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WHY PRICE INDEX NUMBER FORMULAS DIFFER: ECONOMIC THEORY AND EVIDENCE ON PRICE DISPERSION

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Abstract

The results from different index number formulas can differ and substantially so. The main criteria for explaining such differences, and governing choice between them, are their ability to satisfy desirable test properties—the axiomatic approach—and their correspondence to plausible substitution behavior as predicted from economic theory. Yet the numerical differences between index number formulas have been shown to be related to the extent of, and changes in, the dispersion of prices. Explaining differences between index number formulas in terms of price dispersion benefits from the existence of economic theories of price dispersion, including search cost, menu cost, and signal extraction models. This paper outlines the nature of the relationships between index number formulas in terms of price dispersion and economic theories of price dispersion. Theoretical (law-of-one-price) and empirical work on price-level dispersion require homogenous items. Yet many products are highly differentiated. As such the empirical analysis in this study of television sets explains price dispersion first, in terms of variation in the brands, characteristics and sales environment of the individual models of television sets by estimating heterogeneity-controlled prices. Second, by modeling heterogeneity-controlled price variation within a period by recourse to search cost theory and over time by drawing on search cost, menu cost, and signal extraction theory. The (scanner) data for television sets used is from retailer’s bar-code readers and amounts to about 73,000 observations over 51 months. The novelty of the paper is threefold: (i) its use of economic theories of price dispersion to explain differences between index number formulas, (ii) its derivation of heterogeneity-controlled prices which not only serves to explain some of the price variation and difference between formulas, but (iii), also allows the economic theory and empirical modeling of price dispersion to be applied to differentiated products. The paper concludes by considering the implications for index number construction.

JEL classifications: C43, C81, D11, D12, D83, E31, L11, L15.

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1. INTRODUCTION

Choice of formula for the measurement of inflation does matter. In January 1999 the formula principally used for aggregating price changes for the U.S. consumer price index (CPI) at the lower level of aggregation was changed from an arithmetic to a geometric mean. The effect of the change has been estimated by the Bureau of Labor Statistics (BLS) (2001) to have reduced the annual rate of increase by approximately 0.2 percentage points. Following estimates from the Boskin Commission’s Report on the U.S. CPI (Boskin et. al, 1996 and 1998), this implied a cumulative additional national debt from over-indexing the budget of more than $200 billion over a twelve year period up to the mid-1990s. These ‘lower’ level aggregation formulas are applied to samples of prices from outlets of finely-defined goods such as varieties of apples and are the building blocks of a CPI.1 The differences between these formulas are seen below to be primarily determined by changes in price dispersion while the desirability of the geometric (Jevons) index against the arithmetic (Dutot) index will be shown to be determined by the absolute level of price dispersion.

The Schultze and Mackie (2002) report on the U.S. CPI recommended the use of a trailing superlative index instead of the Laspeyres index since it would capture weighted upper level substitution effects. One such superlative index which has much to commend it (Diewert, 1997 and 2003a) is the Fisher index, a geometric mean of Laspeyres and Paasche. Boskin estimated that upper level substitution accounted for 0.15 percentage points bias in the U.S. CPI. Changes in price dispersion will be seen to account for some of the differences between these formulas.

Although the Laspeyres formula is commonly thought to be the formula used for the U.S. and other CPIs at the upper level, the expenditure weights for a comparison, say between periods 0 and t, relate to a previous time period b, as opposed to period 0, since it takes time to compile the information from an expenditure survey for the weights. A resulting practically-used index is a Young index which is shown below to be biased (Diewert, 2003a), the extent of which depends again on changes in price dispersion.2

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1 The resulting lower level indexes of price changes are combined at the higher level using a base-period weighted arithmetic mean of price changes to form the overall index.

2 There are other index number issues whose probity are dictated by price dispersion. For example, Ehemann, Katz and Moulton (2002) identify price dispersion as leading to negative values for a subaggregate composed of positive values in a proposed additive system of national accounts by Hillinger (2002).
In spite of the importance of price dispersion in explaining difference between these key formulas, choice between formulas is mainly governed by their satisfaction of axiomatic tests and their correspondence to plausible substitution behavior as predicted from economic theory. The contribution of this paper lies in its analysis of the differences between index number formulas in terms of price dispersion. Economic theories of price dispersion have at their roots the law-of-one-price and its failure. Yet such theories relate to homogenous items. The empirical part of this paper is concerned with the price variation of models of television sets (TVs). It first, explains price variability in terms of the heterogeneity of the characteristics and brands of models, and the outlet-types in which they are sold. Estimates of heterogeneity-controlled prices and price dispersion are then modeled with regard to frameworks not usually associated with index number work, including search cost and menu cost theories and signal extraction models. More particularly, the empirical work on lower level indexes provides a hitherto neglected focus on product heterogeneity to explain the bias in the ratio of arithmetic means (Dutot) as an index. The empirical section uses detailed scanner data from retailers’ bar-code readers which amounts to about 73,000 observations over 51 months on prices and characteristics of models of TVs sold in different outlet types.

Section 2 considers in turn elementary, lower level, unweighted index number formulas and upper level, weighted index number formulas. In each case it outlines the main formulas and their current justification from axiomatic considerations, economic and sampling theory. Section 3 and the Annexes show how differences between index number formulas are functions of the absolute level or changes in price dispersion. In section 4 consideration is given to search cost theory, menu cost theory and signal extraction models as a basis for explaining price dispersion and its changes. Section 5 commences the empirical work, based on extensive retail bar-code scanner data for television sets (TVs) with an outline of data, variables and measures and section 6 provides the results. Section 7 focuses on a specific issue relating to the Dutot index and compares it with results using heterogeneity-controlled prices. Section 8 concludes with implications for index number compilation. The analysis shows how economic theory rooted in the failure of the law of one price and the persistence of price dispersion can provide insights into differences between index number formula at upper and lower levels. This novel approach\(^3\) complements the still valuable analysis previously considered only in terms of axioms and consumer substitution theory.

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\(^3\) Balk (2001:2) comments that some insights have been obtained by looking at changes in variances, but only using an approximate “…more or less intuitive economic reasoning.” There has been, to the author’s knowledge, no formal examination of changes in dispersion in this context.
2. INDEX NUMBER FORMULAS AND THEIR RATIONALE: AXIOMS AND CONSUMER SUBSTITUTION THEORY

a. The lower level

i) formulas

The main lower level formulas used in practice (see Dalen (1992) and Diewert (1995 and 2003b) for details of other such indexes)\(^4\) for \(m=1,..M\) matched items with prices and quantities in period \(t\), \(p_m^t\) and \(q_m^t\) respectively, and in period 0, \(p_m^0\) and \(q_m^0\) respectively, are:

The arithmetic mean of price relatives—the Carli price index \(P_C\):

\[
P_C = \frac{1}{M} \sum_{m=1}^{M} \left( \frac{p_m^t}{p_m^0} \right)
\]  \hspace{1cm} (1)

The ratio of the arithmetic means—the Dutot price index, \(P_D\):

\[
P_D = \left( \frac{\sum_{m=1}^{M} p_m^t / M}{\sum_{m=1}^{M} p_m^0 / M} \right) = \sum_{m=1}^{M} \left( \frac{p_m^t}{p_m^0} \right) \cdot \left( \frac{p_m^0}{\sum_{m=1}^{M} p_m^0} \right)
\]  \hspace{1cm} (2)

which can be seen to be a base-period price share weighted Carli index.

The geometric mean of price relatives (which is also equal to the ratio of geometric means)—the Jevons price index \(P_J\):

\[
P_J = \prod_{m=1}^{M} \left[ \frac{p_m^t}{p_m^0} \right]^{1/M} = \left( \prod_{m=1}^{M} p_m^t \right)^{1/M} / \left( \prod_{m=1}^{M} p_m^0 \right)^{1/M}
\]  \hspace{1cm} (3)

The use of the geometric mean is not a novel idea. It was first proposed in 1922 on axiomatic grounds by Irving Fisher, though its adoption was prompted by the Boskin Commission’s Report in 1996 based on the economic theory of consumer substitution behavior. These three simple formulas are widely-used for calculating lower level, aggregate, unweighted price changes of

\(^4\) The main alternative formulas are the harmonic mean of price relatives \(P_H\) and the Carruthers, Sellwood, Ward and Dalen (\(P_{CSDW}\)) index = geometric mean of \(P_C\) and \(P_H\).
matched items over time; weights are then used to aggregate these lower level indexes to higher level aggregates.

ii) Lower level formulas, their rationale: the axiomatic approach, price dispersion and commensurability, substitution

The axiomatic approach identifies which formulas are desirable on the basis of their satisfaction of reasonable test properties. The Carli index fails the time reversal test such that $P_c(p^0, p') \times P_c(p', p^0) \geq 1$; it is upwards-biased.\(^5\) The Jevons index satisfies all of the main tests. The Dutot index satisfies all of the main tests with the important exception of the Commensurability Test, i.e., if the units of measurement for each model in each outlet change, then the elementary index remains unchanged (Diewert, 1995 and 2003b). The Dutot index is only advisable when there is item homogeneity.

