

# Enterprise Risk Management, Insurer Pricing, and Capital Allocation\*

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## Abstract

For insurers and reinsurers, economic capital has become central to enterprise risk management and is used in financial decision-making including by-line pricing and capital allocation. The Value-at-Risk (VaR) measure is widely used for determining economic capital. In this paper we use a shareholder and total firm value maximizing model of an insurer incorporating taxes, agency costs, financial distress costs, policyholder preference for financial quality, and by-line price elasticities. We determine optimal value maximizing pricing strategies and capitalizations for an insurer under varying assumptions for a realistic model of a multi-line insurer. We evaluate the performance of VaR-based methods for allocating capital and incorporating the cost of capital into pricing insurance. We find that with imperfectly elastic policyholder demand, capitalization, and pricing are strongly influenced by policyholder sensitivity to price and preferences for financial quality. Incorporating costs of capital using VaR must be carefully implemented if it is to be consistent with value maximizing pricing strategies.

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# 1 Introduction

Economic capital has become an important focus of enterprise risk management for insurers and reinsurers. Risk based measures of capital are used to determine the capitalization of the insurer, usually to meet a minimum level of solvency often based on a Value-at-Risk (VaR) criteria (Hitchcox et al. 2006 [14]). Insurers, particularly reinsurers, are concerned with their credit ratings and financial quality since this must be of the highest standing in order to successfully compete for business. Capitalization impacts pricing since insurance premiums must reflect expected losses and expenses as well as include an allowance for risk and the cost of capital, sometimes referred to as the profit margin. The cost of capital is allowed for in pricing by using an allocation of capital to lines of business and charging an expected return on allocated risk capital. Costs of insurance capital include the allowance for frictional costs of capital such as taxes, agency costs, and costs of financial distress (Hancock et al. 2001 [12]). Conceptually the relationship between capitalization and pricing appears straightforward, however, in practice there are many unresolved issues including the theoretical basis for the profit margin, the methodology for allocating capital to line of business, the expected cost of capital to use for pricing, and the extent to which policyholders and shareholders bear the frictional costs of capital.

Traditional actuarial pricing methods, which can be regarded as a supply-side or insurer approach, assume that insurance prices are expected costs plus a risk or profit loading. More recently, insurance pricing models have developed based on financial pricing theory, including the capital asset pricing model and option pricing models. These models mostly assume that competitive market forces will determine insurance premiums and that insurers are price-takers in the insurance market. As a result, premium profit loadings reflect systematic risk factors. Empirical studies, including Fairley (1979) [8], D'Arcy and Garven (1990) [7], Cummins and Phillips (2005) [5], focus on insurer cost of capital and provide some support for financial pricing models, however, frictional costs such as taxes, agency costs, and costs of financial distress are not readily incorporated into these models, especially for pricing by line of business. Discounted cash flow pricing models, as originally proposed by Myers and Cohn (1987) [19], are often used for pricing insurance for different risks. They require an allocation of capital to a line of business since they determine expected cash flows for insurance losses, expenses, and taxation. Premiums are based on an expected return on allocated equity for each line of business.

Approaches used in practice are often based on Dynamic Financial Analysis models along with a risk measure to determine capitalization and the impact on pricing. Nakada et al. (1999) [21] develop P&C RAROC. They generate loss distributions and aggregate risks allowing for correlation incorporating credit, market, insurance, and operating risk before allocating capital to lines of business. They use VaR as the risk measure, although they recognize the theoretical importance of the insolvency put option value as a risk measure. Capital is allocated to risk types and lines of business using a continuous marginal contribu-

tion approach, similar to the Myers and Read (2001) [20] methodology, and the risk adjusted return on allocated capital is estimated for different lines of business. After estimating economic capital for the total US industry, they conclude that the U.S. property and casualty (P&C) industry is significantly overcapitalized, indicating that insurers may be holding capital for reasons other than insolvency risk. Cummins and Danzon (1997) [6] and Zanjani (2002) [33] incorporate policyholder price elasticity and the policyholder default put option into their models of insurance pricing and capitalization, allowing a trade-off between price and quantity of insurance business reflecting policyholder demand. These models are more realistic and representative of insurance markets in contrast to the assumption of perfectly competitive financial asset markets.

We focus on P&C insurance and issues of pricing, capitalization, and the cost of capital in imperfect markets for a multi-line insurer. We evaluate optimal capitalization and pricing strategies in a single-period model of a multi-line insurer allowing for frictional costs, imperfectly competitive demand, and policyholder preferences for financial quality. The model assumptions are based on those of Zanjani (2002) [33]. The insurer model is calibrated to be representative of an Australian general insurance company although it is representative of insurers in many countries. Optimal strategies are determined by maximizing two types of insurer value. First, we consider shareholder value, where strategies are determined in order to maximize enterprise value added (EVA) using a value based measure allowing for frictional costs of capital. This measure of value added is different to the economic value added (EVA) measure of financial performance developed by Stern Stewart and Co. and similar to that of Panning (2006) [22]. Second, we consider strategies that maximize the total asset value of the firm by considering both shareholder and policyholder values.

We evaluate the performance of VaR-based methods for quantifying and allocating economic capital commonly used to price individual lines of insurance by comparing pricing margins with our EVA maximizing insurer. Economic capital is determined at the 99.5% confidence level, which is consistent with international solvency requirements and the Australian Prudential and Regulation Authority (APRA) minimum solvency requirements. Capital is allocated using the proportional method as described by Venter (2004) [31]. We consider different commonly used approaches for the cost of capital for lines of business.

We find that VaR-based methods for determining capital and prices are generally not consistent with EVA maximization. VaR-based methods overstate the optimal level of capitalization, resulting in additional frictional costs of capital and, more importantly, impacting on optimal pricing strategies. In order to be consistent with value maximizing pricing margins, the cost of capital will differ across lines of business to account for the differences in the elasticity of policyholder demand across individual lines of business. We find in our model that elasticity of policyholder demand and preferences for financial quality impact on profitability more than risk characteristics when allowing for an imperfectly competitive insurance market.

When total firm value is maximized the optimal structure for the insurer is that of a mutual insurer, where policyholders pay premiums including a capital

subscription. The total firm value maximizing mutual insurer, in comparison to a shareholder value maximizing insurer, has lower default risk and writes a larger volume of business at lower premiums reflecting the best interests of policyholders.

## 2 Insurer Pricing, Capital Allocation, and the Cost of Capital

An important issue in insurance pricing is the premium loading over expected losses since this will be an important factor in determining the expected return on capital for the insurer. The standard actuarial approach to insurance pricing determines premiums as the expected present value of losses plus a loading for profit and expenses. This approach does not explicitly take into account insurer competition or policyholder preferences. Goovaerts, de Vylder and Haezendonck (1984) [9] provide detailed coverage of well known actuarial premium principle models including those based on assumptions of policyholder risk aversion.

