Other-regarding Uzawa preferences and living standard catch-up

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Abstract

This paper provides a new rationale for Uzawa preferences; instead of positing that poor people are more patient because they are poor, it posits that poor people should be more patient if they wish their living standards to catch-up with richer people. To provide a setting for this new rationale, the paper studies the socially optimal choice of living standards over time by social planners for countries which, from low levels of total factor productivity (TFP), experience a gradual catch-up of their TFP level with that of the leader country. In the TFP catch-up scenario, the socially optimal choice of consumption and saving based on time additive preferences leads to no catch-up of living standards. To generate living standard catch-up we propose other-regarding Uzawa preferences (ORUP), in which the rate of time preference is influenced by the gap in living standards between follower country and leader country. This other-regarding specification is consistent with recent findings emphasised in behavioural economics. The ORUP form is illustrated quantitatively by simulating a model of the world economy.

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1. Introduction

There is a large literature on the catch-up of labour productivity by low-income countries with high-income countries. However, the nature of the paths that living standards, that is consumption per person, may follow during a period of labour productivity catch-up has not been addressed. In this paper we study living standard catch-up from the perspective of social optimality. We investigate the socially optimal choice of living standards over time by social planners from countries which, from low levels of total factor productivity (TFP) in the initial period, experience a gradual catch-up of their TFP level with that of the leader country. It will be seen that our approach of assuming the speed of TFP catch-up is exogenous and focusing on the different concept of living standard catch-up reveals insights for the development process about the choice of preference structures in the social welfare function.

The conventional approach of basing a social welfare function on time additive preferences (TAP) does not generate, for countries experiencing TFP catch-up, paths along which living standards will catch-up with the living standards of the leader. Consider the benchmark case of a model of the world economy with a perfect world capital market. Assume that countries have, along the lines of Lucas (2000), begun a sustained period of economic growth at different times. With a common rate of time preference across all countries of the world and a constant elasticity, instantaneous utility function, social planners in the late-starting, low income countries will “front-load” consumption in anticipation of higher incomes in the future, borrowing from the high income countries early on and then servicing these debts later. As the optimal choice of the social planner, living standards will remain below those of the leader. This result, demonstrated below, is consistent with the well known properties of time
additive preferences, see Becker (1980). Alternatively, assuming instead that low income countries have lower rates of time preference than the leader generates optimal paths of living standards that overtake the leader. Thus again, catch-up of living standards does not occur.¹

An alternative to TAP suggested by the literature is Uzawa preferences, that is to specify the rate of time preference as a positive function of living standards, see Uzawa (1968). With the Uzawa specification, social planners in low income, low living standard countries would choose to be more patient than those in high living standard, high income countries and so would choose to save more. Assuming that TFP catches up, the greater rate of saving of low income countries will lead to their living standards catching up.

Uzawa preferences have been criticised for being counter-intuitive, because of their property that poor people are more patient than rich people.² To counter this criticism, we propose a new form of Uzawa preferences. In this new form, the rate of time preference in the social welfare function of the follower country is influenced by the gap in living standards between the follower country and the leader country. In contrast with Uzawa, who assumed the rate of time preference to be a function of the level of living standards, our version is an other-regarding specification. This version highlights the role that international comparisons of living standards may play in encouraging people in low-income countries whose TFP is expected to catch-up, to choose paths of consumption and saving that ensure their living standards catch-up. We show that the optimal choice by social planners in low income, follower countries using other-regarding Uzawa preferences (ORUP) does indeed generate living-standard catch-up, that is paths of living standards that catch-up eventually with the high-income, leader country.

By specifying the rate of time preference to depend on the gap in living standards between leader and follower, we are departing from conventional economic assumptions. The convention has been not to base preferences on comparisons with

¹ There could be some degree of catch up under TAP in the case of an imperfect world capital market because countries that are catching up in terms of technical progress face higher overseas borrowing to finance their investment, which may imply higher interest rates and therefore higher saving rates and higher future consumption. The conditions under which imperfect capital markets can lead to living standard catch up may be worthy of study. In any event, the context of a perfect world capital market, a canonical case, has special reason to be considered.
² The positive function is usually argued to be helpful, although not necessary, to avoid instability, see Das (2003).
others. However, as the growth of behavioural economics shows, there is increasing dissatisfaction with the conventional approach, see Frey and Stutzer (2002), Ljungqvist and Uhlig (2000). We advocate ORUP to study the issue of living-standard catch-up, because it embodies explicitly the idea that people, and thus their social planners, are concerned with relative standards of living, a concern that lies at the heart of living-standard catch-up. The idea that people in low income countries that are enjoying TFP catch-up may be prepared to make the consumption sacrifices that will yield to them or their descendents a catch-up in living standards with the leader country is plausible, or at least worthy of consideration.3

An additional contrast between ORUP and the existing literature on Uzawa preferences is that we argue for ORUP on normative grounds, as a useful structure for a social welfare function, rather than as a positive economics description of how people actually behave. In particular, saving for the living standards of people in the future has an altruistic component. “Self-centred inequity aversion”, see Fehr and Schmidt (1999), may induce citizens to save more for the future, that is, to choose a faster speed of living standard catch-up, if they believe other citizens may also save more. By using ORUP to calculate paths of living standard catch-up, the implied values for the rate of time preference can also be calculated. Thus the weight placed on the welfare of people in the future along a path of living standard catch-up is revealed. This weight may help people to judge whether a particular path of living standards into the future is socially desirable.

Our specification of ORUP introduces additional parameters into the social welfare function. In the paper we discuss the principles that may be thought desirable for setting the values of these parameters. The main principle is the principle of protection – that is, in an optimal process of living standard catch-up, people should not have to suffer reduced living standards. This rule is suggested by the greater loss that people suffer from reductions in living standards, as implied by the concept of loss aversion, and by the future growth in living standards for people in countries whose TFP is catching up with a leader country. It seems unreasonable that people should suffer now in the interests of even higher economic welfare in the future.

3 The alternative approach of specifying a loss function on the living standard gap to guide the social planner’s decisions suffers relative to ORUP from not using basic concepts of inter-temporal decision making of the rate of time preference and the inter-temporal elasticity of substitution.
To support our discussion of the setting of the values of the ORUP parameters, we use in this paper the results of simulation analysis of a model of the world economy. These simulations project scenarios of living standard catch-up from an initial position for the world economy that reflects the actual levels of saving and TFP into a future where TFP catch-up occurs. In the simulations, optimising the ORUP social welfare function generates a path of living standard catch-up consistent with equilibrium in the world capital market. The projected future also takes into account projected demographic change throughout the world.

The paper proceeds as follows. In Section 2 the world model is described. In Section 3 basing the social welfare function on time additive preferences is shown to imply no living standard catch-up for countries experiencing TFP catch-up. In Section 4 other-regarding Uzawa preferences are introduced and discussed. In section 5 simulations of the world model provide further knowledge about ORUP. Section 6 concludes the paper.

2 A world model of optimal consumption planning

A framework through which the implications for living standard catch-up of the specification of the social welfare function can be discussed is provided by the model of the world economy described in this section.

We assume a world model with J countries. The J countries trade in goods and capital in a world goods market and a world capital market. For each country a social planner is assumed to choose paths of saving, investment, current account balance and consumption to solve the country-specific problem (the country subscript is suppressed)

Maximise \( \Gamma = V(C_1, C_2, ..., C_J, W_h) \)
\[
+ \sum_{t=1}^{h} \lambda_t [Y_t + D_t - I_t - C_t - (1 + r_t)D_{t-1}]
\[
+ \sum_{t=1}^{h} \psi_t [(1 - \delta)Y_{t-1} + A_{t-1} I_t^a - Y_t]
\[
+ \phi_h \sum_{t=1}^{T} (1 - \delta)^{t-1} I_{h,t} - D_h - W_h \]  \hspace{1cm} (1)
by choice of $C$ (consumption), $I$ (investment), $D$ (foreign liabilities), $Y$ (GDP), and $W_h$ (terminal wealth). All these variables are aggregate quantities that combine private and public quantities.