Consumer substitution theory holds that utility-maximizing consumers substitute away from items with relatively high prices. An index that ignores such substitution effects in its weighting of price changes is open to substitution bias. However, lower level indexes do not include information on weights so such theory may be argued to not be relevant. Yet Balk (2002) has shown that if the items are selected with probability proportionate to size (pps) (say base period expenditure shares), then a sample unweighted index (say Jevons) is an estimator of a population weighted target index (base period expenditure-share weighted geometric mean, or geometric Laspeyres), for which economic theory applies. Balk (2002) has shown a geometric Laspeyres index to correspond to consumer substitution behavior consistent with an elasticity of substitution of unity. The incorporation of such substitution effects was the main justification for the BLS switch to the Jevons index. More innocuous support for a government’s use of the Jevons index is that by taking account of substitution effects, inflation will be lower than its arithmetic counterparts, as will be public (index-linked) debt (Hulten, 2002).

b. The upper level

i). formulas

Laspeyres and Paasche indexes are fixed basket indexes measuring the aggregate price change of baskets of goods whose quantities are either fixed in period 0, Laspeyres:

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\(^5\) Fisher (1922) famously commented: “In fields other than index numbers it is often the best form of average to use. But we shall see that the simple arithmetic average produces one of the very worst of index numbers. And if this book has no other effect than to lead to the total abandonment of the simple arithmetic type of index number, it will have served a useful purpose.” Irving Fisher (1922; 29-30).
or period $t$, Paasche

$$P_b = \frac{\sum_{m=1}^{M} p'_m q'_m}{\sum_{m=1}^{M} p'^0_m q'_m}$$

A geometric mean of the two is the Fisher index:

$$P_F = (P_L P_P)^{1/2}$$

Paasche and Fisher cannot be used in real time, say between a price reference period 0 and period $t$, because it takes time to conduct and compile the quantity/value information from an expenditure survey in a weight reference period, say $b$. While the CPI is often considered to be a Laspeyres index, this is not the case in practice. The weight reference period $b$ will precede the price reference period 0. The resulting Young index is:

$$P_Y = \sum_{m=1}^{M} s^b_m \left( \frac{p'_m}{p^0_m} \right)$$

where $s^b_m = \frac{p^b_m q^b_m}{\sum_{m=1}^{M} p^b_m q^b_m}$; $m = 1,\ldots,M$.  

ii). The axiomatic approach and consumer substitution theory

Fisher (1922) defined (6) as ‘ideal’ in terms of its satisfaction of desirable axioms (see also Diewert, 1997). In seminal work first Konüs (1924) showed how Laspeyres and Paasche price indexes provided upper and lower bounds on (economic) theoretical cost-of-living indexes. Diewert (2003a) has shown the Fisher index to be a reasonable average of these bounds on axiomatic grounds. Second, Diewert (1976 and 1978) defined a class of superlative formulas, one of which was the Fisher index, which corresponded to flexible functional forms for the underlying consumer’s utility function and thus incorporated substitution effects. A Fisher price index is thus superior to a fixed basket Laspeyres price index with regard to its ability to incorporate substitution effects and thus as a better approximation to a cost-of-living index.

3. NUMERICAL RELATIONSHIPS BETWEEN FREQUENTLY USED INDEX NUMBER FORMULAS – THE IMPORTANCE OF DISPERSION

a. Lower level indexes
These relationships were developed by Marks and Stuart (1971), Carruthers et al., (1980), Dalen (1992), and Diewert (1995 and 2003b). We borrow in this section primarily from Diewert (2003b) for the exposition\(^6\) and consider the numerical relationships between these three formulas.

**b. The relationship between Dutot and Jevons indexes**

Carruthers, et al. (1980:25) show an approximate relationship between the Dutot \(P_D\) and Jevons \(P_J\) indexes—see also Diewert (1995a:27-28) and Balk (2002: 23-4) for more detail. Consider individual prices in each period as deviations from their means:

\[
p_m^i = \bar{p}'(1 + \epsilon_m^i) \quad \text{and} \quad p_m^0 = \bar{p}^0(1 + \epsilon_m^0) \quad \text{where} \quad \sum_{m=1}^{M} \epsilon_m^i = 0, \quad \sum_{m=1}^{M} \epsilon_m^0 = 0 \quad (8)
\]

and \(\bar{p}'\) and \(\bar{p}^0\) are the arithmetic means of prices in periods \(t\) and \(0\) respectively. Since the Dutot price index is \(P_D = \bar{p}' / \bar{p}^0\) it follows that the Jevons price index is given by:

\[
P_J = \left[ \prod_{m=1}^{M} \left(\frac{\bar{p}'(1 + \epsilon_m^i)}{\bar{p}^0(1 + \epsilon_m^0)}\right) \right]^{1/M} = P_D \prod_{m=1}^{M} \left(\frac{1 + \epsilon_m^i}{1 + \epsilon_m^0}\right) \quad (9)
\]

Expanding \(\prod_{m=1}^{M} \left(\frac{1 + \epsilon_m^i}{1 + \epsilon_m^0}\right) \quad (9)\) using a second order Taylor series around \(\epsilon \bar{p}^0\) and \(\epsilon \bar{p}'\) results in the following second order approximation:\(^7\)

\[
P_J = P_D \left(1 + 1/2 \left(\text{var}(\epsilon^0) - \text{var}(\epsilon^i)\right)\right) \quad (10)
\]

Diewert notes that “Under normal conditions, the variance of the deviations of the prices from their means in each period is likely to be approximately constant and so under these conditions, the Jevons price index will approximate the Dutot price index to the second order.” He footnotes ‘normal conditions’ with the caveat that: “If there are significant changes in the overall inflation rate, some studies indicate that the variance of deviations of prices from their means can also change. Also if \(M\) is small, then there will be sampling fluctuations in the variances of the prices from period to period.” Our concern is with former.

We noted in section 2 that the Dutot index failed the commensurability test and was only advised when there was limited price dispersion arising from quality differences in the item itself or the

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\(^7\) Similar results can be found in relation to \(P_C\), \(P_{CSDW}\) (see ff. 5), \(P_H\) and the Balk-Walsh index, the approach having a wider application than to the more widely used Carli, Dutot and Jevons indexes (Balk, 2002: 22).
services provided by the outlet at the time of sale. Quality differences—be they associated with branding, technical specifications, or level of service in outlets—amount to a change in the (utility flow) unit the price is measured in. This can be seen by identifying the Dutot index in terms of an arithmetic average of base-period-price-weighted price changes as in (2). Models with more features have higher prices and their price changes are given more weight. Diewert (2003b) notes:

"...in actual practice, there will usually be thousands of individual items in each elementary aggregate and the hypothesis of item homogeneity is not warranted. Under these circumstances, it is important that the elementary index satisfy the commensurability test, since the units of measurement of the heterogeneous items in the elementary aggregate are arbitrary and hence the price statistician can change the index simply by changing the units of measurement for some of the items." [Diewert’s emphasis].