Financial pricing approaches determine insurance prices for competitive insurance markets and fair pricing assumptions. These approaches assume that the capital structure of an insurer is analogous to that of a corporation consisting of equity and debt. Insurance companies issue risky debt by underwriting insurance contracts and raise equity from shareholders who take on insurance risk. Merton and Perold (1993) [18] highlight several distinct features of insurance companies. They note that the debt holders of an insurer are also the firm's customers, and for this reason they are more credit-sensitive than traditional debtholders. Cummins (2000) [3] notes that unlike traditional debtholders, policyholders do not hold a diversified portfolio of insurance contracts to reduce exposure to the insolvency risk of an insurer. Insurance contracts are purchased from a single insurer for a particular line of business. Insurance companies are also complex and their operations less well understood by investors and policyholders. In order to meet policyholder expectations, insurance companies hold high levels of equity capital to reduce insolvency risk. As a result, agency costs and other frictional costs of capital are significant.

Cummins and Danzon (1997) [6] develop a simple multi-period option pricing model with two lines of business. They assume that demand for insurance is determined by price and the financial quality of the insurer as reflected in the insolvency put option value. They maximize the value of the firm and determine optimal prices and equity. The first order conditions express the profit margin as a percentage of the premium and the insolvency put option in terms of the elasticities of demand with respect to price and the insolvency put. Their results are consistent with the intuition that insurers with higher financial quality, as reflected in the insolvency put option value, will command higher premiums. An important contribution is that the optimal capital is determined by the insurer's demand function rather than exogenously, and that an optimal value maximizing price and capital structure exists in the absence of frictional costs.

Taylor (1995) [29] develops an equilibrium model for insurance pricing incorporating the insolvency put option. He extends an existing single-period equilibrium model with individuals, productive firms, and insurance companies. In the model individuals seek to maximize their consumption and terminal wealth by investing in real assets, buying shares in productive firms and insurance companies, and purchasing insurance contracts from insurers to cover the value of their real assets. Productive firms invest in assets and are fully financed by equity, while insurers are financed by equity and issue policyholder debt. Sherris (2003) [24] discusses the model and implications for optimal capitalization and capital allocation.

Zanjani (2002) [33] develops and analyses an equilibrium model where capital is costly to hold and individuals are concerned with financial quality. This framework extends the existing literature by allowing for imperfectly elastic demand for insurance and frictional costs of capital.

Cummins (2000) [3] and Venter (2004) [31] review the main practices for allocating firm-wide capital to individual lines of business. Standard allocation techniques include risk-based regulatory requirements, employing the CAPM, and using the value at risk (VaR) measure. Merton and Perold (1993) [18] develop a marginal allocation method based on the value of the insurer's insolvency put option. This approach is important in that it captures the correlations between the marginal line of business and the existing portfolio of risks. One limitation is that capital does not necessarily "add up" and it is economically desirable to allocate the full value of equity capital to all lines of business. Myers and Read (2001) [20] consider capital allocation based on the insolvency put option by considering infinitesimal changes in the put option value for infinitesimal changes in a line of business.

Sherris (2006) [25] extends Myers and Read (2001) [20] to evaluate the by-line payoffs allowing for insolvency and discusses capital allocation in a framework that ensures capital allocation will be consistent with fair valuation of liability cash flows. Sherris and van der Hoek (2006) [26] derive analytical results for the by-line valuation of the insurer insolvency put option for use in fair pricing and valuation of liabilities. They demonstrate the wide range of capital allocations that result from commonly used methods including VaR and TailVaR. Incorrectly allocating the insolvency put option can have a significant impact on capital allocation and pricing margins.

Phillips, Cummins, and Allen (1998) [23] use data from 1988 to 1992 for 90 publicly traded property-liability insurers in the U.S. and find significant evidence that prices vary inversely with insolvency risk, as measured by the insolvency put option, for both short and long tail lines of business. For the average insurer from the sample, a 1% increase in the insolvency put ratio lowers the premiums by 2.0% and 11.5% for long and short tail lines of business respectively. In a more recent study, based on U.S. data between 1997 and 2004, Cummins, Lin, and Phillips (2006) [4] find strong evidence that insurance prices are inversely related to insolvency risk as measured by A.M. Best's financial ratings.

In contrast to the fair value pricing of insolvency risk, which implies re-

duced premiums for insurers with low capitalization, the capacity constraint theory predicts that a shortage of capital will lead to higher premiums in order to attract capital to support insurance risk. Gron (1994a, 1994b) [10] [11] finds empirical evidence to support the capacity constraint theory. Using A.M. Best's property-liability data for the period 1952-1986, underwriting profit margins were regressed on insurer surplus as a proxy for capacity. After controlling for inflation and changes to the present value of expected losses, their results indicate that short tail lines of business are impacted by capacity constraints with only weak evidence supporting capacity constraints in long tail lines. Cummins and Danzon (1997) [6] analyze firm specific and industry levels of capital and the relationship between prices and "loss shocks". Contrary to the capacity constraint theory they find prices are positively related to capital supply and weak evidence that prices increase in response to an internal negative reduction to capital.

Kielholz (2000) [15] uses the CAPM to estimate the cost of capital for non-life insurers over the period 1991 to 1998. His estimates for the after-tax cost of capital were 14.12% and 15.51% for the U.S. and U.K. respectively. The average cost of capital was 10.5% for the property-liability industry in the U.S., U.K., Switzerland, France, and Germany. Cummins and Phillips (2005) [5] estimate much higher costs of capital when including factors to measure firm size and costs of financial distress. They use both the CAPM and the Fama-French 3-factor model as well as the full information industry beta methodology to estimate cost of capital by line of business. They estimate returns to equity capital in the U.S. property-liability industry by line of business for the period 1997-2000. The authors findings suggest a cost of capital of 18.1% for market value weighting and 21.0% for equal weighting over this period. For insurers on average, they estimate a cost of capital that is higher in personal lines than in commercial lines, with cost of capital estimates of 23.1% and 19.5% respectively. Overall, the property-liability industry experiences a lower cost of capital of 18.1% compared to the 21.1% for other financial institutions.

Swiss Re (2005) [28] discuss the importance of the frictional costs of capital. These frictional costs of capital include taxes, agency costs, and financial distress costs. They are the costs of holding capital in the firm and are insurer specific costs of capital. In Australia, the taxation system results in very limited taxation for insurers. Swiss Re (2005) [28] indicates that agency costs of 2% may be a reasonable level for an insurer. Altman (1984) [1] estimates the costs of financial distress for a diverse sample of U.S. firms and finds that the direct and indirect costs of financial distress are on average 11.54% three years prior to bankruptcy and 17.40% in the year prior to bankruptcy.

### 3 Insurer Value Maximization

Value maximization as a firm objective is consistent with modern corporate financial theory and the economic foundation of risk management. In perfect markets, the Fisher Separation Theorem implies that investors with diverse risk

preferences will invest capital into firms and delegate production decisions to management, whose objective is to maximize firm value regardless of investor risk preferences. MacMinn (2005) [17] provides excellent and rigorous coverage of financial theory and applications to insurer risk management. Smith (1996) [27] demonstrates that Fisher Separation also holds in the case of incomplete markets when investor preferences, as given by their utility, satisfy conditions of additivity and constant relative risk aversion. Without frictions, value maximizing firms should act as if they were risk neutral. Frictional costs create convexity in the after-tax profits of the firm. They impact financial decision making by creating incentives to reduce risk and volatility in order to maximize firm value. Explicitly, modelling frictional costs allows the quantification of the costs and benefits of holding too much or too little capital.

