Disclosure Quality, Diversification and the Cost of Capital

Greg Clinch
University of Melbourne
clinchg@unimelb.edu.au

June 2013

I thank Cynthia Cai, Kevin Li, and Sorabh Tomar for helpful comments and suggestions on an earlier (incomplete) draft of this paper.
1. **Introduction**

In this paper I discuss several aspects of the link between disclosure quality and cost of capital, with a particular focus on how diversification influences this link.\(^1\) The extent to which uncertainty inherent in disclosure about a firm (sometimes described as ‘information risk’) is reflected in a firm’s cost of capital and whether it might be diversifiable is, understandably, a question of interest to accounting researchers. It is also a question I do not aim to provide a definitive answer to. Rather, my approach is to employ a simple and highly stylized model to illustrate several related results from some relatively recent analytical research regarding disclosure, cost of capital, and diversification. My objective is not to provide new results – though the model does, I think, yield one or two minor new observations – but rather to clarify the role of disclosure quality for a firm’s cost of capital in a ‘large’ economy.

Although my primary focus is on the role of diversification for disclosure quality and the cost of capital, I first consider (in section 2) a simple single firm setting. My purposes here are twofold. First, the single firm setting provides a potentially more accessible entrée into the details of the model that can be obscured in a more complex multi-firm setting. Second, I use the single firm setting to explore the impact of disclosure quality on cost of capital in a setting where both fundamental risk and price risk play important roles. Short, finite-period models generally employed in the related literature are imperfect vehicles within which to capture price risk since, by construction, the final payoff involves no price risk. This means that an investor who holds shares for the entire period spanned by the model will face no price risk, potentially influencing model conclusions regarding disclosure quality and cost of capital. For example, Christensen, De la Rosa and Feltham (2010) employ such a finite period model and show that in their model *ex-ante* cost of capital – that is, cost of capital relating to the entire finite model period but assessed prior to that period – is unaffected by the quality of future disclosure quality, and only reflects underlying cash flow (fundamental).

---

\(^1\) My paper and the paper by Terry Shevlin (Shevlin (2013)) share a similar topical interest, though his paper focuses largely on empirical research and a broader range of issues, while I concentrate on narrower and analytical-related questions. His section 5 provides a good overview and discussion of the analytical literature related to my paper (and beyond), so I do not replicate this here. Also, I restrict my attention to a setting where there is no information asymmetry among investors (in contrast to, for example, Easley and O’Hara (2004) and Lambert, Leuz and Verrecchia (2012)) and no ‘real effects’ of disclosure, *i.e.*, a pure exchange economy (in contrast to, for example, the last section in Lambert, Leuz, and Verrecchia (2007), Gao (2010), and Cai (2013)). For recent comprehensive surveys of analytical research regarding a broad range of disclosure-related research issues see Beyer, Cohen, Lys and Walther (2010) and Berger (2011), as well as Verrecchia (2001) regarding slightly earlier research.
risk. In contrast, the single firm setting I employ here involves an infinitely-lived firm but with finitely-lived investors. Thus investors are unable to avoid price risk. In this setting, I show that the result of Christensen et al (2010) does not hold, and that (ex ante) cost of capital is decreasing in the quality of disclosure.

In the main part of the paper (section 3) I extend the infinite period model to include multiple firms. Thus the multi firm setting slightly extends the existing related research (e.g., Easley and O’Hara (2004), Lambert, Leuz and Verrecchia (2007), and Hughes, Liu and Liu (2007), among others) to incorporate both fundamental and price risk. I employ this setting as the basis for discussing and clarifying several issues raised in the prior research regarding diversification and the link between disclosure and cost of capital. Specifically, I focus on three aspects of a ‘large economy’ that influence how disclosure quality affects cost of capital: (1) the number of firms across which risk is distributed; (2) the number of investors among whom this risk is shared; and (3) the number of information signals (disclosures) available to investors from which to extract information. Consistent with the familiar CAPM, the model illustrates that in a large economy investors price only systematic (or covariance) risk relating to their end of period payoffs. Nevertheless, as pointed out by Lambert, Leuz and Verrecchia (2007), disclosure can affect investors’ assessment of this risk and, as a result, the quality of disclosure can ‘matter’ for cost of capital. This occurs even though the quality of disclosure is due only to idiosyncratic, firm-specific, noise in the disclosure – the idiosyncratic ‘information risk’ does not diversify away. However, the model also indicates that if there are many disclosures in a large economy (for example, there is a disclosure for each firm) then the quality of those disclosures no longer affects cost of capital. Because there are many disclosures available to investors that assist in assessing a firm’s end of period payoff, the quality of each individual signal becomes immaterial to risk assessment - the large number of available signals swamps the influence of disclosure quality.