There is thus a concern with the absolute level of price dispersion and why it arises as well as the changes in dispersion.

b. The relationship between Carli and Jevons indexes

It follows from Schlömilch’s inequality that \( P_J \leq P_C \) (Diewert, 1995), but this inequality does not tell us by how much the Carli index exceeds the Jevons index. Let the price relatives for a comparison between periods 0 and \( t \) be given by:

\[
r_m^t = \frac{p_m^t}{p_m^0} = \bar{r}^t (1 + \nu_m^t)
\]

where \( \nu_m^t \) is the deviation from the mean \( \bar{r}^t \) of the price relative \( r_m^t \) for item \( m \) in period \( t \) and \( \sum \nu_m^t = 0 \). Consider each index number formula as a function of the deviations \( \nu_m^t \) and take second order Taylor series approximations around the point \( \nu_m^t = 0 \) for \( m = 1, ..., M \) to obtain:

\[
P_C \equiv \frac{1}{M} \sum_{m=1}^{M} \left( r_m^t \right) / M = \bar{r}^t \sum_{m=1}^{M} \left( 1 + \nu_m^t \right) / M = \bar{r}^t
\]

\[
P_J = \prod_{m=1}^{M} \left( \frac{r_m^t}{M} \right)^{1/M} = \bar{r}^t \prod_{m=1}^{M} \left( 1 + \nu_m^t \right)^{1/M} \approx \bar{r}^t \left( 1 - 1/2 \text{var}(\nu_m^t) \right)
\]

Differences between Dutot and Carli can also in part be explained by changes in the variance of price relatives (Diewert, 1995) and Annex 1.

b. Upper level indexes
i. The relationship between Laspeyres and Paasche
Annex 2 shows the extent of the divergence between these Laspeyres and Paasche indexes, and by extension between Laspeyres and Fisher, to depend in part on the extent of the dispersion in price relatives where dispersion is considered in terms of the coefficient of variation.

ii. The bias in the Young index
It was noted in section 3 that in practice the Laspeyres index is not used at the upper level for a price comparison between periods 0 and $t$ since expenditure weights are unavailable in period 0 due to the time taken to compile them. Instead the weight reference period refers to an earlier period $b$, the resulting index being the Young index defined by equation (7). Annex 3 follows Diewert (2003a) and shows the Young index to be biased, the extent of the bias depending on the dispersion in price relatives.

iii. A note concerning the Taylor expansion/approximation
The results from the above sections and annexes show the differences between formulas in terms of variances, usually arising from a Taylor expansion around zero. It is an approximation and Annex 4 considers the expansion in more detail.

4. SOME ECONOMIC THEORIES OF PRICE DISPERSION AND ITS CHANGE
The fact that the decomposition of the differences between index number formulas can be identified, at least partially, in terms of price dispersion and its changes requires recourse to economic theory concerned with price dispersion. There is a burgeoning theoretical literature that explains price dispersion even for homogeneous goods. There is a related literature that links changes in (relative) price dispersion to changes in the mean (unanticipated and otherwise) of such prices. Much of this theory, which includes search cost, menu cost and signal extraction models, relates to micro-economic behavior. Lach (2002), in his useful contribution using micro data, notes a dearth of related empirical studies blaming it on problems with access to micro-level data. This study considers the more practical situation where the items are not homogenous, but economic models of price dispersion are applied after correcting for, or explaining, that part of price variability due to product homogeneity.

a. Search Costs and the law of one price: cross-sectional price dispersion
Stigler (1961) argued that optimizing consumers with imperfect information search for additional information such that the (rising) marginal search cost equals the (falling) marginal search benefits. Even in markets with symmetric firms selling homogeneous goods, product prices may differ in equilibrium if there is a positive, but uncertain, probability that a randomly chosen
customer knows only one price. This would result in imperfect information that the firm in question can exploit by charging a higher price (Sorensen, 2002). It is in the interests of firms to adopt strategies which increase search costs, the effect of which is to increase price dispersion and thus the difference between formulas.

Electronic consumer durables (ECDs) have particular characteristics with regard to search costs. First, they are highly differentiated by brand and features. Cohen (2002) argues that while more brand selection increases rivalry and stimulates price competition, it also increases the volume of information on prices and features providing scope for poorly informed customers which can dampen price competition.

Second, it may be argued that the extensive media advertising and door-to-door flyers for ECDs reduce search costs. Yet such advertising is usually on a small selection of branded-models. Koch and Cebular (2002) distinguish between advertising expenditure which reduces consumer search costs and decreases mean prices and advertising that focuses on branding to diminish price elasticities of demand and thus increase mean prices and their dispersion.

Third, price dispersion may be due to outlet heterogeneity as well as feature and brand heterogeneity. ECDs are sold in electrical multiple chains (EM), mass merchandisers (department stores) (MM), independents (IND), and mail order catalogues (MAIL). The different types of outlets provide different types of service: EMs are often large out-of-town (easy parking), specialist warehouses while MMs are usually in-town department outlets selling a much wider range of goods. Sorensen (2000) did not find outlet-types to be a source of price variation for pharmaceutical products. In contrast, Lach (2002) found outlet-type to explain some of price variability for chicken, flour, coffee and refrigerators.

Fourth, Sorensen (2000) found the prices of repeatedly purchased prescriptions to be lower and less dispersed than irregularly purchased ones. He argued that the search benefits from repeat prescriptions were higher since the savings could be repeatedly realized. While infrequently purchased items such as ECDs provide less incentive to accumulate information, they are higher priced which provides more incentive to reap search benefits (Lach, 2002).

8 It is not even required that search costs vary across buyers. Heterogeneity of beliefs about the (cumulative distribution function of) prices for buyers with identical search costs is sufficient (Rauh, 2001).
9 ECD online purchases are rare and while differences in prices, between on-line and regular outlets have been studied for books by Brynjdfsson and Smith (1999) and Clay et al. (2002), the results of the studies differ as to which is the cheapest on average.
10 The prices were for a single identical model of refrigerator (size, brand type and so forth), size and type of chicken, coffee and flour, being collected from on average 38, 37, 14 and 15 outlets respectively. The study by Lach (2000) controlled for product heterogeneity by looking at only one item. This paper covers virtually the whole market, as required by theory, with the heterogeneity of the items being controlled for by the hedonic regressions.
Finally, price dispersion can also be explained simply as a result of price discrimination Yoskowitz (2002) and the Schultze Panel’s (2002) Report on the U.S. CPI. Basic marketing teaches managers to segment their markets according to differences in price elasticity by offering products of different quality to different segments and, more particularly, targeting brands to different segments to exploit any consumer surplus. The analysis in this study by quality characteristics and brands picks up some such price dispersion.

b. The persistence of price dispersion

Over time consumers will build up their inventory of knowledge from routine shopping trips. With stable prices (no depreciation in knowledge) price dispersion should diminish. Varian (1980) explains the empirical persistence of price dispersion by distinguishing between ‘shoppers’ who pay the lowest price and the remaining consumers with search costs who shop randomly. Price dispersion persists because outlets change their prices (randomly) so as to prevent consumers with search costs from becoming fully informed. Lach (2002) found an intensive process over time of re-positioning prices across outlets consistent with Varian’s (1980) random pricing model.

c. Price dispersion and its mean

The persistence of price dispersion can also be explained by search cost theory in terms of a relationship between (relative) price dispersion and its mean over time. Classical economic theory at the aggregate level argues that inflation is a monetary phenomenon and should have no effect on the relative price distribution. Van Hoomissen (1988) argues that as prices increase, the value of the consumer’s inventory of existing search information is eroded, and with higher price increases it is eroded faster. Price dispersion thus persists and varies directly with inflation (Stigler and Kindahl, 1970). However, for infrequently purchased items the storage of information should be minimal11 and any relationship between the dispersion and mean of prices (or relative price changes) requires an alternative theoretical framework, of which there are several.

Signal extraction models hold that relative price variability will increase with inflation as consumers become less able to distinguish between unanticipated inflationary price variation and relative price changes – they fail to properly extract the relative price signals (see Barro (1978), Lucas (1973) and Friedman (1977); extensive empirical work includes Vining and Elwertowsky

11 Since outlets sell a range of infrequently purchased items including fridges, washing machines, dishwashers, stereos, television sets and the like, it might be argued that search information is accumulated on the outlet, as opposed to the item, giving some credence to the theory.
(1976), Parks (1978), Balk (1983), Domberger (1987), Debelle and Lamont (1993), Reinsdorf (1994) and Silver and Ioannidis (2001)). At higher rates of (unanticipated) mean prices (or relative prices) higher rates of (relative) price dispersion are expected.

Menu cost models find price dispersion occurs when a firm’s nominal prices are held constant due to the costs involved in undertaking the price changes. There will come a point when the extent of the change in its real price exceeds that of the costs of making the adjustment and a nominal price adjustment is made. The resulting staggered price changes give rise to a positive relationship between price dispersion and inflation (Sheshinski and Weiss (1977), Bénabou and Gertner (1993), Ball and Mankiw (1994 and 1995) and Levy and Bergen (1997)).

