Soft Budget Constraints in Public Hospitals

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VERY PRELIMINARY DRAFT

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1. Introduction

A soft budget constraint exists if (i) an enterprise exhausts its budget before the end of the budget period and the funding source refinances the enterprise so that it can continue to operate for the remainder of the budget period and (ii) the enterprise knows that this refinancing will occur and makes choices based on this knowledge as does the funding source, Maskin (1996).

Soft budget constraints are prevalent in the public hospital sector. Kornai (2009) argues that public hospitals often exhaust their budgets before the end of the budget period or run up debts by the end of the budget period knowing that the government will "bail them out" by providing additional funding or paying off debts.

There is small literature on soft budget constraints in public hospitals. Shen and Eggleston (2009) and Brekke, Siciliania, and Straume (2012) theoretically analyse the effect of soft budget constraints on the incentives for cost reductions and on quality provision in publicly funded hospitals. They find that soft budget constraints weaken the incentives public hospitals have for cost efficiency and have ambiguous effects on public hospital quality. Shen and Eggleston (2009) empirically examines these predictions using US data and find that softer budget constraints are associated with less cost efficiency and higher quality. Eggleston and Shen (2011) find that not-for profit and public hospitals are less cost efficient and have higher quality than for profit hospitals after controlling for the softness of budget constraints. All of these papers incorporate soft budgets into their analysis by assuming there is some probability that the government will bailout the public hospital if it exhausts its budget before the end of the budget period.

This paper differs from the existing literature by examining the conditions under which a bailout occurs and by specifically incorporating the response of both the public hospital and the government to the existence of a soft budget constraint. The model has a simple structure. The government chooses a budget for the public hospital to maximise total patient benefit. Given this budget, the public hospital chooses the number of non-elective and elective patients to treat to also maximise total patient benefit. If the budget constraint is hard, the government gives the public hospital a budget which is just sufficient to fund the welfare maximising number of patients and the public hospital chooses to treat the welfare maximising number of patients. This is unsurprising as both the government and the public hospital are maximising total patient benefit, though with different constraints.

On the other hand, if the budget constraint is soft, the government and the public hospital play a two period game over the budget period. In the first period, the government chooses a budget to maximise total patient benefit. The public hospital takes this budget as given and chooses the number of patients to treat and the proportion of the budget period in which to exhausts this budget. As a result, the length of the first period is chosen by the public hospital. In the second period, the government chooses a budget to maximise total patient benefit, but now there are political costs of funding any bailout. The public hospital then chooses the number of patients to treat to maximise total patient benefit. The budget is soft if it is exhausted before the end of the budget period.

Proposition 1 states that if the government's cost of funds in the second period is greater than some critical level, then the welfare maximising budget is a hard budget and the public hospital chooses to treat the welfare maximising number of patients. On the other hand, if the government's cost of funds in the second period is less than the same critical level, then the government's optimal budget is a soft budget and it is exhausted before the end of the budget period. In this latter case, the optimal period one budget might be greater or less than the welfare maximising budget. This result is stated Proposition 2 and is verified in Example 1 where it is also shown that (i) if the government's costs of funds in period 2 is close to zero, then the optimal period 1 budget is less than the welfare maximising budget and the budget is soft and (ii) if the government's costs of funds in period 2 is significantly greater than zero, then the optimal period 1 budget is greater than the welfare maximising budget and the budget appears hard (in the sense that it is not exhausted before the end of the budget period). The intuition seems clear, if the cost to the government of re-optimising in period 2 is large, the government gives the public hospital a budget greater than the welfare maximising budget so that the public hospital chooses not to exhaust its budget in period 1. Interestingly, there is a soft budget constraint problem even if the budget is not exhausted in the budget period.

Example 2 extends the analysis by allowing the government cost of funds in period 2 to be a decreasing function of the length of period 1. This captures the idea that having to "bailout" the public hospital early in the budget period is more costly to the government than "bailing out" the public hospital later in the budget period. Although it is shown that the budget is always soft, the optimal period 1 budget can be greater or less than the welfare maximising budget depending on the marginal cost of treating patients. In particular, if the marginal cost of treating patients is relatively large, then the marginal benefit of increasing the budget in period 1 is relatively low and the optimal period 1 budget is below the welfare maximising budget. Proposition 2 and Examples 1 and 2 make it clear that in the face of a soft budget constraint problem, the relationship between the optimal period 1 budget and the welfare maximising budget depends on the parameters of the model.

By optimally choosing the period 1 budget the government is able to somewhat offset the welfare reducing effect of the soft budget constraint. In the final section of the paper, the role that institutions play in hardening public hospital budget constraints and increasing welfare are explored. In particular, it is shown that the presence of a private hospital that treats elective patients not treated by the public hospital hardens the budget constraint of the public hospital and increases welfare.

