Hedonic Price Indexes for High Tech Products

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EMG Workshop '13

Current practices of hedonic methods

- Time-Dummy method (TD):

$$lnp_n^t = \beta(time\_dummy) + \sum_{k=1}^{K} \delta_k f_k \left(z_{k,n}^t\right) + \epsilon_n^t, \quad t = 0, \ldots, T \quad (1)$$

- Hedonic imputation method:

$$lnp_i^0 = \sum_{k=1}^{K} \delta_k^0 z_{k,i}^0 + \epsilon_i^0 \qquad \forall i = 1, \ldots, I \quad (2)$$

$$lnp_v^1 = \sum_{k=1}^{K} \delta_k^1 z_{k,v}^1 + \epsilon_v^1 \qquad \forall v = 1, \ldots, V \quad (3)$$

- Disappearing and new items:

$$\frac{?}{p_d^{(0)}}, \qquad \frac{p_n^{(1)}}{?}$$

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Hedonic imputation price relatives for disappearing and new items

- Single imputation (SI) price relatives:

$$\frac{\widehat{p}_d^{(1)}}{p_d^{(0)}}, \qquad \frac{p_n^{(1)}}{\widehat{p}_n^{(0)}} \tag{4}$$

- Double imputation (DI) price relatives:

$$\frac{\widehat{p}_d^{(1)}}{\widehat{p}_d^{(0)}}, \qquad \frac{\widehat{p}_n^{(1)}}{\widehat{p}_n^{(0)}} \tag{5}$$

- Different authors use different terms, following Triplett (2006); Hill and Melser (2008)
- Imputation methods: more flexible (Diewert, Heravi and Silver 2008); address temporal fixity problem; have advantage in using weights

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Consistency of Imputed Price Relatives

- Hedonic equations (1), (2) and (3) produce unbiased and consistent estimates given the assumptions of classical linear regression model are satisfied: $plim(\widehat{\delta}^0) = \delta^0$ and $plim(\widehat{\delta}^1) = \delta^1$. Let i be a disappearing item. Using *plim* rules,

$$plim\left(\frac{\widehat{p_i^1}}{p_i^0}\right) = plim\left(\frac{\widehat{p_i^1}}{\widehat{p_i^0}}\right) = \frac{exp\left(\theta_i^1\right)}{exp\left(\theta_i^0\right)} = exp(\Theta_i), \tag{6}$$

where, $\theta_i^0 = \sum_{k=1}^{K} \delta_k^0 z_{k,i}^0$ and $\theta_i^1 = \sum_{k=1}^{K} \delta_k^1 z_{k,i}^0$

- Hedonic regression of period 1 (from eq (3)):

$$lnp_v^1 = \sum_{k=1}^{s} \delta_k^1 z_{k,v}^1 + \sum_{k=s+1}^{K} \delta_k^1 z_{k,v}^1 + \epsilon_v^1 \tag{7}$$

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SI Price Relatives with Omissions

- Proposition 1: *If relevant characteristics are omitted, the single imputation method estimates inconsistent price relatives for disappearing items.*

$$plim\left(\frac{\widetilde{p_i^1}}{p_i^0}\right) = exp\left(\Theta_i\right) \times exp\left(\sum_{k=s+1}^{K} \delta_k \left(\sum_{j=1}^{s} \gamma_{j,k}^1 z_{j,i}^0 - z_{k,i}^0\right)\right) \quad (8)$$

- Similarly, SI price relatives for new items are inconsistent.
- In terms of solution to omitted variable bias problem, SI is no better than time-dummy method.
- Under certain conditions, the time-dummy index is the same as the SI index (de Haan 2007).

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- Proposition 2: *Given that* $\gamma^0_{j,k} = \gamma^1_{j,k}$, $\forall j = 1, \ldots, s$ *and* $k = s+1, \ldots, K$ *and that relevant characteristics are omitted, double imputation method estimates consistent price relatives for both new and disappearing items, iff* $\delta^1_k = \delta^0_k$, $\forall k = s+1, \ldots, K$.
- Have implications for data requirement: only a few characteristics drive price changes though a lot more determine price levels.
- What happens if we omit z_k where $\delta^1_k \neq \delta^0_k$ and size $(z^1_k) \neq$ size (z^0_k) (examples: RAM size, processor speed and brand name)
  - Can we still say DI is better than SI and TD?

