

Purchasing Power Parities, Price Levels and Measures of Regional and Global Inflation

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Abstract

The International Comparison Program (ICP) has nearly completed the 2011 phase covering 180 countries and results are due to be released in December of 2013. Purchasing power parities of currencies and real incomes are the headline figures from the ICP that are of considerable interest to economists, international organizations, international businesses and policy makers at the national level. Publication of the results for 2011 brings to the fore the problem of reconciling the new results with the ICP results published in 2005. The main objective of the paper is to provide a method of linking the 2005 and 2011 benchmark comparisons using Gini-Eltetö-Kovács-Szulc (GEKS) method and fixity that is consistent with the basic ICP methodology. Based on this approach, the paper proposes a new measure for regional and global inflation. Analytical properties of the new method are established and empirical estimates of regional and global inflation are presented using the 2005 ICP results for the Asia-Pacific and the 2009 update of the ICP for the Asia-Pacific. Measures proposed in the paper can be applied to results from the global ICP as well as results from the regions including the OECD/EUROSTAT regions.

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INTRODUCTION

In a globalised world there is a growing need for reliable and timely data on internationally comparable national income aggregates such as GDP and its components: Household Consumption; Government Consumption Expenditure and Gross Fixed Capital Formation. The International Comparison Program (ICP) is a global statistical initiative designed to provide timely and reliable measures of purchasing power parities of currencies, price levels and real GDP. The ICP under the auspices of the UN Statistical Commission has been responsible the compilation of PPPs and real incomes since 1970. Started as a small project covering 10 countries, the ICP has grown to a truly global project in 2005 covering 146 countries. The ICP is bringing to conclusion the latest round of the ICP with 2011 as the benchmark year where an increased coverage to 182 countries has been achieved. The World Bank has assumed responsibility for the more recent ICP rounds starting from the 1993 round and the Global Office for the ICP is located in the World Bank. The 2005 and 2011 rounds of the ICP are regionalized comparisons. At the regional level comparisons are conducted by the regional coordinating agencies such as the Asian and African Development Banks. Since 1990, OECD and Eurostat have been conducting international comparison exercises at more regular and frequent intervals. Currently the OECD comparisons are conducted every three years whereas the Eurostat comparisons are made every year.

Despite the enormous popularity enjoyed by the ICP largely due to the purchasing power parity data it generates, there is very little appreciation among researchers and users about the conceptual framework that underpins the ICP. In addition most of the ICP related publications tend to be in the form of reports and consulting reports and papers prepared for the meetings of various expert and advisory groups. A major consequence is the limited dissemination of the vast range of concepts, terminology and methods to the researchers, statisticians working in national statistical offices and users from the wider community. A few exceptions are the recent publications of Balk (2008), Rao (2009) and World Bank (2013). The basic framework for ICP, the methodology employed in the compilation of PPPs and real GDP aggregates, and various applications of the ICP are presented in the recently published *ICP Book* (World Bank, 2013).

With the imminent release of the results for the 2011 benchmark year, expected early in 2014, the World Bank and the regional coordinating agencies such as the ADB, AfDB, ECLAC and OECD-Eurostat are presented with the problem of reconciling the new results for 2011 with results from the 2005 ICP benchmarks. This is an area of major research focus for the ICP.

The main objective of our paper is to contribute to a better understanding and appreciation of the results from the benchmarks and offer a way of linking results from two consecutive benchmarks so that international comparisons across countries and over time can be made. The focus of the paper is on

purchasing power parities and *price level indices* which are easily the two important sets of results from the ICP. The paper draws on the recent work of Rao and Balk (2013) on the concepts and terminology used in the ICP and the work of Rao and Rambaldi (2013) on compiling PPPs for comparisons over space and time which are designed to maintain fixity of benchmark results on PPPs and price levels. Using the main result in Rao and Rambaldi (2013), we develop measures of regional and global inflation and illustrate how these measures are linked with the price level measures commonly used in international comparisons.

The paper is organized as follows. In Section 2 we provide an informal statement of the problem which is designed to motivate the reader to the material in the ensuing sections of the paper. This section presents results from the Asia-Pacific to motivate our work. Section 3 establishes the notation used in the paper. Section 4 describes PPPs and the concept and measures of price level indices. In particular focus of the section is on the measurement of price level indices with regional or global average equal to 100. Section 5 briefly describes the GEKS method for deriving of a set of transitive PPPs for comparisons over time and space which maintains spatial fixity. This section illustrates how results, PPPs and price level indices, from two different benchmarks can be combined leading to consistent spatial-temporal comparisons. In Section 6, we present measures of regional and global inflation based on the results reported in Section 5. The new measures are computed using the 2005 and 2009 comparison results from the Asia-Pacific region.

2. The Problem

We illustrate the problem of spatial-temporal price comparisons using results from the Asia-Pacific region. The 2005 ICP Asia-Pacific region included 23 economies from the region including China and India. Subsequent to the release of the 2005 results, the Asian Development Bank which is the coordinating agency for the region conducted a research study to compile updated results for the benchmark year 2009 using reduced information methods. Thus, we have results for two benchmark years 2009 for the same set of 23 economies. The following table shows the results for a selected set of countries from the region. Results for the full set of countries are shown in the Appendix.

Table 1: PPPs and Real Incomes for Selected Asia-Pacific Countries, 2005 and 2009
Reference Currency – Hong Kong dollar

Country	2005				2009			
	PPP	XR	PLI*	Real per capita GDP	PPP	XR	PLI*	Real per capita GDP
Bangladesh	3.98	8.27	86	7,215	4.16	8.91	76	10,888
China, PR	0.61	1.05	103	23,938	0.64	0.88	117	40,706
Hong Kong	1	1	180	202,941	1	1	162	231,611
India	2.58	5.67	82	12,090	2.70	6.25	70	19,500
Malaysia	0.31	0.49	112	65,217	0.29	0.45	104	82,524
Thailand	2.80	5.17	97	39,070	2.47	4.42	91	52,753
Vietnam	829	2039.12	73	12,185	950.68	2201.95	70	20,389

Note: (*) PLIs are defined relative to regional average with Asia = 100.