2. Basic Model - Hard Budget Constraint

2.1. Patients

In the budget period, there are E patients seeking elective treatment and N patients seeking non-elective treatment at a public hospital. These patients arrive at a constant rate throughout the period. The benefit any non-elective patient receives from treatment is b_n whereas the benefit an elective patients receives from treatment is patient specific and given by b_i $i = 1, ..., E$.

2.2. The Public Hospital

The cost to the public hospital of treating non-elective patients is constant and given by c_n whereas for elective patients this constant cost is c_e. Elective patients are ordered from highest to lowest benefit of treatment to create a benefit function $b_e(e)$, where $b_e'(e) > 0$ and $b_e''(e) < 0$. The public hospital is given a budget B by the government to treat patients and the hospital chooses the number of non-elective patients, n, and elective patients, e, to maximise total patient benefit, subject to the cost of treating patients being no greater than B and the number of non-elective and elective patients being no greater than N and E, respectively. It is assumed that the public hospital knows the benefit a particular patient derives from treatment and so if constrained by its budget to treat less than $N + E$ patients, it treats those patients with the highest benefit. For simplicity assume $c_n = c_e = c$. The public hospital's problem is

$$\max_{n,e} \; H \equiv b_n \cdot n + b_e(e) \tag{1}$$

subject to

$$c \cdot n + c \cdot e \leq B \tag{2}$$

and

$$e \leq E, \quad n \leq N. \tag{3}$$

Let the solution to this problem be given by n^h and e^h.

2.3. The Government

The government chooses the public hospital's budget, B, to maximise total patient benefit. It is instructive to solve for the welfare maximising choice of n and e. The welfare maximising problem is

$$max_{n,e} \; W \equiv b_n \cdot n + b_e(e) - (c \cdot n + c \cdot e) \tag{4}$$

subject to

$$n \leq N, \quad e \leq E. \tag{5}$$

Some additional assumptions are now made to help solve this problem.

Assumption 1: $b_n - c > b_e(0) - c > 0$. This assumption ensures that all non-elective patients are treated as treating them yields more net welfare than treating any elective patient.

Assumption 2: $b_e'(E) - c < 0$. Together with Assumption 1, this ensures an interior solution for e, that is, $e < E$.

Let the solution to the welfare maximising problem be n^* and e^*. This solution is given by

$$n^* = N \tag{6}$$

and

$$b_e'(e^*) - c = 0. \tag{7}$$

This last equation is the familiar condition that welfare is maximised where marginal benefit equals marginal cost.

Let $B^* = c \cdot n^* + c \cdot e^*$. If B^* is given to the public hospital then it will choose $n^h = n^* = N$ because of Assumption 1. It will then spend $B^* - c \cdot N$ (the remaining budget) on treating elective patients and because $b_e(e)$ is an increasing function it will choose $e^h = e^*$. Therefore, under a hard budget constraint the government can achieve the welfare maximising solution by providing the public hospital a budget of B^*.

3. Soft Budget Constraint

In this paper, as in Dewatripont and Maskin (1995), a soft budget constraint arises through a failure of commitment by the funding agency. In particular, if the public hospital budget is spent before the end of the budget period, the government finds it optimal to provide more funding to the public hospital in order for the public hospital to keep treating patients as they arrive. The public hospitals understands this incentive for re-optimisation and behaves differently than it would if the budget was hard. Understanding the incentives faced by the public hospital, the government adjusts its initial budget accordingly.

A soft budget is modeled by assuming that the public hospital can choose to exhaust the budget before the end of the budget period, In particular, it chooses the proportion ϕ of the budget period in which it exhausts the budget. ϕ divides the budget period into two sub-periods. Period 1 is the sub-period in which the budget is exhausted and Period 2 is the period after the budget is exhausted.

3.1. Period Two

With a soft budget constraint, in Period 2, the government re-optimises and provides the public hospital with an additional budget, B_2, in order for the hospital to keep treating patients as they arrive. It is assumed that the cost of funding B_2 is $(1+\gamma) \cdot B_2$, because it can be politically damaging to "bail out"the public hospital.

It is instructive to solve for the government's optimal choice of the number of non-elective patients to treat, n_2, and the number of elective patients to treat, e_2. The government's problem is

$$\max_{n_2, e_2} W_2 \equiv b_n \cdot n_2 + b_e(\frac{e_2}{1-\phi}) \cdot (1-\phi) - (1+\gamma) \cdot (c \cdot n_2 + c \cdot e_2) \tag{8}$$

subject to

$$n_2 \leq (1-\phi) \cdot N, \quad e_2 \leq (1-\phi) \cdot E. \tag{9}$$

Equivalent assumptions to Assumptions 1 and 2 above are made with c scaled up by $(1+\gamma)$. Let the solution to the government's problem be $\hat{n}_2$ and $\hat{e}_2$. This solution satisfies

$$\hat{n}_2 = (1-\phi) \cdot N \tag{10}$$

and

$$b_e'(\frac{\hat{e}_2}{1-\phi}) - (1+\gamma) \cdot c = 0. \tag{11}$$

Let $\hat{e}$ solve $b_e'(\hat{e}) = (1+\gamma)\cdot c$, then $\hat{e}_2 = (1-\phi)\cdot\hat{e}$. As expected, the Period 2 solution involves the government choosing the number of non-elective and electives patients to be the fraction $(1-\phi)$ of what it would have chosen for the whole period, given γ.