The main contributor to value added in an insurer is the profit margin by line of business. Determining optimal pricing strategies that maximize profit margins by line, taking into account frictional costs and imperfect policyholder demand, has important implications for optimal capitalization and hence for enterprise risk management. The optimization approach does not require the allocation of capital or frictional costs of capital to line of business for pricing since optimal prices are determined directly taking into account policyholder demand and preferences for financial quality.

The optimal insurer balance sheet is determined by selecting the insurer capital subscribed and the by-line prices that maximize shareholder value added or EVA. The objective is not to maximize shareholder value but to maximize the value added from writing insurance business over and above the value of the equity subscribed. Value is added by writing insurance business at profit loads above the risk adjusted expected value of claims and costs, allowing for insolvency risk. These profit loads reflect the price elasticity of policyholder demand as well as preferences for financial quality. Frictional costs of capital reduce shareholder value and holding too much capital increases these costs. However, holding too low a level of capital reduces policyholder demand for insurance. This is the important trade-off created by frictional costs and insolvency risk, and contributes to the determination of an optimal level of capitalization and an optimal pricing strategy.

The EVA used is formally defined as the difference between the value of equity at time 0 and the amount of initial capital subscribed allowing for frictional costs and insolvency. For shareholders, allowing for corporate tax and agency costs, the time 1 model payoff is

$$E_1 = (V_1 - L_1 + D_1)(1 - \tau_1) + (\tau_1 - \tau_2)R_0,$$

where

- $V_1$  is the time 1 payoff from the assets of the insurer accumulated at random rate  $r_V$ ,
- $L_1$  is the contractual time 1 liability payoffs of the insurer,

- $D_1$  is the contingent reduction in liability payoffs from insolvency with  $D_1 = \max [L_1 - V_1, 0]$ ,
- $R_0$  is the initial shareholder cash capital subscribed at time 0,
- $\tau_1$  is the tax rate applied to corporate insurer profits at time 1, and
- $\tau_2$  is proportion of shareholder cash capital subscribed that is absorbed as agency costs at time 1.

The time 0 shareholder payoff value is

$$E_0 = (V_0 - L_0 + D_0)(1 - \tau_1) + e^{-r}(\tau_1 - \tau_2)R_0,$$

where

- $V_0 = R_0 + P_0 - c_0$  is the net cash available to invest at time 0 from shareholders and policyholders
- $c_0$  is the production cost for all policies issued
- $L_0$  is the fair value of the insurer liability obligations
- $D_0$  is the fair value of the insurer insolvency put option.

The insurer's objective is to maximize EVA by selecting the by-line prices and capital subscribed so that the formal optimization is

$$\begin{aligned} \max_{R_0, p_{i,0}} \{EVA_0\} &= \max_{R_0, p_{i,0}} \{E_0 - R_0\} \\ &= \max_{R_0, p_{i,0}} \left\{ \begin{array}{l} (P_0 - c_0 - L_0(1 - d_0))(1 - \tau_1) \\ - ((1 - e^{-r})\tau_1 + e^{-r}\tau_2)R_0 \end{array} \right\}, \end{aligned}$$

where  $d_0$  is the reduction in the time 0 fair value of the liabilities resulting from insolvency risk as a proportion of the liability.

The model requires numerical techniques to determine the optimal capitalization and prices because of the non-linear relationships and the interdependencies in the model. For a given set of values for capital structure and prices it is possible to determine the balance sheet structure. The financial quality of the insurer, measured by the default put option, will influence the premium income through the demand for insurance. The value of the default put option is also a function of balance sheet items. Because of this, an iterative approach is required to construct an internally consistent balance. A direct search method was used for the optimization. Direct search is a non-derivative based method and details for the method are given in Lewis and Torczon (2000) [16] and Torczon (1997) [30].

Allowance is made for the number of policies sold with premium revenue at time 0 for sales from the  $N$  lines of business determined by

$$P_0 = \sum_{i=1}^N p_{i,0} q_{i,0}$$

where  $p_{i,0}$  is the premium for a policy in the  $i$ th line and  $q_{i,0}$  is the quantity sold in the  $i$ th line. We assume policies in each line of business are homogeneous with respect to the loss distribution with the time 1 random loss payoff for a policy in the  $i$ th line denoted by  $L_{i,1}$  and the total random losses at time 1 denoted by  $L_1$  with

$$L_1 = \sum_{i=1}^N L_{i,1} q_{i,0}.$$

The fair value of total liabilities at time 0 are valued using a market based risk neutral valuation assumption. We assume that there exists a risk-neutral  $Q$  probability measure that values all cash flows in the model. This is consistent with financial pricing theory under the assumption of arbitrage free markets as covered in standard texts such as Cochrane (2005) [2]. We have

$$L_0 = e^{-r} \sum_{i=1}^N \mu_{i,1} q_{i,0},$$

where

$$\mu_{i,1} = E^Q [L_{i,1}]$$

is the fair value of the insurance loss per policy for the  $i$ th line of business and  $r$  is the continuous compounding risk free rate of interest.

We denote the value of the insurer default put option by

$$D_0 = e^{-r} E^Q [D_1].$$

The default ratio,  $d_0$ , is the default risk per dollar of liabilities with  $D_0 = L_0 d_0$ . The default ratio can be valued as a put option on the asset-liability ratio

$$d_0 = e^{-r} E^Q [d_1],$$

where

$$d_1 = \max [1 - \Lambda_1, 0]$$

and the asset-liability ratio is

$$\Lambda_1 = \frac{V_1}{L_1}.$$

Shareholder profit at time 1 is the shareholder payoff  $E_1$  less the initial capital invested,

$$E_1 - R_0 = V_1 - L_1 + D_1 - R_0.$$

Corporate taxes, including the tax benefit from losses, are assumed to be

$$\tau_1 (E_1 - R_0) = \tau_1 (V_1 - L_1 + D_1 - R_0).$$

Shareholder agency costs of capital arising from management are assumed to be proportional to the amount of capital initially subscribed and equal to

$$\tau_2 R_0.$$

Bankruptcy or financial distress costs are assumed to be zero if the insurer is solvent at time 1 otherwise they are assumed to be a percentage of the shortfall of assets over liabilities reflecting the size of the insolvency so they are

$$0 \text{ if } V_1 \geq L_1$$

or

$$f(L_1 - V_1) \text{ if } V_1 < L_1.$$

with value at time 0 equal to

$$fD_0.$$

Total firm value is the sum of shareholder value and the value of policyholder claims or

$$FV_0 = EV A_0 + H_0.$$

The value of policyholder claims at time 0 allowing for bankruptcy costs is

$$H_0 = L_0 - (1 + f)D_0.$$

Bankruptcy costs reduce the value of policyholder claims and do not reduce shareholder payoffs. The objective function to maximize total firm value added at time 0 is

$$\begin{aligned} \max_{R_0, p_{i,0}} \{FV_0\} &= \max_{R_0, p_{i,0}} \{E_0 - R_0 + H_0\} \\ &= \max_{R_0, p_{i,0}} \left\{ \begin{array}{l} (P_0 - c_0 - L_0(1 - d_0))(1 - \tau_1) \\ - ((1 - e^{-r})\tau_1 + e^{-r}\tau_2)R_0 - fL_0d_0 \end{array} \right\}. \end{aligned}$$

We note that the objective of a firm value maximizing insurer only differs from the objective of an EVA maximizing insurer by the term  $-fL_0d_0$  since bankruptcy costs lower the total value of the firm but not shareholder value directly.