The utility function, $V(C_1, C_2, \ldots C_h, W_h)$ is defined on levels of consumption, $C$, into the far future, to terminal time $h$, and by terminal wealth (=capital stock less foreign liabilities) at the end of the terminal period, $W_h$. The explicit form of this function is specified in later sections.

The first constraint, associated with the Lagrangian multiplier $\lambda$, is the national accounting identity. This constraint balances consumption, investment and foreign borrowing against output (GDP) produced.

The second constraint specifies the production function. The specification chosen is an approximation of the putty-clay vintage production function. Reflecting the finite horizon, the third constraint defines terminal wealth, $W_h$, as the sum of the domestic capital stock and foreign assets. The importance of the putty-clay vintage specification and the finite horizon is for the simulations discussed in Section 5. They do not influence the discussion of TAP and ORUP in sections 3 and 4.

The world rate of interest is determined by the equilibrium condition for the world capital market that world saving equals world investment, that is

$$ \sum_{j=1}^{J} S_j = \sum_{j=1}^{J} I_j $$

where $j$ is the country.

3 The social welfare function with time additive preferences

The social welfare function for a country based on time additive preferences is specified as

$$ V = \sum_{t=1}^{h} \left\{ NU \left( \frac{C_t}{N} \right) (1+\rho)^{-t} \right\} + NU \left( \frac{W_h}{N} \right) (1+\rho)^{-h} $$

$^4$ The reasons for choosing the vintage form of the production function are given in the Appendix along with an outline of the model.
where N is the constant population and so C/N is consumption per person, that is the living standard. It is assumed that U'>0 and U''<0 for all C/N>0.

The first order conditions with respect to C and D for the problem (1), using the TAP social welfare function (3), imply the well-known Euler equation

\[
\frac{U'(C_{t+1}/N)}{U'(C_t/N)} = \frac{1 + \rho}{1 + r_t}
\]

(4)

Given the assumption in the model of a common utility function, a common rate of time preference and a world rate of interest faced by all social planners, (4) implies that follower countries, or late starters in the economic growth race, will choose paths of living standards that never catch-up with the leader country. To see this note first that secular growth in TFP implies that \( r_t > \rho \)\(^5 \). Thus all countries will choose paths of increasing living standards. Second, consider a follower country whose living standard happens to be very close to that of the leader such that in period t+1 its living standard will be equal to the leader’s living standard in period t. When period t+1 arrives, the leader will have increased its living standard and so the follower will not have caught up.

The failure of living standard catch-up is well-illustrated by using the constant elasticity form for the instantaneous utility function. That is, write the TAP social welfare function, as

\[
V = \sum_{t=1}^{\infty} \left[ N\left(\frac{C_t}{N}\right)^{-\beta} \frac{(1 + \rho)^{-t}}{(1 - \beta)} \right] + N\omega \left(\frac{W_h}{N}\right)^{-\psi} \frac{(1 + \rho)^{-h}}{(1 - \psi)}
\]

(5)

\( \beta \) is restricted to be greater than zero with the logarithmic utility function for \( \beta = 1 \).

Optimising (1) using the time-additive social welfare function (3) implies that living standards along the socially optimal path grow according to

\(^5\) At the world level, the model is a closed economy. In balanced growth equilibrium in a closed Ramsey model, the marginal product of capital, and thus the world interest rate, exceeds the rate of time preference. The statement in the text assumes away the unlikely possibility that a huge excess of capital throughout the world has pushed, temporarily, the marginal product of capital below the rate of time preference.
From (6) it can be seen that with TAP if countries face the same world rate of interest and have the same rates of time preference then their optimal growth rates of living standards will be the same. Thus the optimal choice for the future for those countries who happen to have lower living standards than the leader at the beginning of the planning period is for their living standards to never catch-up with the leader. Instead, the ratio of their living standard to that of the leader country will remain constant over time.

Under TAP, the social planner in a low-income country will choose a path of living standards that will reach that of the high-income country if the social planner in the former has a lower rate of time preference. But then the low income country will not only draw level with the leader but also overtake. This is not catch-up.

4 Other-regarding Uzawa preferences

A lack of living standard catch-up violates our intuitive notions of people’s preferences in that it may be felt that people in low income countries in which labour productivity is catching-up with that of the leader country will want their living standards to catch-up. Given that under our assumptions of TFP convergence it is feasible for living standards to catch-up, how can the social welfare function be specified such that the optimal path is for the living standards of the followers to catch-up with those of the leader? This section explores how basing the social welfare function on other-regarding Uzawa preferences (ORUP) can lead to a socially optimal decision by followers to catch-up.

Our intuitive notions that social planners in follower countries desire the living standards of their people to catch-up with those of people in the leader country may be based on the idea that relative consumption levels are important determinants of people’s happiness. This is of course an old idea that has recently received an impetus from research in behavioural economics. At the world level, the trend towards the global village, recently speeded up by the explosion in communications, especially television, is increasing the awareness of people around the world of the living
standards in the high-income countries. These may be the living standards to which people in low income countries either aspire or will aspire and thus should enter the calculations of a social planner.

In this paper we propose a modification of Uzawa preferences. According to Uzawa preferences, the level of consumption influences the rate of time preference. To generate catch-up paths we specify a modified version of Uzawa preferences, ORUP, in which the rate of time preference for a follower country is influenced by the difference between that country’s living standard and the leader country’s living standard such that, relative to the leader country, a low living standard is associated with a low rate of time preference. As living standards catch-up, the rate of time preference increases towards that of the leader country.

An attraction of the modified Uzawa approach is that it operates through the rate of time preference, a concept at the heart of inter-temporal choice. As will be seen, the modified Uzawa preferences calculates a time series for the rate of time preference associated with the degree of living standard catch-up along a particular path chosen by maximising the ORUP social welfare function. Thus the approach reveals the patience for living standards assumed by the social planner along the catch-up path. In this way the catch-up speed is related to the measure of inter-temporal weighting of welfare, the rate of time preference. This relation helps in evaluating the attractiveness of the path. For example, a very fast speed of living standard catch-up will imply a path with, for sections, a negative rate of time preference. This implies a higher weighting on utility enjoyed by people in the future relative to people today. One may regard this as ethically indefensible and so a reason for avoiding such high speeds of living standard catch-up.

4.1 Incorporating Other-regarding Uzawa preferences (ORUP) into the model

To generate living-standard catch-up for the follower countries, we assume that the social planner in each follower country sets the rate of time preference for that country to be initially less than that of the leader country but to increase as the living standard gap is closed. Thus, the larger the living standard gap with the leader, the lower does the social planner in the follower country set the rate of time preference used for the follower country’s optimal program. Specifically, we define the living
standards adjustment factor, $\ell$, as the following function of the living standard gap, that is

$$\ell_i = \left[ \frac{\left(\frac{C}{N}\right)_L - \left(\frac{C}{N}\right)_I}{\left(\frac{C}{N}\right)_1 - \left(\frac{C}{N}\right)_I} \right]^2 (1 + x)^{-t}$$  \hspace{1cm} (7)$$

where the ratio in the square brackets is a measure of the living standard gap. As this gap closes, $\ell$ decreases. Using $\ell$, the rate of time preference is determined by the weighting of $\rho^F_i$ and $\rho^L$ according to

$$\rho^F_i = \ell_i \rho^F_i + (1 - \ell_i) \rho^L \equiv \rho \left[ \frac{C_i}{N} \right]$$  \hspace{1cm} (8)$$

where $\rho^F_i$ is the rate of time preference for the follower country in the initial year of the catch-up process (a rate chosen by the social planner for the follower country), $\rho^L$ is the rate of time preference of the leader country and $\rho[.]$ is the function capturing (7) and (8). To generate the pattern of an increasing $\rho^F$ as the living standard gap closes, we assume that $\rho^F_i$ is set such that $\rho^F_i < \rho^L$. For the leader country, we assume the leader’s social planner optimises using TAP and thus has a constant rate of time preference over the catch-up period.