Let $\hat{B}_2 = c\cdot\hat{n}_2 + c\cdot\hat{e}_2$, Once again it is clear that if the public hospital is given $\hat{B}_2$ at the start of Period 2, then its optimal choices are given by $n_2^h = \hat{n}_2$ and $e_2^h = \hat{e}_2$. Therefore, under a soft budget constraint, the government can achieve its optimal solution, given γ, by providing the public hospital a budget of $\hat{B}_2$.

It should be noted that if $\gamma = 0$, then $\hat{n}_2 = (1-\phi)\cdot n^*$ and $\hat{e}_2 = (1-\phi)\cdot e^*$.

3.2. Period One

The public hospital is given a budget of B_1 and chooses n_1^h, e_1^h, and ϕ to maximise total patient benefit taking into account the effect its choice of ϕ has on total period 2 patient benefit. Its problem is,

$$\max_{n_1^h, e_1^h, \phi} H_1 \equiv b_n\cdot n_1^h + b_e(\frac{e_1^h}{\phi})\cdot\phi + b_n\cdot N\cdot(1-\phi) + b_e(\hat{e}_2)\cdot(1-\phi) \quad (12)$$

subject to

$$c\cdot n_1^h + c\cdot e_1^h \leq B_1 \quad (13)$$

and

$$n_1^h \leq \phi\cdot N, \quad e_1^h \leq \phi\cdot E. \quad (14)$$

As long as $B_1 < c\cdot N + c\cdot E$ constraint (13) binds as H_1 is increasing in n_1^h and e_1^h. Given (13) binds, $n_1^h = \phi\cdot N$ otherwise n_1^h could be increased and e_1^h decreased so that (13) continues to bind and yet H_1 would be greater. Given these two binding constraints, $e_1^h = \frac{B_1 - c\cdot\phi\cdot N}{c}$. Substituting these equalities into H_1 yields

$$H_1(\phi, B_1) = b_n\cdot N + b_e(\frac{B_1}{c\cdot\phi} - N)\cdot\phi + b_e(\hat{e})\cdot(1-\phi). \quad (15)$$

Differentiating with respect to ϕ yields

$$\frac{\partial H_1}{\partial \phi} = b_e(\frac{B_1}{c \cdot \phi} - N) - b_e(\hat{e}) - b_e'(\frac{B_1}{c \cdot \phi} - N) \cdot \frac{B_1}{c \cdot \phi} \tag{16}$$

In the Appendix it is shown that $\frac{\partial^2 H_1}{\partial \phi^2} < 0$. Let the solution to the public hospital's choice of ϕ be denoted by ϕ_1^h.

Proposition 1 Given $\gamma \geq 0$, if $B_1 = B^*$ and $\gamma \geq \gamma_c$, then $\phi_1^h = 1$. On the other hand, if $B_1 = B^*$ and $\gamma < \gamma_c$, then $\phi_1^h < 1$.

Proof At $(B_1 = B^*, \phi = 1)$, $\frac{\partial H_1}{\partial \phi} = b_e(e^*) - b_e(\hat{e}) - b_e'(e^*) \cdot (e^* + n^*)$. Given $\gamma \geq 0$, then $e^* \geq \hat{e}$. Define e_c by $b_e(e^*) - b_e(e_c) - b_e'(e^*) \cdot (e^* + n^*) = 0$. Let γ_c be the γ that yields $\hat{e} = e_c$. If $\gamma \geq \gamma_c$, then $\hat{e} \leq e_c$ and at $(B_1 = B^*, \phi = 1)$, $\frac{\partial H_1}{\partial \phi} \geq 0$. As a result $\phi_1^h = 1$. On the other hand, if $\gamma < \gamma_c$, then $\hat{e} > e_c$ and at $(B_1 = B^*, \phi = 1)$, $\frac{\partial H_1}{\partial \phi} < 0$. As a result $\phi_1^h < 1$.

Proposition 1 establishes that if the government gives the public hospital the welfare maximising hard budget B^*, then as long as its cost of funds, $1 + \gamma$, is below some critical level, then the public hospital exhausts the budget before the end of the budget period, $\phi_1^h < 1$, and the budget is soft. Essentially the public hospital exhausts the budget by treating more non-electives patients in period 1 than the welfare maximising number of patients, that is $e_1^h = e^* + (n^* - \phi \cdot N)$. It achieves this by using the funds not spent on non-electives patients $c \cdot (n^* - \phi N)$ to treat elective patients. In period 2, the public hospital gets further funding, $\hat{B}_2$, to treat all the remaining non-elective patients, $n_2^h = (1 - \phi) \cdot N$, and additional electives patients as well, though less than the welfare maximising number of electives patients, $e_2^h < (1 - \phi) \cdot e^*$.

