of the exchange option allocated to line of business k be denoted by D_k, then this is given by the value of the pay-off to the line of business in the event of insurer default. This is

$$D_k = \frac{1}{1+r} E^Q \left[L_k \max \left[1 - \frac{A}{L}, 0 \right] \right]$$

The total company level insolvency exchange option for the insurer is

$$\begin{aligned} D &= \frac{1}{1+r} E^Q \left[\max \left[L - A, 0 \right] \right] \\ &= \frac{1}{1+r} \sum_{k=1}^{K} E^Q \left[L_k \max \left[1 - \frac{A}{L}, 0 \right] \right] \end{aligned}$$

The value of the insolvency exchange option for each line of business "adds up" to the total insurer value. This is easily shown since

$$\begin{aligned} \sum_{k=1}^{K} D_k &= \frac{1}{1+r} \sum_{k=1}^{K} E^Q \left[L_k \max \left[1 - \frac{A}{L}, 0 \right] \right] \\ &= \frac{1}{1+r} E^Q \left[\sum_{k=1}^{K} L_k \max \left[1 - \frac{A}{L}, 0 \right] \right] \\ &= \frac{1}{1+r} E^Q \left[L \max \left[1 - \frac{A}{L}, 0 \right] \right] \\ &= \frac{1}{1+r} E^Q \left[\max \left[L - A, 0 \right] \right] \\ &= D \end{aligned}$$

It remains to allocate the assets to lines of business in order to determine an allocation of capital. There is no unique way to do this since the allocation of assets to line of business is an internal insurer allocation that will have no economic impact on the payoffs or risks of the insurer since assets are available to meet the losses of all lines of business. Note that an allocation of assets to line of business is implicit in Myers and Read (2001) [10].

Assume that the proportion of the assets allocated to line of business k is α_k. We wish to determine the α_k for $k = 1, \ldots, K$ such that $\sum_{k=1}^{K} \alpha_k = 1$. Note that the allocation of assets to line of business does not affect the allocation or value of the insolvency exchange option in any way. In Myers and Read (2001) [10] the allocation of surplus to lines of business, which involves an implicit allocation of assets to line of business, affects the value of the insolvency exchange option for each line of business. We have shown that this can not be the case since the insolvency exchange option for a line of business is determined by the line of business payoff in the event of insurer insolvency. In fact

$$D_k = \frac{1}{1+r} E^Q \left[L_k \max \left[1 - \frac{A}{L}, 0 \right] \right]$$

and this depends only on the insurer total surplus and not the allocation of the surplus to line of business. The value is determined by the distribution of the overall balance sheet $\frac{A}{L}$ ratio and the amount of claims for line of business k in the event that the insurer becomes insolvent.

Since the allocation of assets is not unique, it is possible to allocate the assets to lines of business using a range of criteria. Lines of business with higher asset allocations will be allocated a higher proportion of investment returns. The expected return on allocated equity by line of business will reflect the implied leverage by line of business resulting from the allocation of the assets. A different level of leverage assumed for different lines of business will mean the expected return on equity by line of business will be different. The allocation may have economic significance if it impacts on decision making and it may be of economic significance in a model that includes market frictions that differ by line of business or in an incomplete insurance market model.

Two possible methods of allocating assets to lines of business are considered here. Surplus could be allocated to lines of business so that each line of business has the same solvency ratio as for the total insurer. In this case the solvency ratio for line of business k would be

$$s_k = \frac{\alpha_k V_A - V_{L_k}}{V_{L_k}}$$

and for the total insurer

$$s = \frac{\sum_{k=1}^{K} \left(\alpha_k V_A - V_{L_k} \right)}{\sum_{k=1}^{K} V_{L_k}} = \frac{V_A - V_L}{V_L}$$

Thus we would select α_k so that $s_k = s$ for all k such that $\sum_{k=1}^{K} \alpha_k = 1$.