Finally (in section 4) I extend the model in a more speculative direction to incorporate some investors who adopt ‘non-rational’ trading rules. Specifically, these investors follow a simple heuristic (or rule of thumb) trading rule tied to each firm’s disclosure without reference to current price – if the disclosure has increased (decreased) compared with the prior period they buy (sell) shares. I show in this setting that the existence of such heuristic traders results in an additional disclosure-contingent factor in equilibrium price that causes price to deviate from rational investors’ expected payoffs. That is, ‘cost of capital’ in this setting comprises two factors: one related to covariance or systematic risk, and the other
related to a ‘heuristic trader’ effect. Moreover, the heuristic trader effect does not disappear in a large economy (where the number of firms and traders - both rational and heuristic - are large), that is, it does not diversify away. This is despite the heuristic trades being tied to firm-specific idiosyncratic disclosures.

In summary, the model I employ, though highly stylized and simplified, does highlight some useful insights into the links between disclosure quality, diversification, and the cost of capital.

2. The single firm setting

2.1 The model

The model I employ in this section is based on a very simple infinite period, single firm setting. The firm generates a payoff, \( x_t \), which follows a random walk, that is:

\[
x_t = x_{t-1} + \varepsilon_t,
\]

where \( \varepsilon_t \) is normally distributed with mean zero, variance \( \sigma^2 \), and independent over time.

There is also a risk-free security which pays \( 1 + r \) each period. Thus \( r \) represents the risk-free rate of return. In each period \( t \) there are \( n \) identical and price taking investors each with negative exponential utility with risk aversion parameter \( \rho \). Investors are assumed to exist for only a single period, at the end of which they liquidate their positions and consume the proceeds. Their utility functions are defined over these proceeds which comprise the firm’s payoff \( x_t \) plus the price, \( P_t \), at which they sell their shares. Finally, supply of the firm’s security is normalized to one (and constant over time).

Given this structure, it is straightforward to derive the following expression for price:\(^2\)

\[
P_t = \frac{1}{1+r} \left( E_t(P_{t+1} + x_{t+1}) - \frac{\rho}{n} \text{Var}_t(P_{t+1} + x_{t+1}) \right),
\]

where \( E_t(P_{t+1} + x_{t+1}) \) and \( \text{Var}_t(P_{t+1} + x_{t+1}) \) denote investors’ expected proceeds and variance of proceeds conditional on whatever information is available to them at time \( t \). Equation (1)

\(^2\) All primary derivations relating to the paper are provided in the appendix. Other details are available on request from the author.
indicates that price is equal to the present value of investors’ end of period expected proceeds less a discount for the assessed risk of, or uncertainty attached to, the proceeds. It is natural to interpret the discount as a cost of capital metric, and I use it as such throughout the paper.

Equation (1) illustrates that the discount depends on investors’ risk aversion (\( \rho \)), the number of investors amongst whom risk is shared (\( n \)), and investors’ assessed risk (\( \text{Var}_t(P_{t+1} + x_{t+1}) \)). The first two factors together reflect the market’s appetite for risk, while the variance term reflects assessed risk. It is only this third factor that might be affected by disclosure.

### 2.2 Disclosure quality and cost of capital

To investigate the impact of disclosure on equation (1) I assume that investors observe a public report each period: \( y_t = x_{t+1} + \varepsilon_t \), where \( \varepsilon_t \) is normally distributed with mean zero, variance \( \sigma_t^2 \), and independent both over time and of \( \varepsilon_t \). Given this information structure, and assuming linear and stable prices over time, it is possible to derive the following equilibrium price expression:

\[
P_t = \frac{1}{r} \left[ x_t + b(y_t - x_t) - \frac{P}{n} s^2 \right]
\]

where \( b = \frac{\sigma_t^2}{\sigma_t^2 + \sigma_e^2} \), and \( s^2 = \text{Var}_t(P_{t+1} + x_{t+1}) = \frac{1}{r^2} \left[ (1+r)^2 - (1+r)^2 - 1 \right] \sigma_e^2 \). Thus price in period \( t \) is equal to a factor, \( 1/r \), multiplied by two terms. The first term, \( x_t + b(y_t - x_t) \), is the expected value of \( x_{t+1} \) given the signal \( y_t \). The multiple times this expectation represents the expected present value of the perpetuity of future payments by the firm and is the price that would prevail in a risk neutral setting. The second term, \( \frac{P}{n} s^2 \), represents a combination of the economy’s appetite for risk (\( \frac{\rho}{n} \)) and investors’ assessed uncertainty/risk (\( s^2 \)). As noted previously, the discount in price captured by this second term represents the firm’s cost of capital due to risk/uncertainty.

Based on (2) it is straightforward to assess the impact of disclosure quality on the firm’s cost of capital in this simple single firm economy. Disclosure quality in the model is represented by \( \sigma_e^2 \), the variance of the error term in the disclosure \( y_t \). Its impact on the
discount in price is via the $b$ coefficient, which is decreasing in $\sigma_\epsilon^2$. Since $s^2$ is also decreasing in $b$, the firm’s cost of capital is unambiguously decreasing in disclosure quality (i.e., increasing in $\sigma_\epsilon^2$).