Proposition 1 also establishes that as long as the government's cost of funds is greater than some critical level, then the welfare maximising solution is achievable and the budget is hard, $\phi_1^h = 1$. Essentially, the number of electives patients that would be treated in Period 2 if the budget was exhausted before the end of the budget period is so far below the welfare maximising number that the public hospital finds it too costly to exhaust the budget even though it obtains extra funding by doing so.

Corollary 1 Given $\gamma \geq 0$, if $B_1 = B^*$, $\gamma < \gamma_c$, and $b_e(E) - b_e(\hat{e}) - b'_e(E) \cdot (N+E) > 0$, then $\phi_{min} < \phi_1^h < 1$. On the other hand, if $b_e(E) - b_e(\hat{e}) - b'_e(E) \cdot (N+E) \leq 0$, then $\phi_{min} = \phi_1^h < 1$.

Proof It was shown above that $e_1^h = \frac{B_1 - c \cdot \phi \cdot N}{c}$. Given B_1, the public hospital's choice of ϕ must satisfy, $e_1^h \leq \phi_1^h \cdot E$. Rearranging this inequality yields $\phi_1^h \geq \frac{B_1}{c \cdot (E+N)} = \phi_{min}$. At ϕ_{min}, $\frac{\partial H_1}{\partial \phi} = b_e(E) - b_e(\hat{e}) - b'_e(E) \cdot (N+E)$. If at ϕ_{min}, $\frac{\partial H_1}{\partial \phi} > 0$, then $\phi_{min} < \phi_1^h$, while if $\frac{\partial H_1}{\partial \phi} \leq 0$, then $\phi_{min} = \phi_1^h$.

Given budget B_1, the total number of patients that arrive in the budget period, E and N, and the fact that total patient benefit is increasing in the number of patients treated, the pubic hospital finds it optimal to spend all the budget. This means it will never choose a $\phi_1^h < \phi_{min}$ because some of its budget would go unspent and ϕ and the number of patients treated could be increased. Corollary 1 states the conditions under which the public hospital chooses the corner solution $\phi_1^h = \phi_{min}$.

Proposition 1 and Corollary 1 established conditions under which $B_1 = B^*$ would be exhausted by the public hospital before the end of the budget period. The question to be addressed now is what is the optimal budget from the point of view of the government. The solution for when the public hospital exhausts its budget is a function of the size of the budget, that is, $\phi_{min} \leq \phi_1^h(B_1) \leq 1$. Substituting this into the government's objective function yields

$$W_1(B_1) = b_n \cdot N + b_e(\frac{B_1}{c \cdot \phi_1^h(B_1)} - N) \cdot \phi_1^h(B_1) + b_e(\hat{e}) \cdot (1 - \phi_1^h(B_1))$$
$$- B_1 - (1+\gamma) \cdot \hat{B}_2, \quad (17)$$

where $\hat{B}_2 = c \cdot (1 - \phi_1^h(B_1)) \cdot N + c \cdot (1 - \phi_1^h(B_1)) \cdot \hat{e}$. Assuming an interior solution for $\phi_1^h(B_1)$, differentiating with respect to B_1, applying the envelope theorem ($\frac{\partial H_1}{\partial \phi} = 0$), and rearranging yields

$$\frac{dW_1}{dB_1} = \frac{b_e'(\cdot)}{c} - 1 - (1+\gamma) \cdot \frac{d\hat{B}_2}{d\phi_1^h} \cdot \frac{d\phi_1^h}{dB_1} \quad (18)$$

Let the solution to the government's problem be denoted $\hat{B}_1$.

At B^*, $\frac{b_e'(\cdot)}{c} - 1 < 0$ because $(\frac{B^*}{c \cdot \phi_1^h(B_1)} - N) > e^*$ and $b_e(\cdot)$ is strictly concave. In the Appendix it is shown that $\frac{d\phi_1^h}{dB_1} > 0$ while $\frac{d\hat{B}_2}{d\phi_1^h} < 0$, therefore the sign of $\frac{dW_1}{dB_1}(B^*)$ is ambiguous. If this sign is positive, then $\hat{B}_1 > B^*$, while if it is negative $\hat{B}_1 < B^*$. The above is summarised in the following Proposition

Proposition 2 If $\gamma > 0$ and $\gamma \geq \gamma_c$, then the budget is hard ($\phi_1^h = 1$), $\hat{B}_1 = B^*$, and the welfare maximum is achieved. On the other hand, if $\gamma < \gamma_c$, then at $B_1 = B^*$ the budget is soft ($\phi_1^h < 1$), $\hat{B}_1 \geq B^*$ or $\hat{B}_1 < B^*$ depending on the parameters of the model, and the welfare maximum is not achieved.