Source: ADB (2007) and ADB (2012)

The first column shows the PPPs of currencies of selected countries with Hong Kong dollar as the reference currency. For example in 2005, the results show that 2.58 Indian rupees have the same purchasing power as one HK dollar with respect to the goods and services in GDP. However, in 2009 the PPP for Indian rupee is 2.70. Can we conclude that prices in India are higher in 2009? A commonly used tool to assess price levels in different countries is to use the *price level index* (PLI) of the country (the ratio of PPP to the exchange rate). It is a common practice to express PLI relative to a regional average which is set at 100. In 2005, PLI in Hong Kong is 80 percent higher than the regional average whereas it is only 62% above the regional average in 2009. Can these two PLI's be compared?

The results reported in Table 1 facilitate comparison across countries in a given year. The relative levels of living reflected by real per capita GDP in 2005 and 2009 are not comparable. Essentially the results in Table 1 are cross-sectional comparisons at a given point in time and cannot be compared across time periods. How can we obtain PPPs that can be used on making comparisons across time and space? What is the PPP for India in 2009 with HK dollar in 2005 as the reference currency? In measuring PLI for a given country in a given year, the regional average is set to 100. Note that the regional average is set to 100 in both 2005 and 2009. However, price levels in the region are not the same in 2005 and 2009. How can we adjust regional average price levels to reflect the overall price change observed in the region. The main objective of the paper is to consider these issues and offer methods that can be used in making comparisons across time and space.

3. Notation and definitions

We consider the problem of making price comparisons across M countries (or regions) based on price and quantity data on N commodities. Let $j = 1, 2, \dots, M$ with ($M \geq 3$) denote countries and $n = 1, 2, \dots, N$ denote commodities. The price vector for country j , expressed in its own currency, is denoted by $p^j \equiv (p_1^j, p_2^j, \dots, p_N^j) \in \mathfrak{R}_{++}^N$. In contrast, the corresponding vector of quantities is denoted by $x^j = (x_1^j, x_2^j, \dots, x_N^j) \in \mathfrak{R}^N$ ($j = 1, 2, \dots, M$). We assume that the price and quantity vectors refer to a particular period of time: without loss of generality this is assumed to be a particular year.¹ This means that our main purpose to make price and quantity comparisons across countries at a given point of time.

Gross domestic product - Value aggregate

The value aggregate for country j is²:

$$V_j = p^j \cdot x^j = \sum_{n=1}^N p_n^j \cdot x_n^j \text{ for } j = 1, 2, \dots, M \quad (1)$$

The value aggregate in (1) is expressed in national currency units and, therefore, not comparable across countries. Given that the quantity vector is allowed to have negative values, we assume that

$$p^k \cdot x^j = \sum_{n=1}^N p_n^k x_n^j > 0 \text{ for all } k, j = 1, 2, \dots, M$$

This assumption implies that the value aggregate obtained when the quantity vector of x^j of country j is evaluated at prices p^k of country k is always positive³. We note here $p^k \cdot x^j$ is a value aggregate expressed in the currency units of country j .

In general, the value aggregate in (1) may refer to total output or total input or in some cases value added. As the principal aggregate used in the ICP is gross domestic product (GDP) we use GDP as the aggregate in our exposition.⁴

¹ At this stage we do not include a time subscript. We will introduce it when we discuss comparisons over time and space.

² From here on, the dot denotes inner product between two vectors.

³ This assumption ensures that the standard measures like the Laspeyres and Paasche indices are strictly positive.

⁴ Though we use GDP as our aggregate, the results and interpretation hold for sub-aggregates like private consumption, government consumption and gross fixed capital formation as well.

Exchange rates⁵

Exchange rates are generally used in converting V_j into a common currency unit. Let XR_j denote the exchange rate of currency of country j with respect to a reference currency (which is, unless stated otherwise, the US dollar); thus XR_j is the amount of j currency per 1 US\$. We assume that exchange rates are transitive between countries.⁶ This means that XR_k/XR_j is the amount of k currency per unit of j currency.

GDP in national currency units

We let GDP_j represent the GDP of country j expressed in its own currency unit.

Nominal GDP

In the international comparison literature, the term *nominal GDP* represents GDP after conversion into a common currency unit using exchange rates. Thus nominal GDP is defined as

$$NGDP_j = \frac{V_j}{XR_j} = \frac{p^j \cdot x^j}{XR_j} = \frac{GDP_j}{XR_j}. \quad (2)$$

Nominal GDP is expressed in the units of the reference currency and therefore additive across countries.

Here the term *nominal* indicates that these value aggregates are not adjusted for price level differences across countries even though they are converted into a common currency.

4. Concepts for international comparisons

Purchasing Power Parities

The purchasing power parity (PPP) of the currency of a country j represents the number of currency units of currency j required to purchase a given basket of goods and services that can be purchased with one unit of currency of a reference country. If the PPP of Indian rupee is 2.50 with respect to the Hong Kong dollar, it means that what can be purchased with one dollar in HK requires 2.50 rupees to purchase the same goods and services within India.

⁵ Exchange rates refer to the market exchange rates.

⁶ Thus we assume that there is no arbitrage in currency conversion.

The PPPs are similar to the price index numbers in the time series context. However, PPPs cannot be directly interpreted as spatial price index number except in the case where all the countries, or the countries for comparison, have the same currency unit. Each PPP measures the number of currency units of a certain country that have the same purchasing power of as one currency unit of the reference country. A few points can be made here:

- PPP is made up of two components viz. the price level component and the currency unit component.
- If the currency units of two countries j and k are the same, then PPP_{jk} can be interpreted as a spatial price index between the two countries. For example, PPP_{jk} between two countries from the Euro-zone can be interpreted as a price index.
- If PPP_{jk} 's are computed using price data that is converted into a common currency unit using exchange rates thus adjusting for differences in currency units, the resulting PPP_{jk} 's can be interpreted as spatial price index numbers.