As noted in the introduction, this contrasts with the result in Christensen, De la Rosa and Feltham (CDF) (2010) where ex ante cost of capital is unaffected by a change in disclosure quality. In CDF, a change in disclosure quality has two effects: (1) it decreases investors’ assessed uncertainty/risk regarding the terminal payment by the firm, and (2) it increases the volatility of price at the forthcoming release of the disclosure. This second effect arises because higher quality disclosure causes investors to change their expectations to a greater extent which flows through to greater magnitude price movements. In CDF these sources of (ex ante) uncertainty exactly offset, resulting in no change in the ex ante cost of capital.

In contrast, in the infinite period setting employed here although the first of these effects is present the second is not. Specifically, an improvement in disclosure quality decreases the investors’ assessed variance of the firm payoff at the end of the period, $x_{t+1}$, which acts to decrease the firm’s cost of capital. However, the assessed variance of end of period price is not affected by disclosure quality: $\text{Var}_i(P_{t+1}) = \frac{1}{r^2} \text{Var}_i [x_{t+1} + b(y_{t+1} - x_{t+1})] = \frac{1}{r^2} \sigma_\epsilon^2$. The reason why is because prices are affected in two offsetting ways by disclosure quality – improved disclosure quality causes end of period prices to be more volatile as investors react to better information, but also improves investors current information about those likely future price movements which reduces investors current assessment of future price volatility. The two effects exactly offset causing price volatility to be unaffected by disclosure quality. As a result, the net effect of disclosure quality is driven solely by its effect on investors’ assessment of the uncertainty regarding $x_{t+1}$, the end of period cash payment by the firm.

To summarize, although the finite period model employed in CDF suggests no role for disclosure quality for ex ante cost of capital, the infinite period model I employ here does admit such a role, with higher disclosure quality leading to lower cost of capital.

\[3\text{ Note that this also implies that the unconditional variance of the change in price also is unaffected by disclosure quality. This can easily be verified using equation (2).}\]
3. The multi firm setting

In this section I extend the single firm, infinite period model to incorporate multiple firms. Specifically there are $m$ firms who each generate a periodic cash flow to investors of $x_i = x_{i-1} + \varepsilon_i$, where each $\varepsilon_i$ is mean zero normally distributed with variance $\sigma^2_i$, and is independent over time but may be correlated across firms with covariance $\sigma_{ij}$. With these assumptions together with the same assumptions for investors as in section 2, it is straightforward to show that the expression for equilibrium price in the multi-firm setting that corresponds to equation (1) (in the single firm setting) is:

$$P_t = \frac{1}{1 + \rho} \left[ E_t \left( P_{t+1} + x_{it+1} \right) - \frac{\rho}{\mu} \text{Cov}_t \left( P_{t+1} + x_{it+1}, P_{Mt+1} + x_{Mt+1} \right) \right],$$

where $P_{M_{t+1}} = \sum_{j=1}^{m} P_{jt+1}$ and $x_{M_{t+1}} = \sum_{j=1}^{m} x_{jt+1}$ represent the aggregate price and cash payment from the market portfolio of all firms in the economy. Again, it is natural to interpret the discount term in equation (3) as representing cost of capital. In this case, and consistent with the familiar CAPM, the relevant risk measure is the covariance term between the proceeds from holding firm $i$ and from holding the market portfolio, $M$:

$$\text{Cov}_t \left( P_{t+1} + x_{it+1}, P_{M_{t+1}} + x_{M_{t+1}} \right),$$

given all available information to investors at time $t$.

Equation (3) is quite general, allowing each firm’s periodic cash payments to have different variances, and for covariances across firms to also differ. I simplify this in order to focus on issues relating to diversification by assuming that variances and covariances are the same across all firms, that is $\sigma^2_i = \sigma^2$ and $\sigma_{ij} = \tau \sigma^2$ for all firms, where $\tau$ represents the correlation between the cash payments of each pair of firms in the economy. Finally, similar to the single firm setting, the link between disclosure and cost of capital occurs via its effect on investors’ assessed covariance between the proceeds from holding a firm’s shares and from holding the market portfolio of all firms, $\text{Cov}_t \left( P_{t+1} + x_{it+1}, P_{M_{t+1}} + x_{M_{t+1}} \right)$. I consider two cases in the following subsections: (1) first where there is only a single disclosure regarding one firm’s likely future cash payment, and (2) where there is a disclosure regarding each firms’ future cash payments. This allows a consideration of how the link between disclosure quality and cost of capital is influenced by three factors: (1) the number of investors among

---

4 Many of the results discussed in this section mirror similar results in Lambert, Leuz and Verrecchia (2007).
whom risk is shared, (2) the number of firms across which the economy’s risk is spread, and (3) the number of disclosures available to investors upon which they base their risk assessments.