With ORUP the social welfare function for a follower country, with the F superscript dropped, is, for the first period being period 1,

$$V_1 = V(C_1, C_2, \ldots C_h, W_h) = \sum_{i=1}^{h} \left[ NU(C_i / N) / \prod_{i=1}^{h} [1 + \rho(C_i / N)] \right] + NU(W_h / N) / \prod_{i=1}^{h} [1 + \rho(C_i / N)]$$  \hspace{1cm} (9)$$

Using this specification of the social welfare function in problem (1) implies from the first order conditions that the optimal rate of growth of C/N will be determined by the Euler equation
\[
\frac{NU'(C_{t+1}/N) - \rho'(C_{t+1}/N)V_{t+1}}{NU'(C_t/N) - \rho'(C_t/N)V_t} = \frac{1 + \rho(C_{t+1}/N)}{1 + r_t}
\]  \hspace{1cm} (10)

with \(V_t, V_{t+1}\), the present value at times \(t\) and \(t+1\) of the future utility flow, defined by (9) and

\[
\rho'(C_t/N) = 2 \left[ \left( \frac{C}{N} \right)_t^{L} - \left( \frac{C}{N} \right)_t^{F} \right] \left( \rho^{L} - \rho^{F} \right) \left( 1 + x \right)^{-1} > 0 \text{ if } \rho^{F} < \rho^{L}
\]  \hspace{1cm} (11)

Note that the catch-up of \((C/N)^{F}\) to \((C/N)^{L}\) and, if \(x>0\), the passage of time tend to reduce \(\rho'(C_t/N)\).

Equation (10) reduces to the Euler equation under TAP if \(\rho'(C_t/N) = \rho'(C_{t+1}/N) = 0\). The modification to the Euler equation caused by ORUP is that, under ORUP, a variation in consumption influences the discounted value of social welfare through the direct effect on the instantaneous utility function, as in TAP, and the indirect effect of changing the rate of time preference. For the latter effect, an increase in consumption at time \(t\) will increase the rate of time preference at time \(t\) and thereby reduce the discounted value of the future flow of social welfare. Thus under ORUP the impact on social welfare of a change in consumption is smaller than under TAP, because of the offsetting effect on the rate of time preference.

The living standards adjustment factor is equal to one in the initial period, period 1. This implies that the rate of time preference in period 1 is equal to the exogenously set rate, \(\rho_t^L\). Assuming \(\rho_t^L\) is set below the rate of time preference in the leader country then, as (10) shows, a level of saving will be chosen by the social planner in period 1 that generates growth in \((C/N)^{F}\) that exceeds the growth in \((C/N)^{L}\). If this reduces the living standard gap then \(\rho^F\) will increase. If the gap widens then \(\rho^F\) will fall further. Either way, eventually, the living standard gap will close and by (7) and (8) as living standards catch-up with the leader, the rate of time preference will

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6 The derivative of the numerator of the left hand side of (10) with respect to \(C_{t+1}\), which is \(U''(C_{t+1}/N)\)-\(\rho_{t+1}V_{t+1}/dC_{t+1}\), is negative and so for given \(C_t\) the lower is \(\rho_{t+1}\) the greater is \(C_{t+1}\) relative to \(C_t.\)
increase towards and catch-up with the leader country’s rate of time preference. Furthermore, the social welfare function will approach the TAP form. Thus the ORUP modification dies out.

The lower the initial value of the rate of time preference in follower countries, the faster their speed of living standard catch-up because their initial rate of saving is higher. Of course, as implied by the higher initial saving, this faster speed of catch-up comes with the cost of lower initial living standards.

The speed of catch-up of living standards of a follower country also depends on the parameter $x$. It can be seen from the definition of the living standards adjustment factor, $h$, that, for $t>0$ the smaller is $x$ the larger is $\ell$ and so the greater the weight, at given values of the living standard gap, on $\rho$. Since for catch-up, $\rho_1^F < \rho_L$, the smaller is $x$ the faster is the speed of living standard catch-up.

4.2 Choosing parameter values in the ORUP social welfare function

In the ORUP social welfare function, there are three parameter values to be determined, $\rho_1^F$, $x$ and $\rho_L$. In discussing the choice of values for these parameters, our frame of analysis is that of a social planner choosing a socially optimal path of living standards from a given situation, and assuming a particular rate of TFP catch-up. The given situation is the actual initial situation of the country, with its existing saving rate, capital stock etc. This given situation is not necessarily and should not be constrained to be a steady state situation. (Indeed the existence of the possibility of TFP catch-up implies that the initial situation is not a steady state.)

We begin our discussion under the assumption that $\rho_L$ is exogenous to the follower country, chosen for example by the social planner in the leader country. Given $\rho_L$, the values of $\rho_1^F$ and $x$ will determine the speed of living standard catch-up, as discussed above. The speed of living standard catch-up determines the inter-temporal distribution of living standards. For example, a faster speed of catch-up will benefit living standards at later dates to the detriment of living standards at earlier dates. So the choice of these parameters implicitly introduces (inter-temporal) distributional objectives.

Given that any catch-up of living standards, indeed any positive growth of living standards, is raising living standards in the future to the detriment of living
standards today, one appealing normative rule is that the socially optimal catch-up path should not require a reduction in living standards relative to the actual level. There is a lot of evidence from behavioural economics that people are loss averse in that they dislike intensely reductions in their living standards. Thus reducing living standards imposes a great loss of utility. This suggests a case for setting a lower limit to \( \rho^F_i \) as the value that yields an initial level of living standards equal to the actual level of living standards in the initial year. This normative rule we call the principle of protection.

It is possible that for a particular country at a particular time the principle of protection will be inconsistent with living standard catch-up as determined by our specification of ORUP. Consider the case of a country in which the rate of saving is so low and the level of living standards so “high” that a rate of time preference equal to the rate of time preference for the leader country may imply higher saving and thus lower initial living standards than the actual levels. In these circumstances there is an alternative possibility under which the ORUP adjustment process for the follower rate of time preference as given by (7) and (8) is over-ridden for a period of years and instead the level of living standards is frozen at its initial level. Then, because TFP is growing according to the TFP catch-up formula, the share of saving in GDP will be increasing. This implies that the rate of time preference that “justifies” the frozen level of living standards will be falling. At some point after the time when this rate has fallen below the rate of the leader country the social planner can switch to the adjustment process given by ORUP.\(^7\)

For guidance in the choice of \( x \), the ORUP form has the valuable characteristic that it calculates the rates of time preference along the paths of living-standard catch-up. Thus the weight that is being placed on the welfare of people in the future for a particular speed of catch-up is revealed. In principle, this weight can be used to judge whether catch-up is planned to proceed at a desirable rate. In the simulations reported in Section 5, this is explored further.

The choice of values of \( \rho^F_i \) and \( x \) will determine, along with the value of \( \rho^L \) and the implied paths of the world interest rate and living standards in the leader country, a time-path of \( \rho^F_i \). To aid in the welfare interpretation of the path of living standard

\(^7\) Formally, choose \((C/P)_t, t=1, 2...h, as max[(C/P)_i, (C/P)*] where (C/P)_i is actual C/P in the initial year of the plan and (C/P)* is C/P as determined by (10)
catch-up, a future path of the rate of time preference can be summarised by being expressed as the implied weight placed on social welfare of people in the future, say in H years time, relative to the present can be calculated. This social welfare weight is

\[
SWW_t = \frac{1}{\prod_{t=1}^{t+H} (1 + \rho_t^F)}
\]

(12)

The SWW is a summary measure of H years of rates of time preference. It allows for the variation in the rate of time preference during a process of living standard catch-up. Using the SWW, the social planner can calculate the weight put on living standards in the future. For countries to catch-up the SWW has to be greater than the weight for the leader country.