Full details of the model are available in Yow and Sherris (2007) [32].

## 4 Model Calibration

The model insurer we use for our analysis aims to be representative of a diversified multi-line insurer writing business in the Australian general insurance industry. Although representative of an Australian insurer, the implications for risk based capital and pricing apply to property-casualty or general insurance companies more broadly. The model incorporates frictional costs of capital, policyholder price elasticities and insurer financial quality and quantifies their impact on profit margins and capitalization.

The distributional assumptions used for the asset and liabilities of the model insurer are similar to those in Sherris and van der Hoek (2006) [26]. Asset and liability values are assumed to be log-normally distributed. The data used to calibrate the model insurer were derived from the following sources.

- Australian Prudential Regulation Authority (APRA), Half Yearly General Insurance Bulletin, December 1998 to December 2001.
- Tillinghast-Towers Perrin, Research and Data Analysis Relevant to the Development of Standards and Guidelines on Liability Valuation for General Insurance, November 2001.
- Reserve Bank of Australia (RBA), S&P/ASX 200 Accumulation Index, January 1979 to September 2006.
- ABN AMRO, Government and Semi-Government Bonds Total Return Index, January 1990 to September 2006.

The model insurer is assumed to write business in the five largest individual lines by net premium revenue: domestic motor, household, fire & ISR, public and product liability, and CTP. These lines represent 68% of industry gross premium revenue in 2005. They include business lines with claims of a variety of different tail lengths, and include classes of business that are personal, commercial, and compulsory as in Table 1.

Lines	Category	Type	Gross Premium Revenue
Motor	Short tail	Personal	21.7%
Household	Short tail	Personal	14.5%
Fire & ISR	Intermediate	Commercial	12.2%
Liability	Long tail	Commercial	8.6%
CTP	Long tail	Compulsory	10.6%

Table 1: The business lines of the model insurer and industry weightings.

The insurer is assumed to hold a diversified portfolio invested in cash, bonds, and stocks similar to that of a typical insurer. We do not optimize over the asset portfolio. The returns, volatilities, and correlations are estimated from historical investment data. We assume no correlation between asset and liability payoffs. Log-returns on the asset portfolio are assumed to have an expected value of  $\mu_V = 10.09\%$  and a standard deviation of  $\sigma_V = 5.04\%$ .

The asset mix assumed for the representative model insurer is based on APRA industry data is shown in Table 2.

Assets	Portfolio Weight
Cash	15%
Bonds	65%
Stocks	20%

Table 2: The asset mix of the model insurer.

Claims are assumed to be log-normally distributed. The log-normal distribution is commonly used by practitioners to model general insurance liability

distributions and is also used in Hitchcox et al. (2006) [14]. Parameters of the log-normal distribution for each line of business are determined from the Tillinghast estimate of the coefficient of variation (CV) for outstanding claims liability by line of business based on industry data between 1997 and 2001 and the average outstanding claim liability for each line from APRA’s Half Yearly Bulletin for the period December 1997 to December 2001. The Tillinghast correlation matrix assumed for the dependence between liabilities by line of business is given in Table 3.

Lines	Motor	Household	Fire & ISR	Liability	CTP
Motor	1.00	0.75	0.40	0.00	0.55
Household	0.75	1.00	0.35	0.00	0.00
Fire & ISR	0.40	0.35	1.00	0.00	0.00
Liability	0.00	0.00	0.00	1.00	0.35
CTP	0.55	0.00	0.00	0.35	1.00

Table 3: Tillinghast correlation matrix.

The properties of the log-normal distribution allow the variance for each line of business to be determined directly from the CV using

$$CV_i = \left( e^{\sigma_i^2} - 1 \right)^{\frac{1}{2}}.$$

For our model assumptions, the average outstanding claims liability for each line of business, the CV by line of business from the Tillinghast report, and the by line  $\sigma_i$  are given in Table 4.

Line	Expected Outstanding Claims Liability per Policy	Tillinghast CVs	Lognormal $\sigma_i$
Motor	203	11.1%	0.1107
Household	105	13.2%	0.1314
Fire & ISR	201	14.1%	0.1403
Liability	256	19.0%	0.1883
CTP	249	23.5%	0.2318

Table 4: Expected outstanding claims and Tillinghast CVs by line of business.

Underwriting risks are assumed to have low systematic risk and all expectations are discounted at the risk-free rate.

We assume constant per policy underwriting expenses typical of a large general insurer and ignore any potential benefits of economies of scale. For each line we estimate these by the sample mean of the average expense per policy for the general insurance industry between December 1997 and December 2001 sourced from APRA’s Half Yearly Bulletin. Table 5 gives the by-line per policy underwriting expenses assumed for the model insurer.

Lines	Underwriting Expense
Motor	66.6
Household	65.6
Fire & ISR	152.4
Liability	125.2
CTP	44.7

Table 5: Underwriting expenses per policy.

The default value under the log-normal assumptions is given in Sherris and van der Hoek (2006) [26]. The default put option is equivalent to an exchange option or a put option on the asset liability ratio of the insurer and is given by

$$d_0 = \Phi(z) - \Lambda_0 \Phi(z - \sigma)$$

with

$$\Lambda_0 = \frac{V_0}{L_0},$$

$$z = \frac{-\ln(\Lambda_0)}{\sigma} + \frac{1}{2}\sigma.$$

The volatility is given by

$$\sigma = \sqrt{\sigma_L^2 + \sigma_V^2 - 2\sigma_{LV}}.$$

where

$$\sigma_L^2 = \sum_{i=1}^N \sum_{j=1}^N x_i x_j \sigma_i \sigma_j \rho_{ij},$$

$$\sigma_{LV} = \sum_{i=1}^N x_i \sigma_i \sigma_V \rho_{iV},$$

$\rho_{ij}$  is the loss correlation between the log growth rate of the lines of business,  $\rho_{iV}$  is the correlation between the log asset return and the log growth rate of each line of business, and  $x_i$  is the proportion of value of liabilities in the  $i$ th line

$$x_i = \frac{e^{-r} \mu_{i,1} q_{i,0}}{\sum_{j=1}^N e^{-r} \mu_{j,1} q_{j,0}}.$$

The by-line default value ratios are given by

$$d_{i,0} = \Phi(z_i) - \Lambda_0 \Phi(z_i - \sigma_{\Lambda_0})$$

where

$$z_i = \frac{-(\ln \Lambda_0 + \mu_{i\Lambda_0})}{\sigma} + \frac{1}{2}\sigma$$

and

$$\mu_{i\Lambda_0} = \sigma_L^2 - \sigma_L\sigma_V\rho_{LV} + \sigma_i\sigma_V\rho_{iV} - \sigma_i\sigma_L\rho_{iL}.$$

We assume zero corporate taxes since in Australia the imputation tax credit system provides investors with tax credits for corporate taxation. We assume that agency costs of capital are 2%, reflecting Swiss Re (2005) [28], and bankruptcy costs are 25%. A more complete analysis of the impact of varying assumptions for frictional costs on capital and pricing strategies is given in Yow and Sherris (2007) [32].