As discussed above, if gamma is large enough, $\gamma \geq \gamma_c$, it is too costly for the hospital to exhaust its budget before the end of the budget period and the budget is hard. Proposition 2 establishes that if gamma is small enough, $\gamma < \gamma_c$, then the budget is soft and the government responds to this softness by choosing an initial budget that may be greater than or smaller than the welfare maximising budget. This possibility is demonstrated in the following example.

Example 1a:

$b_e(e) = 100e - \frac{1}{2}e^2$, $c = 20$, $b_n = 120$, $E = 100$, $N = 50$, $\gamma = \frac{1}{4}$

First note Assumptions 1 and 2 are satisfied. $b_n = 120 > b_e(0) = 100 > c = 20$ and $b_e'(E) = 0 < c = 20$

Welfare Maximum - Hard Budget: Calculation yields $n^* = 50$, $e^* = 80$, $B^* = 2600$, and $W^* = 8200$.

Soft Budget - Period 2: Calculation yields $n_2^h = \hat{n}_2 = (1-\phi) \cdot 50$, $\hat{e} = 75$, $e_2^h = \hat{e}_2 = (1-\phi) \cdot 75$, $\hat{B}_2 = (1-\phi) \cdot 2500$, and $\hat{W}_2 = (1-\phi) \cdot 7562.5$.

Soft Budget - Period 1: Calculation yields $e_c = 25.17$ and $\gamma_c = 2.74$. Now $\gamma = \frac{1}{4} < \gamma_c$, so with $B_1 = B^*$, $\phi_1^h < 1$. Setting $\frac{\partial H_1}{\partial \phi}(B^*) = 0$, yields $\phi_1^h = .87896 > \phi_{min} = .867$. Calculation yields $W_1(B^*, \phi_1^h = .87896) = 7981$.
At B^*, $\frac{dW_1}{dB_1}(B^*) = .161343 > 0$ so $\hat{B}_1 > B^*$. Therefore, the optimal period 1 budget is greater than the welfare maximising budget.

It turns out that $W_1(B_1)$ is a linear function of B_1. Calculation reveals that the optimal period 1 budget is $\hat{B}_1 = 2958 > B^* = 2600$. This induces $\phi_1^h = 1$. That is, the cost of having $B_2 > 0$ is so large that the period 1 budget is set so that the public hospital does not exhaust the budget before the end of the period. Further calculation yields welfare to be $\hat{W}_1 = 8039.75$ which is greater than welfare at B^*.

Example 1b:

The parameters are the same as in Example 1a except $\gamma = .05$.

At B^*, calculation yields $\phi_1^h = .875287$ and $W_1(B^*, \phi_1^h = .875287) = 8033.7$ Further calculation gives $\frac{dW_1}{dB_1}(B^*) = -.0141546 < 0$ so $\hat{B}_1 < B^*$. Therefore, the optimal period 1 budget is less than the welfare maximising budget.

The optimal period 1 budget is $\hat{B}_1 = 0 < B^* = 2600$ which induces $\phi_1^h = 0$. That is, the cost of having $B_2 > 0$ is so small that the period 1 budget is set so that the public hospital exhausts the budget immediately and $\hat{B}_2 = 2580$. In this case, calculation reveals welfare is $\hat{W}_1 = \hat{W}_2 = 8070.5$ which is greater than welfare at B^*.

The tradeoff seems clear. The greater is γ, the greater is the cost to the government of having the public hospital exhaust the budget before the end of the budget period, that is, the greater is the cost of having $B_2 > 0$. Therefore, the government increases B_1 above B^* so that ϕ_1^H increases. On the other hand, the smaller is γ the smaller is the cost to the government of having $B_2 > 0$ and so it reduces B_1 below B^*. The linearity of the example drives the optimal period 1 budget and ϕ_1^h to corner solutions so that the budget is either so large that it is not exhausted or it is so small that it is immediately exhausted.

Proposition 2 states that the optimal period 1 budget may be greater or less than the welfare maximising budget depending on the parameters of the model. Example 1 verified this result but it was found that the optimal period 1 budget was either so large that it was not exhausted in period 1 (although there is still a soft budget problem) or it was so small it was immediately exhausted. Although the example is instructive its knife-edge result seems too stark. In the next section, the public costs of funds is made endogenous to determine whether the knife-edge result of Example 1 can be overturned.

3.3. Endogenous Public Costs of Funds

In the subsections above and Example 1, the government's cost of funds in period 2 was constant and equal to γ. This cost was greater than where the budget was hard because of the political costs of "bailing out" the public hospital if it exhausts its budget before the end of the budget period. In this section, it is assumed that the government's costs of funds is a function of ϕ, that is, $\gamma(\phi)$, where $\gamma'(\phi) < 0$ and $\gamma(1) = 0$. The rationale for these restrictions is that the political cost of the bailout is greater the longer is the bailout period and as the length of this bailout period approaches zero, the costs of funds approaches 0.