This conceptual difference between PPPs and spatial price index numbers has led the practitioners to the concept of price level indexes.

Real value aggregates or Real GDP

Since PPPs provide amounts of currencies that have purchasing power equivalent to one unit of currency of reference currency, we can use PPPs to convert the GDP of each country to an aggregate that can be compared across countries. This aggregate is referred to as the *real GDP* in ICP terms. Its definition is⁷

$$\text{Real GDP of country } j = RGDP_j = \frac{GDP_j}{PPP_{1j}} \quad (6)$$

As PPPs reflect the purchasing power of currencies reflecting the prices prevailing in different countries, real GDP is GDP adjusted for price level differences across countries. Therefore it is possible to talk about relative GDP levels between countries and also to compare real per capita GDP across countries and use it as a measure of standard of living.

We note here that the PPPs used in (6) are derived using price data collected from countries participating in the ICP along with National Accounts weights for aggregating those data.⁸ The fact that these PPPs

⁷ Strictly speaking we need to include a superscript to indicate that country 1 is the reference country.

⁸ For details of the process of price collection and subsequent aggregation see Rao (2013a)'s paper on the ICP framework.

refer to a particular year⁹ in which price data are collected implies that real GDP according to (6) also refers to a particular year and therefore is not comparable over time. We return to this in section 4.

Price Level Indexes (PLIs)

The price level index for country j relative to a certain reference country, say country 1, is defined as the ratio of PPP to the exchange rate. It is given by¹⁰

$$PLI_{1j} = \frac{PPP_{1j}}{XR_{1j}} \quad (7)$$

PLI is seen as a measure of price level relative to the level at which the currency can be converted at the exchange rate. Consider the case of Australian dollar versus the US dollar. The current exchange rate between is 0.97 AUD per 1 USD. Suppose a BigMac costs 2.25 USD and 2.75 AUD in these countries respectively. Then the PPP for AUD with USD as the reference currency is $2.75/2.25=1.22$. The PLI based on the price BigMac alone is then $1.22/0.97=1.26$.

We note the following properties of the PLIs:

- By definition, the PLI of the reference country is always equal to 1.
- When the PLI of a country is high, it is difficult to know whether prices in the reference country are low or prices in the country under consideration are high.
- The PLIs for a certain country based on two different reference countries are not simply comparable.
- PLIs are transitive. This follows from the transitivity of PPPs and XRs.

The following table shows PLI's for a selected set of countries from the global comparisons reported in World Bank (2008, p.23).

Table 2: Price Level Indexes for Selected Countries, 2005

(Price level in USA = 1.0)

Country	PPP	XR	PLI with USA = 1	PLI with world average. =1
Australia	1.39	1.31	1.06	1.32
Denmark	8.52	5.95	1.42	1.76

⁹ This is usually referred to as the benchmark year in ICP.

¹⁰ We deviate from the standard convention where the reference country is not shown in the price level index formula. For example ADB (2007) defines it in Appendix C simply as $PLI_{1j} = PPP_{1j}/XR_{1j}$ suppressing the reference country.

India	14.67	44.10	0.33	0.41
Brazil	1.36	2.43	0.56	0.69
South Africa	3.87	6.36	0.61	0.76
USA	1	1	1	1.24

Source: World Bank (2008)

Price Level Indexes relative to World Average =1

A standard practice to address the interpretation problem associated with PLIs¹¹ in the table above is to express them relative to the world average. The idea is simply to compute the world average PLI and then measure PLI's relative to this world average. Essentially the process will result in PPPs for each country that are defined against a basket of world currencies such that the nominal world GDP at exchange rates equals the real world GDP which is the sum total of real GDP for each country in the comparison. The method can be described as below¹²:

Start with PLI of country j with reference country 1:

$$PLI_{1j} = \frac{PPP_j}{XR_j} \text{ for } j=1,2,\dots,M$$

Define weighted average of PLI's from all the M countries in the international comparison. Given that the PLI's are based on country 1 as the reference country, we denote the weighted average as μ_1 . We have

$$\text{World average of } PLIs = \mu_1 = \sum_{j=1}^M PLI_{1j} \frac{RGDP_{1j}}{\sum_{j=1}^M RGDP_{1j}} \quad (8)$$

where the weights reflect the relative sizes of real GDP in different countries. It is useful to find the following equivalent expression. Substituting the definition of PLI in (7) and the expression of real GDP, we have

$$\text{World average of } PLIs = \mu_1 = \sum_{j=1}^M \frac{PPP_{1j} GDP_j / PPP_{1j}}{XR_{1j} \sum_{j=1}^M RGDP_{1j}} = \frac{\sum_{j=1}^M GDP_j / XR_{1j}}{\sum_{j=1}^M GDP_j / PPP_{1j}} \quad (9)$$

Here μ_1 can be seen as an adjustment factor for PPP's such that we transform them from PPPs with country 1 as the reference to country to PPPs with world average basket, $PPP_{Wj} = PPP_{1j} / \mu_1$. Then we have the world average PLI equal to 1 by construction and the world real GDP using the new PPPs equal to world nominal GDP expressed in the currency of country 1. The following remarks are useful:

1. PPPs at the world basket: $PPP_{Wj} = PPP_{1j} / \mu_1$;
2. PLI for country j against world average = $PLI_{Wj} = PPP_{Wj} / XR_{1j}$;
3. World average PLIs at new PPPs =

¹¹ For example, give a high PLI for a certain country relative to the reference country it is difficult to know whether the prices in comparison country are high or the prices are low in the reference country.