The demand function for the  $i$ th line of business determines the quantity of insurance sold for each of the  $N$  lines of business. For a price of  $p_{i,0}$  per unit of insurance the demand,  $q_{i,0}$ , is assumed to be a function of price, default risk, and bankruptcy costs

$$q_{i,0} = q(p_{i,0}, d_0, f).$$

where  $\frac{\partial q_{i,0}}{\partial p_{i,0}} < 0$ ,  $\frac{\partial q_{i,0}}{\partial d_0} < 0$ , and  $\frac{\partial q_{i,0}}{\partial f} < 0$ . Higher prices are assumed to result in reduced sales, as do higher levels of default risk and bankruptcy costs. The capitalization of the insurer determines the financial quality of the insurer. The demand for different lines of business is assumed to be sensitive to firm-wide default risk so that policyholders care about financial quality. We use the value of the insolvency put option for the financial quality of the insurer. Policyholder losses in the event of insolvency include bankruptcy costs and it is assumed that the size of these frictional costs influence the demand for insurance.

In a market where the demand for insurance is imperfectly elastic and policyholders care about financial quality, the demand function faced by the insurer will be downward sloping with respect to both price and default risk. We assume a linear demand function for the  $i$ th line of business,

$$q_{i,t} = q(p_{i,t}, d_t, f) = \alpha_i \max[1 + \beta_i p_{i,t} + \gamma_i (1 + f) d_t, 0].$$

The demand function  $\max[1 + \beta_i p_{i,t} + \gamma_i (1 + f) d_t, 0]$  ranges in value from zero to one. It can be interpreted as a measure of policyholder preference for purchasing an insurance policy in the  $i$ th line from the model insurer given its price and financial quality. The total demand is the product of a scale parameter  $\alpha_i$  that determines the maximum volume of business demanded for the  $i$ th line for the model insurer and the policyholder preference for the insurer given its characteristics. As prices rise, the demand for insurance will fall and  $\beta_i < 0$ . Similarly, as default risk increases policyholders will demand less insurance and  $\gamma_i < 0$ . The sensitivity to default risk is also assumed to reflect the bankruptcy costs that policyholders bear in the event of insolvency on a proportionate basis.

The demand function is calibrated to be consistent with the volume of business for different lines of business representative of a large and diversified insurer. Values assumed for  $\alpha_i$  are shown in Table 6. The model insurer represents approximately 10% of the industry.

There are no published studies that we are aware of that provide estimates of general insurer price elasticities of demand for the lines of business in our model insurer. In order to capture market imperfections and reflect reasonable

Lines	Average No. of Policies	$\alpha_i$ (thousands)
Motor	9,962	19,923
Household	10,384	20,768
Fire & ISR	2,441	4,883
Liability	3,083	6,165
CTP	5,972	11,944

Table 6: Assumed scale parameters by line of business.

price elasticities for each line we calibrate the policyholder demand function based on an assumed margin above per policy expected claims and costs at which policyholder demand is assumed to be zero. The assumed margins,  $m_i$ , at which the insurer is priced out of the market are shown in Table 7.

Lines	$m_i$	Max Price
Motor	14%	296.56
Household	14%	187.66
Fire & ISR	8%	371.07
Liability	8%	400.26
CTP	20%	337.76

Table 7: Maximum profit margins allowed over per policy expected cost.

These assumptions are representative of a reasonably price competitive insurance market. In a perfect and fully competitive market any increase in price over and above expected cost would drive demand to zero.

We assume that policyholder demand for insurance will fall to zero when,  $d_t = 1$  since insurance policies will then be worthless. Since at this point demand will be zero we assume that  $\gamma_i = -1$ . This means that the assumed percentage change in demand will be proportional to the by-line default option value as a percentage of the fair value of liabilities.

Full details of the model calibration are available in Yow and Sherris (2007) [32].

## 5 Results and Discussion

Given the wide-spread use of VaR in banking and insurance, it is of interest to compare standard economic capital models with strategies for pricing and capitalization derived from value maximization. The alternative approaches, which we will refer to as strategies, considered for pricing and capitalization are summarized in Table 8.

The first strategy is the maximization of EVA for our model insurer. The

Strategy	Approach	Method of Allocation	Assumed Cost of Capital
1	Maximize EVA	None	None
2	VaR at 99.5%	Proportional	15%
3	VaR at 99.5%	Proportional	20%
4	VaR at 99.5%	Proportional	15% (Commercial) and 25% (Personal/Compulsory)
5	Maximize Firm Value	None	None

Table 8: Strategies for insurer pricing and capitalization.

second assumes that capital is determined at the insurer level based on VaR at a 99.5% probability of solvency over a one year horizon and is allocated to line of business in proportion to the VaR of individual lines of business. A constant cost of capital of 15% is assumed across all lines in order to determine premium loadings for the cost of capital. Premium loadings also include frictional costs. These are added to fair values for expected losses and an allowance for the insolvency put is included. The quantity sold for these prices is determined based on policyholder demand and the financial quality of the insurer, and the resulting balance sheet determined. Strategy 3 is the same as for Strategy 2 except that a higher cost of capital is used based on an assumption that frictional costs increase the expected costs of capital. Strategy 4 uses different costs of capital by line reflecting the empirical results from Cummins and Phillips (2005) [5].

Strategy 5, the final strategy, maximizes total firm value including both shareholder value added and policyholder obligations. We will consider this after considering the shareholder value added strategy compared to the VaR approaches.

All of the strategies using VaR at the 99.5% confidence level produce a higher capitalization than the value maximizing optimal level of capitalization in terms of both the amount of capital subscribed and as a percentage of liabilities as illustrated by Figure 1. Results demonstrate that incorrectly specifying the cost of capital leads to suboptimal capitalization. All VaR-based strategies produce high levels of economic capital above 40.0% of liabilities, whereas optimal capitalization is 29.8%. For Strategy 2, with a cost of capital of 15%, capital levels are roughly 2.5 times the optimal level.

The direct cost of holding too high levels of capital erodes shareholder value as shown by Table 9. Frictional costs of capital incurred are proportional to capital levels due to model assumptions. VaR-based strategies all overstate optimal capital holdings and result in frictional costs being between 1.65 and 3.05 times optimal levels.