An alternative basis could be to allocate the assets so that the expected return on allocated capital will be equal across all lines of business and the same as for the total insurer. Denote the allocation of capital to line of business k by V_{X_k} where

$$V_{X_k} = \left(\alpha_k V_A - V_{L_k} + D_k \right)$$

The expected return on capital for line of business k will be

$$\begin{aligned}
E^P\left[R_{X_k}\right] &= E^P\left[\frac{\alpha_k\left(A-V_A\right)+\left(V_{L_k}-D_k\right)-L_k\left(1-\max\left[1-\frac{A}{L},0\right]\right)}{V_{X_k}}\right] \\
&= \frac{1}{V_{X_k}}E^P\left[\alpha_k A - L_k\left(1-\max\left[1-\frac{A}{L},0\right]\right)\right]-1 \\
&= \frac{\alpha_k E^P\left[A\right]-E^P\left[L_k\left(1-\max\left[1-\frac{A}{L},0\right]\right)\right]}{V_{X_k}}-1 \\
&= \alpha_k r-(1+r)\left[cov^P\left(m,R_{X_k}\right)\right]
\end{aligned}$$

The expected return on equity for the company is

$$\begin{aligned}
E^P\left[R_X\right] &= E^P\left[\frac{\left(A-V_A\right)+\left(V_L-D\right)-L\left(1-\max\left[1-\frac{A}{L},0\right]\right)}{V_X}\right] \\
&= r-(1+r)\left[cov^P\left(m,R_X\right)\right]
\end{aligned}$$

We would then select α_k so that $E^P\left[R_{X_k}\right]=E^P\left[R_X\right]$ for all k such that $\sum_{k=1}^{K}\alpha_k=1$.

Note that regardless of how the assets are allocated, the capital allocated to lines of business will always “add up”, provided the insolvency exchange option is allocated to lines of business correctly, since

$$\begin{aligned}
V_X &= \frac{1}{1+r}E^Q\left[A-L+\max\left(L-A,0\right)\right] \\
&= \frac{1}{1+r}E^Q\left[\sum_{k=1}^{K}\left[\alpha_k A-L_k+L_k\max\left(1-\frac{A}{L},0\right)\right]\right] \\
&= \frac{1}{1+r}\sum_{k=1}^{K}E^Q\left[\alpha_k A-L_k+L_k\max\left(1-\frac{A}{L},0\right)\right] \\
&= \sum_{k=1}^{K}\left(\alpha_k V_A-V_{L_k}+D_k\right) \\
&= \sum_{k=1}^{K}V_{X_k}
\end{aligned}$$

provided $\sum_{k=1}^{K}\alpha_k=1$, but with no other restrictions on α_k.

As long as the insurer is solvent at the start of the period, so that $s>0$, the first of these methods will ensure that capital allocations to lines of business will be positive for each line of business since we have

$$\begin{aligned}
V_{X_k} &= \left(\alpha_k V_A-V_{L_k}+D_k\right) \\
&= s_k V_{L_k}+D_k \\
&= sV_{L_k}+D_k
\end{aligned}$$

and with $s > 0$ and $D_k \geq 0$ we have $V_{X_k} > 0$ for all k. However, in general, it is possible to have negative allocations of capital to lines of business since the allocation of assets is not unique. This does not imply that the line of business generates capital since this is an internal allocation of capital that has no economic impact.

4 Market Premiums by Line of Business

Phillips, Cummins and Allen (1998) [12] and Sherris (2003) [13], amongst others, derive an equilibrium pricing result for individual lines of business and individual policies, allowing for the insurer level insolvency exchange option value. In the Phillips, Cummins and Allen (1998) [12] paper, on page 605, the market value of the policyholders claim on the firm in line of business i, and therefore the premium they are willing to pay, is given by their equation (14) as

$$PH_i(\tau) = L_i e^{-\left(r_f - r_{L_i}\right)\tau} - w_{L_i} I(A, L, \tau)$$

where τ is the time to the end of the period (in the single period case we would have $\tau = 1$), L_i is the market value of the insurer's loss liabilities to policyholder class i at τ, r_f is the continuous compounding risk free rate of interest, r_{L_i} is the expected growth rate of liabilities to policyholder class i, $w_{L_i} = \frac{L_i}{\sum_i L_i}$ is the proportion of the liabilities to policyholder class i to total liabilities, and $I(A, L, \tau)$ is the same as the insolvency exchange option in this paper. We can rewrite this equation using our notation as

$$V_{L_k} - D_k = V_{L_k} - \frac{V_{L_k}}{\sum_k V_{L_k}} D$$

In the Sherris (2003) [13] paper, again using our notation, the payoff for the liability k is given as

$$L_k - L_k \max\left[1 - \frac{A}{L}, 0\right]$$

and, using the assumptions for the Margrabe [6] (1978) exchange option value, the fair price of the insurance contract is given as