¹² Material in this subsection draws from a short note prepared by the first author for the Asian Development Bank which appeared as Appendix C in the 2005 ICP Asia-Pacific Final Report published by ADB (2007).

$$\sum_{j=1}^M \frac{PPP_{Wj} GDP_j / PPP_{Wj}}{XR_{1j} \sum_{j=1}^M RGDP_{Wj}} = \sum_{j=1}^M \frac{(PPP_{1j} / \mu_1) GDP_j / (PPP_{1j} / \mu_1)}{XR_{1j} \sum_{j=1}^M GDP_j / (PPP_{1j} / \mu_1)} = \frac{(PPP_{1j} / \mu_1) GDP_j / PPP_{1j}}{XR_{1j} \sum_{j=1}^M GDP_j / PPP_{1j}} = 1$$

4. The world real GDP at the new PPPs is equal to world nominal GDP at exchange rates with country 1 as the reference,

$$\sum_{j=1}^M \frac{GDP_j}{PPP_{Wj}} = \sum_{j=1}^M GDP_j / XR_{1j}$$

5. The PLIs defined using the world average are invariant to the choice of the reference currency. This means that if we were to repeat the whole process using country 2 as the reference and define PLIs using (10) we will get the same price levels as using country 1 as the reference country (due to transitivity of PPPs and XRs).
6. We note that PPP_{Wj} is difficult to explain to practitioners as this PPP does not correspond to the currency of any particular country. So our recommendation is that PPPs are published using one of the comparison countries as reference but use PLIs according to the world average.
7. See Table 1 for PLIs with USA and World averages equal to 1 respectively. In the last column it can be seen that prices in USA are 24% above world average. All the PLIs are adjusted accordingly.

5. A Constrained GEKS Approach to Consistent Panels of PPPs and Real Incomes at Constant Prices

This section draws material from Rao and Rambaldi (2013). In this section we pursue a totally different strategy and present a new approach and method of compiling PPPs with time-space dimensions. As we introduce time dimension to international comparisons, we need to introduce additional notation. In this section we assume that PPP matrices (for comparisons at current prices) are available for each of the periods. In this case we assume that a procedure similar to Rao et al (2010) is already implemented and thus a panel of PPP_{it} $i=1, \dots, M$ and $t=1, \dots, T$ is available to start the proposed procedure.

In view of the space-time nature of the approach, we introduce further notation to what has been introduced in Section 2. Let the time periods be indexed by $t = 1, 2, \dots, T$ and countries be indexed by $j = 1, 2, \dots, M$. Let PPP_{jk}^s denote the PPP for country k in period s expressed relative to the reference country j and reference period t . Let Π represent a (TM x TM) matrix of PPPs over space and time. Then we can write Π as

$$\Pi = \begin{bmatrix} \Pi^{11} & \Pi^{12} & \dots & \Pi^{1T} \\ \Pi^{21} & \Pi^{22} & \dots & \Pi^{2T} \\ \dots & \dots & \dots & \dots \\ \Pi^{T1} & \Pi^{T2} & \dots & \Pi^{TT} \end{bmatrix} \quad (11)$$

where Π^{ts} represents a (M x M) matrix showing PPPs for countries period s with countries in period t used as reference countries.

5.1 Elements of block-diagonal matrices

The matrix in equation (11) involves two types of information. The first refers to the block-diagonal matrices, Π^t for $t = 1, 2, \dots, T$. For example if t is the year 2005, then this matrix provides PPPs for all pairs of countries in the benchmark year. Thus PPPs in this block diagonal matrices represent the PPPs as defined in Section 4. These are often referred to as PPP in current prices as they are not referred to a base year. We assume that the block diagonal matrices satisfy transitivity property. Transitivity of Π^t implies the existence of a vector of constants, say $\pi^t = [\pi_1^t, \pi_2^t, \dots, \pi_M^t]$ such that

$$PPP_{jk}^t = \frac{\pi_k^t}{\pi_j^t} \quad (12)$$

The source of information for these block diagonal matrices is the ICP for different benchmark years and studies like Rao et al (2010) or the PWT which provide extrapolations of PPPs from the benchmark years to non-benchmark years. Without loss of generality we can assume that all the Π^t matrices satisfy transitivity and, therefore, can be expressed in a form similar to (12).

5.2 Elements of off-diagonal matrices

Elements of the off-diagonal matrices are not directly observed nor are available from any of the standard extrapolation studies. These matrices refer to PPP for a country k in given period s relative to a reference country j in the reference period t . We propose the following procedure to fill these elements. Let us for example consider PPP_{jk}^{12} . This is PPP for country k in period 2 relative to country j in period 1. We can derive this comparison either using a comparison between j and k in period 1 or in period 2. We can update the period 1 comparison, PPP_{jk}^{11} using the implicit deflator d_k^{12} which represents movements in prices of country k from period 1 to 2. In this case, we have

$$PPP_{jk}^{12} = PPP_{jk}^{11} \cdot d_k^{12} \quad (13)$$

Alternatively, we could start with comparisons in period 2 and adjust PPP_{jk}^{22} backwards using d_j^{21} representing the implicit price deflator in country j measuring change from period 2 to period 1. This in turn gives an alternative to (13) in the form:

$$PPP_{jk}^{12} = PPP_{jk}^{22} / d_j^{21} \quad (14)$$

As both of these are equally satisfactory, we make use of the geometric mean of (13) and (14) to measure PPP_{jk}^{12} .

$$PPP_{jk}^{12} = \left[\left(PPP_{jk}^{11} \cdot d_k^{12} \right) \left(PPP_{jk}^{22} / d_j^{21} \right) \right]^{1/2} \quad (15)$$

Substituting (12) into (15), we can express the general element in the off-diagonal matrices in logarithmic form as:

$$\ln PPP_{jk}^{ts} = \frac{1}{2} \left[(\ln \pi_k^s - \ln \pi_j^s) + (\ln \pi_k^t - \ln \pi_j^t) + (\ln d_k^{ts} - \ln d_j^{st}) \right] \quad (16)$$

Using the form in (15) we can fill all the off-diagonal blocks thus completing the matrix Π . However, Π is not transitive. To solve this we propose to use the standard GEKS approach with a slight modification.

