On the Aggregation of Total Factor Productivity Measures*

by

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1. Background

In recent years, policy makers and economic analysts have exhibited growing interest in the measurement of productivity. Some analysts are interested mainly in measuring the performances at firm, plant or division levels while others are concerned in the productivity growth of particular industries, sectors or the whole economy. A great deal of concern among economists and statistician over the last years is the relation between aggregate and firm level productivity measures. This includes: the extent to which these unit level productivity growth measures can be consistently aggregated; the validity of the underlying assumptions in aggregate analyses; the search for a possible aggregation approach with nice properties; and, the choice of the weights.

Over the decades, a large number of published studies have investigated aggregation of efficiency measures. Most common are the measurement of technical efficiency, allocative inefficiency as well as overall economic efficiency. A very important issue on the measurement of productive efficiency initially raised by Farrell (1957) is on the computation of an industry performance measure consistent with the aggregates of individual firm performance measures. This has motivated some researchers to investigate further. Several issues on aggregation have been discussed and examined by: Li and Ng (1995); Blackorby and Russel (1999); Fox (1999); Bogetoft (1999); Briec, Dervaux and Leleu (2002) and Fare and Grosskopf (2002).

A more recent paper of Fare and Zelenyuk (2003) demonstrated a new approach in aggregating Farrell efficiency measures and discover conditions under which firm level efficiencies can be aggregated to obtain an industry level efficiency. This industry overall revenue efficiency measure is an output value (observed)-weighted average of the firms’ revenue efficiencies. Moreover, Fare and Zelenyuk (2003) made use of the Li and Ng (1995) results to decompose the said industry overall efficiency measure into aggregate allocative and aggregate technical efficiency measures. In addition, an industry technical efficiency measure is also introduced which is a multi-output generalisation of the Farrell (1957) measure of structural efficiency of an industry.
Fare and Zelenyuk (2003) stated: “To define the industry revenue function and obtain an aggregation theorem, it is crucial that all firms face the same output price vector.” In the real world, firms belonging to a certain industry producing the same commodity may have different output prices. Moreover, they may also face different prices for their inputs. So, it is useful to examine the effects of relaxing the output price condition (i.e. same for all firms) on their aggregation and decomposition procedures.

Reallocation of industry resources among firms also plays an important role in any aggregation exercise. Farrell (1957) in his seminal paper disclosed that “two firms which, taken individually, are technically efficient, are not perfectly technically efficient when taken together. In general, this implies that the industry outputs may not be on the industry production frontier, even if each firm of the industry is efficiently operating on its production frontier. This brought Fare et al (1992) to extend the Johansen approach and formulate an industry production model that allows some input(s) to be firm specific (not reallocatable) and some to be allocatable across firms in an industry performance measurement. Their finding is that when all inputs are reallocated, industry is at least as large as when only some inputs are, and that when no input is reallocatable, industry output is smallest. Li and Ng (1995) in their measurement of productive efficiency of a group of firms shows that shadow revenue of a group of firms can indeed be further increased through reallocation of inputs among firms. Labour, energy, materials and other services inputs in an industry can be reallocated across firms while the possibility of reallocating capital resources is very little or none at all in the short-run.

To be able to effectively reallocate resources across firms, one has to know industry technology. One can derive industry technology by summing individual firm technologies, just like what Fare and Zelenyuk (2003) adopted in their study. Moreover, they assumed that each firms faces different technology. Though summing of individual firm’s technology to define the industry technology make sense, it make more sense to talk about industry and firms efficiency measures if we imposed the same technology to all firms in the industry.

In this paper, we first aim to examine and evaluate the behaviour of all the efficiency measures developed by Fare and Zelenyuk (2003) when we relax the assumptions
that: (a) firms face the same price; and, (b) no reallocation of resources among firms. In fact, the paper numerically examines the behaviour of the aggregate efficiency measures that allow some inputs (capital and others) to be firm specific (not reallocatable) and labour, energy, materials or other services inputs to be allocatable across firms in an industry. In addition, the paper also looks at the efficiency measures when same technology is imposed for each individual firm.

With a good result in the first objective, this paper will then attempt to investigate the dynamic behaviour of TFP index with the aim of defining an industry Malmquist TFP index. The present paper will first consider the output oriented Malmquist TFP. If successful, a parallel application will be done using the input-oriented Malmquist TFP index later.

This paper was motivated by a desire to measure industry TFP change from the firm level TFPs. Using the traditional TFP index methods, Fox (2002) aggregation method, and Balk (2001) method, the third objective of the paper is to empirically examine the different industry level TFP measures. Fox (2002) provides alternative aggregation method which satisfies monotonicity property while Balk (2001) measures productivity level and productivity change based on real profitability. With the availability of extensive micro data bases for market industry which provide input, output and price data at the firm level, we assess the sensitivity of the aggregate productivity results. A very important issue on aggregation which the paper looks at is on the calculation of the relative size of each firm (weights) used in each of the aggregation procedures. These include methods based on revenue and cost shares. A part of this study is the investigation of the use of shadow shares, an objective that will be pursued in future research.

This paper is organised into sections. In Section 2 we show how the Farrell output oriented efficiency indexes could be aggregated when firms face different output prices. We also investigate the effect of reallocation of allocatable inputs across firms in the industry in the aggregation process. A numerical illustration ends the section making use of the Australian Bureau of Statistics (ABS) microdata from the 1998 ABS Confidentialised Unit Record File (CURF). In Section 3, we briefly review the different firm and industry productivity measures and start to define the industry
Malmquist TFP index in Section 4. We also present selected results from the empirical applications of the aggregation methods using the Australian Bureau of Statistics (ABS) microdata from the 1994-95 to 1997-98 ABS Confidentialised Unit Record File (CURF). We consider 100 firms from the Clothing, Textile, Footwear and Leather manufacturing industry (ANZSIC Code 22). Finally, some concluding remarks are presented in Section 5.

2. Aggregating the Farrell output orientated efficiency indexes when firm faces different output prices

In this section, we re-examine the Fare and Zelenyuk (2003) methods. We relax the assumption that firms face the same output price and show how industry output (revenue) efficiency can be derived from the member firms’ output efficiency. At the latter part, we relate this to the industry output oriented technical and allocative efficiency measures. The section will take into account the output (or revenue) oriented measures of Farrell (1957) efficiency framework. A parallel investigation was also executed and applied to the input oriented case but for the present paper it will not be presented and discussed.