Many ECDs are imported or assembled from imported components. If prices are set in the consumer’s local currency then changes in nominal exchange rates do not affect prices; there is zero pass-through of exchange rate changes. Feenstra and Kendall (1997) and Engel and Rogers (2001) found a significant proportion of price dispersion to be due to some exchange rate pass-through. If nominal exchange rates fluctuate with inflation, so too might price dispersion.

Serial correlation in price dispersion may arise from sales, in which prices are marked down for a short period only to return to their preceding levels. Hong et al. (2002) argued that serial correlation will be induced for fast-moving-goods (fmgs) such as paper towels as consumers build up inventories at sale prices. While this is not applicable to ECDs, prices are reduced at well known sale times and consumers may delay their purchases until such times.

In conclusion, there are reasons to expect price dispersion for ECDs. First, we explain such cross-sectional price dispersion in terms of the characteristics and branding of the models of ECDs and outlet heterogeneity, the sheer variety of which has been argued to induce search costs. Use is made of hedonic regressions which relate prices to the quality characteristics of these differentiated models, their brands and outlet-types. The residuals are heterogeneity-controlled prices which are related to a more direct search cost variable. Second, our concern is with explaining the variation over time in the dispersion of the heterogeneity-controlled (residual) prices in terms of variables from search cost, menu cost and signal extraction theories.

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12 Levy and Bergen (1997) show such costs can be substantial.
13 More recently the focus of such work has been on the relationship between the skewness of relative price changes and inflation (Ball and Mankiw, 1995); for empirical work on skewness see Rodger (2000) and Silver and Ioannidis (1996). While Balke and Wynne (1996) have argued for a similar relationship using a multisector real-business-cycle model, Bryan and Cecchetti (1999) have dismissed the relationship as a statistical artefact due to small sample bias (see also Debelle and Lamont (1997) and Verbrugge (1998)). Our use of the population of observations argues against any such bias-induced, spurious relationship.
5. EMPIRICAL WORK: DATA AND MEASURES

a. Data

The empirical work utilized monthly scanner data for television sets (TVs) from January 1998 to March 2002. The scanner data were supplemented by data from price collectors from outlets without bar-code readers, though this was negligible. Each observation is a model of a TV in a given month sold in one of four different outlet types: multiple chains, mass merchandisers (department stores), independents and catalogue stores. For example, an observation in the data set for January 1998 was the unit value (price) (£275.80), volume (5,410 transactions) and quality characteristics of the Toshiba 2173DB 21 inch TV sold in multiple chain outlets only. For the 51 months of January 1998 to March 2002 there were 73,020 observations which covered 10.8 million transactions worth £3.9 billion.

b. Variables

The variable set on each observation included: price, the unit value of a model across all transactions in a month/outlet-type (see Balk, 1996 for the statistical properties of unit values) and volume, the number of transactions during the period in the outlet type - many of the models sold had relatively low sales. There were 38 brands—37 dummy variables benchmarked on Sony; the characteristics included (i) size of screen—dummy variables for about 19 screen sizes; possession of (ii) Nicam stereo sound; (iii) wide screen; (iv) on-screen text retrieval news and information panels from broadcasting companies, in order of sophistication: teletext, fastext and top fastext—3 dummy variables; (v) 6 types of reception systems—5 dummy variables; (vi) continental monitor style; (vii) Dolby Pro, Dolby SUR/DPL, Dolby Digital sound—3 dummy variables; (viii) Flat & Square, Super-Planar tubes—2 dummy variables; (ix) s-vhs socket; (x) satellite tuner, analogue/digital—2 dummy variables; (xi) digital; (xii) DVD playback or DVD recording—2 dummy variables; (xiii) rear speakers; (xiv) without PC-internet/PC+internet; (xv) real flat tube; (xvi) 100 hertz, doubles refresh rate of picture image; (xvii) vintage and (xviii) DIST—the percentage of outlets in which the model was sold. Outlet types are multiple chains, mass merchandisers (department stores), independents and catalogue stores.\(^{14}\)

c. Measures of dispersion and average prices

i. Weights

\(^{14}\) There was some variability in the number of variables used over time with DVD, rear speakers, top fastext, Dolby digital and SUR/DPL (as opposed to just Dolby Pro sound), 100 hertz and integrated PC not being used until January 2000; 11 variables excluded as not being relevant.
Since Annexes 2 and 3 have shown that any differences between the upper level formulas Laspeyres and Paasche and, thus, Fisher indexes and the bias in the Young index to be at least in part dictated by differences in the weighted dispersion of prices over time, results for weighted dispersion measures are calculated and analyzed in the empirical section.

We also have a concern with differences between (unweighted) lower-level indexes and this gives rise to an empirical concern with the change in unweighted variances. The empirical micro-economic literature on price dispersion and the law-of-one price is dominated by the use of unweighted measures and includes Beaulieu and Mattey (1999), Cohen (2000), Sorensen (2000), Engel and Rodgers (2001), Clay et al. (2002), Lach (2002) and Hong et al. (2002). Theories of price dispersion also apply at the unweighted level: menu cost theory predicts that retailers will have costs of price adjustment and not undertake such adjustments unless the price change is outside of some bounds, thus leading to price dispersion—a case that can be argued for each model of TV irrespective of its sales quantity. Similarly if a proportion of the population has search costs some retailers can enjoy a surplus on their models which will again lead to price dispersion irrespective of sales quantities. Finally, mistakes in anticipating inflation will lead to erroneous decisions by consumers irrespective of the quantities demanded and supplied and this may lead to price dispersion. In all cases the welfare effects require quantities to be taken into account, but this is not our concern here. Furthermore, prices at the lower level are sampled for the U.S. CPI with probability proportionate to value share in the base period. Thus even for lower level indices it may be the weighted results that matter (Balk, 2002), though we consider weighted and unweighted measures in the empirical section.

ii. Parametric measures of absolute and relative dispersion

The weighted standard deviation, $SD_w^t$, and coefficient of variation, $CV_w^t$, are given as absolute and relative parametric measures respectively for the prices $p_m^t$ of $m = 1, \ldots, M$ models sold in a given outlet-type in a given month $t$ by:

$$SD_w^t = \left\{ \sum_{m=1}^{M} s_m^t \left( p_m^t - \bar{p}_w^t \right)^2 \right\}^{1/2} \quad \text{and} \quad CV_w^t = \frac{SD_w^t}{\bar{p}_w^t}$$  \hspace{1cm} (14)

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15While such measures dominate the literature they are not advised outside this specific application. If relative price increases are accompanied by decreases in expenditure, unweighted measures would exaggerate price dispersion. Spurious correlations between dispersion and its mean have been argued by Bryan and Cecchetti (1999) to arise when unweighted measures are used.

16Though the unanticipated means used are still weighted ones.
where $s_{m}^{t}$ is their expenditure share and $\bar{p}_{m}^{t}$ the mean weighted price $= \sum_{m=1}^{M} s_{m}^{t} p_{m}^{t}$. Unweighted measures are similarly defined. Non parametric/robust measures are not used since the index number formulas relationships are based on parametric measures.

6. **EMPIRICAL WORK: RESULTS**

For the empirical work two things are of interest. First, is an explanation of price dispersion in any period. Price collectors are required to collect prices of similar models. The more similar the models, the less the dispersion in prices and the closer together the results of different formulas. Search cost theory tells us that the product heterogeneity may itself be a device to increase search costs. Thus product heterogeneity can generate dispersion over and above that due the technical characteristics and conditions of sale of the models. An understanding of the differences between formulas thus requires an understanding of the ‘technical’ heterogeneity of prices, the derivation of estimates of heterogeneity-controlled prices and an explanation of their dispersion in relation to search costs. Second, is the explanation of changes in dispersion over time and search cost, menu cost and signal extraction models will be used to underlie this empirical work.

a. **Explaining price variation**

Table 1 provides summary statistics on raw price dispersion. First, the extent of dispersion in raw prices is substantial; the unweighted and weighted coefficients of variation for the 51 months averaged around 0.85. Second, price dispersion increased substantially: by just over 20 percent, for the unweighted SD and by 40 percent for the weighted SD. Commonly purchased models have quite disparate price movements. Third, the unweighted CV was relatively stable; much of the increase in unweighted raw price dispersion can be accounted for by inflation. Bear in mind that for this raw data the dispersion and ‘inflation’ in TV prices includes changes in the composition of TVs purchased towards higher priced models. Finally, the weighted CV increased substantially over the first three years and subsequently fell, inflation not being an immediately obvious explanation for the changes in the dispersion of raw prices.