The analysis of Period 2 is identical to that above except γ is replaced with $\gamma(\phi)$. As a result, $\hat{e}$ is now a function of ϕ, that is, $\hat{e}(\gamma(\phi)$ and $\hat{B}_2(\phi) = c \cdot \hat{n}_2 + c \cdot \hat{e}(\gamma(\phi)$.

The public hospital's problem in Period 1 is

$$H_1(\phi, B_1) = b_n \cdot N + b_e(\frac{B_1}{c \cdot \phi} - N) \cdot \phi + b_e(\hat{e}(\gamma(\phi))) \cdot (1 - \phi). \tag{19}$$

Differentiating with respect to ϕ yields

$$\begin{aligned} \frac{\partial H_1}{\partial \phi} &= b_e(\frac{B_1}{c \cdot \phi} - N) - b_e(\hat{e}(\gamma(\phi))) - b'_e(\frac{B_1}{c \cdot \phi} - N) \cdot \frac{B_1}{c \cdot \phi} \\ &+ (1 - \phi) \cdot b'_e(\hat{e}(\gamma(\phi))) \cdot \frac{d\hat{e}}{d\gamma} \cdot \frac{d\gamma}{d\phi}, \end{aligned} \tag{20}$$

where $\frac{d\hat{e}}{d\gamma} < 0$ and $\frac{d\gamma}{d\phi} < 0$. The last term in (20) is positive as an increase in ϕ reduces the government's cost of funds and increases the number of elective patients treated in period 2.

At $(B_1 = B^*, \phi = 1)$, $e^* = \hat{e}$ and so $\frac{\partial H_1}{\partial \phi} = -b'_e(e^*) \cdot (e^* + n^*) < 0$. Therefore $\phi_1^h < 1$ and the budget is soft. This contrasts with Proposition 1 where depending on the size of γ the budget might be hard. However, it is always soft in this case because $\gamma(1) = 0$ and there is no cost to reducing ϕ.

Period 1 government welfare is identical to (17) above except $\gamma(\phi(B_1))$. Similarly, differentiating welfare with respect to B_1 yields an expression identical to (18) except with a additional term. In particular,

$$\frac{dW_1}{dB_1} = \frac{b_e'(\cdot)}{c} - 1 - (1 + \gamma(\phi(B_1))) \cdot \frac{d\hat{B}_2}{d\phi_1^h} \cdot \frac{d\phi_1^h}{dB_1} - \hat{B}_2(\phi(B_1)) \cdot \frac{d\gamma}{d\phi} \cdot \frac{d\phi}{dB_1}. \quad (21)$$

In the Appendix it is shown that $\frac{d\phi}{dB_1} > 0$. Therefore, the last term in (21) is positive, that is, there is an additional benefit to increasing B_1 compared to the subsections above, namely, an increase in B_1 decreases the length of the bailout period and reduces the cost of funds.

In Example 2 below it is shown that the optimal Period 1 soft budget can be greater or less than the welfare maximising hard budget depending on the parameters of the model.

Example 2a:

$b_e(e) = 100e - \frac{1}{2}e^2$, $c = 20$, $b_n = 120$, $E = 100$, $N = 50$,

$\gamma(\phi) = \gamma_0 - \gamma_0 \cdot \phi$, $\gamma_0 = 6$.

First note that $\frac{d\gamma}{d\phi} = -\gamma_0 < 0$ and $\gamma(1) = 0$. So $\gamma(\phi)$ satisfies the conditions given in the text.

The welfare maximum solutions are the same as in Example 1.

Soft Budget - Period 2: Calculation yields $n_2^h = \hat{n}_2 = (1-\phi) \cdot 50$, $\hat{e} = 120 \cdot \phi - 40$, $\phi > \frac{1}{3}$, $e_2^h = \hat{e}_2 = (1-\phi) \cdot \hat{e}$, $\hat{B}_2 = (1-\phi) \cdot 200 \cdot (1 + 12 \cdot \phi)$, and $\hat{W}_2 = (1-\phi) \cdot 200 \cdot (-1 + 6 \cdot \phi + 36 \cdot \phi^2)$, where $\phi > \frac{1}{3}$.

Soft Budget - Period 1: At $B^* = 2600$, calculation reveals that $\phi_1^h = .902429 < 1$ as demonstrated in the text. Further calculation yields $W_1(B^*) = 7969.03$. Setting $\frac{dW_1}{dB_1} = 0$, yields the optimal period one budget, $\hat{B}_1 = 2870.54 > B^*$. With this budget, $\phi_1^h = .972116 < 1$ and $\hat{W}_1 = 8036.71 > W_1(B^*)$ as expected.

It should be noted that with these parameter values $\hat{B}_1 > B^*$, that is, the Period 1 optimal budget is greater than the welfare maximising budget yet the budget is still soft because $\phi_1^h < 1$. In contrast to Example 1 the solution for ϕ_1^h is interior. This is expected given the properties of $\gamma(\phi)$.