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- The quality change is brought about by changes in a few characteristics ("unstable" characteristics)
- A new product is a better quality product, includes more of given characteristics and/or adds new characteristics (eg. RAM, hard disk size and processor speed in computers)
- Similarly, a disappearing product is a lower quality product.
- Assumptions hold for high tech products
- Assumptions do not hold for housing market, may not hold for some supermarket products

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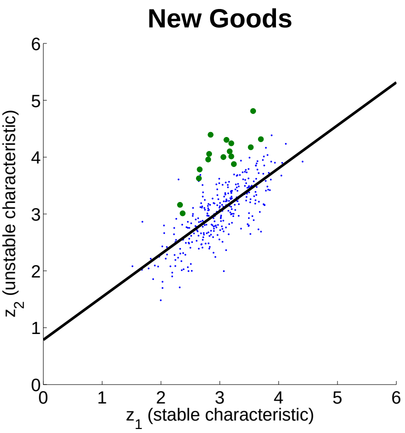

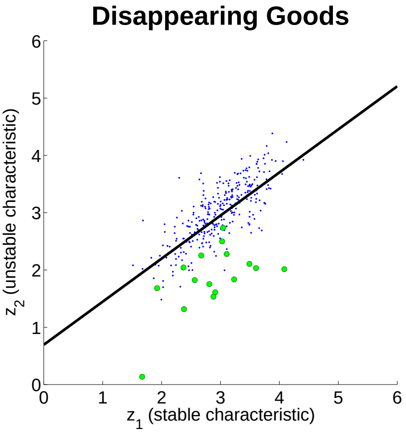

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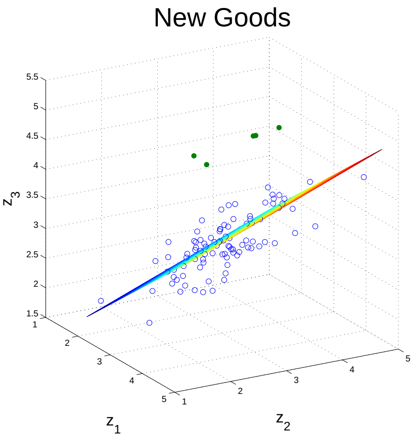

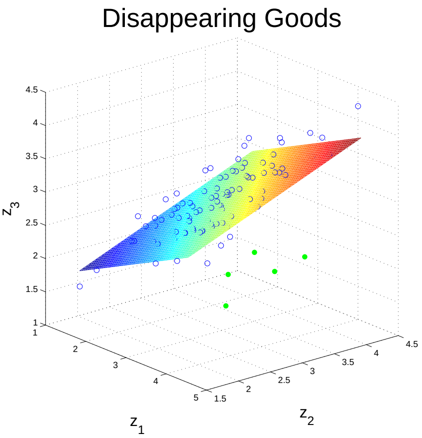

Propositions on the Direction of SI and DI Biases

- SI Bias:
  - Proposition 3: *If* $\gamma^1_{j,k} \geq \gamma^0_{j,k}$ $\forall j = 1, \ldots, s$ *and* $k = s+1, \ldots, K$*, then the single imputation method estimates* **upward biased price relatives for disappearing items***; otherwise, the direction of bias is ambiguous.*
  - Proposition 4: *If* $\gamma^1_{j,k} \geq \gamma^0_{j,k}$ $\forall j = 1, \ldots, s$ *and* $k = s+1, \ldots, K$*, then the single imputation method estimates* **upward biased price relatives for new items***; otherwise, the direction of bias is ambiguous.*
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Propositions on the Relative Magnitude of SI and DI Bias

- Proposition 7: *The double imputation bias is lower than the single imputation bias for disappearing items, irrespective of whether the single or double imputation bias is upward or downward.*

$$Rbias^{dis}_{DI/SI} = \frac{1}{exp\left(\sum_{k=s+1}^{K} \beta_k \left(\sum_{j=1}^{s} \gamma^0_{j,k} z^0_{j,i} - z^0_{k,i}\right)\right)} \quad (9)$$