2.1 Technology sets

Consider the case of an industry with $K$ firms, $k=1, 2, 3, ..., K, \ K>1$. Each $k^{th}$ firm has an input vector $x^k = (x_{k1}, ..., x_{kN}) \in \mathbb{R}^N_+$, $N$ inputs, and output vector $y^k = (y_{k1}, ..., y_{kM}) \in \mathbb{R}^M_+$, $M$ outputs.

Note that as a notation for the present paper, vectors without superscripts (but sometimes with primes) will be utilised as variables. Vectors with bars will represent observations, thus for instance $(\overline{x}^k, \overline{y}^k)$ denotes the input and output quantities of firm $k$. 
Each $k^{th}$ firm has a production technology defined by its output sets $P^k(x^k)$ which represents the set of all output vectors, $y^k$, which can be produced using the input vector, $x^k$. That is,

$$P^k(x^k) = \{ y^k : x^k \text{ can produce } y^k \text{ at firm } k \}. \quad (2.1)$$

Each $k^{th}$ firm technology as represented by $P^k(x^k)$, is assumed to satisfy standard properties (see Fare and Primont (1995)).

The industry output (technology set) is defined as the sum of the $K$ firm’s technologies. That is,

$$P^I(x) = \sum_{k=1}^K P^k(x^k) = \left\{ y = \sum_{k=1}^K y^k : y^k \in P^k(x^k), k = 1, \ldots, K \right\}, \quad (2.2)$$

where $x = (x^1, x^2, \ldots, x^K)$. These industry technologies possess properties similar to those of the firm technologies. Also, there is also no reallocation of inputs among the firms in the industry.

### 2.2 Revenue Efficiencies

We start by defining the firms’ $k$ price output vector, $p^k = (p_{k1}, p_{k2}, \ldots, p_{km})$, $p^k \in \mathbb{R}^m_+$. The firm faces a strictly positive vector of output prices. The $k^{th}$ firm revenue function is given by

$$R^k(x^k, p^k) = \max_{y^k} \{ p^k y^k : y^k \in P^k(x^k) \}. \quad (2.3)$$

Let $p^k y^k$ be the firm $k$’s observed revenue. Since $P^k(x^k)$ satisfies certain properties then $R^k(x^k, p^k)$ is well defined. Following Fare et al (1985, p.95), the firm revenue efficiency can be expressed as the ratio of the firms’ maximal to its observed revenue, that is,

$$\frac{R^k(x^k, p^k)}{p^k y^k}. \quad (2.4)$$

Again, this is well defined for all $p^k \in \mathbb{R}^m_+$ as the output set $P^k(x^k)$ is compact and $p^k y^k > 0$.

Similarly, the industry revenue function can be defined as

$$R^I(x^1, \ldots, x^K, p^1, p^2, \ldots, p^K) = \max_{y^1, \ldots, y^K} \left\{ \sum_{k=1}^K p^k y^k : y^k \in P^k(x^k) \right\}, \quad (2.5)$$
and the *industry revenue efficiency* as

\[
R^I\left(x^1, \ldots, x^K, p^1, p^2, \ldots, p^K\right) / \sum_{k=1}^{K} p_k \bar{y}_k
\]  

(2.6)

Note that \( R^I\left(x^1, \ldots, x^K, p^1, p^2, \ldots, p^K\right) \) is well defined as the industry faces strictly positive vector of output prices given by \( p=(p^1, p^2, \ldots, p^K) \in \mathbb{R}^M_+ \) and seek to maximize the revenue \( \sum_{k=1}^{K} p_k y_k, y_k \in P^k(x^k), k=1,2,\ldots,K \). Note that \( \sum_{k=1}^{K} p_k \bar{y}_k > 0 \).

Using the revenue version of the Koopmans’ Theorem (Fare and Grosskopf, 2002, p.106) which states that “the industry maximal revenue is the sum of the firms’ maximal revenues,” (2.5) is then equal to the sum of (2.3), that is,

\[
R^I\left(x^1, \ldots, x^K, p^1, \ldots, p^K\right) = \sum_{k=1}^{K} R^k(x^k, p^k), p^k \in \mathbb{R}^M_+.
\]  

(2.7)

The proof is straightforward.

Thus, relating (2.4) and (2.6) combined with (2.7), we define the *industry overall output (or revenue) efficiency* as the share-weighted average of the firms’ revenue efficiencies. This is given by

\[
RE^I_O = \frac{R^I\left(x^1, \ldots, x^K, p^1, \ldots, p^K\right)}{\sum_{k=1}^{K} p_k \bar{y}_k} \times \bar{s}_\bar{o}^k,
\]  

(2.8)

where \( \bar{s}_\bar{o}^k = \frac{p_k \bar{y}_k}{\sum_{k=1}^{K} p_k \bar{y}_k} \) is the \( k \)-th firm’s observed revenue share. Note that \( \sum_{k=1}^{K} \bar{s}_\bar{o}^k = 1 \)

and \( 0 < RE^I_O \leq 1 \).

Also, note that (2.4) and (2.6) by definitions are bigger than or equal to one, hence (2.8) satisfies the Aggregation Indication Axiom (AI) of Blackorby and Russel (1999), which states that “the industry is considered to be efficient if and only if each of its firms is efficient.” Clearly,

\[
\frac{R^I\left(x^1, \ldots, x^K, p\right)}{\sum_{k=1}^{K} p_k \bar{y}_k} = 1 \quad \text{if and only if} \quad \frac{R^k(x^k, p)}{p_k \bar{y}_k} = 1, \text{for all } k.
\]
2.3 Distance functions

We now focus at the distance functions for our firm technologies defined in Section 2.1. Firstly, we define an output-oriented distance function on the output set $P^k(x^k)$, $k=1,2,...,K$, as

$$D_0^k(x^k, y^k) = \inf \left\{ \delta^k : \delta^k > 0, \left( y^k / \delta^k \right) \in P^k(x^k) \right\}$$

(2.9)

Since the outputs are disposable, we then have

$$y^k \in P^k(x^k) \text{ if and only if } D_0^k(x^k, y^k) \leq 1,$$

(2.10)

which means that the distance function, $D_0^k(x^k, y^k)$, will take a value less than or equal to one if the output vector, $y^k$, is an element of the feasible production set, $P^k(x^k)$. It will also be increasing in $x^k$ and linearly homogeneous in $y^k$.