5.3 GEKS Methodology and transitivity

The GEKS methodology involves the minimisation of sum of squared logarithmic differences between observed PPPs and the PPPs solved out of the system. Let Π^* be the solution of the GEKS method. Then the typical elements of Π^* , PPP_{jk}^{*ts} are obtained by minimising

$$\sum_{t=1}^T \sum_{s=1}^T \sum_{j=1}^M \sum_{k=1}^M \left[\ln PPP_{jk}^{ts} - \ln PPP_{jk}^{*ts} \right]^2 \quad (17)$$

Subjecting to the transitivity of PPP_{jk}^{*ts} . In implementing GEKS we reparametrise the objective function by noting that the matrix Π^* is transitive if and only if there exists a vector π^* of order $(TM \times 1)$ with a typical element, π_k^{*s} associated country k and period s such that

$$\Pi_{jk}^{*ts} = \frac{\pi_k^{*s}}{\pi_j^{*t}} \text{ for all } j, k \text{ and } t, s \quad (18)$$

Substituting (18) into (17) yields the GEKS objective function in terms of the new parameters:

$$\sum_{t=1}^T \sum_{s=1}^T \sum_{j=1}^M \sum_{k=1}^M \left[\ln PPP_{jk}^{ts} - \ln \pi_k^{*s} + \ln \pi_j^{*t} \right]^2 \quad (19)$$

Minimisation of (19) yields the standard EKS solution to the problem. In the process we get a transitive matrix of PPPs which are time-space consistent. In this paper we improve this process further by improving additional restrictions on the solutions to ensure consistency of the time-space PPPs from (19) and the observed PPPs for each of the time periods, a form of *fixity*.

5.4 GEKS with fixity condition

The main problem with a straightforward minimisation of (19) is that comparisons between countries at a given period of time obtained from GEKS will not be equal to the PPP's matrix in the block diagonal of matrix Π in equation (11). Suppose we have international price comparisons in the form of PPPs for pairs of countries for a given year, say 2005. These comparisons are essentially price comparisons at the prices observed in 2005. When we minimise (19), the resulting comparisons between countries for the year 2005 will not be the same as those observed for 2005 as the new comparisons are affected by comparisons for all pairs of countries for all periods in the exercise. This basically means that price and real income comparisons for 2005 will differ at current 2005 prices and constant 2005 prices. So we implement a

refined GEKS by imposing the condition that the price (and hence real income) comparisons for a given year are the same at the current and constant prices. This can be achieved by minimising (19)

$$\sum_{t=1}^T \sum_{s=1}^T \sum_{j=1}^M \sum_{k=1}^M \left[\ln PPP_{jk}^{ts} - \ln \pi_k^{*s} + \ln \pi_j^{*t} \right]^2$$

Subject to additional restrictions:

$$\pi_k^{*s} = \delta^{*s} \pi_k^s \text{ for all } k \text{ and } s. \quad (20)$$

If we incorporate restrictions (20) into (19), the GEKS with *fixity* requirement simplifies to one of minimising

$$\sum_{t=1}^T \sum_{s=1}^T \sum_{j=1}^M \sum_{k=1}^M \left[\ln PPP_{jk}^{ts} - \delta^s - \ln \pi_k^s + \delta^t + \ln \pi_j^t \right]^2 \quad (21)$$

with respect to $\{\delta^1, \delta^2, \dots, \delta^T\}$ where $\delta^s = \ln \delta^{*s}$.

We note that if $\{\delta^1, \delta^2, \dots, \delta^T\}$ is a solution to the problem, $\{c\delta^1, c\delta^2, \dots, c\delta^T\}$ for any $c > 0$ is also a solution to the problem. Hence we minimise (21) after imposing an identifying restriction. In the discussion below we impose the restriction $\delta^1 = 0$ which is the same as $\delta^{*1} = 1$. This means that all the comparisons are anchored on the reference period 1. The final solution can then be considered as price and real income comparisons at *constant year 1 prices*.

The first order conditions for optimisation after imposing $\delta^1 = 0$ yield the following system of (T-1) linear equations:

$$\begin{bmatrix} 1 & -\frac{1}{T-1} & \dots & -\frac{1}{T-1} \\ -\frac{1}{T-1} & 1 & \dots & -\frac{1}{T-1} \\ \dots & \dots & \dots & \dots \\ -\frac{1}{T-1} & -\frac{1}{T-1} & \dots & 1 \end{bmatrix} \begin{bmatrix} \delta^2 \\ \delta^3 \\ \dots \\ \delta^T \end{bmatrix} = \begin{bmatrix} \frac{1}{(T-1)M(M-1)} \sum_{s=1(\neq 2)}^T \sum_{j=1}^M \sum_{k=1}^M \left[\ln PPP_{jk}^{2s} - \ln \pi_k^s + \ln \pi_j^2 \right] \\ \dots \\ \dots \\ \frac{1}{(T-1)M(M-1)} \sum_{s=1(\neq T)}^T \sum_{j=1}^M \sum_{k=1}^M \left[\ln PPP_{jk}^{Ts} - \ln \pi_k^s + \ln \pi_j^T \right] \end{bmatrix}$$

This leads to the following solution for the unknown constants.

$$\begin{bmatrix} \delta^2 \\ \delta^3 \\ \dots \\ \delta^T \end{bmatrix} = \left(\frac{T-1}{T} \right) \begin{bmatrix} 2 & 1 & \dots & 1 \\ 1 & 2 & \dots & 1 \\ \dots & \dots & \dots & \dots \\ 1 & 1 & \dots & 2 \end{bmatrix} \begin{bmatrix} \frac{1}{(T-1)M(M-1)} \sum_{s=1(\neq 2)}^T \sum_{j=1}^M \sum_{k=1}^M \left[\ln PPP_{jk}^{2s} - \ln \pi_k^s + \ln \pi_j^2 \right] \\ \dots \\ \dots \\ \frac{1}{(T-1)M(M-1)} \sum_{s=1(\neq T)}^T \sum_{j=1}^M \sum_{k=1}^M \left[\ln PPP_{jk}^{Ts} - \ln \pi_k^s + \ln \pi_j^T \right] \end{bmatrix} \quad (22)$$