Example 2b: The parameters are the same as in Example 2a, except $c = 60$.

Welfare Maximum - Hard Budget: As marginal cost has changed so does the welfare maximum. Calculation yields $n^* = 50$, $e^* = 40$, $B^* = 5400$, and $W^* = 3800$.

Soft Budget - Period 2: Calculation yields $n_2^h = \hat{n}_2 = (1-\phi)\cdot 50$, $\hat{e} = 360\cdot\phi - 320$, $\phi > \frac{8}{9}$, $e_2^h = \hat{e}_2 = (1-\phi)\cdot\hat{e}$, $\hat{B}_2 = 5400\cdot(-3+7\cdot\phi-4\cdot\phi^2)$, and $\hat{W}_2 = (1-\phi)\cdot 200\cdot(1-\phi)\cdot(181-486\phi+324\cdot\phi^2)$, where $\phi \geq \frac{8}{9}$.

Soft Budget - Period 1: At $B^* = 5400$, calculation reveals that $\phi_1^h = .920415 < 1$ as demonstrated in the text. Further calculation yields $W_1(B^*) = 3599.58$. Setting $\frac{dW_1}{dB_1} = 0$, yields the optimal period one budget, $\hat{B}_1 = 5259.75 < B^*$. With this budget, $\phi_1^h = .917518 < 1$ and $\hat{W}_1 = 3601.84 > W_1(B^*)$ as expected.

It should be noted that with these parameter values $\hat{B}_1 < B^*$, that is, the Period 1 optimal budget is less than the welfare maximising budget and the budget is soft because $\phi_1^h < 1$.

Example 2 demonstrates where the cost of government funds in period 2 is decreasing in the length of period 1 and $\gamma(1) = 0$, that the budget is exhausted before the end of the budget and the period 1 budget can be greater than or less than the welfare maximising budget. This makes it clear that the government can respond to the public hospitals incentives to exhaust the budget before he end of the budget period, by either increasing the budget if the political cost of re-optimising are too great or reducing the budget if re-optimising is near costless.

4. A Public and a Private Hospital

It was seen above that in the case where the cost of government funds, γ, was below some critical threshold, the government anticipates its lack of commitment in period 2 and, even though it optimally chooses its period 1 budget, it is not able to achieve the welfare maximum. In this section, the role of a private hospital in hardening the government budget constraint is examined. Intuitively it seems that the presence of a private hospital that treats patients who are left untreated by the public hospital weakens the incentives the government has to bailout the public hospital in period 2 if the public hospital exhausts its budget before the end of the budget period. This intuition is now examined.

4.1. The Private Hospital

A monopoly private hospital treats elective patients who are left untreated by the public hospital. Assume the marginal cost of treatment is constant and equal to c. Given, the number of patients treated by the public hospital in period 2, e_2, and the length of period 2, $(1-\phi)$, the residual demand curve of the private hospital is given by $p(e_{m2}) = b'_e(\frac{e_2+e_{m2}}{1-\phi})$. Let the profit maximising choice for the number of elective patients treated by the private hospital be $e_{m2}(e_2)$, where $e'_{m2}(\cdot) < 0$. That is, the more elective patients that are treated by the public hospital in period 2 the less elective patients are treated by the profit maximising private hospital.

4.2. Period Two

The government's period two problem is given by

$$\max_{n_2,e_2} W_2 \equiv b_n \cdot n_2 + b_e(\frac{e_2 + e_{m2}(e_2)}{1-\phi}) \cdot (1-\phi) - (1+\gamma) \cdot (c \cdot n_2 + c \cdot e_2) - c \cdot e_{m2}(e_2) \tag{22}$$

subject to

$$n_2 \leq (1-\phi) \cdot N, \quad e_2 + e_{m2}(e_2) \leq (1-\phi) \cdot E, \tag{23}$$

which is similar to that is section 3.1 except $b_e(\frac{e_2}{1-\phi})$ is replaced by $b_e(\frac{e_2+e_{m2}(e_2)}{1-\phi})$ as the public and private hospitals now both treat patients and the costs of treating e_{m2} patients in the private hospital must be accounted for. Differentiating (22) with respect to e_2 yields

$$\frac{\partial W_2}{\partial e_2} = b_e'(\frac{e_2 + e_{m2}(e_2)}{1-\phi}) - (1+\gamma) \cdot c + \big(b_e'(\frac{e_2 + e_{m2}(e_2)}{1-\phi}) - c\big) \cdot \frac{de_{m2}}{de_2} \tag{24}$$

Setting (24)equal to zero and solving yields the solution to the government's problem for the number of elective patients to treat. It is assumed that $b_e(\cdot)$ is sufficiently concave that $W_2(\cdot)$ is also concave. This solution is denoted by $\tilde{e}_2$.