Following Fare and Zelenyuk (2003), we define an output-oriented distance function for the industry technology as

$$D_0(x^1,...,x^K, \sum_{k=1}^K y^k) = \inf \left\{ \delta : \delta > 0, \left( \sum_{k=1}^K y^k / \delta \right) \in P^I(x) \right\}$$

$$= \max_{\delta} \left\{ \inf \left\{ \delta^k : y^k / \delta^k \in P^k(x^k), \delta^k > 0 \right\} \text{ for all } k = 1,2,...,K \right\}$$

(2.11)

$$D_0^I(x^1,...,x^K, \sum_{k=1}^K y^k) \leq 1 \text{ as } \sum_{k=1}^K y^k \in P^I(x).$$

2.4 Technical Efficiencies

Following Fare et al (1985), the Farrell output-oriented technical efficiency for the $k^{th}$ firm is given by

$$TE_0^k = D_0^k(x^k, y^k), \quad TE_0^k \leq 1.$$  

(2.12)

If $TE_0^k = 1$, then firm $k$ is said to be output technically efficient. This technical efficiency measure satisfies the following desirable properties (see Fare et al (1985)).

The industry output-oriented Farrell technical efficiency measure is given by
\[ TE^I_O = D^I_O(x^I, x^2, ..., x^K, \sum_{k=1}^K y^k), \quad TE^I_O \leq 1. \]  

The right-hand side of the equation is obtained using (2.11). Equation (2.13) also satisfies the desirable properties and they are independent of the output prices.

Following Fare and Zelenyuk (2003), we can define a \textit{share-weighted} output oriented industry technical efficiency measure using industry member's individual technical efficiencies (2.12) as

\[ TE^{I*}_O = \sum_{k=1}^K D^I_O(x^k, y^k) \times s\bar{\sigma}^k = \sum_{k=1}^K TE^k_O \times s\bar{\sigma}^k, \quad TE^{I*}_O \leq 1, \]  

where \( s\bar{\sigma}^k \) is the \( k^{th} \) firm's observed revenue share. Fare and Zelenyuk (2003) named this output-oriented overall industry technical efficiency measure as the multi-output generalization of the Farrell single output "structural efficiency of an industry". Note that instead of output shares it uses observed revenue shares. \( TE^{I*}_O \) satisfy the technical efficiency aggregation indication axiom (TEAI) formulated by Fare and Zelenyuk (2003, p. 617) based on Blackorby and Russell (1999), that is

\[ TE^{I*}_O = 1 \text{ if and only if } TE^k_O = 1, k = 1, 2, ..., K. \]  

It is important to note that:

i) Fare and Grosskopf (2003) justify the use of revenue shares as weight;

ii) \( TE^{I*}_O \) is not a good measure of technical efficiency since it contains value information and is not just a function of inputs and outputs;

iii) if we have a single output, then it is price independent, assuming all firms face same output price; and,

iv) \( TE^{I*}_O \) can use price independent weights (see Fare & Zelenyuk 2003).

### 2.5 Allocative Efficiencies

Recall that Farrell (1957) proposed the efficiency of a firm that consists of two components, namely, the technical efficiency (TE), which we have been looking earlier, and the allocative efficiency (AE), which reflects the ability of a firm to
optimally use the resources, given their respective input prices and the production technology. The product of these two efficiency components provides the measures of overall economic efficiency. Khumbhakar and Lovell (2000, p.57) also proposed this decomposition to revenue efficiency. The measure of output revenue efficiency \(RE_O\) decomposes into output oriented technical efficiency \(TE_O\) and output allocative efficiency \(AE_O\) as,

\[
RE_O = TE_O \times AE_O
\]  

(2.16)

In addition, they defined the measure of output allocative efficiency as a ratio of \(RE_O\) and \(AE_O\). Knowing that a measure of revenue efficiency can be decomposed multiplicatively into technical and allocative components, the \(k^{th}\) firm revenue efficiency can be expressed as,

\[
RE^k_O = \frac{R^k(x^k, y^k)}{p^k y^k} = TE^k_O \times AE^k_O = D^k_O(x^k, y^k) \times AE^k_O,
\]  

(2.17)

where the firm \(k\)'s allocative efficiency measure, \(AE^k_O\), following (2.17) can be obtained as a residual, that is,

\[
AE^k_O = \frac{R^k(x^k, y^k)}{p^k y^k} / D^k_O(x^k, y^k), \quad AE^k_O \leq 1 .
\]  

(2.18)

Suppose we relax the assumption of allocative efficiency for each firms. Following Li and Ng (1995), we can define an aggregate industry allocative efficiency, \(AE^I_O\), as a share-weighted average of firms’ output oriented allocative efficiencies, that is,

\[
AE^I_O = \sum_{k=1}^{K} AE^k_O \times so^{k*}, \quad AE^I_O \leq 1,
\]  

(2.19)

where the firm \(k\)'s allocative efficiency, \(AE^k_O\), is obtained using (2.18).

The weights in (2.19) are now based on potential outputs rather than the observed outputs, defined as

\[
so^{k*} = \left( \frac{p^k y^k / D^k_O(x^k, y^k)}{\sum_{k=1}^{K} (p^k y^k / D^k_O(x^k, y^k))} \right) = \left( \frac{p^k y^{k*}}{\sum_{k=1}^{K} (p^k y^{k*})} \right),
\]  

(2.20)

where \(y^{k*}\) is unobserved potential output vector.

Following Fare and Zelenyuk (2003, p.618), we can decompose the industry overall output (revenue) efficiency (2.8) into an aggregate allocative efficiency (2.19) and an aggregate technical efficiency equivalent to (2.14). Algebraically, we have
2.6 Reallocation of inputs across firms in an industry

Again, we use the notation and assumptions in section 2.1. Now assume all firms face output price vector denoted by \( p = (p_1, p_2, \ldots, p_m) \in \mathbb{R}^M_+ \).

What would happen to the method developed by Fare and Zelenyuk (2003) when we are given an industry technology and reallocate the allocatable inputs endowment across firms holding the other inputs fixed or as firm specific? For this exercise we assumed that all firms face same output prices.

Under this set up, we can define a new industry technology as,

\[
P^I(x^*) = \left\{ y^*: x^* = \sum_{k=1}^{K} x^k, \ y^* = \sum_{k=1}^{K} y^k, \ x^* \in \mathbb{R}^N_+, \ y^* \in \mathbb{R}^M_+, \ y^k \in P^k(x^k) \right\}
\]

(2.22)

This industry output set is the maximal output that can be obtained from each firm given the total amount of input resources available in the industry. Allowing for the reallocation of inputs, we could define an industry maximal potential revenue function as,

\[
R^{I*}(x, p) = \max_{y^*} \left\{ p y^*: x^* = \sum_{k=1}^{K} x^k \text{ can produce } y^* = \sum_{k=1}^{K} y^k \in P^I(x^*) \right\}
\]

(2.23)

This is a revenue version of the Johansen industry model where the maximum is found over all feasible input allocations.