The optimal pricing strategy produces profit margins that are roughly 3% for personal lines, 6% for compulsory lines, and lower margins of approximately 1% apply for commercial lines. These prices are labeled Strategy 1 in Figure 2.

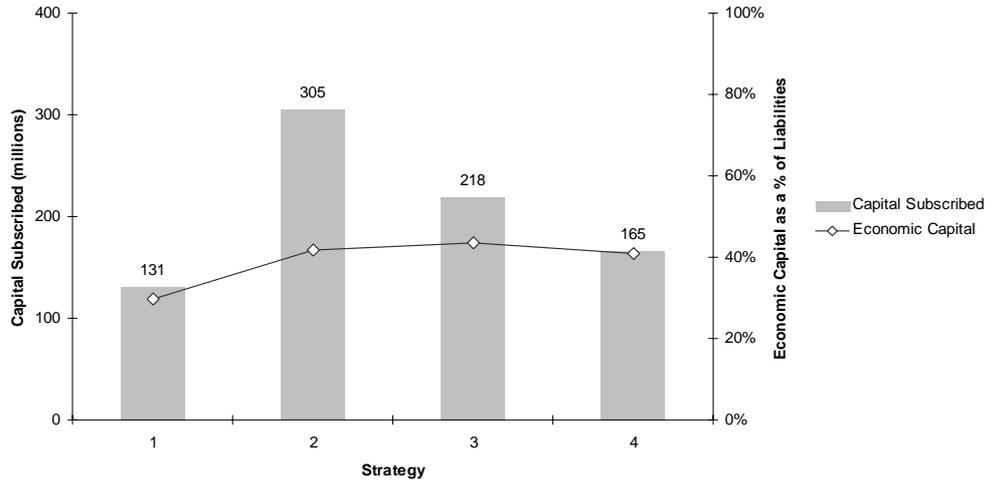


Figure 1: Capital structure for different pricing strategies.

Strategy	Frictional Costs (thousands)	%
1	2,489	100
2	6,413	305
3	5,171	218
4	4,299	165

Table 9: Differences in frictional costs of capital by strategy.

The relative profit margins reflect the assumptions for the elasticities of demand for each line of business. This is given by the second column of Table 10. Price elasticities are the percentage change in number of policies written for a 1% change in price. While the demand for insurance is price elastic across all lines of business, fire & ISR, and liability lines are particularly sensitive to price changes with elasticities of 26.1% and 24.8% respectively. This reflects the assumption that commercial lines are more price elastic than personal and compulsory lines. The sensitivity of demand in motor and household lines have lower elasticities of 15.6% and 16.4% respectively. The demand for CTP insurance is the least elastic at 11.7%. These price elasticities indicate a relatively high level of assumed competitiveness at the optimum in the model. Low elasticity lines, such as personal and compulsory, yield higher profit margins at the optimum, as demand is less sensitive to increases in premiums.

Commercial lines also display higher elasticities of demand with respect to default risk when compared to personal and compulsory lines. The default risk elasticities of demand are the percentage change in the number of policies written for a 1% change in the firm-wide default ratio. Default risk elasticities evaluated at the optimum are shown in the third column of Table 10. The sensitivity of policyholder demand to default risk is highest in commercial lines. A 1% increase in the insurer's default ratio would see a reduction in policies of 0.78% and 0.74% in fire & ISR and liability lines respectively. The demand for CTP insurance is the most inelastic with respect to default risk with an elasticity less than half that of commercial lines at 0.37%. These results indicate that policyholder demand at the optimum, while elastic to changes in price, is relatively inelastic to the default ratio. This is expected, as at the optimum the value of the default put option is low at 0.12% of liabilities.

The level of profit margins generated for this assumption are reasonably consistent with, although not the same as, the tariff margins mentioned by Hill [13] and US empirical data in D'Arcy and Garven [7]. This indicates that the demand elasticities are reasonable assumptions for assessing the impact of frictional costs on profit margins. To make the model more realistic the elasticities could be calibrated to market or historical profit margins, however, this would assume that these market based profit margins were optimal. Since we are interested in the relative impact of these costs in the optimal strategy and the relative differences by line it is important to incorporate a range of elasticities in the model assumptions. Market or historical margins will be also expected to vary across insurers.

Figure 2 displays the profit margins produced by VaR-based strategies, labelled Strategies 2 to 4. Strategy 2 uses a cost of capital of 15% across all lines of business and understates the margins on all lines of business except liability insurance. The mispricing is most significant in personal and compulsory lines, as this approach fails to consider the lower price elasticity of demand with margins more than 3% below optimal levels. The relatively high assumed price elasticity of demand in liability insurance results in a 15% cost of capital producing prices which overstate optimal levels for that line by 1.3%.

Strategy 3 assumes a higher cost of capital of 20% across all lines of business.

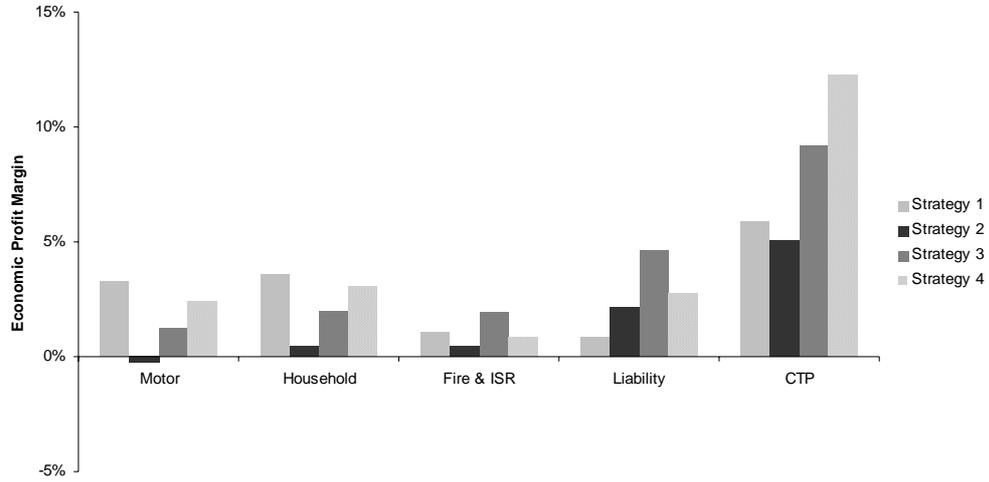


Figure 2: Pricing by line of business for different strategies.

Lines	Price Elasticity of Demand	Default Risk Elasticity of Demand
Motor	15.6	0.48
Household	16.4	0.50
Fire & ISR	26.1	0.78
Liability	24.8	0.74
CTP	11.7	0.37

Table 10: Elasticities of demand by line of business.

This leads to prices being closer to optimal levels in personal lines, however, this strategy overstates commercial and compulsory lines of business. Profit margins are 3.3% and 3.8% higher than optimal levels in CTP and liability insurance respectively. The results of Strategies 2 and 3 demonstrate that assuming a uniform cost of capital for all lines of business results in pricing inconsistencies due to the varying characteristics of policyholder demand.