3.1 Disclosure by a single firm

In this subsection I assume only a single firm, firm 1, provides a periodic disclosure, \( y_{it} = x_{it+1} + e_{it} \), regarding its end of period cash payment. \( e_{it} \) is normally distributed with mean zero and variance \( \sigma_{e_i}^2 \), and independent over time and of \( e_{it} \) for all \( i \) and \( t \). With this signal available to all investors the equilibrium price expression in (3) becomes for the disclosing firm (firm \( i = 1 \)):

\[
P_t = \frac{1}{r} \left[ x_{it} + b(y_{it} - x_{it}) - \frac{\rho}{n} s_t^2 \right]
\]

where \( b = \frac{\sigma_e^2}{\sigma_e^2 + \sigma_{ei}^2} \) and

\[
s_t^2 = \text{Cov}_i(P_{1t+1} + x_{it+1}, P_{mt+1} + x_{mt+1}) = \frac{1}{r} \left[ (1 + r)^2 - \left( (1 + r)^2 - 1 \right) b \right] \sigma_e^2 \left( 1 + (m - 1)\tau \right).
\]

Equation (4) has the same form as in the single firm setting (equation (2)) but the relevant risk measure, \( s_t^2 \), represents the firm’s assessed covariance with proceeds from holding the market portfolio (systematic risk), rather than the firm’s variance. Moreover, this assessed covariance term contains two primary components. One component, \( \sigma_e^2 \left( 1 + (m - 1)\tau \right) \), is equal to \( \text{Cov}_i(x_{it+1}, x_{mt+1}) \) in the absence of any disclosure and so represents the covariance or systematic risk associated with the firm’s underlying cash payments. The other component, \( \left[ (1 + r)^2 - \left( (1 + r)^2 - 1 \right) b \right] \), reflects the impact of the disclosure on investors’ assessment of risk. As in the single firm setting this component is decreasing in \( b \), which itself is increasing in disclosure quality (i.e., decreasing in \( \sigma_{ei}^2 \)), and so the discount, or cost of capital, is decreasing in disclosure quality. That is, an improvement in disclosure quality causes investors to assess the firm’s systematic risk to be lower and results in a lower cost of capital.

An important question is to what extent this effect would remain in a ‘large’ economy. In a large economy \( n \), the number of investors, and/or \( m \), the number of firms, are
likely to also be large. A larger $n$ allows for more risk sharing among investors, while a larger $m$ allows for greater diversification opportunities to investors but also greater aggregate economy-wide risk to be shared among investors. However, these effects have no impact on the component of risk that reflects the quality of disclosure by firm $1$, \[
\left((1+r)^2 - ((1+r)^2 - 1)b\right).\] Thus the extent to which disclosure quality effects (or information risk) will be ‘diversified away’ in a large economy will simply mirror the extent to which risk in general is diversified away. If risk, in general, is detectable in prices via a non-zero discount, then disclosure quality will also be detectable to the extent there is sufficient variation in disclosure quality.

Moreover, as equation (4) makes clear, it is the effect of an increased $n$ (number of investors) that causes the discount to decrease in a large economy, not the increased number of firms, $m$. This can most easily be seen by considering an economy where all firms are independent, i.e., $\tau = 0$. In this case, $s_i^2$, the assessed covariance risk that is priced for firm $i = 1$, collapses to simply \[
\frac{1}{r^2}\left(1 \left((1+r)^2 - ((1+r)^2 - 1)b\right)\right)\sigma^2_{1},\] irrespective of the number of firms in the economy. Thus despite a large number of independent firms across which investors are able to (and in equilibrium do) diversify, the assessed risk component that is priced does not change. This is because although the greater number of firms offer increased diversification opportunities, there is also greater economy wide risk (due to more firms) that must be absorbed by investors. These two effects offset each other in equilibrium. It is only if there is also a greater number of investors amongst whom to share this risk will the discount in price decrease. Thus, in effect, the ‘diversification effect’ in a large economy might more accurately be described as a risk-sharing effect.

A further implication of the definition of the price discount in equation (4),
\[
\frac{1}{n r^2}\left(1 \left((1+r)^2 - ((1+r)^2 - 1)b\right)\right)\sigma^2_{1}\left(1 + (m-1)\tau\right),\] is that in a large economy where both $n$ and $m$ are large, the underlying systematic risk per investor, \[
\frac{\sigma^2_{1}\left(1 + (m-1)\tau\right)}{n},\] approaches \[
\frac{m}{n}\tau\sigma^2_{1}\]. Thus, as observed in Lambert, Leuz and Verrecchia (2007), as long as $\tau$ is positive

---

5 This (reasonably) assumes that the average covariance between firms’ cash payments, represented by $\tau$, is non-negative.
approaches a finite constant, the discount will be positive and the impact of firm 1’s disclosure quality on cost of capital will not ‘diversify away’. This is despite disclosure quality being solely related to disclosure noise, \( \sigma_{i1}^2 \), that is idiosyncratic to firm \( i = 1 \). That is, the disclosure noise itself has no ‘systematic’ component and yet the degree of this noise (i.e., disclosure quality) impacts the discount in price in a large economy even when any unsystematic component of a firm’s underlying cash payments does not. The reason this occurs is that the disclosure is being used by investors to assess underlying systematic risk that is important to pricing. Whether noise in a firm’s disclosure is idiosyncratic or not does not affect this.