The industry potential revenue efficiency can now be written as
\[
RE^{I*}_O = \frac{R^{I*}(x, p)}{p \sum_{k=1}^K \bar{y}^k}.
\] (2.24)

Li and Ng (1995) shows that given fixed input allocations for individual firms, reallocating resources among firms may raise the industry output even if all firms are efficient individually. This means that even if each firm is technically efficient, but since output of industry can increase via reallocation implies that the industry is inefficient. Hence, using (2.7) and (2.23), it can be shown that

\[
R^{I*}(x, p) \geq R^I(x^1, ..., x^K, p) = \sum_{k=1}^K R^k(x^k, p), p \in \Omega^I
\] (2.25)

With the above results, we can then easily establish that (2.25) is greater than or equal to (2.8), that is

\[
\frac{R^{I*}(x, p)}{p \sum_{k=1}^K \bar{y}^k} \geq \frac{k \cdot R^k(x^k, p)}{p \bar{y}^k} \times \delta^k.
\] (2.26)

There is an interesting story behind the inequality in (2.26). By reallocating inputs among firms within an industry, we realize a potential gain in total maximal revenue and this is given by the difference between the right and left hand side of (2.26). We can then call this as an efficiency gain.

Another significant result out of (2.26) is that the industry maximal revenue efficiency (2.8) is a lower bound for the industry maximal revenue allowing for reallocation of inputs.

We now find a measure for this efficiency gain. First, we define an output orientated distance function for our industry technology allowing for reallocation of inputs. This can be expressed as

\[
D^I_0(x^*, y^*) = \inf \left\{ \delta^* : \frac{y^*}{\delta^*} \in P^I(x^*), \delta^* > 0 \right\}.
\] (2.27)

We can now formulate the output oriented industry potential revenue efficiency using (2.17) as

\[
RE^{I*}_O = D^I_0(x^*, y^*) \times AE^{I*}_O, \quad RE^{I*}_O \leq 1.
\] (2.28)

Following Li and Ng (1995), we can define the output (revenue) oriented industry measure of reallocative efficiency as

\[
RE^{I*}_O = REA^I_0 \times RE^I_0
\] (2.29)
where \( RE_A^I \) is obtained as a residual from the ratio of \( RE^I_* \) and \( RE^I_O \). The \( RE^I_O \) is the aggregate output oriented industry revenue efficiency (2.21) defined in Section 2.5. \( RE^I_O \) capture the maximum total revenue after the technical and allocative inefficiencies of firms have been eliminated while \( RE^I_* \) is the maximum potential total industry revenue after reallocation of allocatable inputs across firms in an industry. A related graphical relationship between reallocate efficiency, allocative efficiency and technical efficiency measures is illustrated in Li and Ng (1998, p. 384).

### 2.7 Empirical Illustration

In this subsection, we use the records of individual firms in the Australian textile, clothing, footwear and leather manufacturing industry, which are taken from the Australian Bureau of Statistics confidentialized unit record file (ABS-CURF), to illustrate the effects of the reallocation of inputs in the revenue maximization. We also show how the overall industry efficiency can be derived from the individual firm efficiencies. We also examine also the sensitivity of the industry efficiencies by comparing it with the simple arithmetic, geometric and harmonic mean of the firm specific efficiencies.

**The data**

The author as a part of his main research project constructed a micro (firm)-level database for efficiency and productivity measurement using the ABS-CURF. The raw data, from which the micro database is constructed, are obtained from the results of the Australian Business Longitudinal Survey conducted by the ABS. The database includes one output, three inputs, an output price and a set of input prices for each individual firm. The output \( y \) is measured as real gross output where the output price is the corresponding producer price index \( p \) of the industry commodity at 2-digit ANSZIC Code. Capital input \( x_1 \) is the estimate of firm’s real capital stock. Labour input \( x_2 \) is the firm’s total employment while the real intermediate input \( x_3 \) is derived from the firm’s purchases of raw materials, fuel, water and others. User’s cost of capital \( w_1 \), price of labour \( w_2 \) and materials price index \( w_3 \) are also obtained from the ABS data.
The linear program

To be able to produce firm and industry efficiency measures discussed in the earlier sections, we applied standard data envelopment analysis (DEA) models assuming variable returns to scale (VRS) (see Coelli et al, 1998). We use the output orientated DEA models to derive individual firm’s technical and allocative efficiencies scores. While for the firm revenue efficiency measurement, the revenue maximization DEA problem in Coelli et al (1998, p. 162) is then solved using the Shazam software.

For the measurement of the industry potential revenue efficiency (2.24), we solve the revenue maximization DEA problem with reallocation, given by

$$\max_{\lambda, \lambda^*, \lambda^*, \cdots, \lambda^*, \cdots, \lambda^*} \quad \text{subject to}$$

$$py^* + py^2 + \cdots + py^K$$

$$-y^* + Y\lambda_i \geq 0 \quad i = 1, 2, \ldots, K$$

$$x_i^* - X\lambda_i \geq 0 \quad i = 1, 2, \ldots, K,$$

$$v = n_f + 1, \ldots, N$$

$$x_f^* - X\lambda_i \geq 0 \quad i = 1, 2, \ldots, K,$$

$$f = 1, 2, \ldots, n_f$$

$$x_v^* + \cdots + x_v^K \leq x_v \quad v = n_f + 1, \ldots, N$$

$$1'\lambda_i = 1 \quad i = 1, 2, \ldots, K$$

$$\lambda_i \geq 0$$

In model 2.30, we assume that there are data on $N$ inputs and $M$ outputs for each of $K$ firms. For the $i^{th}$ firm these are represented by the column vectors $x^i$ and $y^i$ respectively. The output matrix, $Y (M \times K)$ and the $N \times K$ input matrix, $X$, represent the data for all $K$ firms. The column vectors $x^i$ can be partitioned into a column vector of firm non-allocatable inputs $x^i$ and a vector of allocable inputs $x^i_v$. Now, $x_v$, is the sum of the allocatable input vectors while $1$ is a $K \times 1$ vector of ones. We also assumed a fixed vector of output prices, $p$, for all firms.

The aggregation and reallocation results

Subject to the limitation of the computer software used in the analysis, the numerical illustration takes only 10 firms in the Australian textile industry. We assumed that each firm uses three inputs to produce one output, which are defined above. We
assume the following: each firm faces same output prices; labour and material inputs are allocatable across firms in the industry; and capital input is non-allocatable in each firm. We compute the individual firm’s revenue efficiencies, the share-weighted industry output (revenue) efficiencies, the industry potential revenue efficiency, and the efficiency gain due to reallocation.