$$V_{L_k}\left\{1 - \left\{N(d_1) - \frac{V_A}{V_L} N(d_2)\right\}\right\}$$

Both these papers implicitly assume that

$$D_k = \frac{V_{L_k}}{\sum_k V_{L_k}} = \frac{V_{L_k}}{V_L} D$$

which implies that

$$\begin{aligned} D_k &= \frac{1}{1+r} E^Q \left[L_k \max \left[1 - \frac{A}{L}, 0 \right] \right] \\ &= \frac{1}{1+r} E^Q \left[\frac{L_k}{L} \max [L - A, 0] \right] \\ &= \frac{V_{L_k}}{V_L} D \\ &= \frac{1}{1+r} \frac{V_{L_k}}{V_L} E^Q [\max [L - A, 0]] \end{aligned}$$

Now

$$\begin{aligned} D_k &= \frac{1}{1+r} E^Q \left[\frac{L_k}{L} \max [L - A, 0] \right] \\ &= \frac{1}{1+r} \left\{ E^Q \left[\frac{L_k}{L} \right] E^Q [\max [L - A, 0]] - cov^Q \left(\frac{L_k}{L}, \max [L - A, 0] \right) \right\} \end{aligned}$$

and for the above result to hold we require

$$cov^Q \left(\frac{L_k}{L}, \max [L - A, 0] \right) = 0$$

and

$$E^Q \left[\frac{L_k}{L} \right] = \frac{V_{L_k}}{V_L}$$

This would hold if the liabilities were deterministic, in other words, risk free. Since this is not the case in these models, the line of business pricing results in the above papers are based on an assumption that does not hold.

5 A Discrete State Model and Example

We will now develop a single period discrete state complete markets model including assets and insurer liabilities and use a simple version of the model to illustrate our results with a numerical example. Assume that the market consists of J assets, including a risk free asset, and K distinct insurance risks that are regarded as separate lines of business. For a complete market discrete state model we require $J+K$ states. We number the states $\omega = 1, 2, \ldots, J+K$. Assume that the real world probabilities of the states are given by p_ω; $\omega = 1, 2, \ldots, J+K$, and the risk-neutral Q probabilities are given by q_ω; $\omega = 1, 2, \ldots, J+K$. The risk free asset, which we assume to be asset $j = 1$ has initial value 1 and payoffs in all states equal to $1 + r$ where r is the risk free (and default free) interest rate. The liabilities are the actual claim costs and ignore the effect of insurer insolvency. Insurance policies are options on these underlying liabilities that allow for insurer insolvency risk. In this model insurance policies are contingent securities whose payoff depends on the underlying insurance risks.

Consider an insurer holding a portfolio of assets and underwriting insurance risks by issuing insurance policies on a number of lines of business. For this insurer the value of the assets will be denoted by V_{A_j}; $j = 1, \ldots, J$, and the value of the insurance risks (liabilities) will be denoted by V_{L_k}; $k = 1, \ldots, K$. The payoffs to asset j in state ω will be denoted by $A_{j\omega}$; $\omega = 1, 2, \ldots, J + K$, and the liability payoff for liability k in state ω will be denoted by $L_{k\omega}$.

We then have

$$V_{A_j} = \frac{1}{1+r} \sum_{\omega=1}^{J+K} q_\omega A_{j\omega} \quad \text{for } j = 1, \ldots, J$$

and

$$V_{L_k} = \frac{1}{1+r} \sum_{\omega=1}^{J+K} q_\omega L_{k\omega} \quad \text{for } k = 1, \ldots, K$$

Consider a particular insurer that writes these lines of business with total liability value, ignoring the insolvency exchange option, given by

$$V_L = \sum_{k=1}^{K} V_{L_k}$$

and with a target solvency ratio of s for its surplus. Thus the total value of the assets of the insurer will be

$$V_A = (1 + s) V_L$$

and the weight of asset j in the insurer asset portfolio is then

$$w_j = \frac{V_{A_j}}{V_A}$$

The policyholders in a particular line of business will pay a market based premium equal to the fair value of the payoff on the insurance contracts, allowing for the reduction in payment of claims in the event that the insurer becomes insolvent. The value of this reduction is equal to

$$D_k = \frac{1}{1+r} \sum_{\omega=1}^{J+K} q_\omega \left[L_{k\omega} \max \left[1 - \frac{A_\omega}{L_\omega}, 0 \right] \right]$$

where

$$A_\omega = \sum_{j=1}^{J} A_{j\omega}$$

and

$$L_\omega = \sum_{k=1}^{K} L_{k\omega}$$

Thus at the start of the period, the policyholders for line of business k will pay premiums of

$$V_{L_k} - D_k$$

and shareholders will invest capital amounting

$$V_A - \sum_{k=1}^{K} [V_{L_k} - D_k] .$$

The insurer balance sheet will have the required solvency ratio and this will be reflected in the market price of the premiums for the lines of business.