Finally, equation (4) describes equilibrium price for the disclosing firm. It is also possible to show that equilibrium price for non-disclosing firms in the economy is:

\[
P_{jt} = \frac{1}{r} \left[ x_{jt} + \tau b(y_{jt} - x_{jt}) - \frac{b}{n} s_j^2 \right]
\]

where \( j \neq 1, b \) is the same as in equation (4), and

\[
s_j^2 = \frac{1}{r^2} \left[ (1+r)^2 - (1+r)^2 - \tau^2 b \right] \sigma_{jt}^2 \left( 1 + (m-1)\tau \right).
\]

Thus firm 1’s disclosure is used by investors in pricing other firms as long as the firms’ cash payments are correlated, i.e., \( \tau \neq 0 \). In particular, disclosure quality for the disclosing firm is negatively associated with non-disclosing firms’ cost of capital. Moreover, all the implications discussed above regarding disclosure quality and cost of capital continue to hold for the non-disclosing firms.

To summarize the results of this subsection, in a ‘large’ economy where there is a single disclosure regarding one firm the quality of that disclosure will influence cost of
capital as long as underlying (fundamental) risk is not diversified away. This occurs even though disclosure quality relates to noise in the disclosure that is idiosyncratic to the single firm.

3.2 Disclosure by all firms

Following the previous subsection, I continue to assume that the $\epsilon_t$ terms (that is, the periodic innovations in each firm’s cash payment) are identically distributed across firms with covariance equal to $\tau \sigma^2$. However, in this subsection I assume that each of the $m$ firms discloses a signal, $y_{it} = x_{it+1} + \epsilon_t$, where $\epsilon_t$ is normally distributed with mean zero and variance $\sigma^2$. The $\epsilon_t$’s are independent across firms and periods, and also independent of $\epsilon_t$ across firms and periods. In this setting investors will use the disclosures of all firms to assess $E_t(P_{it+1} + x_{it+1})$ and $\text{Cov}_t(P_{it+1} + x_{it+1}, P_{jt+1} + x_{jt+1})$ in forming prices according to equation (3). In particular, it is straightforward to show that in this case the pricing equation (3) becomes:

$$P_{it} = \frac{1}{n} \left[ x_{it} + \sum_{j=1}^{m} b_y (y_{jt} - x_{jt}) - \frac{P}{n} s^2 \right]$$

(6)

where the $b_y$’s represent the coefficients for firm $i$ attached to the disclosures by each firm, $y_{jt}$, in determining $E_t(x_{jt+1})$, and

$$s^2 = \text{Cov}_t(P_{it+1} + x_{it+1}, P_{jt+1} + x_{jt+1}) = \frac{1}{r^2} \left[ (1 + r)^2 - (1 + r)^2 - 1 \right] \sum_{j=1}^{m} b_y \sigma^2 (1 + (m - 1)r).$$

As in the previous sections, the effect of disclosure quality on the discount in price is captured via the $\sum_{j=1}^{m} b_y$ term in the expression for $s^2$, which is generally a quite complicated expression of the model parameters. For this reason determining the effect of an increase in disclosure quality is impractical except under very strict restrictions on the parameters in the model.

---

8 It is possible, at some cost in terms of algebraic complexity, to allow for correlation across firms in the disclosure noise terms, $\epsilon_t$, without changing the results discussed in this subsection.

9 As shown in the appendix, if $B$ denotes the $m \times m$ matrix of $b_y$ coefficients, then $B = \Sigma^e (\Sigma^e + \Sigma^e)^{-1}$, where $\Sigma^e$ and $\Sigma^e$ are the variance-covariance matrices of $\epsilon_t$ and $\epsilon_e$, respectively. The quality of disclosures is captured by $\Sigma^e$. The $\sum_{j=1}^{m} b_y$ term is then obtained by summing across the $i$’th row in $B$. 

10
example, it is possible for \( \sum_{j=1}^{m} b_{ij} \) (and thus cost of capital) to be *increasing* in the quality of disclosure by firm \( i \) for specific parameter values.