Table 1 presents the summary of the efficiency estimates including the observed and maximal revenues for each firm. From the table, we can deduce that four firms show improvements in their output as manifested by the calculated individual revenue efficiencies. The industry exhibits a 9.69 percent improvement in the revenue relative to the observed revenue. After subjecting the industry to reallocation of variable inputs, it reveals a hefty 18.22 percent increase in revenue efficiency relative to the actual or observed revenue. When we reallocate the two inputs among our firms in the industry, we achieved a potential gain of 8.53 percent. This demonstrates our relationship in (2.26).

Based on our results in the output-oriented case, we have verified that if output or revenue of the industry can increase via reallocation, then the total industry is in fact inefficient. The industry’s output oriented reallocative efficiency is found to be 1.0778. This implies that after reallocation, a further improvement of 7.78 percent in revenue efficiency is observed relative to the industry’s maximal revenue. Looking at the sensitivity of the estimates when compared to the simple averages, it is relatively significant.

Table 1. Revenue efficiencies\(^2\) for firms and industry

<table>
<thead>
<tr>
<th>Firms</th>
<th>Observed revenue ($'000)</th>
<th>Maximal revenue ($'000)</th>
<th>Revenue efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4487.60</td>
<td>4572.63</td>
<td>1.0189</td>
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\(^2\) The computation of efficiency measures are done on Shazam using variable returns to scale.
3. Measures of TFP Growth

A wide variety of techniques have been developed for productivity change measurement. Some are applicable to macro or industry analyses and some to analyses at micro or firm level. The most commonly used technique in the measurement of TFP growth is the non-parametric approach. This section gives a brief review of common non-parametric approaches in firm level productivity measures and how these measures can be used in measuring aggregate industry TFP growth.

3.1 Firm level productivity measures

In this subsection, we revisit the traditional index number methods for firm level performance measurement. Following Balk (2001), we also look at the gross output and value-added approaches in productivity change estimation.

**TFP Growth Measurement using Index Numbers**

We first review the traditional index number formula for measuring productivity changes in the levels of output produced and the levels of inputs used in the production process over two time periods.
By definition, total factor productivity index measures change in total output relative to the change in the usage of all inputs. The $k^{th}$ firm TFP index for two time periods $s$ and $t$ is expressed as

$$\ln TFP_{st}^k = \ln \frac{Output \ Index_{st}^k}{Input \ Index_{st}^k}$$  \hspace{1cm} (3.1)$$

In most empirical applications, the Tornqvist index formula is used for both the output and input index calculations. Diewert (1992) shows Fisher index satisfies more properties as compared to Tornqvist index. Fisher index is referred to as an exact and superlative index, hence it could also be use as another suitable formula to compute firm’s TFP change. This is given by

$$TFP_{st}^k = \frac{Fisher \ output \ index_{st}^k}{Fisher \ input \ index_{st}^k}$$ \hspace{1cm} (3.2)$$

Both Tornqvist and Fisher indices possess important properties and satisfies basic axioms. In most practical applications, both these indices when used in TFP measurement yield very similar numerical values. Most statistical agencies involved in TFP measurement, preferred to use Tornqvist formula.

**Gross output and value-added based productivity measures**

We now consider the productivity concepts and measures discussed in Balk (2001). In his study, the firm is considered as an input-output system. The commodities produced by the firm, such as goods and/or services are at the output side while the commodities consumed such as capital, labour, energy, materials and services inputs are at the input side.

For any firm $k$, we let the value of the nominal gross output (GO) during the period $t$ be equal to the observed revenue. Its production cost (PC) is also equal to the observed cost. The firm’s value added (VA) can be defined as gross output minus the cost of intermediate inputs (II). Two important assumptions considered here are: (1) firm operates in a market environment, so that every commodity comes with a value
(in monetary terms) and (2) intermediate inputs are acquired from other firms or imported.

Balk (2001) defines the firm’s profitability as its revenue divided by its production cost, that is

\[
\text{Firm profitability} = \frac{GO}{PC} \quad (3.3)
\]

According to his paper, there is a link between the measurement of productivity levels and the measurement of productivity growth at the firm level. This link is provided by the concept of real profitability. If \( GO \) is the firm’s total revenue and \( PC \) is the total cost, then \( GO \) divided by \( PC \) is the firm’s nominal profitability. Hence,

\[
\text{Firm’s Real Profitability} = \frac{\text{deflated} GO}{\text{deflated} PC} \quad (3.4)
\]

The above expression is then equivalent to real productivity of the firm. Based on this, the index of productivity change can be expressed as either a/an:

i) ratio of two productivity levels;  
ii) index of real profitability;  
iii) index of deflated revenue relative to index of deflated cost; and/or,  
iv) output quantity index divided by an input quantity index.

The productivity level of firm \( k \) in period \( t \) is then defined as

\[
\text{TFP}_t^k = \frac{\text{Real output}_t^k}{\text{Real input}_t^k}. \quad (3.5)
\]

The gross output based TFP index for firm \( k \) at period \( t \) is measured as

\[
\text{TFP}_{t, \text{GO}}^k = \frac{\text{Real gross output}_t^k}{\text{Real input (production cost)}_t^k}. \quad (3.6)
\]

The corresponding firm level value added based TFP index is obtained by
The Output-Oriented Malmquist TFP (CCD Approach)

We glance at the general Malmquist productivity index based on Caves, Christensen and Diewert (CCD) (1982). In here, the Malmquist index is measured using distance functions. It measures the TFP change between two data points by calculating the ratio of the distances of each data point relative to common technology. Following Caves, Christensen and Diewert (1982) approach, under the assumption of technical and allocative efficiency, we can define a $k^{th}$ firm output-based Malmquist productivity growth from period $s$ to period $t$ as,

$$TFP^k_{st} = \frac{\text{Real value added}^k_i}{\text{Real capital and labour inputs}^k_i}.$$  \hspace{1cm} (3.7)

This would mean that given the assumption that each firm is technically and allocatively efficient and adding CRS technology then $M^k_O(y^k_s, y^k_t, x^k_s, x^k_t)$ can simply be computed using the traditional Tornqvist TFP index number formula. The above result provides a justification for the use of the standard index number measure for firm level TFP growth analyses.

3.2 Industry level productivity measures

We examine different approaches available for aggregating firm level productivity indices leading to industry-level measures. We also look at the different methods to calculate the relative sizes of the firms (weights). For all these measures, we assume...
that all firms in the industry exist during both the comparison and base periods, and there is no entry or exit of firms.