To illustrate these results consider the following simple example where we assume a single risky asset and two lines of business. The numbers used are not meant to be realistic, but are designed to highlight the key results. The assumed payoff for a unit of the risky and risk free asset and the payoff to the liabilities as well as the P and Q probabilities are given in Table 1.

Table 1: Probabilities and Payoffs for Example Insurer

			Time 1 Payoffs			
State	P-probs	Q-probs	Risky Asset	Risk Free Asset	Liability 1	Liability 2
1	0.1	0.1	0.6	1.05	200	40
2	0.6	0.4	1.1	1.05	4	10
3	0.2	0.4	1.0	1.05	2	4
4	0.1	0.1	1.5	1.05	0	310
Time 0 Value			1.0	1.0	21.3333	38.6667

The time 0 values are expected values determined using the Q probabilities and discounted at the risk free rate. For the insurer who underwrites both Liability 1 and Liability 2 with the payoffs as given in Table 1 the total value of liabilities will be $60 = 21.3333 + 38.6667$. We assume that the insurer has a target surplus ratio of 2.3333. Thus the total assets of the insurer will be $(1 + 2.3333) \times 60 = 200$ and the surplus is 140.

To determine the premium for the liabilities allowing for the default option we need to specify the payoffs on the insurer balance sheet for each line of business allowing for insurer default. Table 2 gives the payoffs for the assets and liabilities as well as the amount of liabilities not met because of insufficient assets. Note that the insurer defaults in both State 1 and State 4.

Table 2 Insurer Balance Sheet Payoffs

	Time 1 Insurer Balance Sheet Payoffs				
State	Assets	L_1	L_2	Total L	$max(L - A, 0)$
1	120	200	40	240	120
2	220	4	10	14	0
3	200	2	4	6	0
4	300	0	310	310	10
Time 0 Value	200	21.3333	38.6667	60	12.381

The payoffs for each line of business are reduced by their share of any asset shortfall in the event of insolvency. In this example the shortfalls for each line of business are given in Table 3.

Table 3 Liability Shortfalls in the Event of Insolvency

	Time 1 Liability Shortfalls	
State	$D_1 = L_1 \max(1-\frac{A}{L}, 0)$	$D_2 = L_2 \max(1-\frac{A}{L}, 0)$
1	100	20
2	0	0
3	0	0
4	0	10
Time 0 Value	9.5238	2.8571

In State 1 both lines of business are paid less than their full outstanding claims because of a shortfall of assets. In State 4 only line of business 2 will have less than the full outstanding claims paid. The premium for each line of business is determined allowing for the insurer insolvency exchange option value. For line of business 1 the premium will be $21.3333 - 9.5238 = 11.8095$ and for line of business 2 it will be $38.6667 - 2.8571 = 35.8095$. The allocation of the insurer shortfall of assets over liabilities is based on equal priority of the policyholders to the assets for each line of business. Thus in State 1 the shortfall of 120 is allocated in proportion to the outstanding liabilities so that $\frac{200}{240} \times 120 = 100$ is the shortfall for line of business 1 and $\frac{40}{240} \times 120 = 20$ is the shortfall for line of business 2.

The ratio of the insolvency exchange option value to the value of the liabilities for the insurer and for each line of business is

$$d = \frac{D}{V_L} = \frac{12.381}{60} = 0.2063$$

$$d_1 = \frac{D_1}{V_{L_1}} = \frac{9.5238}{21.3333} = 0.4464$$

and

$$d_2 = \frac{D_2}{V_{L_2}} = \frac{2.8571}{38.6667} = 0.0739$$

The surplus ratio for the insurer is

$$s = \frac{S}{L} = \frac{200 - 60}{60} = 2.3333$$

The economic capital of the insurer at time 0 will be the value of the assets less the value of the liabilities ignoring the insolvency costs and plus the value of the insolvency exchange option which is $200 - 60 + 12.381 = 152.381$. This can be allocated to lines of business by allocating the individual components that make up the capital. The allocation of the liabilities and the insolvency exchange option value is already determined so that it remains to allocate the assets.