Of particular interest here is the behaviour of \( \sum_{j=1}^{m} b_{ij} \) as \( m \) becomes large. Again, to simplify matters I focus on two highly stylized situations: (1) where one firm (firm 1) has a disclosure quality different to that of all other companies who have equal disclosure quality; and (2) where all firms have the same disclosure quality. The first situation allows varying disclosure quality for a single firm while holding the disclosure quality of other firms constant, as would be the case for example where a specific firm changed its disclosure quality. The second situation allows varying all firms’ disclosure qualities simultaneously, for example by a change in accounting standards applied to all firms (e.g., the adoption of IFRS).

In the appendix I show that in both of these settings \( \sum_{j=1}^{m} b_{ij} \) approaches 1 as \( m \) becomes large.

An immediate implication of this is that for large \( m \) disclosure quality (for either a single firm, or for all firms) becomes irrelevant to risk assessment, and thus cost of capital. That is, for large \( m \) the collection of disclosures for all firms provides maximal information \( (\sum_{j=1}^{m} b_{ij} = 1) \) regarding investors’ end of period payoffs irrespective of the quality of the individual disclosures involved. The intuition is straightforward, in a large economy each individual signal provides little incremental information regarding relevant systematic risk beyond what is available from the total collection of disclosures across all firms, and so changing the quality of each individual disclosure matters little for risk assessment.\(^{10}\)

Finally, it is interesting to note that this result is not due to diversification of portfolios by investors across many firms, but rather by ‘diversification’ of information sources for investors in a large economy. The same effect would occur if there were only a single firm but many disclosures.

\(^{10}\) Note that this does not mean that disclosure quality does not affect each individual disclosure response coefficient, \( b_{ij} \). In fact, it is straightforward to show that these vary with disclosure quality even for large \( m \). However, for large \( m \) these effects offset for the purposes of risk assessment by investors that is reflected in the discount in price. Thus changing disclosure quality affects how each firms’ disclosure moves price in equation (6), but has an immaterial effect on investors’ assessment of risk.
To summarize the results from this section, in a large economy three things occur: (1) there are more investors among whom to share the economy’s risk; (2) there are more firms generating economy-wide risk and across which the economy’s risk is spread (and can be diversified); and (3) there are more disclosures/signals from which investors can obtain information to help assess that risk. Each of these affects the level and assessment of risk that is reflected in prices and thus cost of capital. In an economy where investors’ payoffs are correlated across firms, the first two factors result in cost of capital reflecting systematic risk consistent with the CAPM. If only a limited number of disclosures are available, then the quality of those disclosures will be reflected in investors’ risk assessments (and thus in cost of capital) even though quality relates to firm-specific idiosyncratic noise in the disclosures. But if many disclosures are available, disclosure quality has a minimal impact on risk assessments and cost of capital.

4. The impact of heuristic traders

The analysis above is based on a setting where all investors are assumed to be fully rational, that is they correctly process all information available in making their demand decisions to maximize expected utility. In this section, I modify the model slightly to also include investors who are not fully rational, but instead follow a simple heuristic trading rule linked to firm disclosure. My objective is to develop some initial thoughts on how the link between disclosure, diversification and the cost of capital might be affected by the presence of less than fully rational investors, albeit in a very simple and stylized setting.

The model is the same as in the previous section except that in addition to $n$ fully rational traders each period, there are also $\lambda n$ heuristic traders, where $\lambda \geq 0$. Thus when $\lambda = 0$ (i.e., there are no heuristic traders) the model reverts to that used in the previous section. Like the rational investors, heuristic traders exist for a single period. However, heuristic traders set their demand for each firm’s shares by following a simple trading heuristic, or rule of thumb, and set demand equal to $h\Delta y_{it}$, where $h$ is a positive multiple and $\Delta y_{it} = y_{it} - y_{it-1}$, the change in firm $i$’s disclosure from the previous period. This simple

---

11 The analysis in this section draws upon ongoing research with Matt Pinnuck.
12 A large and growing body of research similarly considers the impact of less than fully rational trading in a variety of capital market settings. See, for example, Kothari (2001), Lee (2001), Richardson, Tuna and Wysocki (2010), Shleifer (2000), and Thaler (2005) for some recent surveys that discuss relevant research.
trading rule represents a setting where heuristic traders tie their trading decisions to whether a firm’s disclosed $y_{it}$ has increased or decreased relative to the prior period (i.e., represents either good or bad news.) The parameter $h$ captures how aggressive heuristic traders are in following this trading rule.

Finally, I only investigate the simplest setting where all firms’ cash payments and disclosures are independent, i.e., $e_{it}$ and $e_{ie}$ are independent across firms (as well as over time). With this assumption the multi-firm setting simplifies considerably to one where each firm’s price is independent of all other firms’ prices and disclosures. Although this is a considerable simplification it turns out that the resulting analysis still yields several potentially interesting insights.