Measuring price dispersion under product differentiation requires controls for the brand and technical characteristics of the models and, since different outlet-types provide different services, the outlet-types in which the model is sold. Table 2 is based on a pooled hedonic regression of the prices, $p_{m}^{t}$, for observations on $m$ individual models of TVs sold in a specific outlet-type over the period $t$ 51 months of January 1998 to March 2003—just over 73,000 observations. The explanatory variables are month dummies, trend, 37 brand dummies, 3 outlet-type dummies, 19
screen size dummies, 6 tube-type dummies and 23 further characteristics as outlined in sections 5a and 5b above.

\[
\ln p'_m = \beta_0 + \sum_{i=1}^{11} \beta_i \text{Month}'_m + \beta_{12} \text{Trend}'_m + \sum_{i=13}^{61} \beta_i \text{Charac}'_m + \\
\sum_{i=62}^{99} \beta_i \text{Brand}'_m + \sum_{i=100}^{103} \beta_i \text{Outlet-type}'_m + \epsilon'_m
\]  

(15)

Table 2 provides a nested decomposition of price variation to explain some of the existing price dispersion and identify the remaining dispersion—the heterogeneity-controlled prices—in terms of the residuals of the regression. The coefficients were almost invariably statistically significant and their signs accorded with a priori expectations. The \( R^2 \) for the estimated equation (15) shows that over 90 percent of variation in price was explained by the regression. Month and time provided little explanatory power. The product heterogeneity, vis-à-vis product characteristics, brand and outlet-type variation, accounted for most of the price variation irrespective of what was dropped in the specification. Multicollinearity precluded our assigning variation separately to brands, characteristics or outlet-type, however, the variables on characteristics did most of the work: a regression on month, trend and brands only accounted for 0.35 percent of variation—\( \bar{R}^2 = 0.0035 \)—\( \bar{R}^2 \) was similarly low for month, trend and outlet-type. The regression successfully controlled prices for the heterogeneity of their features. The residuals, \( |\hat{\epsilon}'_m| \), are estimates of heterogeneity-controlled prices. Mean variation in prices was reduced by over 50 percent by the regression, and the standard deviation of the heterogeneity-controlled prices was about one-fifth that of the actual price dispersion (Table 2). Bear in mind it is not just the variation in technical characteristics, brands and the services from outlet-types that explain the price dispersion. If this were the case the law-of-one-price should hold for the residuals. The sheer multiplicity of models hinders search giving rise to price dispersion, something we now turn to.

We now consider a more explicit modeling of search to help explain the residual price dispersion within months. In section 3(b) DIST was defined for each model as the percentage of (television) stores with inventories or sales of that model. A model sold in fewer stores is less likely to have a comparable model available in any search carried out, thus restricting direct price comparisons and allowing some premium margin to be charged, an expected negative sign to help explain the remaining variation. To establish whether heterogeneity–controlled price variation can in part be

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17 Details available from authors.
explained by this search cost variable we first estimated separate hedonic regressions of the form in equation (15) for each month. The mean $\bar{R}^2$ for the 24 regressions was 0.89 with a minimum of 0.85 (and for weighted regressions 0.96 and 0.95 respectively), normality of residuals (Jacques-Bera) was by and large rejected for the OLS and WLS estimators,\textsuperscript{18} though the null of homoskedasticity was generally not rejected\textsuperscript{19} for both estimators (Breusch-Pagan).\textsuperscript{20} We second, regressed heterogeneity-controlled prices on DIST:\textsuperscript{21}

$$\ln \nu'_m = \alpha'_o + \alpha'_t DIST'_m + \varpi'_m \quad (16)$$

where $\nu'_m$ are the (exponents of the) residuals from hedonic regressions akin to equation (15), but run separately for each month. The results are in Table 3, though only for the first 24 months the variable DIST not subsequently being part of routine data provision by the data supplier.\textsuperscript{22} For an OLS estimator Table 3 shows a consistently positive, statistically significant relationship. The more stores a model is sold in the higher its heterogeneity–controlled price. This runs counter to our expectation from search cost theory. One explanation for the positive sign is that the OLS estimator gives the same weight to all models, including the many models with very few sales at the end of their life cycle. Such models are no longer sold in all the stores they were previously sold in and have been shown (Silver and Heravi, 2002) to have low prices relative to their specifications. The WLS estimator for equation (16) virtually ignores the low selling old models to give in Table 3 consistently negative coefficients on DIST, which become statistically significant towards the end of the period.

\textsuperscript{18} Though for the WLS estimator the null of the (component) test for skewness (Davidson and MacKinnon, 1993) was not rejected in one-third of the months—details available from authors.

\textsuperscript{19} The Breusch-Pagan test failed to reject the null of homoskedasticity for all but three months using the OLS estimator, though rejected in one-third of the months using WLS (by value)—details available from authors.

\textsuperscript{20} This deviation from the normality assumption and some heteroskedasticity may not permit correct inferences to be drawn on the coefficients. However, a heteroskedasticity-consistent covariance matrix estimator (HCCME) was used following White (1980) to allow asymptotically correct tests to be undertaken. A wild bootstrap estimator is usually advised for estimating models with heteroskedastic and skewed residuals due to small-sample bias in the HCCME. Davidson and Flachaire (2001) show that the wild bootstrap is only necessary to alleviate small-sample bias; the HCCME estimator is appropriate for the large sample tests in this study.

\textsuperscript{21} DIST might well have been included in the hedonic regression equation except for a priori expectations that the residuals, heterogeneity–controlled prices, may be correlated with it, as indeed we found. Modern outlets hold relatively little inventories. Advanced replacement ordering systems work in a way such that inventory levels are unrelated to price. The level of inventories was used each month as an instrument for instrumental variable hedonic regressions that included DIST. The results from such regressions found DIST significant at a 5\% level in 6 of the 24 months and at a 10\% level in 8 months. In all such regressions the coefficients on DIST had a positive sign. A series of Hausman test for each month found the null of no relationship between the errors and DIST was rejected in 50\% of the months using OLS but in only 20\% of the months using WLS.

\textsuperscript{22} Even for this period data on DIST were more limited than the rest of the data, comprising nearly 22,000 observations (as opposed to 29,000).
Thus three things have been found to explain price dispersion within months: (i) the use of weights (Tables 1 and 2), (ii) the technical features, brand and outlet type of the models (Table 2), and (iii) the extent to which models are sold in different stores, the nature of the relationship depending on whether our concern is with weighted or unweighted dispersion.

b. Changes in price variation

Section 2 showed that our concern with differences between formulas also required consideration of changes in dispersion. Table 4 and Figure 1 show the unweighted and weighted standard deviations of residuals over time. Note that the residuals are from separately estimated regressions each month and thus are normalized by their semi-log functional form to have means of unity in each month. As such the standard deviation of the residuals is the de facto coefficient of variation; the standard deviation normalized by the mean. This is our measure of dispersion. The derivations of the differences between index number formulas in the section 3 and the Annexes are based on normalized dispersion and the measurement and modeling naturally follows from this. Table 4 and Figure 1 show an increase in residual, unweighted dispersion of nearly 100 percent over the 51 months, and this is after being controlled for heterogeneity and mean (quality-adjusted) inflation. Note that quality-adjusted mean prices were falling over this period by about 30 percent, and that this should contribute to a fall in the standard deviation. So this relative normalized concept of dispersion is increasing even after abstracting from the fall in the mean. Also shown in Table 4 and Figure 1 is substantial volatility in the series. So dispersion is increasing accompanied by volatility, but so too must the difference between the formulas and this is after we control for heterogeneity and average price changes. So can we explain such changes?