Assets can be allocated so the same solvency ratio will apply to each line of business and for the total insurer. For this to hold we would allocate 71.1111 of the asset value to line of business 1 and 128.8889 to line of business 2. This would then give a capital allocation of $71.1111 - 21.3333 + 9.5238 = 59.3016$ to line of business 1 and $128.8889 - 38.6667 + 2.8571 = 93.0794$ to line of business 2. The solvency ratios for each line of business are then

$$\frac{71.1111 - 21.3333}{21.3333} = 2.3333$$

and

$$\frac{128.8889 - 38.6667}{38.6667} = 2.3333$$

confirming that this allocation of assets produces line of business solvency ratios equal to the insurer solvency ratio as required.

We can also allocate the capital to lines of business to equate the expected return to capital by line of business and to the insurer expected return to equity. If we allocate 50.3544 of the asset value to line of business 1 and 149.6456 to line of business 2 then this will equate the expected return to capital (equity) for each line of business. This would give a capital allocation of $50.3544 - 21.3333 + 9.5238 = 38.5449$ to line of business 1 and $149.6456 - 38.6667 + 2.8571 = 113.8361$ to line of business 2.

In order to confirm this we need to determine the expected return to equity. To do this we need to specify the equity payoffs in each state and use the P-probabilities to determine the expected return. Table 4 gives the insurer equity payoffs.

Table 4 Insurer Equity Payoffs

	Time 1 Insurer Equity Payoffs			
State	P-probs	Assets	Total L	Equity $= max(A - L, 0)$
1	0.1	120	240	0
2	0.6	220	14	206
3	0.2	200	6	194
4	0.1	300	310	0
Time 0 Value		200	60	152.3810

The expected return to equity for the insurer is

$$\frac{0.1 \times 0 + 0.6 \times 206 + 0.2 \times 194 + 0.1 \times 0}{152.3810} - 1 = 0.06575$$

Based on the capital allocation to line of business the payoffs to each line of business are given in Tables 5 and 6.

Table 5 Allocated Equity Payoffs for Line of Business 1

	Time 1 Line of Business 1 Allocated Payoffs				
State	P-probs	Assets	L_1	D_1	Equity
1	0.1	30.2126	200	100	-69.7874
2	0.6	55.3898	4	0	51.3898
3	0.2	50.3544	2	0	48.3544
4	0.1	75.5316	0	0	75.5316
Time 0 Value		50.3544	21.333	9.5238	38.5449

Table 6 Allocated Equity Payoffs for Line of Business 2

	Time 1 Line of Business 2 Allocated Payoffs				
State	P-probs	Assets	L_2	D_2	Equity
1	0.1	89.7874	40	20	69.7874
2	0.6	164.6102	10	0	154.6102
3	0.2	149.6456	4	0	145.6456
4	0.1	224.4684	310	10	-75.5316
Time 0 Value		149.6456	38.6667	2.8571	113.8361

The expected return to equity for each line of business is

$$\frac{0.1 \times -69.7874 + 0.6 \times 51.3898 + 0.2 \times 48.3544 + 0.1 \times 75.5316}{38.5449} - 1 = 0.06575$$

for line of business 1 and

$$\frac{0.1 \times 69.7874 + 0.6 \times 154.6102 + 0.2 \times 145.6456 + 0.1 \times -75.5316}{113.8361} - 1 = 0.06575$$

for line of business 2. This confirms that this allocation of assets results in expected returns to equity that are equal by line of business.

Since the allocation of capital to lines of business is not unique, many possible approaches could be adopted. As long as the value of the assets, liabilities and the insolvency exchange option are determined using an arbitrage-free or fair market value basis, then the allocation of capital will be irrelevant to the economic operation of the business. For any given allocation of capital, the expected return to equity by line of business and for the insurer will be a fair rate of return. Altering the allocation of capital by line of business does not change the ratio of the insolvency exchange option value to the liability value for that line of business.

6 Diversification and Lines of Business

We now consider the impact of adding a line of business. Similar considerations apply in the case of exiting an existing line of business. Denote the new line of business payoffs by L_{K+1} with actuarial value, ignoring insurer default, of $V_{L_{K+1}}$ determined using the Q-probabilities.