In this setting it is possible to show (see the appendix) that equilibrium price for firm $i$ is:

$$P_{it} = \frac{1}{r} \left[ x_{it} + (b_i - \gamma_i (1-b_i))(y_{it} - x_{it}) + (1+r)\gamma_i \Delta y_{it} - \frac{P}{n}s^2_{it} \right]$$

(7)

where $b_i = \frac{\sigma^2_{ei}}{\sigma^2_{ei} + \sigma^2_{e}}, \gamma_i = \frac{r}{(1+r)^2} \lambda \rho \sigma^2_{ei},$ and

$$s^2_{t} = \text{Var}(P_{it+1} + x_{it+1}) = \frac{1}{r^2} \left[ (1+\gamma_i)^2 \left( (1+r)^2 - (1+r)^2 - 1 \right) b_i \right] + ry_i \left( \frac{r\gamma_i}{b_i} + 2(1+\gamma_i) \right) \sigma^2_{ei}.$$

There are several aspects of equation (7) that are notable. First, as would be expected, if there are no heuristic traders, i.e., $\lambda = 0$, the equation collapses to the corresponding pricing equation from the previous section (equation (6)) in the uncorrelated firm case (i.e., when $\tau = 0$). Second, as is the case in the previous sections, the effect of disclosure quality is captured via $b_i$, the coefficient that rational investors place on the firm’s disclosure when setting their expectations of the end of period firm payment; a higher $b_i$ represents higher disclosure quality. Third, since the definition of $s^2_{t}$ includes $\gamma_i$ which itself depends on $s^2_{t}$, it is determined implicitly by this expression. Because the expression is quadratic in $s^2_{t}$ it is straightforward to solve but yields an ‘algebraically messy’ solution.\(^{13}\) Nevertheless it is

\(^{13}\) Because the expression is quadratic it is possible that no, one or two solutions exist. It is possible to show that no solution exists (and thus no equilibrium occurs in the market) if there are ‘too many’ heuristic traders, or there is ‘too much’ risk. Also, when two solutions exist only one is ‘economically plausible’. That is, only one
possible to show that the resulting value for $s_i^2$ is increasing in the fraction of heuristic traders ($\lambda$), and the underlying risk associated with the firm’s cash payments ($\sigma_i^2$), as would be expected. It is also decreasing in disclosure quality (i.e., decreasing in $\sigma_i^2$), indicating that, as in previous sections, for a given ‘size’ of the economy (i.e., $n$ and $m$) greater disclosure quality results in a lower assessed risk by rational investors.

It is straightforward to rearrange the price expression in (7) to the following:

$$P_{it} = \frac{\mathbb{E}_t(P_{t+1} + x_{t+1})}{1+r} + \frac{1+r}{r} \gamma_i \Delta y_{it} - \frac{1}{1+r} \frac{\mathbb{P}}{n} s_i^2.$$  \hspace{1cm} (8)

This indicates that price is equal to the present value of the rational investors’ expected value of their end of period payoff from holding firm $i$ plus a factor relating to the effect of heuristic traders on price less a second factor relating to rational investors’ risk assessment associated with their end of period payoff. These two factors combined represent the divergence of price from the present value of investors’ payoffs, and so also represent the firm’s cost of capital. The second factor mirrors the risk-related discount in the previous sections, although as indicated above, the assessed risk will be greater in the presence of heuristic traders. The first factor indicates that heuristic traders generate additional price pressure which causes prices to move away from the discounted expected value, and that this price pressure is contingent on the disclosure realisation each period, reflecting the heuristic traders’ rule of thumb trading rule. That is, the discount, or ‘cost of capital’ has a period and disclosure specific component which is not due to risk but rather to the price pressure brought about by the heuristic traders.

Importantly, the two components of cost of capital are affected differently in a ‘large economy’, i.e., as $m$ and $n$ become large. The risk-related component, $\frac{1}{1+r} \frac{\mathbb{P}}{n} s_i^2$, behaves in the same manner as in the previous sections. Specifically, because firms are assumed independent here, assessed risk ($s_i^2$) does not change with either $m$ or $n$, but because $n$ is large it gets shared among more investors and so has less effect on price (or is ‘diversified away’). In contrast, the disclosure-contingent component of the discount, $\frac{1+r}{r} \gamma_i \Delta y_{it}$, is not
‘diversified away’ in a large economy because it is unaffected by either \( m \) or \( n \). Despite increased firms across which to diversify and increased numbers of rational investors among which to share risk, heuristic traders generate price pressure which is reflected in equilibrium prices. This occurs due to the assumption that when the number of rational investors (\( n \)) is large, so too is the number of heuristic traders (\( \lambda n \)). Only if heuristic traders are presumed to become less prevalent in a large economy would their influence on price be ‘diversifiable’. This is despite their actions being ‘idiosyncratic’, i.e., uncorrelated across firms.