The SSW suggests another criterion for choosing values of \( \rho_t^F \) and \( x \). The social planner may wish to avoid weights on future welfare, that is values of the SSW implied by the choice of \( \rho_t^F \) and \( x \), that are too high. An ethical view in economics, expounded famously by Ramsey (1928), is that a rate of time preference above zero is ethically indefensible. This view translates into the dictum that an SWW less than one is ethically indefensible. The same logic, that it is unethical to discount the welfare of future generations surely implies it is unethical to discount the welfare of the current generation. This implies that an SWW greater than one is ethically indefensible. Thus only SWW’s equal to one are “Ramsey-ethical”.

If one stuck rigidly to the Ramsey-ethical value of SWW, then the ORUP process would be ruled out. However, it may be that one would not want to rule out faster, or slower, speeds of living standard catch-up.

Finally, turning to the value of \( \rho_L^L \), first note that this value is the indicator of the future path of living standards of the leader country. In as far as that path is exogenous, and therefore \( \rho_L^L \), is exogenous, to the follower country, an exogenous standard or reference point is created. For the normative issues of catch-up, this is conceptually straightforward. However, as is usual when considering normative

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*8 It is usually argued that uncertainty about the future at the society-wide level implies a higher value of the rate of time preference than zero without violating the thrust of Ramsey’s argument. However, in the model of this paper uncertainty is excluded.*
questions in a framework that includes reference dependence, things get less straightforward if the agent can influence the reference standard. Suppose for example, that through a world agreement the follower countries can influence the behaviour of the leader country. If the follower countries can influence $\rho^L$ upwards, that is induce the leader country to act less patiently then, because living standards in the leader country will grow at a slower rate, the target for the follower countries is reduced. This, according to the ORUP social welfare function, would increase their social welfare.9

4.3 Conventional Uzawa preferences in the social welfare function

The other-regarding form of Uzawa preferences is not necessary for a catch-up process. For example, consider the following process. For the leader country, set the rate of time preference by the Uzawa form

$$\rho^L_t = \bar{\rho} + \theta \left( \frac{C}{N} \right)_t^L \left( \frac{1}{T_t} \right)$$

(14)

where $\theta > 0$, $(C/N)^L$ is the living standard in the leader country and $T_t$ is a trend variable. The latter is necessary in order that the rate of time preference can be constant in a steady state in which living standards are growing. Then set $\bar{\rho}$ and $\theta$ such that the optimal value of $(C/N)^L$ in the initial year is equal to the actual value. For a follower country, set the rate of time preference by

$$\rho^F_t = g_t \rho^F_t + (1-g_t) \left[ \bar{\rho} + \theta \left( \frac{C}{N} \right)_t^F \left( \frac{1}{T_t} \right) \right]$$

(15)

with $g_t = (1+x)^t$ and $x>0$. In this Uzawa form, the rate of time preference for both leader and follower countries tends to a pattern determined by a common process.

9 This suggests a consumption externality and thus a case for taxing the rate of return on saving at the margin in countries with high living standards, similar to the case based on consumption externalities within a country, see Ljungqvist and Uhlig (2000). Of course, our assumption that TFP growth in the leader country is exogenous rules out the possibility that by taxing the return on saving, TFP growth may be reduced. A full treatment of the optimal tax rate would address this possibility.
Thus in the long run, as \( g \) approaches zero the preferences of the countries approach commonality.

As noted earlier, the standard criticism of the Uzawa form, that is (14) and (15), is that it is counter-intuitive to suppose that poor people are more patient than rich people. This carries over to the normative context of a social planner. It is surely unreasonable to suppose that poor people should be more patient simply because they are poor. By comparison, the ORUP form gives a reason for this higher patience in that it is based on the idea of the catch-up of living standards. Poor people are more patient because they desire to catch-up with rich people. The normative basis of ORUP is catch-up. If a poor country wants to experience living standard catch-up then it can specify social preferences according to ORUP. For this reason, for normative planning we advocate the extension to Uzawa preferences embodied in ORUP.

5 Simulations of socially optimal paths of living standards

In this section we use simulations of the world model to illustrate the relation between the speed of living standard catch-up and the weighting of future welfare. This relation can help in evaluating the desirability of a particular program on living standard catch-up. For example, the measure of the weight on future welfare implied by a program of speedy living standard catch-up may cause one to question, if the weight is high, the desirability of that program. Furthermore, we show that what we call the Ramsey ethic would rule out some explicit programs of living standard catch-up.

The simulations also illustrate other characteristics of ORUP living standard catch-up programs, in particular the potential contribution of foreign asset accumulation and the influence of demographic change.

5.1 The world simulation model

The world model described in earlier sections is simulated over a future period of TFP catch-up throughout the countries of the world. In these simulations, countries are grouped into nine regions. These are: Africa, Asia (excluding China, India and Japan), China, Europe, India, Japan, Latin America, North America and Oceania. We use the UN medium fertility demographic projection and specify processes of TFP
catch-up and a convergence across regions of employment-population ratios by age and gender. The bases of these projections are described in the Appendix.

In 2000 North America had the highest level of labour productivity of the nine regions and so is classified as the leader region.\textsuperscript{10} As it happens, in 2000 North America also has the lowest saving ratio to GDP of the nine regions, see Table 1. Across the nine regions there is in 2000 a great variety of saving/GDP ratios, from 17.6\% for North America to 42.1\% for China. This data and the principle of protection are used to set $\rho^L$ and $\rho^F_1$. For the leader region, $\rho^L$, the leader’s constant rate of time preference over the projection period, is set by an iterative technique such that the optimal level of saving in the initial year is equal to the actual level of saving for the leader region. Simultaneously, for the follower regions, $\rho^F_1$, the rate of time preference in the first period of the projection, is set for each region such that the initial level of optimal saving in each region is equal to the actual level saving for that region.

As will be seen below, applying our procedure for setting $\rho^L$ and $\rho^F_1$ across regions yields a set of values such that all the follower regions can, in 2000, adopt a path of consumption and saving that is both optimal according to ORUP and does not require a reduction in living standards, that is does not violate the principle of protection. Because this is the lowest value of $\rho^F_1$ consistent with not violating the principle of protection it also yields the fastest speed of living standard catch-up consistent with not violating the principle of protection, given the value of $x$.

Because of the variety in saving behaviour, our procedure for setting $\rho^F_1$ yields a variety of ORUP paths for regions and thus serves to illustrate some of the properties of ORUP-based living standard catch-up. For example, it implies that regions with high saving ratios will choose paths of faster living standard catch-up. Such high saving regions will also use foreign asset accumulation as part of their program of living standard catch-up. The simulations will illustrate these properties.

Our procedure for setting $\rho^F_1$ also has the advantage of injecting an empirical element into the simulations. One may suppose that regions whose actual saving/GDP

\textsuperscript{10} According to our calculations, see Appendix, in 2000 North America’s level of TFP is less than Japan’s. But labour productivity is a more reliable measure than initial levels of TFP because of the assumptions that have to be made to calculate the latter and so labour productivity is a better measure by which to classify the leader.
ratios are high are regions that place a higher value at the present time on a fast catch-up of living standards. Our procedure for setting $\rho_i^f$ captures this suggestion.

### 5.2 Living standard catch-up

We ran two simulations to capture the ORUP social welfare function based on two values of $x$, 0.015 in the simulation labelled ORUP1 and 0 in the simulation labelled ORUP2. The instantaneous utility function is specified using the constant elasticity form with $\beta=2$, see appendix. Differences in consumption by age were included, see appendix. The parameter determining the speed of TFP catch-up, $\gamma$, was set at 0.02, implying that TFP for all regions catches up substantively with the leader region in 150 years, that is in 2149 the average TFP of the follower regions as a group is 95% of the leader region.