Now we can derive expressions for each of the elements of $\{\delta^2, \dots, \delta^T\}$. We note further that typical elements involved in the summation on the RHS of equation (22) involve terms like:

$$\sum_{s=1(\neq 2)}^T \sum_{j=1}^M \sum_{k=1}^M \left[\ln PPP_{jk}^{2s} - \ln \pi_k^s + \ln \pi_j^s \right] \quad (23)$$

which can be further simplified by noting the procedure used in filling the elements of the off-diagonal blocks of matrices as described in equation (17). We have from (17)

$$\ln PPP_{jk}^{ts} = \frac{1}{2} \left[(\ln \pi_k^s - \ln \pi_j^s) + (\ln \pi_k^t - \ln \pi_j^t) + (\ln d_k^{ts} + \ln d_j^{st}) \right]$$

Inserting this expression in (23) allows us to derive simple closed form solutions for the elements of the vector $\{\delta^2, \dots, \delta^T\}$. After tedious but fairly straightforward algebraic manipulations, Rambaldi and Rao (2013) show that the solution has the following simple expression for δ^s ($s = 2, \dots, T$),

$$\delta^s = \frac{1}{M} \sum_{j=1}^M \left[\frac{1}{T} \right] \sum_{l=1}^T (\ln d_j^{lj} + \ln d_j^{ls})$$

with $\delta^1 = 0$.

However our main interest is in $\delta^{*s} = \exp(\delta^s)$. Thus we have

$$\delta^{*1} = 1$$

$$\delta^{*s} = \exp(\delta^s) = \prod_{j=1}^M \left\{ \prod_{l=1}^T [d_j^{lj} \cdot d_j^{ls}]^{1/T} \right\}^{1/M} \quad (24)$$

The expression in (24) has a fairly intuitive explanation. First, it can be seen as a geometric average of measures of price change from period 1 to s observed in each of the M countries. The price change from period 1 to s is the GEKS measure derived from price indices for different pairs of time periods.

The GEKS formula makes use of measures of price changes from all pairs of time periods. However, in practice most national statistical offices produce either year-on-year chained price index numbers constructed using the Fisher index or produce fixed base price index numbers with frequent revisions in time. Here we assume that the price deflators available from countries are the chained index numbers constructed using a formula that satisfies the time reversal test.

5.6 Two Periods Case with $T=2$

As the main focus of the paper is on the comparison of two benchmark ICP comparisons and the subsequent measurement of global and regional inflation over the two benchmark years, we shift to the special case of two periods. Thus, we have $T=2$. In this case the general form given in (24) simplifies to the following:

$$\delta^{*1}=1 \quad \text{and} \quad \delta^{*2} = \prod_{j=1}^M [d_j^{12}]^{1/M} \quad (25).$$

This means that all the PPPs in period 2 are adjusted using δ^{*2} .

Weighted versus unweighted adjustment factors

Equation (25) is an unweighted average of movements in prices from period 1 to 2 as observed in the M countries involved in international comparisons. As the countries in the comparison vary in the size of their respective economies, we consider a weighted average of the price movements with weights reflected by relative sizes of these economies as measured by the real GDP. In period 1, these shares denoted by s_j^1 ($j=1,2,\dots,M$) are given by:

$$s_j^1 = \frac{RGDP_j^1}{\sum_{k=1}^M RGDP_k^1} = \frac{GDP_j^1 / PPP_j^1}{\sum_{k=1}^M [GDP_k^1 / PPP_k^1]}.$$

Similarly, we can compute real GDP shares in period 2. As there is no objective way to choose between these two sets of shares, we follow an approach similar to that used in a Tornqvist index and use an arithmetic average of these two shares. The resulting weighted adjustment factor is given by:

$$\delta_2^* = \prod_{j=1}^M [d_j^{12}]^{\frac{s_j^1 + s_j^2}{2}} \quad (26)$$

Is the adjustment factor in (26) a reasonable measure? Are there alternatives? In Section 6 we explore this question.

Geometric versus arithmetic averages of national deflators

The solution from the application of GEKS with fixity leads to a geometric average of the national deflators. However, GDP totals for regions and the global economy are often sought. Since these totals are in additive format, it may be useful to consider arithmetic averages of national deflators which is given by:

$$\delta_2^* = \sum_{j=1}^M \left(\frac{s_j^1 + s_j^2}{2} \right) d_j^{12} = \sum_{j=1}^M \left(\frac{\frac{RGDP_j^1}{\sum_{k=1}^M RGDP_k^1} + \frac{RGDP_j^2}{\sum_{k=1}^M RGDP_k^2}}{2} \right) d_j^{12} \quad (27)$$

The expression in (27) is weighted average of the national deflators from period 1 to 2 in countries $j=1,2,\dots,M$. Weights here are the arithmetic averages of shares of real GDP of country j in the region or the world in periods 1 and 2.

In this next section we explore the notion of global inflation and the role of expressions (26) and (27) in measuring global inflation.¹³

6. Measurement of Global Inflation

Global inflation is a term that is used in popular press and also by various international organizations. Eurostat documents refer to inflation in the EU region and similar references are made in various OECD documents. However, it is difficult to find formal definitions of global inflation even though it is commonly measured as a weighted average of inflation (usually GDP deflator) levels observed in the member countries of these organizations with weights reflecting the relative sizes of these economies. The notion of global inflation is also used by researchers. For example, Cicarelli and Mojon (2008) uses weighted and unweighted averages of national deflators as a measure of global inflation which is in turn used in explaining movements in national inflation rates. Ward (2001) is a possible exception where the conceptual issues concerning global inflation and its relationship with international price levels and purchasing power parities are discussed.

In this section we explore systematically the notion of global inflation and its connection with price levels and PPPs discussed in sections 3, 4 and 5. We begin with the notion of inflation at the national level and extend it discuss global inflation.

At the national level, let GDP_j^s and GDP_j^t represents GDP in country j in periods s and t . Even though these aggregates are expressed in the currency unit of j , they cannot be directly compared in assessing the relative levels of GDP in periods s and t . In the national accounts, GDP is expressed at constant prices after deflating the GDP using the GDP deflator. Let $CGDP_j^{st}$ represent the GDP in period t expressed in period s prices. This is given by:

$$CGDP_j^{st} = \frac{GDP_j^t}{d_j^{st}} \quad (28)$$

where d_j^{st} represents the GDP deflator for period t with period s as the base period thus converting GDP into constant prices of year s .