- Proposition 8: *The double imputation bias is lower than the single imputation bias for new items, irrespective of whether the single or double imputation bias is upward or downward.*

$$Rbias^{new}_{DI/SI} = exp\left(\sum_{k=s+1}^{K} \delta_k \left(\sum_{j=1}^{s} \gamma^1_{j,k} z^1_{j,v} - z^1_{k,v}\right)\right) \quad (10)$$

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Monte Carlo and Price Indexes

- DGP for period 0 and 1:

$$lnp_i^0 = 0.9 + 0.9z_{1,i} + 0.9z_{2,i} + \varepsilon_i^0 \tag{11}$$
$$lnp_i^1 = 0.9 + \kappa_1 z_{1,i} + \kappa_2 z_{2,i} + \varepsilon_i^1 \tag{12}$$

where, ε_i^0, $\varepsilon_i^1 \sim N(0,1)$

- Characteristics z_1 and z_2:

$$\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = N\left[\begin{pmatrix} \pi_1 \\ \pi_2 \end{pmatrix}\begin{pmatrix} \omega_1^2 & \rho_{12} \\ \rho_{12} & \omega_2^2 \end{pmatrix}\right] \tag{13}$$

- Distribution of disappearing, matched and new items in each period: Disappearing: 5%; Matched: 90%; New: 5%
- z_1 and z_2 of the matched items: $[\pi_1^m, \pi_2^m, \omega_1^2, \omega_2^2, \rho_{12}] = [10, 10, 1, 1, 0.5]$

Monte Carlo and Price Indexes

- DGP for period 0 and 1:

$$lnp_i^0 = 0.9 + 0.9z_{1,i} + 0.9z_{2,i} + \varepsilon_i^0 \quad (11)$$

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Estimated Price Relatives and the Bias

- Estimated models:

$$lnp_i^0 = \xi_0 + \xi_1 z_{1,i} + e_i^0 \quad (14)$$
$$lnp_i^1 = \eta_0 + \eta_1 z_{1,i} + e_i^1 \quad (15)$$

- Run 10 simulations with $100 \times 3^{k-1}$ observations for $k = 1, \ldots, 10$
  Repetitions: 1000
- We obtain:
  $plim \left(\frac{p_i^1}{p_i^0}\right), \left(\frac{\widetilde{P}_i^1}{P_i^0}\right), \left(\frac{P_v^1}{\widetilde{P}_v^0}\right), \left(\frac{\widetilde{P}_i^1}{\widetilde{P}_i^0}\right), \left(\frac{\widetilde{P}_v^1}{\widetilde{P}_v^0}\right)$
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Simulation Results for DI and SI Price Relatives when the Stable Characteristic is Omitted

- Cost Savings/Technological Progress in z_1: $[\kappa_1, \kappa_2] = [\mathbf{0.7}, 0.9]$; $[\pi_1^d, \pi_1^m, \pi_1^n] = [10, 10, 10]$; $[\pi_2^d, \pi_2^m, \pi_2^n] = [10, 10, 10]$

Method	Sample Size	Disappearing Item Mean†	Disappearing Item Variance†	New Item Mean	New Item Variance
Double	100	1.0075	0.0072	1.0033	0.0080
Imputation	300	1.0020	0.0022	1.0028	0.0022
($\pi_1^d = 10$,	900	1.0013	0.0007	1.0016	0.0007
$\pi_1^n = 10$,	2700	1.0000	0.0002	1.0009	0.0002
$\kappa_1 = 0.7$,	8100	0.9994	0.0001	0.9997	0.0001
$\kappa_2 = 0.9$)	24600	1.0000	0.0000	1.0000	0.0000
	72900	0.9999	0.0000	1.0000	0.0000
	218700	1.0000	0.0000	1.0000	0.0000
Single	100	1.4154	1.6524	1.3830	1.6318
Imputation	300	1.3717	1.8239	1.4233	1.6473
($\pi_1^d = 10$,	900	1.3647	1.5223	1.3660	1.6058
$\pi_1^n = 10$,	2700	1.3515	1.3921	1.3403	1.3495
$\kappa_1 = 0.7$,	8100	1.4069	2.0009	1.3475	1.5770
$\kappa_2 = 0.9$)	24600	1.2687	1.0336	1.3539	1.5380
	72900	1.3571	1.4917	1.3650	1.6874
	218700	1.2687	1.1229	1.3167	1.4186

† Mean and variance of 1000 repetitions.