We ran a third simulation based on TAP preferences, labelled TAP, in which the rate of time preference was set equal to 0.9 per cent. The other functions and exogenous values were as for the ORUP simulations. (Note in particular that the speed of TFP catch-up was the same.) The rate of time preference of 0.9 % implies, in the initial year, an optimal rate of world saving as calculated by the model roughly equal to the actual value for 2000 of 23.9% and so, given our calibration of the ORUP social welfare function, yields optimal saving ratios equal to actual saving ratios in the initial year.

The paths followed by the living standards of the follower region, the average of the eight follower regions, are shown in Chart 1 as a ratio of living standards in the follower region to living standards in the leader region are plotted for the period 2000 to 2100 along with, for benchmarking, the average labour productivity of the follower region relative to the leader region. Under ORUP1, the living standards of the follower region catches up with the living standards of the leader region at about the same rate as labour productivity catches up. Under ORUP2 living standards in the follower region catch-up more quickly, faster indeed than labour productivity catch-up, but this speeding up occurs in the second half of the 21st century.

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11 There is a case for reporting GDP weighted or population weighted averages of living standards and labour productivity. However, because the GDP shares and population shares of regions change so much over the projection period, this weighting makes the averages harder to interpret and so we chose to report the simple averages.
By comparison with ORUP, under TAP living standards in the follower region do not catch-up, see Chart 1. They remain throughout the projection period at a constant ratio of 45% of the leader region.

The evolution of the rates of time preference for all nine regions implied by the ORUP1 simulation is shown in chart 2. In 2000, all follower regions have lower rates of time preference than the leader country, reflecting their higher rates of saving. It can be seen that five regions, China, Japan, Africa, India and Asia have negative rates of time preference in 2000, equal to –5.4%, -5.1%, -1.4%, –1.3% and -0.6% respectively. As time progresses the rates of time preference for all follower regions approach that of the leader region.

The speed with which the rate of time preference adjusts to that of the leader region varies substantially across regions. These speeds are faster for those regions with a higher rate of actual saving in 2000, because, according to our setting of $\rho_1$, those regions are posited to choose faster speeds of living standard catch-up.

We have emphasised that a valuable property of using our ORUP specification to study living standard catch-up is the central role given to the rate of time preference. With the help of Chart 3, we illustrate this advantage by considering the living standard catch-up paths of two of the poorest regions who begin, in 2000, at the bottom end of living standards, that is China and India. Their levels of living standards in 2000 are very similar, being 0.09 and 0.08 relative to the leader country. Their speeds of living standard catch-up are very different, reflecting their very different saving/GDP ratios in 2000.

In Chart 3, the horizontal axis measures the SWW, defined by equation (12), and the vertical axis measures the living standard in 100 years time, relative to the leader region. Relations from ORUP1, ORUP2 and two other simulations are reported. To understand Chart 3 consider first the most south-westerly observation of the three linked observations labelled “India”. This is the combination of the SWW for 100 years time, that is the value in 2000 of welfare in 2100, and the living standard for 100 years time, for 2100, for India calculated from the ORUP1 program. It shows that the attainment by India of a living standard of 0.59 of the leader by 2100 under the ORUP1 simulation is associated with an implicit weight today on welfare enjoyed in 100 years time of 1.10. Compare this combination with the similar south-west observation of the China cluster. China experiences a substantially greater amount of
living standard catch-up, getting to 0.80 of the leader by 2100, but this is associated with a higher weight on future welfare of 1.64.

Relative to the growth in labour productivity, the two regions have different experiences under ORUP1. Labour productivity in 2100 relative to the leader is shown by the horizontal line. Living standards in India almost grow with labour productivity. For China living standard catch-up gets ahead of labour productivity catch-up. In the China program, this “excess” growth of living standards is associated with a massive build up of foreign assets, see below.

Because of our use of the principle of protection in calibrating ORUP, it is the large level of saving in 2000 for China that generates the higher weighting on future welfare than the weighting for India for the same value of x. Of course, regions can increase the weighting on future welfare by using a lower value of x. Suppose, for example, that India sets x=0 for itself while all other regions set x=0.015. Then the outcome for India is the easterly point on the upper arm of the India cluster. That is, the living standard for India is 0.80 of the leader in 2100 and the SWW associated with this achievement is 1.98. By almost doubling the weight on future welfare, India is able to reduce over 100 years the living standard gap by 21 percentage points.

If all regions attempt faster catch-up by choosing a lower value for x then for a particular follower region the living standard gain is not so great for a given SWW. There is a pecuniary externality, because the investment opportunities at the world level are limited ultimately in our model of exogenous population change by the size of the world’s working age population. Thus for ORUP2, where all regions set x=0, the outcomes for India and China are the easterly points on the lower arms of their respective clusters. The reported observation shows that the attainment by India of a living standard of 0.75 of the leader by 2050 under the ORUP2 simulation is associated with an implicit weight today on welfare enjoyed in 50 years time of 3.11.\[12\]

5.3 Foreign asset accumulation

\[12\] The attempt to attain faster living standard catch-up represented by ORUP2 leads to a world rate of interest about one percentage point lower over the 21st century than under ORUP1. The direction of the effect of a lower world rate of interest on a region depends partly on its ambitions and its demographic structure. Those regions seeking rapid catch-up are penalised, because the return on their foreign lending is depressed somewhat. Also ageing regions, for whom foreign lending is important, are penalised. On the other hand, those regions seeking slower catch-up will be rewarded by the lower return because they will be borrowing. And of course the young regions, who tend to borrow, will be advantaged.
The speed of living standard catch-up has implications for the evolution of foreign assets and thus the importance of the world capital market. Demographic change also influences foreign asset accumulation. Our simulations cannot separate out the effects of these two factors. However, using Chart 4, the nature of these effects can be discussed. We discuss the effect of the speed of living standard catch-up in this section and demographic influences in the next section.

In Chart 4, from the ORUP1 simulation, the foreign asset position for all nine regions is measured along the horizontal axis and the living standard relative to the leader region along the vertical axis. The paths over the 21st century for foreign asset accumulation and living standards for each of the nine regions are described by three data points, for 2000, 2050 and 2100. The movement over time for any region, except by definition the leader, is up.

The ORUP1 optimal path for China is supported by a massive build up of foreign assets. China’s foreign asset position is projected to increase from four per cent of China GDP in 2000, the lowest point in Chart 4, to a massive 870 per cent in 2050 and then to come back to 570 per cent of GDP by 2100. In part, this massive increase in foreign assets is to support the very fast speed of living standard catch-up implied by its choice of such high SWW’s. (There is also a demographic effect discussed in the next section.)

A slower catch-up of living standards is associated with less foreign asset accumulation. Consider India and Africa. Living standards in India get to 59 per cent of the leader country by 2100 along a path that requires foreign assets to decrease by 2050, to –82 per cent of GDP, and then to increase to 50 per cent of GDP by 2100. Africa, with the slower catch-up of living standards, which reach 56 per cent of the leader by 2100, experiences a large decrease in its foreign asset position to –372 per cent of GDP in 2050 coming back to -70 per cent by 2100.