**Simple Averages**

An easy way to aggregate firm level TFP indices is by the use of simple averaging procedure. In this aggregation procedure, we are assuming the same (equal) weights for all the firms in the industry. This is an approach used in some of the DEA program such as DEAP.

A simple arithmetic, geometric and harmonic mean of all the firm’s TFP growths between period \( t \) and \( s \) are given by the following formulas:

\[
T_{FP}^{st} = \frac{\sum_{k=1}^{K} TFP_{st}^{k}}{K} \quad ; \quad T_{FP}^{st} = \prod_{k=1}^{K} (TFP_{st}^{k})^{\frac{1}{K}} \quad ; \quad \text{and} \quad T_{FP}^{st} = \frac{1}{K} \sum_{k=1}^{K} \left( \frac{1}{TFP_{st}^{k}} \right).
\]  

**Balk (2001) Aggregation Method**

Disregarding interactions between firms in an industry, Balk (2001) defines a natural measure of \( T_{FP}^{st} \), that is, the aggregate productivity level of all firms existing at period \( t \), as the sum of firm specific real output divided by the sum of firm-specific real input. This is given by

\[
T_{FP}^{st} = \frac{\sum_{k=1}^{K} \text{Real Input}^{st}_{k}}{\sum_{k=1}^{K} \text{Real Input}^{st}_{k}}.
\]  

Balk (2001) rewrites the above formula as a weighted-arithmetic mean of the firm-specific productivity levels, \( TFP^{st}_{t} \), as

\[
T_{FP}^{st} = \frac{\sum_{k=1}^{K} \text{real input} \times \text{real output}}{\sum_{k=1}^{K} \text{real input} \times \text{real output}} \equiv \sum_{k=1}^{K} \theta^{k} \times TFP^{st}_{t}.
\]
where the weights $\theta^k_t$ are firm-specific real input shares in period $t$. Note that
\[ \sum_{k=1}^{K} \theta^k_t = 1. \]

The aggregate productivity change between two periods $s$ and $t$, can be measured as a difference of $TFP^I$ corresponding to those periods, that is,
\[ \Delta TFP^I_{st} = TFP^I_t - TFP^I_s . \]  (3.13)

A percentage change is obtained by dividing $\Delta TFP^I_{st}$ by $TFP^I_s$.

**OECD (2001)**

Following the OECD (2001) Productivity Manual, the aggregate productivity level is measured as a weighted geometric mean, that is
\[ TFP^I_t = \prod_{k=1}^{K} (TFP^k_t)^{\theta^k_t} , \]  (3.14)

where $TFP^k_t$ and $\theta^k_t$ are defined just like in (3.5) and (3.12) respectively. Taking the natural logarithmic form of the above expression we can rewrite (3.14) as
\[ \ln TFP^I_t = \sum_{k=1}^{K} \theta^k_t \times \ln TFP^k_t . \]  (3.15)

The aggregate productivity change for the industry between periods $s$ and $t$, is given by the logarithmic difference
\[ \Delta \ln TFP^I_{st} = \ln TFP^I_t - \ln TFP^I_s . \]  (3.16)

**Fox (2002) aggregation method**

Assuming that the TFP scores have been calculated using any index number formula, Fox (2002) constructed a Tornqvist aggregation function to aggregate TFP change. This is given by
\[
\Delta TFP^{st}_k = \exp\left[\sum_{k=1}^{K} \left(\frac{1}{2}(\theta^k_s + \theta^k_t)(\ln TFP^k_t - \ln TFP^k_s)\right)\right],
\]

where \( \theta^k_s \) is the share of firm \( k \) in total industry for period \( s \). It could be noted that the arithmetic mean of the shares in two periods \( (s, t) \) are used to weight (log) changes in TFP. The share could take on any form (i.e., output share, input share, revenue share, cost share or valued-added share). Equation (3.17) has a nice property of being additive, such that the weighted productivity growths for each firm can simply be added. Moreover, this aggregate measure can be decomposed into the contribution from each firm and it satisfies the monotonicity property. The only limitation of this aggregation function is that it is only applicable to an industry where the same set of firms exists in the two periods.

**Weighted averages of firm level productivity**

We use averaging techniques (3.10), with weights, defined as the arithmetic mean of the shares of firm \( k \) in two periods \( (s, t) \), being compared. We can re-express (3.10) by:

- weighted geometric mean of TFP change indices
  \[
  TFP^{st}_k = \prod_{k=1}^{K} (TFP^k_s)^{\frac{\theta^k_s + \theta^k_t}{2}};
  \]
- weighted arithmetic mean of TFP change indices
  \[
  TFP^{st}_k = \sum_{k=1}^{K} \left(\frac{\theta^k_s + \theta^k_t}{2}\right) TFP^k_s; \text{ and,}
  \]
- weighted harmonic mean
  \[
  TFP^{st}_k = \frac{1}{\sum_{k=1}^{K} \left(\frac{\theta^k_s + \theta^k_t}{2}\right) TFP^k_s}. \]

**4. The Firm and Industry Malmquist TFP index**

A non-parametric measure that has become popular over the last decade is the data envelopment analysis (DEA) method. This calculates the Malmquist productivity
index based on Fare et al (1994). Subject to availability of suitable panel data, one can use this frontier estimation method to estimate firm level TFP growth without requiring any price information. Further, it does not require the assumption that all the firms are fully efficient, cost minimisers and revenue maximisers. This is of importance when we are to analyse non-market sectors or non-profit institutions performances. Another important aspect of DEA is that it permits Malmquist TFP index to be decomposed into technical efficiency and technical change components.

In this section, we first define the firm level Malmquist TFP index. We assume that: the firms face the same output prices; all firms face the same technology; all the firms exist (continuing firm) during all comparison periods; and there is no reallocation of inputs among firms in the industry. The discussion will be based on the results in Section 2. We then show how an industry output-oriented Malmquist TFP index can be defined and derived from firm-specific indexes.

**Notation**

Note that as a notation for this section, vectors with subscripts will represent observations, thus for instance \((x_t^k, y_t^k)\) denotes the input and output quantities of firm \(k\) in period \(t\). In the analysis, we will use \(t\) to denote the ‘comparison period’ and \(s\) for the ‘base period’.

Assuming that all firms face the same technology for any given period \(t\) or \(s\), that is (say at period \(t\)), then a firm and industry output (technology set) for period \(t\) is simply defined by \(P^x_t(x^t)\) and \(P^y_t(x)\), respectively. Similarly, we can define a period-\(s\) technology for any firm \(k\) and industry as \(P^x_s(x^s)\) and \(P^y_s(x)\). The above output sets also satisfies the standard properties.