Now let us turn to the global level. First we need measures of global GDP in a given period. How do we measure global GDP. Intuitively, global GDP should be the sum of GDPs of all the countries in the world. Given that GDP_j represents GDP in country j , global GDP need to sum GDP_j over all the countries $j=1,2,\dots,M$. However, two problems arise in the process. First is that GDP_j is expressed in the currency units of country j . Second, even if GDP_j is expressed in the same currency unit for all the countries (as is the case in the Euro zone countries where the currency unit is euro), it is still not permissible to sum GDP_j over all the countries as price levels are different in different countries. Following section 3, we

¹³ The discussion and the methods described apply for regional or any grouping of countries. Without loss of generality we focus only on global inflation in the ensuing sections.

can address these two issues using purchasing power parities to convert GDP_j across all countries.

Recall that real GDP in country j is denoted by $RGDP_j$ which is given by equation (6) repeated here for convenience:

$$RGDP_j = \frac{GDP_j}{PPP_j} \quad (29)$$

All the GDP_j 's are converted into the common reference currency (used in PPP_j) and are also adjusted for price level differences, it is now possible to sum $RGDP_j$ across all the countries. Let $RGDP$ denote the global GDP which is defined as:

$$RGDP = \sum_{j=1}^M RGDP_j = \sum_{j=1}^M GDP_j / PPP_j \quad (30)$$

Equation (30) gives a measure of the size of the world economy and is often used as the world GDP.

PPP_j , $RGDP_j$ and regional and global totals of real GDP are compiled in the ICP and published by the World Bank on a regular basis. Results of the 2005 ICP covering 146 countries were published by the World Bank (2008).

Now we raise the question of making comparisons of global GDP in two different time periods. Let $RGDP^s$ and $RGDP^t$ represent global GDP in periods s and t . Global GDP in period s is given by:

$$RGDP^s = \sum_{j=1}^M RGDP_j^s = \sum_{j=1}^M GDP_j^s / PPP_j^s \quad (31)$$

$RGDP^t$ can be defined analogously.

World GDP at period t relative to period s is

$$\frac{\sum_{j=1}^M GDP_j^t / PPP_j^t}{\sum_{j=1}^M GDP_j^s / PPP_j^s} = \exp \left\{ \sum_{j=1}^M \Psi^j \ln \left(\frac{GDP_j^t / PPP_j^t}{GDP_j^s / PPP_j^s} \right) \right\} \quad (32)$$

$$= \exp \left\{ \sum_{j=1}^M \Psi^j \ln \left(\frac{RGDP_j^t}{RGDP_j^s} \right) \right\} \quad (33)$$

where the weights are defined by $\Psi^j \equiv \frac{L \left(\frac{GDP_j^t / PPP_j^t}{\sum_{j=1}^M GDP_j^t / PPP_j^t}, \frac{GDP_j^s / PPP_j^s}{\sum_{j=1}^M GDP_j^s / PPP_j^s} \right)}{\sum_{j=1}^M L(\dots)}$ ($j = 1, \dots, M$), and $L(\cdot, \cdot)$ is the logmean.

We can write each country's GDP ratio as the product of a price index (deflator) and a quantity index

$$\frac{GDP_j^t}{GDP_j^s} = P_{GDP}^j(t, s) Q_{GDP}^j(t, s) \quad (j = 1, \dots, M). \quad (34)$$

World inflation is then defined by

$$WI(t, s) \equiv \exp \left\{ \sum_{j=1}^M \Psi^j \ln \left(P_{GDP}^j(t, s) \frac{PPP_j^s}{PPP_j^t} \right) \right\} \quad (35)$$

and world growth is defined by

$$WG(t, s) \equiv \exp \left\{ \sum_{j=1}^M \Psi^j \ln Q_{GDP}^j(t, s) \right\}. \quad (36)$$

When the period t PPPs are obtained by extrapolating the period s PPPs, that is, when $PPP_j^t = PPP_j^s P_{GDP}^j(t, s) / P_{GDP}^{j'}(t, s)$ where j' is the numeraire for the PPPs, then the world inflation expression reduces to

$$WI(t, s) \equiv \exp \left\{ \sum_{j=1}^M \Psi^j \ln P_{GDP}^{j'}(t, s) \right\} = P_{GDP}^{j'}(t, s), \quad (37)$$

since the weights add up to 1, $\sum_{j=1}^M \Psi^j = 1$.

Now there are M choices for the numeraire j' , so it makes sense to define mean world inflation as the unweighted geometric mean

$$\bar{WI}(t, s) \equiv \prod_{j=1}^M P_{GDP}^{j'}(t, s)^{1/M}, \quad (38)$$

7. Empirical Illustration using Asia-Pacific data

In this empirical illustration we make use of data on 22 participating economies in the Asia-Pacific ICP in the benchmark year 2005 and results for the same 22 countries for the 2009 from a Research Study reported in ADB (2012). Results for the 2005 benchmark are available from ADB (2007). We present in Table below the basic measures: PPP, PLI and Real GDP for the 22 economies.