At $\hat{e}$ the first two terms in (24) are together negative because $e_{m2}(\hat{e})$ is positive and $b_e(\cdot)$ is concave. In addition, the third term in (24) is also negative because the term in brackets is positive (the monopoly price is greater than marginal cost) while $e_{m2}'(e_2)$ is negative. Therefore, at $\hat{e}$, $\frac{\partial W_2}{\partial e_2} < 0$. Given W_2 is concave, then $\tilde{e}_2 < \hat{e}$. The intuition is clear, from the government's perspective the private hospital treats elective patients at lower cost than the public hospital and so it wants to treat less patients in the public hospital, however, there is a cost to substituting private treatment for public treatment treatment as private treatment involves a monopoly distortion. If the government gives the public hospital a period 2 budget of $\tilde{B}_2 = c \cdot \hat{n}_2 + c \cdot \tilde{e}_2$, then the public hospital chooses $n_2^h = \hat{n}_2$ and $e_2^h = \tilde{e}_2$. Note that $\tilde{B}_2 < \hat{B}_2$, that is, the presence of the private hospital reduces the optimal period 2 budget of the government below what it would have been in the absence of the private hospital.

4.3. Period One

Given B_1, the public hospital's problem for the choice of ϕ is identical to (15) in section 3.2 except $b_e(\hat{e}) \cdot (1-\phi)$ is replaced with $b_e(\tilde{e}) \cdot (1-\phi)$, where $\tilde{e}$ solves $b_e'(e + e_m(e)) - (1+\gamma) \cdot c + (b'(e + e_m(e)) - c) \cdot \frac{de_m}{de} = 0$. Now

$$b_e(\tilde{e}) < b_e(\hat{e}) \tag{25}$$

Therefore, using (16) in section 3.2, at ϕ_1^h, $\frac{\partial H_1}{\partial \phi} > 0$. As a result, the presence of the private hospital leads the public hospital to increase ϕ. In the presence of a private hospital, let the solution to the public hospital's period 1 choice of ϕ be denoted $\tilde{\phi}_1^h$. The argument above is summarised in the following proposition.

Proposition 3 $\tilde{\phi}_1^h \geq \phi_1^h$. That is, the presence of the private hospital hardens the budget constraint of the public hospital in the sense that the public hospital chooses to exhausts the budget later in the budget period.

The intuition seems clear. The presence of the private hospital reduces the size of the bailout in period 2, $\tilde{B}_2 < \hat{B}_2$, as the private hospital treats electives patients left untreated by the public hospital. In turn, as the bailout is smaller, the cost to the public hospital of increasing ϕ is reduced and so it exhausts its budget later in the budget period than in the absence of the private hospital.

5. Conclusion

This paper outlines conditions under which a public hospital faces a soft budget constraint. When these conditions are met, it establishes that the government optimally responds to the soft budget constraint problem by either increasing or decreasing its budget relative to the welfare maximising hard budget. It also shows that the soft budget constraint problem results in "too many" elective patients being treated relative to the welfare maximum. Finally it is shown that the presence of a private hospital hardens the public hospital's budget constraint and increases welfare.

6. References

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7. Appendix

1. Differentiating (16) in the text with respect to ϕ yields

$$\frac{\partial^2 H_1}{\partial \phi^2} = b_e^{''}(\cdot) \cdot \frac{B_1^2}{c^2 \phi^3} < 0 \tag{A-1}$$

by the concavity of $b_e(\cdot)$.

2. Applying the Implicit Function Theorem to (16) in the text yields

$$\frac{d\phi}{dB_1} = \frac{b_e^{''}(\cdot) \frac{B_1}{c^2\phi^2}}{\frac{\partial^2 H_1}{\partial \phi^2}} > 0 \tag{A-2}$$

by the concavity of $b_e(\cdot)$ and the result in 1. above.

3. Differentiating (20) in the text with respect to ϕ yields

$$\begin{aligned} \frac{\partial^2 H_1}{\partial \phi^2} &= b_e^{''}(\cdot) \cdot \frac{B_1^2}{c^2 \phi^3} + \left(-2b_e^{'}(\hat{e}) + (1-\phi) \cdot b_e^{''}(\hat{e}) \frac{d\hat{e}}{d\phi} \right) \cdot \frac{d\hat{e}}{d\phi} \\ &+ (1-\phi) \cdot b_e^{'}(\hat{e}) \cdot \frac{d^2\hat{e}}{d\phi^2} < 0 \end{aligned} \tag{A-3}$$

by the concavity of $b_e(\cdot)$ as long as $\frac{d^2\hat{e}}{d\phi^2}$ is not too positive.

4. Applying the Implicit Function Theorem to (20) in the text yields

$$\frac{d\phi}{dB_1} = \frac{b_e^{''}(\cdot) \frac{B_1}{c^2\phi^2}}{\frac{\partial^2 H_1}{\partial \phi^2}} > 0 \tag{A-4}$$

by the concavity of $b_e(\cdot)$ and the result in 3. above.