Consider the weighted dispersion in Table 4 and Figure 1. The residuals from the WLS hedonic regressions were weighted by their relative expenditure shares to reflect their economic importance. Table 4 shows this weighted heterogeneity-controlled price dispersion to be about two-thirds that of its unweighted counterpart, and it too shows a striking increase, over 75 percent. Figure 1 shows much more volatility in the unweighted (OLS-based) measure, especially

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23 The measurement of the change in mean heterogeneity-corrected prices is problematic since the expected values of the residuals, or their logs, will be zero or unity by construction and not vary over time. As such the measurement of mean quality-adjusted prices used the dummy variable method. A chained index was calculated using hedonic regressions with the same specification as (15), but the regressions were based on sets of two successive monthly stacked data where Month was a dummy variable which took the value of one if it was the second of the two months and zero (see Silver and Heravi,(2001). The estimated coefficients on Month were linked by successive multiplication to form a chained index of the heterogeneity-controlled mean price. The index (results available form authors) fell by about 30 percent over the period.
towards the end of the series. Thus weighting reduced dispersion, the change in dispersion, and thus the differences between the results from index number formulas, and it also reduced the volatility of dispersion. Though for weighted and unweighted measures the evidence remains of substantial increases in dispersion even after we have explained technical, brand and outlet-type variation.

c. Explaining changes in residual price dispersion

In this section we seek to explain changes in residual price dispersion over time in relation to search cost related theories, signal extraction models (unanticipated means) and menu cost theory.

Search cost theory argues that as inflation varies, the value of existing information decreases requiring higher search costs just to return to the previous search equilibrium (Van Hoomissen, 1988). It would predict increased dispersion from an increasing or decreasing mean since in both cases the consumer’s inventory of knowledge would depreciate as average prices change. Bearing in mind average (heterogeneity-adjusted) prices were falling (ff.24), it would predict that dispersion normalized on the mean would rise. We test for trend stationarity in our measures of dispersion. Unit root test results are given in Table 5 and although not conclusive, the weight of evidence is towards accepting I (1). The drift in Figure 1 is upwards in both cases confirming a positive trend in dispersion over the mean. If search was frequent then consumers would become more knowledgeable and dispersion would decrease. However, given the frequency of search for TVs is more limited than for frequently purchased goods the finding is not surprising. Search cost theory would also predict that as the total number of models of TVs on sale increases, there would be higher search costs and thus dispersion, and thus a negative sign on the estimated coefficient for ‘number of models’ in a regression of dispersion on the latter. This would hold in spite of their infrequent purchase.

Signal extraction models require indicators of unanticipated inflation. The first indicator is that of the noise surrounding the purchase choice for models of TVs and the second for purchase decisions of TVs vis-à-vis other goods and services. Anticipated inflation was predicted from ARIMA models for weighted heterogeneity-controlled (H-C) means and the U.K. all items Retail Prices Index. AR(1) processes fitted the series best in both instances. Unanticipated inflation H-C

24 There is also a shift at January 2000 but this may be due to a change in the format of the data and slightly more detailed variable definitions available from this month onwards.

25 We also differenced and subsequently repeated the unit root tests to ensure I(2) was rejected in favor of I(1). The p-values for (augmented) weighted asymmetric tau tests on weighted and unweighted differenced dispersion were 0.00578 and 0.00249 respectively rejecting I(2).
UNAN\textsubscript{TVs} and H-C UNAN\textsubscript{RPI} were generated as the residuals. To test signal extraction models heterogeneity-controlled price dispersion was regressed on these measures of unanticipated inflation. The estimated coefficients on the unanticipated inflation measures should have positive signs following signal extraction theory that increased dispersion arises from the inability of economic agents to properly anticipate inflation, such inability increasing with inflation. However, the evidence of a positive relationship is far from conclusive. For example, Hesselman (1983), and Silver (1988) found negative relationships for the UK. Silver (1988) argued that the coefficient may have a negative sign as economic agents become more cautious in their price-setting and price-taking under uncertainty. Buck (1990), found negative association for Germany (using 19th century data), but positive for the US using data for the same period. Reinsdorf (1994) found a negative relationship for 65 categories of goods in 9 US cities, though this was for price levels. Reinsdorf (1994) found his explanation for a negative relationship from consumer search cost theories with unexpected inflation inducing more search due to consumers’ incomplete information about price distributions. Silver and Ioannidis (2001) found negative relationships for a range of European countries using a consistent methodology.

Menu cost theories can be tested by examining the relationship between the dispersion and the mean for data with a bimonthly frequency and that with a monthly frequency. Larger price changes and dispersion should materialize in the latter, though the limited time series here precludes this study for the time being. More straightforward is to include a variable on the proportion of models (for unweighted dispersion) or expenditure (for weighted dispersion) in catalog outlets (CAT\textsubscript{unw} and CAT\textsubscript{w} respectively), the catalog outlets having higher menu cost than other outlets. Positive signs would be expected on the estimated coefficients.

Table 6 shows the regression results to explain variation in changes in weighted and unweighted dispersion, as measured by heterogeneity-controlled residuals normalized on their means. The null hypotheses of unit roots were rejected for H-C UNAN\textsubscript{TVs} and H-C UNAN\textsubscript{RPI} (Table 5). The H-C UNAN variables were expressed in levels in estimated equations I and II in Table 6 and the remaining variables in first differences.\textsuperscript{26}

For unweighted dispersion the coefficient on the ‘number of models’ sold is positive and statistically significant as predicted from search cost theory. No corresponding result is found for weighted dispersion (Table 6 equations I and II). UNAN\textsubscript{TVs} has the expected negative coefficient for both unweighted and weighted dispersion. Equation III in Table 6 confirms the negative

\textsuperscript{26} Null hypotheses of I(2) were also tested as were Engl-Granger (tau) cointegration tests, the latter finding non-rejection of the null when H-C SD were chosen as the dependent variables.
statistically significant result for UNAN\_TVs, and the expected positive result for ‘number of models’ from search cost theory. In addition it finds a negative statistically significant result for the proportion of sales from catalogue outlets. We argued for a positive relationship from a menu cost stance on the grounds that the cost and resulting delay in making price adjustments would lead to relatively large changes when they took place. Catalog outlets are more prone to such delays. Yet during the print run of the catalog there should be less fluctuations and it may be that this overshadowed the price spikes from the adjustments. Note that unanticipated prices for all items (RPI) had no explanatory power and that the dummy variable which marked the change in data coding (see section 5) had some limited explanatory power.

Having thus explained price variation in terms of its product/outlet heterogeneity, we find that changes in such H-C dispersion over time can be further explained in terms of economic theory, particularly with regard to search cost and signal extraction models.

7. A HETEROGENEITY-CONTROLLED DUTOT INDEX

It was argued in section 2 that the Dutot index is particularly sensitive to product heterogeneity, since although it passed all the other main tests, unlike the Jevons index, it failed the commensurability test. The Dutot index is only advised (Diewert, 2003b) for relatively homogenous items. We compare a Dutot index for heterogeneity-controlled prices as against Dutot, Jevons and Carli indexes for uncontrolled, raw prices. The results are in Table 7. First, formula does matter for the indexes using raw prices; there is 15 per cent fall according to Carli and Dutot, but a 20 percent fall given by the Jevons index. Second, The bias (as measured here) of the Dutot index against the heterogeneity-corrected Dutot index is about 1.5 percentage points upwards over the period.

8. INDEX NUMBER IMPLICATIONS

A number of implications arise from the study.

First, the Dutot index is only advised when there is item heterogeneity. Yet this argument is based on the failure of the Dutot index to satisfy the commensurability test. There is no indication of the extent of any such bias. We showed in Table 1 that price dispersion for TVs was extensive, and in Table 2 that the hedonic regressions accounted for a vast proportion of such price variability. We found that even for this highly price dispersed data, the extensive heterogeneity controls had a relatively minor effect on the index suggesting that such bias may not be as large as generally considered.
Second, there was found to be considerable price dispersion for TVs (Table 1) which is not unusual for highly differentiated consumer durables. Brand, characteristics and outlet-type together explained much of such variation (Table 2). Minimizing price variance for price index number compilation requires either the use of hedonic indexes or detailed specifications for a selection of ‘representative’ items and care in judging replacements to be ‘comparable’ when an item goes missing if its characteristics are different. When a model is missing the price collector may judge another model to be of comparable quality and compare its prices with those of the old model. There is too much price variation associated with product heterogeneity to be lax about any leniency in such selections.

Third, as soon as weights were applied the dispersion was reduced (Table 2) and thus the difference between formulas. The selection of more popular models serves to not only make the index more representative, but also to reduce the disparity between the results from different formulas.

Fourth, for unweighted indexes, product heterogeneity aside, models sold in more outlets (DIST) have higher prices (Table 3). The selection of items for index number compilation should be representative of a model’s coverage of outlets for the sample to be representative.