The impact on the economic balance sheet of the insurer of the line of business will depend on the market premium charged and this will depend on the additional capital added to the balance sheet to support the new line of business. The market premium for the new line of business will take into account the insolvency exchange option value for the balance sheet of the insurer including the impact of the new line of business on assets payoffs, liability payoffs and of additional capital.

Capital should be added to maintain the market value of each existing line of business taking into account the new balance sheet values of assets and liabilities. If this is not the case then there adding the line of business will result in a transfer of wealth between policyholders and shareholders.

The insolvency exchange option value for the new line of business will be given by

$$D_{K+1} = \frac{1}{1+r} E^Q \left[L_{K+1} \max \left[1 - \frac{A + \Delta A}{L + L_{K+1}}, 0 \right] \right]$$

where ΔA is the additional asset payoff resulting from the increase in the assets of the insurer from the additional line of business premium and capital. The market premium for the new line of business will be

$$V_{L_{K+1}} - D_{K+1}$$

and the additional assets, including the additional capital ΔV_X, will be

$$\Delta V_A = \Delta V_X + V_{L_{K+1}} - D_{K+1}$$

As already noted ΔV_X should be such that, for each line of existing business, the insolvency exchange option value remains unchanged so that

$$\begin{aligned} D_k^{new} &= \frac{1}{1+r} E^Q \left[L_k \max \left[1 - \frac{A + \Delta A}{L + L_{K+1}}, 0 \right] \right] \\ &= \frac{1}{1+r} E^Q \left[L_k \max \left[1 - \frac{A}{L}, 0 \right] \right] \\ &= D_k^{old} \quad \text{for all } k = 1, \ldots, K \end{aligned}$$

We illustrate the impact of adding lines of business, usually regarded as a form of diversification of the risk of the insurer, using the previous numerical example with a single risky asset and two lines of business introduced earlier in the paper. To begin with assume that there are two separate companies, Company 1 and Company 2, one writing only line of business 1 and the other writing only line of business 2. The assets of Company 1 and 2 are assumed to equal the amount allocated to line of business 1 and 2 to equate the expected return on equity by line of business in the company writing both lines of business. For Company 1 and Company 2 the balance sheets are given in Tables 7 and 8.

Table 7 Company 1 Payoffs writing only Line of Business 1

	Time 1 Company 1 Payoffs			
State	Assets	L	D	Equity
1	30.2126	200	169.7874	0
2	55.3898	4	0	51.3898
3	50.3544	2	0	48.3544
4	75.5316	0	0	75.5316
Time 0 Value	50.3544	21.3333	16.1702	45.1913

Table 8 Company 2 Payoffs writing only Line of Business 2

	Time 1 Company 2 Payoffs			
State	Assets	L	D	Equity
1	89.7874	40	0	49.7874
2	164.6102	10	0	154.6102
3	149.6456	4	0	145.6456
4	224.4684	310	85.5316	0
Time 0 Value	149.6456	38.6667	8.1459	119.1248

For Company 1 the ratio of the insolvency exchange option value to the liability is $\frac{16.1702}{21.3333} = 0.758$, the ratio of surplus to the liability value is $\frac{50.3544-21.3333}{21.3333} = 1.3604$ and the expected return on equity is 6.343%. For Company 2 the equivalent figures are $\frac{8.1459}{38.6667} = 0.2107$, $\frac{149.6456-38.6667}{38.6667} = 2.8701$ and 6.505%.

Now assume that these two companies are merged. The balance sheet for the combined company is given in Table 9.

Table 9 Balance Sheet Payoffs of Merged Company

	Time 1 Merged Company Payoffs			
State	Assets	L	D	Equity
1	120	240	120	0
2	220	14	0	206
3	200	6	0	194
4	300	310	10	0
Time 0 Value	200	60	12.381	152.381

For the combined Company the ratio of the insolvency exchange option value to the liability is $\frac{12.381}{60} = 0.2064$, the ratio of surplus to the liability value is $\frac{200-60}{60} = 2.3333$ and the expected return on equity is 6.575%. Both the assets and the liabilities for the merged company, ignoring the insolvency exchange option, are the sum of the balance sheet values for the two companies. However the effect of the merger of the two companies is a transfer of wealth from equityholders to policyholders because the insolvency exchange option value for the merged company is lower than the sum of the individual company insolvency exchange option values. Combining the individual companies has benefitted the policyholders since the undertaking by the insurers to pay the liabilities is more valuable in the combined company. The higher expected return to equity of the combined company reflects the riskier position of the equityholders of the combined company. The benefit from diversification has accrued to the policyholders.