Just as some may question the possibility that China can accumulate such large levels of foreign assets, the possibility that Africa can, or will be allowed by non-African lenders, to accumulate such large levels of foreign debt may also be questioned. If, in spite of the feasibility of these paths, their high levels of foreign assets and debt are considered unfeasible for other reasons then the results imply that living standards in China will be slower to catch-up and in Africa will be faster.
5.4 Demographic influences

Demographic change also influences the optimal path of foreign asset accumulation. Chart 4 can also be used to illustrate this influence. However, to see this it is necessary to realise that the nine regions can be classified into ageing regions and young regions. We do this using the support ratio, that is the aggregate employment/population ratio, weighted by labour productivity and relative consumption demands (Cutler et al., (1990)). A decrease in the support ratio implies a diminished capacity to meet a given level of consumption demands per capita. Chart 5 plots the support ratios for the nine regions for the UN medium fertility scenario. The five regions with imminently declining support ratios are Japan, Europe, China, North America and Oceania. These regions will be referred to as the ageing regions. The other four regions: Africa, Asia, India and Latin America, are younger in that their support ratios follow a hump-shape, rising initially then declining. We refer to these as the young regions.

It is optimal for ageing regions to build up foreign assets. This is because ageing and the associated slow growth of the working-age age group limits their domestic investment opportunities. In Chart 4 the paths of foreign asset build-up of the ageing regions, in particular Europe and Japan, show this. One can see the large increase in foreign assets for these regions in the first half of the 21st century. For China, the massive build up of foreign assets is dominated by the effect of a high speed of living standard catch-up, as discussed above, but we can infer from the Japan path that an ageing effect is also playing a role for the China pattern. For the young regions, the early period of decreasing dependency is associated with a high rate of growth of the working age population and thus many domestic investment opportunities. It is optimal to finance some of these by foreign borrowing. This effect is part of the explanation, along with relatively slow catch-up of living standards, of the increased foreign borrowing by the young regions, India, Latin America, Asia and

6 Conclusion

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13 The construction of the support ratios for the regions is described in the Appendix.
14 The decline in the support ratio for Europe and for China does not commence until 2010 and 2005, respectively. However, the initial rise in the support ratio is very small.
For countries that are enjoying a catch-up of their levels of total factor productivity (TFP) with that of the leader country, choosing optimal paths of living standards based on time-additive preferences (TAP) with the same rate of time preference as the leader implies that their living standards will never catch-up. Instead they will choose to borrow massively in the earlier period of TFP catch-up in order to exploit early the fruits of future high levels of TFP. If, with TAP, the follower countries choose a lower rate of time preference than the leader country, then their living standards will eventually overtake the living standards of the leader. These are implications of well-known properties of TAP.

This paper suggests using an alternative specification of preferences, which we call other-regarding Uzawa preferences (ORUP), to study the nature of the socially-optimal degree of catch-up. Under ORUP, the rate of time preference in a follower country is a decreasing function of the gap between the follower’s living standard and the leader’s living standard. Assuming the follower country at the beginning of the living standard catch-up program, uses, to set consumption and saving levels, a rate of time preference lower than that of the leader country, then under ORUP living standards will catch-up during the process of TFP catch-up. Just how fast depends on the quantitative setting of the parameters in the ORUP social welfare function. The catch-up of living standards during the process of TFP catch-up can also occur if consumption and saving decisions are based on optimising with a social welfare function based on Uzawa preferences, that is without the outward-looking aspect. Thus ORUP is not essential for living standard catch-up. However the attraction of the ORUP form is that it explicitly addresses the issue of catch-up. In doing that, it provides a new reason for positing an increasing relation between the rate of time preference and the level of consumption. The traditional rationale for this relation has appeared to many as counter-intuitive, that is the assumption that poor people are more patient than rich people does not seem plausible as an empirical proposition. However the rationale on which the ORUP form is based is that if poor people want their living standards to catch-up with those of rich people then they have to act as if they have lower rates of time preference. This is more plausible in as far as people care about relative living standards.

The outward looking nature of ORUP is consistent with the emphasis of behavioural economics on the importance of comparisons of living standards in human welfare and decision making.
The ORUP form of the social welfare function reveals a relation for a country between the speed of living standard catch-up and the rate of time preference, given the behaviour of the leader country and the rest of the world. This relation is negative—that is by choosing paths with generally lower rates of time preference, a faster speed of living standard catch-up is generated.

By placing the focus of living standard catch-up on the rate of time preference, the ORUP form yields results with a welfare interpretation. The social welfare weights (SSW), that is the weighting of utility in the future relative to utility today, implied by a program of catch-up can be calculated. These weights summarise a path of rates of time preference. From these weights, a judgement can be made about the desirability of a particular catch-up program.

One suggestion from these weights is that paths along which the SWW’s are greater than one should be ruled out as unethical, because that would imply a weighting on welfare in the future greater than the weight on welfare enjoyed today. This inference is consistent, we suggest, with Ramsey’s (1928) famous dictum that one should not under-weight the future. One should not over-weight the future either. These two sides of Ramsey’s dictum imply a rate of time preference equal to zero and an SWW equal to one. We call this the Ramsey ethic.

However, applying Ramsey’s ethic has implications that cast doubt on its acceptability. For all countries to choose a rate of time preference equal to zero would imply that living standards of follower countries will not catch-up with those of the leader country because this choice of time preference implies the TAP social welfare function.

To provide a practical application of ORUP, in the paper simulations are reported for a world model in which eight regions experience TFP catch-up with a leader region, North America. We calibrate ORUP for these simulations by setting the parameter values such that the principle of protection, that is along a living standard catch-up path living standards never fall. These simulations reveal that, following an ORUP program all eight follower regions can enjoy a catch-up of living standards at about the same rate as the catch-up of labour productivity without violating the principle of protection. They also quantify the relation between the speed of living standard catch-up and the weight social planners place on future economic welfare.
Appendix Assumptions underlying the empirical application of the world model

In the simulations, the optimal paths of living standards are projected for regions of the world over a period of TFP catch-up assumed to occur over the 150 year period from 2000. In this period, large demographic changes, in both the size and age structure, are expected to occur. As the discussion of the simulations makes clear, these demographic changes have a significant impact on the outcomes. In this appendix, much of the discussion is how the model is extended to incorporate demographic change.

1 Demographic projections

We divide the world economy into nine regions consisting of the eight regions in the United Nations (2000) long run demographic projections plus Japan as a separate country. The nine regions are: Africa, Asia (excluding China, India and Japan), China, Europe, India, Japan, Latin America, North America and Oceania. The medium fertility scenario of the United Nations projections (up to 2150) assumes that fertility in all major areas stabilizes at replacement level around 2050.

2 The convergence of employment-population ratios

The evolution of the aggregate employment level will be influenced by the speed of population growth and changes in population age structure. These forces will vary across our nine regions. To calculate aggregate employment projections for each country for each year we first multiply age and gender specific employment/population (L/N) ratios by age and gender specific population totals. The former are based on the actual values of employment/population ratios by country. However, as revealed by the International Labour Organisation (ILO (2001)) database: Key Indicators of the Labour Market (KILM), there is variation in the L/N ratios between regions. The greatest variation is for young people. For males and females the L/N for 15-24 year olds for Europe are 0.52 and 0.43 respectively while they are 0.78 and 0.80 for China.

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15 Europe extends to the Urals and so is somewhat heterogenous in levels of GDP per capita. Oceania, comprising Australia, New Zealand as well as Papua New Guinea and Fiji, is also heterogenous. However, in discussion we regard both these regions as ‘high-income’.
and 0.68 and 0.63 for North America. In keeping with the idea of diffusion, we assume that it is unlikely that such divergences will persist as economic development progresses. Hence employment projections for each country are calculated by assuming that the age- and gender-specific L/N ratios partially converge toward those of leader country, North America, indicated by the superscript L for leader, according to the following formula:

\[
(L/N)_t = (L/N)_{t-1} \left( \frac{(L/N)^L}{(L/N)_{t-1}} \right)^\sigma
\]  
(A1)

where \(t\) is the year from 2001 to 2149 and \(\sigma\) is the convergence parameter set equal to 0.025. With this value, the employment/population ratios for 15-24 year olds have substantially converged to the North American values by 2100.