Fifth, we found an increase in dispersion of nearly 100 percent over the 51 months, and this was after being controlled for heterogeneity and mean (quality-adjusted) inflation (Table 4). Such changes lead to formulas differences and require explanation. Some of the explanation could be identified via the trend, the upwards drift in the series. The drift was more volatile and accentuated for weighted dispersion than unweighted dispersion (Figure 1).

Sixth, differences in dispersion over time accord with predictions from search cost theory, menu cost theory and signal extraction models. Such frameworks were shown to explain some of the variation in dispersion over time (Table 6) and thus the results from index number formulas. This applied both to weighted dispersion (formulas) and unweighted dispersion, though more successfully to the former.
Annexes

Annex 1: The relationship between Dutot and Carli indexes: Following Dievert (1995a, 27) and where \( r_m^n = p_m^n / p_m^0 \):

\[
P_D = \frac{\sum_{m=1}^{M} r_m^n p_m^0}{\sum_{m=1}^{M} p_m^0} = \frac{\sum_{m=1}^{M} r_m^n / M + \sum_{m=1}^{M} \left( p_m^0 / \sum_{m=1}^{M} p_m^0 - 1 / M \right)}{w_m} \]

\[
= P_c + \sum_{m=1}^{M} r_m^n \left[ p_m^0 / \sum_{m=1}^{M} p_m^0 - \frac{\left( \sum_{m=1}^{M} p_m^0 / M \right)}{\sum_{m=1}^{M} p_m^0} \right]
\]

\[(A.1)\]

which is \( P_c \) plus the covariance of normalized \( r_m^n \) and \( p_m^0 \). The correlation coefficient between price relatives and base period prices is defined as \( \rho(r_m^n, p_m^0) \) which is equal to the covariance of \( (r_m^n, p_m^0) \) divided by the product of the variances of the individual variables. Therefore:

\[(A.2)\]

\[P_D = P_c + M \left[ \text{var}(r_m^n) \right] \left[ \text{var}(p_m^0) \right]^{1/2} \rho(r_m^n, p_m^0)\]

Since the variances must be positive, the sign of \( \rho \) determines which of these formulas will give results with higher values. The correlation would be expected to be negative as higher base period prices for similar items should have lower price increases. Thus \( P_c \) is expected to exceed \( P_D \). The two formulas will give the same results if the \( \text{var}(r_m^n) = 0 \), that is, all price relatives are the same, or the \( \text{var}(p_m^0) = 0 \), all (normalized) base period prices are the same, or if \( \rho(r_m^n, p_m^0) = 0 \), there is no correlation between price relatives and base period prices. As either of these depart from zero, the difference between the results from the two formulas will increase. Any difference due to the above factors can be seen to be magnified as \( M \), the number of prices increases. Note that the relationships in this section have been phrased as long-run ones, between periods 0 and \( t \). As time progresses it might be expected that the correlation \( \rho(r_m^n, p_m^0) \) weakens though the variance of price relatives \( \text{var}(r_m^n) \) may increase.

Annex 2: The relationship between Laspeyres and Paasche: The results are due to Bortkiewicz (1922, 1924) reproduced in Allen (1975, 62-64). The period 0 weighted means of price and quantity relatives are Laspeyres price and quantity index numbers:

\[(A2.1)\]

\[P_L = \sum_{m=1}^{M} w_m^0 \frac{p_m^t}{p_m^0} / \sum_{m=1}^{M} w_m^0 \quad \text{and} \quad Q_L = \sum_{m=1}^{M} w_m^0 \frac{q_m^t}{q_m^0} / \sum_{m=1}^{M} w_m^0 \quad \text{where} \quad w_m^0 = p_m^0 q_m^0\]

Paasche price and quantity indexes can be similarly defined and denoted as \( P_p \) and \( Q_p \) and the value index as \( V \). It is easily demonstrated that \( P_L \times Q_P = V \) and \( P_p \times Q_L = V \) so that:

\[(A2.2)\]

\[P_P / P_L = Q_P / Q_L\]

This is the ratio of Paasche to Laspeyres formulas we seek to explain. The weighted variances for Laspeyres price and quantity indexes defined in (A2.1) are:

\[(A2.3)\]

\[\sigma_p^2 = \sum_{m=1}^{M} w_m^0 \left( p_m^t / p_m^0 - P_L \right)^2 / \sum_{m=1}^{M} w_m^0 \quad \text{and} \quad \sigma_q^2 = \sum_{m=1}^{M} w_m^0 \left( q_m^t / q_m^0 - Q_L \right)^2 / \sum_{m=1}^{M} w_m^0\]

The weighted covariance times \( \sum_{m=1}^{M} w_m^0 \) is:
\[
\sum_{m=1}^{M} w_m^0 \left( \frac{p_m^t}{p_m^0} - P_L \right) (q_m^t/q_m^0 - Q_L) = \sum_{m=1}^{M} w_m^0 \frac{p_m^t q_m^t - p_m^0 q_m^0}{p_m^0 q_m^0} - P_L \sum_{m=1}^{M} w_m^0 q_m^t - Q_L \sum_{m=1}^{M} w_m^0 p_m^t + P_L Q_L \sum_{m=1}^{M} w_m^0
\]

(A2.3)

\[
= \sum_{m=1}^{M} w_m^0 \frac{p_m^t q_m^t}{p_m^0 q_m^0} - P_L Q_L \sum_{m=1}^{M} w_m^0
\]

We divide by \( \sigma_p \sigma_q \sum_{m=1}^{M} w_m^0 \) to get the weighted correlation coefficient \( \rho \) between price and quantities:

\[
\rho = \frac{\sum_{m=1}^{M} w_m^0 \frac{p_m^t q_m^t}{p_m^0 q_m^0} - P_L Q_L}{\sigma_p \sigma_q \sum_{m=1}^{M} w_m^0}
\]

(A2.4)

Using \( w_m^0 = p_m^0 q_m^0 \) and (A2.1)

\[
= \sum_{m=1}^{M} w_m^0 \frac{p_m^t}{p_m^0} \frac{q_m^t}{q_m^0} = \sum_{m=1}^{M} \frac{p_m^t}{p_m^0} q_m^t = V = P_F Q_L
\]

(A2.5)

This brings in the Paasche price index. Substituting in (A2.4):

\[
\rho = \frac{P_F Q_L - P_L Q_L}{\sigma_p \sigma_q \sum_{m=1}^{M} w_m^0} = \frac{P_F Q_L}{\sigma_p \sigma_q} \left( \frac{P_F}{P_L} - 1 \right)
\]

(A2.6)

Rearrange gives the required common ratio (A2.1) of the Paasche to the Laspeyres index numbers:

\[
\frac{P_F}{P_L} = \frac{Q_P}{Q_L} = 1 + \rho \frac{\sigma_p \sigma_q}{P_L Q_L}
\]

(A2.7)

To interpret (A2.7), note that the operative terms are the coefficient of correlation \( \rho \) between price and quantity relatives, multiplied by two coefficients of variation, i.e. the standard deviations from (A2.3) as ratios of the means (A1.2). The coefficients of variation are positive so that the sign of \( \rho \) fixes the direction of the divergence of the Paasche from the Laspeyres index. The Paasche index is the greater if \( \rho >0 \) and the Laspeyres index if \( \rho <0 \). The extent of the divergence, in whichever direction it is, depends partly on the strength of the correlation \( \rho \) and partly on the dispersion of the price and quantity relatives as shown up in the coefficients of variation.

**Annex 3: The relationship between Young and rectified Young:** While the Laspeyres index is well-known, it is not used in index number compilation by statistical agencies. This is because expenditure weights for the price reference period 0, for a comparison between periods 0 and \( t \), take time to be compiled and relate to an earlier weight reference period \( b \). The resulting Young index is:

\[
P_{Y(0,t)} = \sum_{m=1}^{M} s_m^b \left( \frac{p_m^t}{p_m^b} \right) \text{ where } s_m^b = \frac{p_m^b q_m^b}{\sum_{m=1}^{M} p_m^b q_m^b}; \quad m = 1, ..., M.
\]

(A3.1)

A problem with this index is that it fails the time reversal test: the index between periods 0 and \( t \) exceeds its time antithesis—it has an upwards bias. Let \( e_m \) defined deviations of price changes from their mean:

\[
r_m = \frac{p_m^t}{p_m^b} = \bar{r} (1 + e_m) \text{ for } m = 1, ..., M \text{ where } \bar{r} \text{ is a weighted mean of } r_m \text{ defined by:}
\]
(A3.2) \( \bar{r} = \sum_{m=1}^{M} s^b r_m \)

which is the direct Young index, \( P_{Y(0)} \). Diewert (2003a) compares the direct Young index with its reciprocal to ascertain the bias and finds that to the accuracy of a certain second order Taylor series approximation, the following relationship holds between the direct Young index, \( P_{Y(0,t)} \) and its time antithesis, \( P_{Y(0,0)} \):

(A3.3) \( P_{Y(0,t)} \approx P_{Y(0,0)} + P_{Y(t,0)} \) \( \var{\epsilon} \) \( \approx \sum_{m=1}^{M} s^b \left[ e_m - \bar{e} \right]^2 \) and

(A3.4) \( \bar{e} \equiv \sum_{m=1}^{M} s^b e_m \)

which is equal to 0. Hence the more dispersion there is in the price changes \( p_m^t / p_m^0 \) relative to their mean, to the accuracy of a second order approximation, the more the direct Young index will exceed its counterpart that uses month \( t \) as the initial base period rather than month 0 and thus the greater the bias.