Strategy 4 assumes different costs of capital depending on whether a line of business is personal, commercial, or compulsory. The resulting prices are closer to optimal levels for motor, household, and liability. However, this strategy overstates optimal margins in both liability and CTP. In particular, the profit margin for CTP insurance is overstated by 6.3%.

The VaR-based approaches produce higher capitalizations resulting in significantly lower firm-wide and by-line insolvency risk. All the VaR-based strategies result in a rating above AA, while Strategy 1 has twelve times the default risk, corresponding to a much lower credit rating between A and BBB. Figure 3 summarizes the financial quality of the model insurer under the shareholder value maximizing strategy and the VaR strategies. We assess the financial quality using the default put option value as a percentage of the insurer's liability. The improved financial quality of the insurer for Strategies 2 to 4 benefit policyholders, however, since capital is costly to hold and shareholders bear the frictional costs of capital this is not consistent with shareholder value added maximization. Note that Strategy 4, with multiple cost of capital targets, results in lower levels of default risk compared to the other VaR-based strategies despite lower capitalization. This indicates that incorrectly pricing risk, using a uniform cost of capital, will distort the optimal business mix of the insurer and can lead to an increased risk of insolvency.

Strategic differences in both capitalization and pricing across individual lines of business influence the value of business written as shown in Figure 4. All VaR-based strategies understate optimal prices in personal lines and write excessive amounts of business in these lines, while Strategies 3 and 4 result in significantly reduced volumes of CTP policies being underwritten.

All of the VaR strategies, except for Strategy 4, result in EVA's as a proportion of liabilities much lower than Strategy 1, as shown by Figure 5. Results demonstrate that assuming different costs of capital by line of business produces an increased EVA. More importantly, these results highlight the importance of firms understanding the elasticity of policyholder demand toward price and insolvency risk in individual lines of business. These demand characteristics should be factored into the cost of capital if prices are to be consistent with value maximization.

We now assume that the insurer maximizes total firm value comprising the economic value of policyholder claims and shareholder value added. The optimal balance sheets are given in Tables 11 and 12. The total size of the balance sheet is significantly larger in the total firm value case compared to an EVA maximizing insurer. Assets in the firm value case are approximately double. The fair value of the liabilities as a percentage of total assets is however similar in both cases, representing 80.0% and 77.0% respectively. In the model, insurers

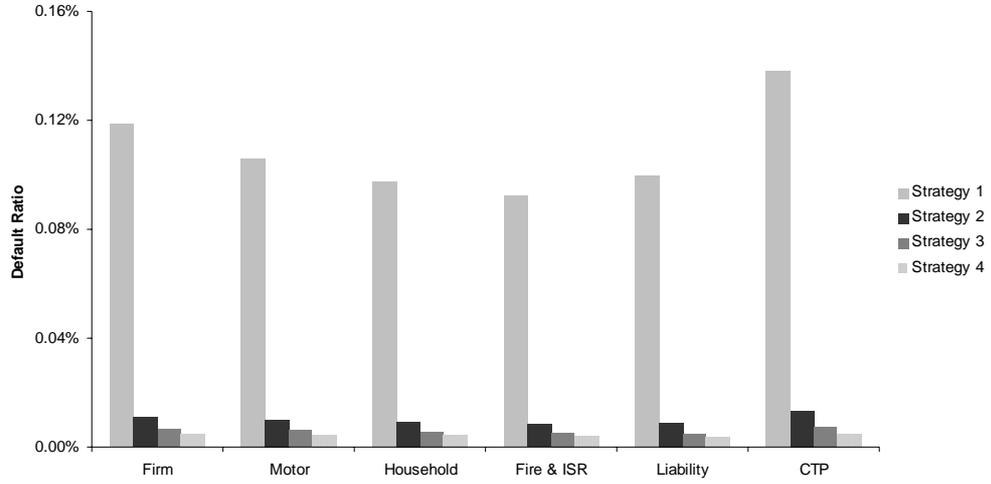


Figure 3: Default ratios for different strategies.

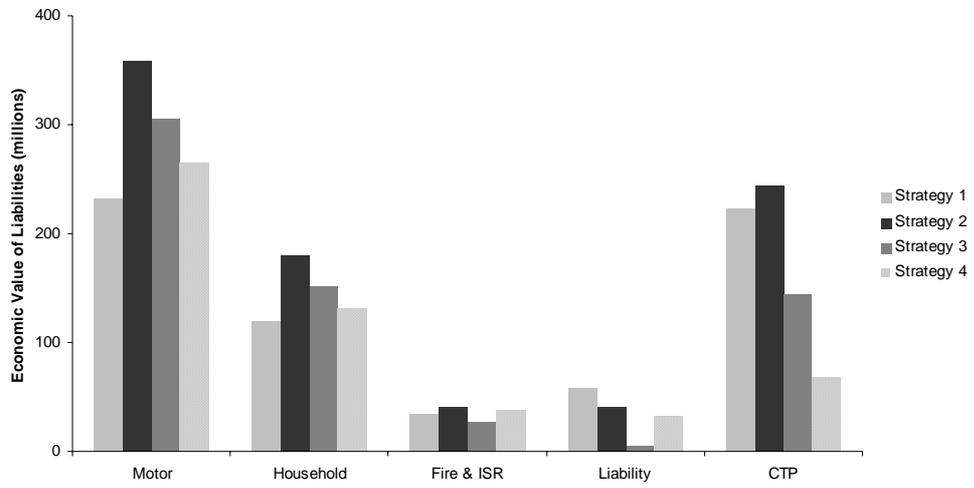


Figure 4: Economic value of liabilities for different strategies.

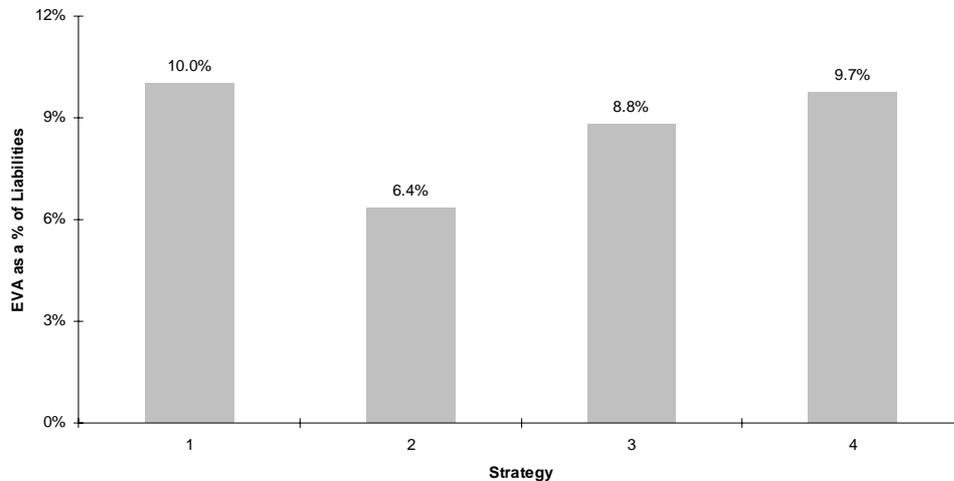


Figure 5: EVA for different strategies.

who maximize shareholder value added do not excessively leverage the financial structure because of policyholder preferences for financial quality.