The calculations described in the previous paragraph provide employment levels by age and gender for each country for each year. To calculate aggregate employment numbers the second step, following Cutler et al. (1990), Miles (1999) and Elmendorf and Sheiner (2000), is to weight the employment numbers to account for age-specific differences in labour productivity. As suggested by relative wages, labour productivity of middle-aged workers is higher than that of younger and older workers. In the absence of reliable data for all regions on labour productivity by age, we adopt as an expediency the age-productivity relation in Miles (1999), where the productivity weight is a quadratic function of age: 0.05age – 0.0006age². This then provides aggregate employment levels by year by country.

3 Consumption demand weights and the social welfare function

Consumption demands vary by age – in particular, education and medical expenses. This suggests that the efficiency of consumption varies by age. To allow for this in the social welfare function, we apply the consumption weights in Cutler et al (1990); that is 0.72 for 0-19 year olds, 1.0 for 20-64 year olds and 1.27 for over 64 year olds to each region. Weighting people by age group by consumption weights and summing over age groups gives the aggregate population in consumption units,
To incorporate into the TAP social welfare function the distinction between $N$, population in natural units, and $P$, population in consumption units, the explicit functional form used in the TAP1 simulation is

$$V = \sum_{h=1}^{h} \left[ N_h \left( \frac{C_t}{P_t} \right)^{1-\beta} \left( \frac{1+\rho}{1-\beta} \right)^{1-h} \right] + N_h \omega \left( \frac{W_h}{N_h} \right)^{1-\psi} \left( \frac{1+\rho}{1-\psi} \right)^{1-h}. \quad (A2)$$

The utils generated by the living standard through the parameter $\beta$ are weighted by population measured in natural units to generate social welfare.\footnote{Cutler et al (1990) do not allow for relative efficiency in consumption to affect social welfare. Elmendorf and Sheiner (2000b, pp.20-22) do, specifying consumption per consumption unit as the measure of individual utility in their social welfare function, as in (8). However, they weight individuals by their consumption unit equivalent, not by their natural units, that is they specify $P_t$ in (8) instead of $N_t$. The specification in this paper is based on Sen (1973, pp.15-23), who adopts the weighting in natural units. Below this specification is compared with that of Elmendorf and Sheiner (2000a,b), Blanchard and Fischer (1989) do not weight by population. However, for an ageing economy that is not appropriate. The size of the population in the future relative to today influences the size of the task of preparing for ageing. Lim and Weil (2003) use consumption per consumption unit and weight by population in natural units, as in (8).}

Optimising (1) using the time-additive social welfare function (A2) implies that living standards along the socially optimal path grow according to

$$\frac{C_{t+1}}{P_{t+1}} = \left[ \frac{N_{t+1}}{N_t} \left( \frac{C_t}{P_t} \right) \right]^{1/\beta} \left( \frac{1+\rho}{1+\rho} \right)^{1/\beta}. \quad (A3)$$
The term in square brackets is the demographic factor. Demographic change causes this term to differ from one temporarily. For example, during the process of ageing \( P \) grows faster than \( N \), implying that the demographic factor is less than one. Non-unit values of the demographic factor will affect the socially optimal path of living standards according to the value of \( 1/\beta \), the inter-temporal elasticity of substitution.

The ORUP social welfare function for a follower region is specified as

\[
\sum_{t=1}^{h} \left[ \frac{N_t \left( \frac{C_t}{P_t} \right)^{1-\beta}}{(1-\beta) \prod_{i=1}^{t} [1+\rho_i]} \right] + \frac{N_h \omega \left( \frac{W_h}{N_h} \right)^{1-\beta}}{(1-\beta) \prod_{i=2}^{h} [1+\rho_i]} \tag{A4}
\]

with

\[
\rho_t^f = \ell_t \rho_t^f + (1-\ell_t) \rho^L \tag{A5}
\]

and

\[
\ell_t = \left[ \frac{\left( \frac{C_t}{P_t} \right)^L - \left( \frac{C_t}{P_t} \right)^F}{\left( \frac{C_t}{P_t} \right)^L - \left( \frac{C_t}{P_t} \right)^F} \right] (1+x)^{-1} \tag{A6}
\]

In the simulations, we assumed for convenience \( \rho'(C_t/P_t)=0 \) in the Euler equation, (10), reducing (10) to (A3). The approximation error introduced by this assumption is tiny, because of the small size of \( \rho'(C_t/P_t) \) and because it appears on the numerator and the denominator of (10). Of course in the simulations the rate of time preference for a follower region is determined by (A5) and (A6).

4 Modelling TFP catch-up

\[\text{It also avoids another issue. With our specification of the instantaneous social welfare function, if } \beta>1 \text{ then the units of social welfare are negative numbers, that is } V<0. \text{ To maintain the sense of (10), for } \beta>1 \text{ the sign on } \rho'(C/P) \text{ would have to be changed to positive.}\]
For each country the initial value of $A$ is set such that the optimal level of investment is equal to the actual level of investment in the initial year. From that base, TFP in the leader country is assumed to grow at the exogenous rate of 1 per cent. For the follower regions, TFP grows according to the catch-up formula based on the specification by Lucas (2000) for the catch-up of labour productivity by countries of the world.

$$A_t = (1 + a)A_{t-1}L_{t-1}^{A_{t-1}}$$

The parameter $\gamma$ determines the speed of catch-up. In the simulations for the base case we set $\gamma=0.02$. This implies that the TFP levels across regions go from a range of 2.3 to 19.0 in 2000 to a range of 65.5 to 72.7 by 2149. Thus over 150 years all the follower regions substantively catch-up with the leader country.

The specification of catch-up reflects mainly the transmission of technology. However it also captures country-specific distortions in the initial level of investment. For example, suppose investment in Japan in the initial year, 2000, is excessive, as argued by many, see Guest and McDonald (2003c). This implies that our estimate of $A$ for Japan for the first year of the simulation, 2000, is too high. However, over time the convergence of TFP in Japan to the level of the leader country implies that this distortion will disappear.\textsuperscript{19} \textsuperscript{20} At the other side, consider Africa. Suppose in the initial year, 2000, that investment in Africa is lower than the true socially optimal level because political uncertainty adds a risk premium to investment decisions. Thus our initial estimate of $A$ for 2000 for Africa will be too low. However, over time the convergence of TFP in Africa to the level of the leader country implies that this distortion will disappear.

5 The putty-clay vintage production function

\textsuperscript{19} This will imply that the growth of GDP in early years is overstated, but this error is small in the context of our long simulation period.
\textsuperscript{20} In 2000 Japan has a higher level of TFP than North America, 19 compared with 16.4. However, we use North America as the leader country. As suggested in the text, our measure may overstate the level of Japanese TFP. Of course as a result the catch-up formula implies that Japanese TFP grows more slowly than that of North America.
There are three advantages of our production function. First, the embodiment of new ideas in new capital goods is realistic. For a country experiencing TFP catch-up, it is of particular importance to include the requirement that new ideas need to be embodied in new investment goods and services because that will allow for the impact of TFP catch-up on the resource constraint. Second, this embodiment determines a gradual adjustment process for the capital stock, and is therefore an attractive alternative, being based on the realistic concept of embodiment, to the practice adopted in the homogenous capital approach of using an arbitrarily determined adjustment cost function. In our simulation method the adjustment process is particularly important because we allow for the possibility that, in the initial year, countries may not have a steady-state capital stock. In consequence, their optimal paths of investment in the early years of the simulation may include a component that adjusts for any initial ‘misallocation’ in their capital stocks. To allow for this is more realistic than to assume, as is often done, that the initial capital stock is the steady-state value of the capital stock. Third, there are pragmatic reasons for adopting the vintage form. It does not require data for the capital stock at the beginning of the projection period. Given the poor quality of this data for many economies this parsimony is an advantage, see Dadkah and Zahedi (1986). Also, it turns out that the computer simulations of the vintage form of the production function solve more easily than do simulations using the homogeneous capital form.