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Simulation Results for DI and SI Price Relatives when the Unstable Characteristic is Omitted

- Cost Savings/Technological Progress in z_2: $[\kappa_1, \kappa_2] = [0.9, \mathbf{0.7}]$; $[\pi_1^d, \pi_1^m, \pi_1^n] = [10, 10, 10]$; $[\pi_2^d, \pi_2^m, \pi_2^n] = [\mathbf{8, 10, 12}]$

Methods	Sample Sizes	Disappearing Items Mean†	Disappearing Items Variance†	New Items Mean	New Items Variance
Double Imputation ($\pi_1^d = 9$, $\pi_1^n = 11$, $\kappa_1 = 0.9$, $\kappa_2 = 0.7$)	100	0.9916	0.0326	1.4651	0.0932
	300	0.9819	0.0300	1.4583	0.0714
	900	0.9773	0.0314	1.4517	0.0670
	2700	0.9743	0.0312	1.4617	0.0619
	8100	0.9810	0.0317	1.4577	0.0636
	24600	0.9718	0.0283	1.4474	0.0661
	72900	0.9689	0.0274	1.4509	0.0655
	218700	0.9777	0.0278	1.4557	0.0694
Single Imputation ($\pi_1^d = 9$, $\pi_1^n = 11$, $\kappa_1 = 0.9$, $\kappa_2 = 0.7$)	100	2.5936	3.3350	3.6530	10.4488
	300	2.5967	3.2510	3.5674	9.1812
	900	2.6037	2.8310	3.5780	9.9699
	2700	2.6374	3.0504	3.6322	8.7721
	8100	2.5990	3.7894	3.6399	9.1956
	24600	2.6112	2.9683	3.5979	11.5583
	72900	2.6437	3.1467	3.6264	12.6707
	218700	2.5398	2.7338	3.7365	14.5494

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Simulation Results for DI and SI Price Relatives when the Unstable Characteristic is Omitted

- Cost Savings/Technological Progress in z_2: $[\kappa_1, \kappa_2] = [0.9, \mathbf{0.7}]$; $[\pi_1^d, \pi_1^m, \pi_1^n] = [10, 10, 10]$; $[\pi_2^d, \pi_2^m, \pi_2^n] = [\mathbf{8, 10, 12}]$

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Conclusion

Are new items on average better than the existing items and disappeared items worse than the surviving items?

Apple releases faster, more powerful iPhone

Apple announced the release of the iPhone 5, a taller, thinner and lighter model powered by the A6 chip that can operate on 4G LTE. Other new features include redesigned headphones, a smaller connector and mapping software.

CATEGORY	iPhone 5	iPhone 4S	iPhone 4	iPhone 3GS	iPhone 3	iPhone
Resolution	1136 x 640; 326ppi	960 x 640; 326 ppi	960 x 640; 326 ppi	480 x 320; 163 ppi	480 x 320; 163 ppi	480 x 320; 163 ppi
Camera size (megapixels)	8	8	5	3	2	2
Video calling	FaceTime	FaceTime	FaceTime	none	none	none
Video recording	HD 1080p	HD 1080p	HD 720p	VGA	none	none
Battery life (talk time in hours)	Up to 8 on 3G	Up to 8 on 3G	Up to 7 on 3G	Up to 5 on 3G	Up to 5 on 3G	Up to 8 on 2G
Wireless carrier	AT&T, Verizon, Sprint, others	AT&T, Verizon, Sprint	AT&T, Verizon	AT&T	AT&T	Cingular (now part of AT&T)
Date released	Sept. 2012	Oct. 2011	June 2010	June 2009	July 2008	June 2007

SOURCE: Apple Inc. AP

If the answer is 'yes', then the omitted variable bias is lower in the DI price indexes compared to the time-dummy and SI indexes.