A significant difference between the two cases is the amount of capital initially subscribed and the net present value of profit margins. The EVA maximizing insurer generates capital from the profit margins on policies whereas the capital generated from premiums is almost negligible for the total firm value maximizing insurer. The returns to the capital subscribers for the total firm value maximizing insurer are close to zero so that this will not be a strategy adopted by shareholders aiming to maximize the return on their investment in the insurer. This will be optimal where the shareholders of a total firm value maximizing insurer are also its policyholders. A mutual insurer will aim to maximize total firm value and policyholders pay a premium and effectively subscribe capital by a participating insurance policy. Figure 6 shows that the total firm value maximizing mutual insurer has substantially lower default risk than the EVA maximizing insurer.

The prices for the mutual insurer, allowing for the frictional costs they have to meet as effective shareholders, are lower than for the EVA maximizing insurer as shown by Figure 7. Lower prices and default risk under a mutual structure result in almost double the amount of underwriting business for the insurer, with a mutual insurer writing 1.94 times the economic value of liabilities of an EVA maximizing insurer.

Assets	(thousands)	%	Economic Liabilities	(thousands)	%
Invested Assets			Loss Reserves		
Cash	130,004	15.0	Motor	231,334	26.7
Bonds	563,351	65.0	Household	119,433	13.8
Stocks	173,339	20.0	Fire & ISR	34,342	4.0
Total Assets	866,695		Liability	58,197	6.7
			CTP	222,481	25.7
			PV of Tax Liability	0	0.0
			PV of Agency Cost Liability	2,489	0.3
			Total Economic		
			Liabilities	668,276	77.1
			Equity		
			Capital Subscribed	130,850	15.1
			NPV of Future Profits	66,777	7.7
			Default Value	791	0.1
			Total Economic		
			Capital	198,418	22.9
			Total Economic		
			Liabilities & Capital	866,695	

Table 11: The balance sheet of an EVA maximizing insurer.

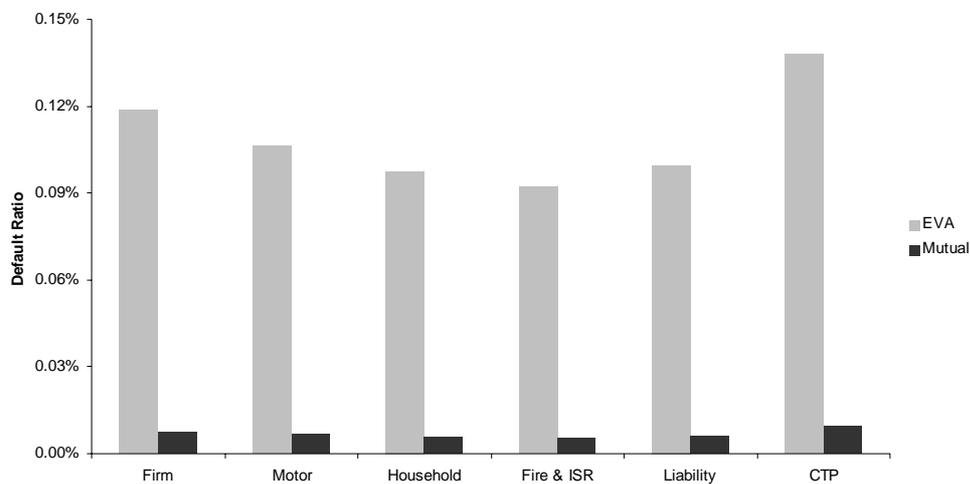


Figure 6: Default values for an EVA maximizing insurer and a mutual insurer.

Assets	(thousands)	%	Economic Liabilities	(thousands)	%
Invested Assets			Loss Reserves		
Cash	240,725	15.0	Motor	373,674	23.3
Bonds	1,043,142	65.0	Household	253,007	15.8
Stocks	320,967	20.0	Fire & ISR	123,249	7.7
Total Assets	1,604,833		Liability	189,806	11.8
			CTP	348,748	21.7
			PV of Tax Liability	0	0.0
			PV of Agency Cost Liability	5,891	0.4
			Total Economic		
			Liabilities	1,294,374	80.7
			Equity		
			Capital Subscribed	309,637	19.3
			NPV of Future Profits	729	0.0
			Default Value	94	0.0
			Total Economic		
			Capital	310,459	19.3
			Total Economic		
			Liabilities & Capital	1,604,833	

Table 12: The balance sheet of a firm value maximizing insurer.

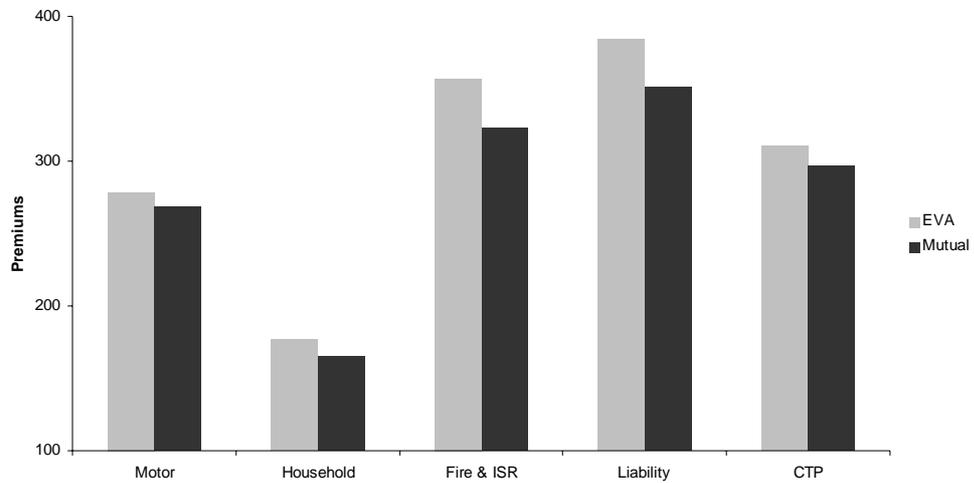


Figure 7: Premiums charged by an EVA maximizing insurer and a mutual insurer.

## 6 Conclusions

We have used a value maximizing model of an insurer including frictional costs, demand elasticity and policyholder preferences for financial quality to assess economic capital approaches for pricing and capitalization. We use economic capital approaches based on VaR and proportional capital allocation to line of business. We find that in general economic capital approaches to pricing are not consistent with shareholder value maximizing strategies if they do not take into account policyholder demand and preferences for financial quality. Direct approaches to capital and pricing using insurer value maximizing models are preferable to economic capital approaches where policyholder preferences for financial quality and demand elasticity impact on pricing. By considering the total firm value, both policyholder and shareholder wealth will be maximized if the insurer is a mutual. Shareholder wealth maximization strategies do not produce optimal wealth outcomes for policyholders. Finally, enterprise risk management should increase its emphasis on value maximization rather than on risk and economic capital.

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