We now define an output-oriented distance function on the output set \(P^y_t(x^t)\), \(k=1,2,...,K\), as

\[
D^t_O(x^t, y^t) = \inf \left\{ \delta^t : \delta^t > 0, \left( y^t / \delta^t \right) \in P^y_t(x^t) \right\} 
\]

Since the outputs are disposable, we then have

\[
y^t \in P^y_t(x^t) \quad \text{iff} \quad D^t_O(x^t, y^t) \leq 1, \tag{4.2}
\]
which means that the distance function, \( D^k_0(x^k, y^k) \), will take a value less than or equal to one if the output vector, \( y^k \), is an element of the feasible production set, \( P^{kt}(x^k) \). It will also be increasing in \( x^k \) and linearly homogeneous in \( y^k \). Similarly, we define an output-oriented distance function for any firm \( k \) in period \( s \) on the output set \( P^{ks}(x^k), k=1,2,...,K, \) as

\[
D^k_0(x^k, y^k) = \inf \left\{ \delta^k : \delta^k > 0, \left( \frac{y^k / \delta^k}{\delta} \right) \in P^{ks}(x^k) \right\}
\]

Using (2.11), we define an output-oriented distance function for the industry technology in period-t as

\[
D^k_0(x^1,...,x^K, \sum_{k=1}^K y^k) = \inf \left\{ \delta : \delta > 0, \left( \sum_{k=1}^K y^k / \delta \right) \in P^s(x) \right\}
\]

\[
= \max_{\delta} \left\{ \inf \left( \frac{y^k / \delta^k}{\delta} \in P^{kt}(x^k), \delta^k > 0 \right) \right\} \text{ for all } k = 1,2,...,K
\]

\[
D^k_0(x^1,...,x^K, \sum_{k=1}^K y^k) \leq 1 \text{ as } \sum_{k=1}^K y^k \in P^s(x).
\]

Similarly, we have an output-oriented distance function for the industry technology in period-s expressed as \( D^k_0(x^1,...,x^K, \sum_{k=1}^K y^k) \).

**Homotheticity**

The period-t technology for any firm \( k \) exhibits output homotheticity if \( P^{kt}(x^k) = G^{kt}(x^k)P^{kt}(1,x^k) \) for all \( x^k \), where \( G^{kt} : \mathbb{R}^N_{+} \rightarrow \mathbb{R}^N_{+} \) is a non-decreasing function consistent with the properties of \( P^{kt}(x^k) \). This is equivalent to saying that \( D^k_0(x^k, y^k) = D^k_0(1_N, y^k) / G^{kt}(x^k) \) (Balk, 1998).

**Implicit Hicks Neutrality**

The sequence of k-firms technologies pertaining to periods \( t=0, 1, 2, 3,... \) exhibits implicit Hicks output neutrality if for all \( x^k \), \( P^{kt}(x^k) = \hat{P}(x^k)B(t,x^k) \) where \( \hat{P}(x^k) \) satisfies the properties of \( P^{kt}(x^k) \) but independent of \( t \). \( B(t,x^k) \) is also a
function satisfying the same properties. It is equivalent to saying that 
\[ D_0^H(x^k, y^k) = \hat{D}_0(x^k, y^k) / B(1_k, x^k) \] (Balk, 1998).

4.1 The Firm level Malmquist productivity index

In this section we look at how an industry Malmquist TFP index can be define using the firm level Malmquist TFP indices. In, short we investigate the possibilities of aggregating firm level TFP growth measure using the Malmquist TFP index. Malmquist index formula can be defined using either the output-orientated approach or the input-orientated approach. This section will also be limited to the output-orientated approach.

The Malmquist TFP change index

Suppose for any firm \( k \), we have sufficient observations in each period \((t, s)\), so that a technology in each period can be estimated using mathematical programming, then one would not require the assumptions of technical and allocative efficiency to be able to calculate the Malmquist TFP index (Coelli et al, 1998). Following Fare et al (1994), the Malmquist output-oriented TFP change index between the base period \( s \) and the comparison period \( t \) for any \( k \) firm is given by,

\[
M_0^k(y_s^k, x_s^k, y_t^k, x_t^k) = \left[ \frac{D_0^k(y_s^k, x_s^k)}{D_0^k(y_t^k, x_t^k)} \times \frac{D_0^k(y_t^k, x_t^k)}{D_0^k(y_s^k, x_s^k)} \right]^{\frac{1}{2}}.
\] (4.5)

This is a geometric mean of two TFP indices. A value of \( M_0^k(y_s^k, x_s^k, y_t^k, x_t^k) \) greater than one indicates positive TFP change from period \( s \) to \( t \) for firm \( k \). If \( M_0^k(y_s^k, x_s^k, y_t^k, x_t^k) \) is less than one then it shows that firm \( k \) has a decline in TFP growth.

Following Fare et al (1994), the \( k^{th} \) firm Malmquist output-oriented TFP is defined as

\[
M_0^k(y_s^k, x_s^k, y_t^k, x_t^k) = \frac{D_0^k(y_s^k, x_s^k)}{D_0^k(y_t^k, x_t^k)} \times \left[ \frac{D_0^k(y_t^k, x_t^k)}{D_0^k(y_s^k, x_s^k)} \times \frac{D_0^k(y_s^k, x_s^k)}{D_0^k(y_t^k, x_t^k)} \right]^{\frac{1}{2}},
\] (4.6)
where \( D^O_k(y^k, x^k) \) is the distance from the period-t observations to the period-s technology for any firm \( k \). The first ratio term in the right-hand side of equation (4.6) measures the change in the output-oriented Farrell technical efficiency of firm \( k \) between period \( s \) and \( t \). We then write this first component as

\[
TEC^k_o = \frac{D^O_k(y^k, x^k)}{D^O_k(y^s, x^s)}, \quad (4.7)
\]

and the second component which is inside the bracket measures the technical change, that is,

\[
TEC^k_o = \left[ \frac{D^O_k(y^k, x^k)}{D^O_k(y^s, x^s)} \times \frac{D^O_k(y^k, x^k)}{D^O_k(y^s, x^s)} \right]^{\frac{1}{2}}, \quad (4.8)
\]

This decomposition is easily illustrated using a constant return to scale technology involving a single output and a single input.

**4.2 The industry Malmquist productivity index**

Can we define an industry Malmquist TFP index measure which is an aggregate of all the firm level technical efficiency changes and technical changes?