In the calculations the labour released from the investment goods of age T+1 which are scrapped is assumed to be negligible, that is \( (1-\delta)^Tl_{t+1,T} = 0 \). Remember that after T years of depreciation, only \( (1-\delta)^T \) of the original capacity of the investment goods remains and so little labour is still working on those investment goods. In the calculations, \( l_t \) is typically about 15 per cent of \( L_t \), \( T \) is more than 15 and the rate of growth of employment is about 1 per cent. This suggests that the amount of labour working on investment goods at the point of being scrapped is about one per cent of aggregate employment. Thus the error introduced is small. Furthermore, it is rendered insignificant by the calibration approach. In particular, the effect of these

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21 However, note that in the long run as a steady state is approached, the vintage production function approaches the homogenous capital production function; see Guest and McDonald (2003b) for a comparison of adjustment and long run behaviour with vintage and homogenous production functions.
approximations on the size of fluctuations in A and I, which is the important consideration, is insignificant.

The number to be employed on the newest vintage of capital is equal to aggregate employment growth plus labour released from old vintages because of depreciation and scrapping. The capacity of all vintages in operation contracts at rate \( \delta \), releasing \( \delta L \) labour per period. Furthermore, in each period the least productive (the oldest) vintage is scrapped, releasing \( (1-\delta)^T l_{t+1-T} \) units of labour, where \( T \) is the age of the capital goods being scrapped and \( l_{t+1-T} \) is the labour working on those capital goods at time \( t \). These two sources of labour give the number to be employed on the newest vintage at time \( t+1 \) as

\[
l_{t+1} = L_{t+1} - (1-\delta)L_t + (1-\delta)^T l_{t+1-T} \tag{A8}
\]

From the maximisation problem, the optimal level of investment is related to employment growth, as measured by \( l \), by

\[
I_t = \left[ \frac{\alpha A_t}{E_t} \right]^{\frac{1}{1-\alpha}} l_{t+1} \tag{A9}
\]

where \( E \), the user-cost of capital, is determined by

\[
E_t = \frac{1}{\sum_{k=1}^{T_t} \frac{(1-\delta)^{k-1}}{\prod_{t+1}^{T_t} (1+r_t)}} \tag{A10}
\]

and \( T \), the age of the oldest capital good in use, is determined by

\[
T = \frac{-(1-\alpha)\ln(1-\alpha)}{\ln(1-\alpha)} . \tag{A11}
\]

6 Parameter settings and solution method
Initial values of GDP by region were calculated from the Penn World Table 6.1, see Table A1. The actual values of saving, used in the setting of $\rho^F$ in ORUP and $\rho$ in TAP, see text, and investment, used in the setting of the initial values of TFP, see text, as ratios of GDP were calculated from World Bank Tables, see Table A1. The terminal wealth-consumption ratios were assumed to be equal to the initial, in 2000, wealth-consumption ratios. Thus the optimal program does not diminish the region’s wealth. These values were calculated from data on capital stocks from the Penn 1990 tables and data on consumption from World Bank Tables, see Table A1. We set initial foreign debt levels equal to zero. It should be realised that the value of these wealth-consumption ratios has a negligible influence on the simulation output. One can double or halve them without changing the results.

The model was solved by a shooting algorithm in which an initial path of world interest rates is chosen arbitrarily and the optimal values for each country are solved according to the equations presented above. Then, for each year in which world saving does not equal world investment (equation (2)) within a degree of tolerance, the world interest rate is changed in those years in the appropriate direction. This process is repeated until (2) holds for all years.

\[22\] To ensure the terminal wealth constraint was satisfied, we chose the value of the parameter $\psi$ in the social welfare function (8) such that the optimal program yielded a terminal W/C equal to the required terminal value.
### Table A1 Data for 2000

<table>
<thead>
<tr>
<th>Region</th>
<th>Y</th>
<th>S/Y</th>
<th>I/Y</th>
<th>W/C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Africa</td>
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<td>19.0</td>
<td>18.6</td>
<td>1.44</td>
</tr>
<tr>
<td>Asia</td>
<td>8997380</td>
<td>27.8</td>
<td>20.7</td>
<td>1.61</td>
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<td>6498806</td>
<td>38.0</td>
<td>34.8</td>
<td>1.88</td>
</tr>
<tr>
<td>Europe</td>
<td>15042543</td>
<td>23.4</td>
<td>21.8</td>
<td>2.72</td>
</tr>
<tr>
<td>India</td>
<td>3555795</td>
<td>20.3</td>
<td>22.9</td>
<td>1.46</td>
</tr>
<tr>
<td>Latin America</td>
<td>5444600</td>
<td>19.5</td>
<td>20.6</td>
<td>1.45</td>
</tr>
<tr>
<td>NAm</td>
<td>13605932</td>
<td>17.7</td>
<td>20.7</td>
<td>2.53</td>
</tr>
<tr>
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<td>21.1</td>
<td>20.7</td>
<td>3.17</td>
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<tr>
<td>Japan</td>
<td>4336957</td>
<td>27.4</td>
<td>25.9</td>
<td>5.35</td>
</tr>
<tr>
<td>World</td>
<td>60466544</td>
<td>23.9</td>
<td>22.9</td>
<td>1.81</td>
</tr>
</tbody>
</table>

Source: Penn 6.2

### Table 1 The ORUP1 simulation

<table>
<thead>
<tr>
<th>Region</th>
<th>NAm</th>
<th>Oceania</th>
<th>Japan</th>
<th>Europe</th>
<th>China</th>
<th>Lat Am</th>
<th>Asia</th>
<th>Africa</th>
<th>India</th>
</tr>
</thead>
<tbody>
<tr>
<td>rho(2000)</td>
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<td>2.8</td>
<td>-5.1</td>
<td>0.6</td>
<td>-5.4</td>
<td>1.5</td>
<td>-0.6</td>
<td>-1.4</td>
<td>-1.3</td>
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<tr>
<td>rho(2000)</td>
<td>72.8</td>
<td>47.3</td>
<td>60.8</td>
<td>37.5</td>
<td>7.4</td>
<td>21.2</td>
<td>14.2</td>
<td>6.5</td>
<td>7.3</td>
</tr>
<tr>
<td>C/P rel NAm(2000)</td>
<td>1.00</td>
<td>0.59</td>
<td>0.66</td>
<td>0.43</td>
<td>0.09</td>
<td>0.25</td>
<td>0.16</td>
<td>0.07</td>
<td>0.08</td>
</tr>
<tr>
<td>C/P rel NAm(2050)</td>
<td>1.00</td>
<td>0.72</td>
<td>0.95</td>
<td>0.69</td>
<td>0.46</td>
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<td>0.37</td>
<td>0.23</td>
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<tr>
<td>C/P rel NAm(2100)</td>
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<td>0.98</td>
<td>0.85</td>
<td>0.80</td>
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<td>0.69</td>
<td>0.56</td>
<td>0.59</td>
</tr>
<tr>
<td>SWW of 2050 in 2000</td>
<td>0.14</td>
<td>0.21</td>
<td>0.30</td>
<td>0.37</td>
<td>3.93</td>
<td>0.39</td>
<td>0.07</td>
<td>1.56</td>
<td>1.42</td>
</tr>
<tr>
<td>SWW of 2100 in 2000</td>
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<td>0.04</td>
<td>0.05</td>
<td>0.08</td>
<td>1.64</td>
<td>0.13</td>
<td>0.40</td>
<td>1.45</td>
<td>1.10</td>
</tr>
</tbody>
</table>

Source: Penn 6.2

For definitions of variables, see text.
Chart 3 The relation between living standard catch up and the weighting of future welfare for China and India

Labour productivity in 100 years

China
India
Chart 4 Living standards and foreign asset accumulation, ORUP1, 2000 to 2100
References