Table 3: PPP, PLI and Real GDP for 22 Asia-Pacific Countries – 2005, 2009

Economy/country	2005				2009			
	Purchasing Power Parity (PPP)	Exchange Rate (LCU/HK\$)	Price Level Index (Asia = 100)	Real GDP (million)	Purchasing Power Parity (PPP)	Exchange Rate (LCU/HK\$)	Price Level Index (Asia = 100)	Real GDP (million)
Bangladesh	3.98	8.27	86	988332	4.16	8.91	76	1570115
Bhutan	2.77	5.67	88	13340	2.53	6.25	66	24213
Brunei Darussalam	0.21	0.21	133	99916	0.14	0.19	121	111994
Cambodia	225	526.21	77	114297	229.74	534.11	70	188422
China, People's Republic	0.61	1.05	103	30334238	0.64	0.88	117	54331621
Fiji	0.25	0.22	288	20167	0.26	0.25	169	21133
Hong Kong, China	1	1	180	1382675	1	1	162	1622203
India	2.58	5.67	82	13315076	2.7	6.25	70	22735794
Indonesia	692	1247.82	100	4026228	730.83	1340.63	89	7667814
Lao PDR	525	1370.03	69	58230	473.18	1098.84	70	100517
Malaysia	0.31	0.49	112	1703958	0.29	0.45	104	2336258
Maldives	1.43	1.65	156	6711	1.42	1.65	139	13297
Mongolia	73.4	154.97	85	38306	94.5	185.52	83	69510
Nepal	3.98	9.18	78	155766	4.71	10.01	76	228090
Pakistan	3.36	7.65	79	2098218	4.15	10.54	64	3319126
Philippines	3.82	7.08	97	1421731	3.58	6.15	95	2240383
Singapore	0.19	0.21	159	1024330	0.18	0.19	155	1498403
Sri Lanka	6.18	12.92	86	389389	7.99	14.83	87	604041
Taipei, China	3.4	4.14	148	3358809	2.98	4.26	113	4190945
Thailand	2.8	5.17	97	2530303	2.47	4.42	91	3663144
Vietnam	829	2039.12	73	1012850	950.68	2201.95	70	1753993
			RGDP(2005)	64092870			RGDP(2009)	108291016

In this illustration, we use periods 2005 and 2009 which are respectively s and t in our exposition in Section 6. We use 2005 as the benchmark or reference year. For example, the real GDP in Bangladesh is 988.332 billion HK\$ in 2005 and the corresponding figure in 2009 is 1,570.115 billion HK\$ in 2009. How can we compare these two figures? As our analysis is restricted to these 22 countries, ie., $M=22$, our global totals refer to these 22 economies. In 2005, the global GDP (RGDP) is 64.092 trillion HK\$ and the corresponding figure in 2009 is 108.291 trillion HK\$. As these figures refer to 2005 and 2009, these are not strictly comparable. In order to make these comparisons we need a measure of global inflation shown in equation 35. In the Table below, we implement equation 35.

Table 4: Computation of Global (Asia-Pacific) Inflation

country	Deflators			
	2005 to 2009	2005	2009	Psi
Bangladesh	1.301482	0.015420311	0.014499	0.01497
Bhutan	1.204442	0.000208135	0.000224	0.000216
Brunei Darussalam	0.977021	0.001558925	0.001034	0.00128
Cambodia	1.282412	0.001783303	0.00174	0.001763
China	1.196693	0.473285687	0.501719	0.487855
Fiji	1.100538	0.000314653	0.000195	0.00025
Hong Kong SAR, China	1.035104	0.021572992	0.01498	0.018095
India	1.297166	0.207746603	0.209951	0.209057
Indonesia	1.623795	0.062818657	0.070807	0.066801
Lao PDR	1.257989	0.000908525	0.000928	0.000919
Malaysia	1.131726	0.026585765	0.021574	0.024017
Maldives	1.398922	0.000104707	0.000123	0.000114
Mongolia	1.684003	0.000597664	0.000642	0.00062
Nepal	1.414327	0.002430317	0.002106	0.002267
Pakistan	1.65756	0.032737152	0.03065	0.031714
Philippines	1.195867	0.022182358	0.020689	0.021448
Singapore	1.100559	0.015981965	0.013837	0.014899
Sri Lanka	1.562833	0.006075387	0.005578	0.005829
Taipei	0.961215	0.052405346	0.038701	0.045253
Thailand	1.153598	0.039478697	0.033827	0.036617
Vietnam	1.503539	0.01580285	0.016197	0.016015

Source: ADB (2012), 2009 Purchasing Power Parity Update for Selected Economies in Asia and the Pacific- A Research Study; ADB (2007), Purchasing Power Parities and Real Expenditures - 2005 ICP in Asia and the Pacific; World Development Indicators, 2013; Key Indicators, 2012.

The deflators data are drawn from the World Development Indicators, 2013. However, WDI does not provide data for Taipei. Taipei data is drawn from the ADB's Key Indicators, 2012.

The global inflation measure computed from Table 3 PPPs for 2005 and 2009, column 1 on deflators ($P_{GDP}^j(2009,2005)$), the computed ψ^j (last column Table 4) and formula in (35). The resulting measure of global inflation is

$$\text{Global (Asia-Pacific) inflation from 2005 to 2009} = 120.25$$

This means that an increase in general price level of 20.25 percent was observed in the region.

A measure of Global (Asia-Pacific) growth from 2005 to 2009 (equation 36) = 1.405 or 40.50%

8. Conclusions

In this paper we bring together several strands of research to provide a measure of global inflation. The work of Rao and Balk (2013) provides the basic framework and the inter-relationships between PPPs, price levels and real GDP with the main focus on the normalization used in measuring price levels relative to the regional or global average equal to 100. The main focus here is the cross-sectional comparisons of prices and PPPs across countries at a given period of time. The second strand of research refers to the computation of PPPs that can be used in temporal-spatial comparisons. Following Rao and Rambaldi (2013) who proposed the use of GEKS with fixity to derive a matrix of PPPs over countries and time periods. Their work provides a framework for comparing PPP in a given country at a given point of time with respect to a reference country in a reference year. Following Rao and Rambaldi (2013), the solution essentially involves the computation of an adjustment factor for each year and the adjustment factor is essentially a geometric average of price movements in all the countries. In this paper, we show that the adjustment factors from Rao and Rambaldi (2013) provide a measure of global inflation and also adjusted price levels anchored on global average price level in a reference year equal to 100. The results in the paper are illustrated using the data drawn from the Asia-Pacific ICP comparisons in 2005 (ADB, 2007) and a subsequent research study (ADB, 2012) which provides results for 2009. These data are used in providing a measure of regional inflation which is 1.20 or a 20.25 percent increase in prices in the region.

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