Suppose we consider an industry with \( K \) firms, each firm \( k \) producing a single output \( y^k_j \) and a single input \( x^k_j \), for any period \( j, j = s, t \). The firms continually exist in the two periods and all firms assume the same technology in each period \( j \). Moreover, the input resources are assumed to be fixed, that is, no reallocation of inputs among firms is allowed in the industry. Then we define an industry Malmquist TFP index by

\[
M^I_O = \frac{D^O_k(x^1, \ldots, x^K, \sum_{k=1}^K y^k)}{D^O_k(x^1, \ldots, x^K, \sum_{k=1}^K y^s)} \times \left[ \frac{D^O_k(x^1, \ldots, x^K, \sum_{k=1}^K y^k)}{D^O_k(x^1, \ldots, x^K, \sum_{k=1}^K y^s)} \times \frac{D^O_k(x^1, \ldots, x^K, \sum_{k=1}^K y^k)}{D^O_k(x^1, \ldots, x^K, \sum_{k=1}^K y^s)} \right]^{\frac{1}{2}}, \quad (4.9)
\]

where,
\[ D^k_O(x^1_i, \ldots, x^K_i, \sum_{k=1}^K y^n_{i_k}) = \inf \left\{ \delta : \delta > 0, \left( \sum_{k=1}^K y^n_{i_k} / \delta \right) \in P^k(x_i) \right\} \]
\[ = \max_{\delta} \left\{ \inf \left( \delta^k : y^n_{i_k} / \delta^k \in P^{k^2}(x^n_i), \delta^k > 0 \right) \right\} \text{for all } k = 1, 2, \ldots, K \] (4.10)

and \[ P^k(x_i) = P^k(x^1_i, \ldots, x^K_i) = \sum_{k=1}^K P^{k^2}(x^n_i) \equiv \left\{ y = \sum_{k=1}^K y^n_{i_k} : y^n_{i_k} \in P^{k^2}(x^n_i), k = 1, 2, \ldots, K \right\} \] (4.11)

Applying the results in Section 2.4, for a single output case, we can measure the first component of the right hand side of the equation (4.9) using (2.13), thus

\[ TEC_{O}^{Ist} = \frac{D^k_O(x^1_i, \ldots, x^K_i, \sum_{k=1}^K y^n_{i_k})}{D^k_O(x^1_i, \ldots, x^K_i, \sum_{k=1}^K y^n_{i_k})} \equiv \frac{TE^k_O}{TE^k_O} \cdot \] (4.12)

However, for a multi-output case, following (2.14) result, equation (4.12) can be defined as

\[ TEC_{O}^{Ist} = \frac{D^k_O(x^1_i, \ldots, x^K_i, \sum_{k=1}^K y^n_{i_k})}{D^k_O(x^1_i, \ldots, x^K_i, \sum_{k=1}^K y^n_{i_k})} \equiv \frac{\sum_{k=1}^K TE_{O}^{k^2} \times \delta_{k^2}}{\sum_{k=1}^K TE_{O}^{k^2} \times \delta_{k^2}} \cdot \] (4.13)

It could be noted that in the above measure, we need prices for the firm revenue shares, otherwise we can use shadow shares.

The question now is how to measure industry technical change?

\[ TC\_{O}^{Ist} = \left[ \frac{D^k_O(x^1_i, \ldots, x^K_i, \sum_{k=1}^K y^n_{i_k})}{D^k_O(x^1_i, \ldots, x^K_i, \sum_{k=1}^K y^n_{i_k})} \times \frac{D^k_O(x^1_i, \ldots, x^K_i, \sum_{k=1}^K y^n_{i_k})}{D^k_O(x^1_i, \ldots, x^K_i, \sum_{k=1}^K y^n_{i_k})} \right]^{1/2} \] (4.14)

The problem of measuring technical change for the industry is currently under investigation. The scope for the use of shadow output shares, in the absence of price data, are also being considered.
4.3 Preliminary aggregation results

In this section, we use the records of 100 individual firms in the Australian textile, clothing, footwear and leather (TCFL) manufacturing industry, which are taken from the Australian Bureau of Statistics confidentialised unit record file (ABS-CURF), to illustrate the sensitivity of the aggregation methods discussed in Section 3. All the 100 firms are continuing firms based on the four-year periods survey results. We also examine the sensitivity of the results to the use of various industry productivity changes when we apply different weights in the aggregation of firm level productivity growths. Lastly, we calculate an aggregate productivity change based on Malmquist TFP index using geometric average. It could be noted that the data series only contains four-year period, hence the TFP changes will be limited to three comparison periods (1996, 1997, and 1998) with 1995 as the base period. Results are presented only in the form of graphs.

Figure 1 exhibits the aggregate MFP index for the total TCFL manufacturing industry calculated using the standard non-transitive Tornqvist index formula. One sees an almost parallel pattern in the different aggregation procedure.

In Figure 2, we compare the aggregate MFP index for the same industry calculated using revenue shares and cost shares for the relative size of the firms. There are only minimal differences in the MFP indices using the Tornqvist formula.
Figure 2. Aggregate MFP
(cost vs. revenue shares)

Figure 3 reveals the aggregate productivity change measured as a weighted arithmetic mean of the firm-specific productivity levels using firm real input shares as weights, except for the OECD and Fox methods which use geometric average.

Figure 4 shows the annual percentage change of value-added TFP for the industry applied with input and value added share weights. The value added share weighted average measure appears to show much higher TFP growth.
Figure 5 depicts the aggregate TFP estimated using the Malmquist TFP index. We compare the results with the aggregate TFP obtained using the standard Tornqvist index. We use also cost and revenue shares in aggregating the firm level Malmquist TFP indices.

5. Concluding Remarks

Three major aspects of aggregation have been investigated in this paper.
First, we re-assess the various firm and industry efficiency measures developed by Fare and Zelenyuk (2003) after we relax the assumption that firm faces identical output prices and when we impose identical technology to each firm. We found out that Fare and Zelenyuk (2003) methods work well when each firm in the industry faces different output prices and it will not affect the aggregation process. We are able to reallocate the allocatable input endowments across firms and obtain an overall industry potential revenue efficiency measures. We have been successful in realizing a potential revenue efficiency gain in the process and come up with an industry revenue measure of reallocative efficiency.

Secondly, we empirically examine the industry productivity growth measure under the different aggregation approaches. We look at the sensitivity of the results. Whether the choice of methods used for calculating aggregate productivity levels are arithmetic average or geometric average, the obtained percentage changes in productivity do not differ much.

Third, we attempt to define an industry productivity growth measure based on the Malmquist TFP index. Further research will be focused on this aspect.
References