Annex 4  A note concerning the Taylor expansion/approximation

The results from the above sections and annexes show the differences between formulas in terms of variances, usually arising from a Taylor expansion around zero. It is an approximation and Annex 4 considers the expansion in more detail. The variances in the expansion following equation (9) contain squares and cross-products and these cross-products are over time across products as well as for a given time period across items.

Say there are only \( m=4 \) items over two periods, 0 and 1, as depicted in Figure A4.1 below. The \( e_j^t \) and \( e_j^0 \) are the normalized errors so that in any period -0.2, for example, is 20% below the unitary mean and +0.2 is 20% above it.

(A4.1) \( \prod_{m=1}^{M=4} (1 + e_m^t) / (1 + e_m^0) \right)^{1/4} \approx 1 + \frac{1}{4} \left[ \sum_{m=1}^{4} e_m^t - \sum_{m=1}^{4} e_m^0 \right] - \frac{1}{4^2} \left[ \sum_{m=1}^{4} e_m^2 - 10 \sum_{m=1}^{4} \left( e_m^0 \right)^2 \right] \)

- \frac{1}{4^3} \left[ \sum_{m=1}^{4} \sum_{m \neq n} e_m e_n - \sum_{m=1}^{4} \sum_{m \neq n} e_m e_n - \sum_{m=1}^{4} \sum_{m \neq n} e_m e_n - \sum_{m=1}^{4} \sum_{m \neq n} e_m e_n \right] + \frac{1}{4^4} \left[ 14 \sum_{m=1}^{4} \left( e_m^0 \right)^3 - 30 \sum_{m=1}^{4} \left( e_m^0 \right)^2 \right] + \ldots \)

Note that the terms in the first square brackets sum to zero by definition in (8). The next two terms denote the difference between the normalized variances. Consider the cross-product term \( e_j^t e_k^t \) for \( m \neq n \) and Figure A4.1 below.

There should be \( 3C_2 = 28 \) cross-products of which 12 are for \( m \neq n \) and \( t \neq 0 \) — these are comparisons over time between different items. Say prices fluctuate around a normalized mean, so that in period 0, items 1 and 3 are above average (+ve) and items 2 and 4 below average (-ve), and the positions are in reversed period 1. For large \( M \) these cross-products will cancel. However, the next term are the 4 changes over time, \( e_1^0 e_1^1, e_2^0 e_2^1, e_3^0 e_3^1, e_1^0 e_1^4, e_2^0 e_2^4, e_3^0 e_3^4, e_4^0 e_4^1, e_4^0 e_4^4 \) for which will all be negative, followed by the 6 cross-products across items in each of period 0 and 1 respectively \( m \neq n \cap t \neq 0 \), which should cancel to zero from Figure A4.1 for large \( M \). Finally there are 8 cubic terms which will naturally be followed by their cross products. The difference between the formulas also depends upon changes in the skewness of the price deviations.

Figure A4.1

<table>
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<tr>
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<tr>
<td>0</td>
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<tr>
<td>(+ve) e_1^0</td>
</tr>
<tr>
<td>(-ve) e_1^1</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>(+ve) e_1^2</td>
</tr>
<tr>
<td>(-ve) e_1^3</td>
</tr>
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</table>

Note: Balk (2002) shows an alternative derivation where the difference is identified as being dependent on the covariance between (the log mean of each period’s) relative prices and price relatives. Such a covariance can be decomposed into the variances of the items on the Taylor expansion below.

\( \bar{r} = \sum_{m=1}^{M} s^b r_m \)
The focus remains on changes in the variance to explain differences in the formulas, though fluctuations in the levels for individual items in the manner depicted in Figure A4.1 will also have an effect, as may changes in the cubic (skewness) terms.

It is not immediately obvious as to how to reconcile (A4.1) with (9). However, if a common denominator of $4^2 = 64$ is used the variances have weights of $-6/64$ and $+10/64$ in periods $t$ and $0$ respectively and the cross products of $(-12/16, -4/16, +6/16$ and $+6/16)$ respectively which sums to $-16/64$. All items at the second order thus have weights summing to $-12/64 = 1/4$, i.e., the $1/M$ in (9).

Note the asymmetry in the weights for the variances in (A4.1). If the variances were the same the higher positive weight given to period 0’s variance would lead to $P_J > P_D$ though the cross-products might ameliorate the situation, especially the negative influence of cumulated price changes under the scenario in Figure A4.1 outlined above. Any increase in the variances over time might tip the expression for the difference between the variances in (A4.1) to be negative, and the expansion to be less than unity so that $P_J < P_D P(\delta^0, \delta^1)$ as is apparent from (9).
REFERENCES


Table 1, Descriptive statistics on average monthly price dispersion*

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*The figures for the standard deviations (SD) and means ($\bar{p}_t^u$) are calculated for each month, the annual figures being simple averages of the 12 monthly measures. The CV annual averages follow accordingly. In 2002 there were only 3 months data the averages being for the months to March 2002.

Table 2, Decomposition of price variation

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* $R^2$ are based on untransformed variables.
Table 3, Results for regression of H-C prices on DIST

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Table 4, Heterogeneity-controlled dispersion

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Figure 1, Heterogeneity-controlled dispersion

Table 5, Unit root tests

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H-C is heterogeneity–controlled. Both SD weighted and unweighted are normalized by the regression.

Table 6, Regression results

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<th>Estimated equation III</th>
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<td>(\Delta H-C) SD(^{\text{weighted}}) (\dagger)</td>
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<td>Coefs. (t)-statistics (\dagger)</td>
<td>Coefs. (t)-statistics (\dagger)</td>
</tr>
<tr>
<td></td>
<td>-0.002 (0.30)</td>
<td>0.002 (0.76)</td>
<td>0.111 (3.64^{***})</td>
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<td>(\Delta)Number of models</td>
<td>0.0001 (1.74^*)</td>
<td>0.000 (0.52)</td>
<td>0.0001 (2.58^{**})</td>
</tr>
<tr>
<td>H-C UNAN(^{\text{TVs}})</td>
<td>-0.771 (2.26^{**})</td>
<td>-0.372 (2.703^{***})</td>
<td>-0.629 (2.94^{***})</td>
</tr>
<tr>
<td>H-C UNAN(^{\text{RPI}})</td>
<td>0.002 (0.83)</td>
<td>0.000 (0.41)</td>
<td>0.001 (0.75)</td>
</tr>
<tr>
<td>(\Delta)CAT(^{\text{unweighted}})</td>
<td>-0.382 (0.79)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\Delta)CAT(^{\text{weighted}})</td>
<td></td>
<td>0.039 (0.37)</td>
<td>-0.541 (3.43^{***})</td>
</tr>
<tr>
<td>Dummy</td>
<td>-0.000 (0.004)</td>
<td>-0.001 (0.39)</td>
<td>0.038 (5.75^{***})</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.13</td>
<td>0.07</td>
<td>0.64</td>
</tr>
</tbody>
</table>

\(\dagger\)Tests are two-tailed and *, **, *** denotes statistically significant at a 10%, 5% and 1% level respectively.

\(\dagger\)H-C is heterogeneity–controlled. Both SD weighted and unweighted are normalized by the regression.
Table 7, Index number formulas results (Jan-98=100.00)

<table>
<thead>
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<th>Period</th>
<th>Heterogeneity-controlled Dutot</th>
<th>Heterogeneity-controlled Dutot</th>
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<tr>
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</tr>
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