7 Covariance allocations

Wang (2002) [17] proposes a method for allocating capital based on the aggregate loss of a portfolio and Esscher transforms, or exponential tilting. Using the notation in our paper, Wang's approach is to take the total liabilities L as the reference portfolio and for each line of business derive another random variable based on the Esscher transform given by

$$L_k^{wang} = L_k \frac{e^{\lambda L}}{E\left[e^{\lambda L}\right]}$$

The total capital for the company is used to calibrate the parameter λ such that

$$V_X = E\left[L \frac{e^{\lambda L}}{E\left[e^{\lambda L}\right]}\right] - E\left[L\right]$$

and capital allocation for line of business k is then

$$V_{X,L_k} = E\left[L_k \frac{e^{\lambda L}}{E\left[e^{\lambda L}\right]}\right] - E\left[L_k\right]$$

This approach effectively treats the capital as equivalent to a premium loading over the expected value of the liability.

Since

$$L = \sum_{k=1}^{K} L_k$$

we have

$$V_X = \sum_{k=1}^{K} V_{X,L_k}$$

and this allocation "adds up" by line of business. Since we can write

$$\begin{aligned} E\left[L_k \frac{e^{\lambda L}}{E\left[e^{\lambda L}\right]}\right] - E\left[L_k\right] &= E\left[L_k \frac{e^{\lambda L}}{E\left[e^{\lambda L}\right]}\right] - E\left[L_k\right] E\left[\frac{e^{\lambda L}}{E\left[e^{\lambda L}\right]}\right] \\ &= cov\left(\frac{e^{\lambda L}}{E\left[e^{\lambda L}\right]}, L_k\right) \end{aligned}$$

this is an example of capital allocation using a covariance principle. The form of this covariance is similar to the expression given for the risk loading derived earlier. However it only explicitly takes account of liabilities and the equivalent of the stochastic discount factor is not based on an economy wide factor.

This method of allocating capital takes no account of the economic value of the balance sheet other than the total capital that is to be allocated. Although this capital allocation "adds up", it does not explicitly allocate the insurer assets nor does it take into account the insolvency exchange option value. Different levels of risk in the insurer assets are not reflected in this approach.

Panjer (2001) [11] develops an approach to capital allocation that uses covariances. Valdez and Chernih (2003) [16] extend Panjer and Wang's approach to elliptical distributions. Neither of these approaches use the economic values of the balance sheet assets and liabilities for the purposes of capital allocation. They also do not explicitly include the insolvency exchange option value or the risk of the insurer assets. The closed form results of Panjer (2001) [11] and Valdez and Chernih (2003) [16] rely on the assumption of a multivariate normal distribution or a multivariate elliptical distribution for insurance losses. Empirical data demonstrate that insurance losses do not have these symmetric distributions.

8 Conclusion

This paper has used economic valuation of an insurer balance sheet to determine the allocation of capital to lines of business taking into account the liability risk, the asset risk and the solvency of the insurer. Using an economic valuation of the balance sheet ensures that liabilities are fairly priced and that equity earns a fair expected rate of return. We have shown how to allocate the insolvency exchange option value to lines of business determined by the ranking of outstanding claim payments in the event of insolvency. This allocation depends only on the insurer total surplus and not the allocation of the surplus to line of business. The value is determined by the distribution of the overall balance sheet surplus ratio and the claims distribution for an individual line of business in the event that the insurer becomes insolvent. The results have been illustrated using a simple numerical example for a single period discrete state model.

In Myers and Read (2001) [10] the allocation of surplus to lines of business affects the value of the insolvency exchange option for each line of business. We have shown that this can not be the case and give allocation results that apply in general, including for the continuous state log-normal and normal assumptions used in Myers and Read (2001) [10]. We have also shown that allocation of capital to lines of business requires an allocation of assets to line of business and that there is no unique way to do this in a complete markets model with no frictions. The alternatives of using a common surplus ratio and a common expected return to equity by line of business were considered as two assumptions that will produce a unique allocation of assets and hence of capital to lines of business.

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