5. Conclusion

Based on a simplified and modified version of models typically employed in related research I highlight several observations concerning disclosure quality, diversification, and the cost of capital. First, in a setting where price risk cannot be avoided by investors, (ex ante) cost of capital is decreasing in disclosure quality. This contrasts with Christensen, de la Rosa and Feltham (2010). Second, as in Lambert, Leuz and Verrecchia (2007), disclosure quality continues to play a role in a ‘large economy’ (i.e., does not diversify away) if the number of available disclosures is not also ‘large’. However, if the number of disclosures is also large, then disclosure quality ceases to be important. Finally, when the model includes investors who follow a heuristic trading rule linked to firms’ disclosures without reference to price, cost of capital includes an additional disclosure-contingent factor beyond the effect of risk. So long as the relative numbers of rational and heuristic traders remain unchanged in a large economy this factor does not diversify away. This is despite the heuristic trader factor being tied to firm-specific idiosyncratic disclosures.
References


Shevlin, T. 2013, ‘Some personal observations on the debate on the link between financial reporting quality and the cost of equity capital’, unpublished paper.


Appendix – Some Derivations

In this appendix I outline a quite general version of the model employed in the various sections of the paper and then employ simplified versions of the model to provide derivations of results discussed in the paper.

Let \( t \mathbf{x} \) denote the \( m \times 1 \) vector of firms’ payments which follow the process
\[
\mathbf{x}_t = \mathbf{x}_{t-1} + \mathbf{\varepsilon}_t,
\]
where \( \mathbf{\varepsilon}_t \) is distributed multivariate normal with mean zero and variance-covariance matrix \( \mathbf{\Sigma}_\varepsilon \). Also let \( \mathbf{y}_t = \mathbf{C}\mathbf{x}_{t+1} + \mathbf{e}_t \) be the \( k \times 1 \) vector of \( k \) disclosures or signals available to investors (in addition to \( \mathbf{x}_t \)) at time \( t \), where \( \mathbf{e}_t \) is distributed multivariate normal with mean zero and variance-covariance matrix \( \mathbf{\Sigma}_e \). \( \mathbf{\varepsilon}_t \) and \( \mathbf{\varepsilon}_t \) are assumed to be independent of each other and over time. The matrix \( \mathbf{C} \) allows for quite general situations. For example, if there is a disclosure available only about firm 1 (as in section 3.1 of the paper) \( \mathbf{C} \) will be a \( 1 \times m \) vector with a 1 as the first element and zeroes elsewhere. Alternatively, if all firms provide a signal equal to \( \mathbf{y}_t = \mathbf{x}_{t+1} + \mathbf{e}_t \) (as in section 3.2 of the paper) then \( \mathbf{C} \) is an \( m \times m \) identity matrix. There is also a risk free security which pays \((1+r)\) at the end of each period.

There are \( n \) identical rational investors each with negative exponential utility with risk aversion coefficient \( \rho \). Each investor lives for one period only, and chooses their demand for securities to maximize expected utility over end of period payoffs: \( \mathbf{P}_{t+1} + \mathbf{x}_{t+1} \), where \( \mathbf{P}_t \) is the \( n \times 1 \) vector of prices. Given these assumptions it is well known that the demands rational traders will choose are given by
\[
\mathbf{D}_{\text{Rational}} = \frac{1}{\rho} \mathbf{S}^{-1} \left( \mathbb{E}_t (\mathbf{P}_{t+1} + \mathbf{x}_{t+1}) - (1+r)\mathbf{P}_t \right),
\]
where \( \mathbb{E}_t (\mathbf{P}_{t+1} + \mathbf{x}_{t+1}) \) denotes expected value of the end of period payoff given all information available to investors at time \( t \), and \( \mathbf{S} = \text{Var}_t (\mathbf{P}_{t+1} + \mathbf{x}_{t+1}) \) is the variance-covariance matrix of end of period payments assessed by investors given information available at time \( t \). There are also \( \lambda n \) heuristic traders whose demands are set according to
\[
\mathbf{D}_{\text{Heuristic}} = \mathbf{H}(\mathbf{y}_t - \mathbf{y}_{t-1}) = \mathbf{H} \Delta \mathbf{y}_t,
\]
where \( \mathbf{H} \) is an \( m \times k \) diagonal matrix of multiples that

\[14\] Because of the normality and linearity assumptions \( \mathbf{S} \) does not vary across different periods as confirmed below.
heuristic traders apply to the components in $\Delta y$, when setting their demands. Finally, I assume supply is equal to $1$, the unit vector, each period.

Given these assumptions the market clearing requirement for equilibrium each period is: $nD_{\text{Rational}} + \lambda nD_{\text{Heuristic}} = 1$. Substituting in for rational and heuristic investors’ demands and rearranging yields the following expression that must be satisfied by equilibrium price:

$$\left(1 + r\right)P_t = E_t\left(P_{t+1} + x_{r+1}\right) + \lambda \rho SH \Delta y_i - \frac{\rho}{n} S1. \quad (9)$$

Equation (9) collapses to equation (1) in section 2.1 of the paper when there is only one firm ($m = 1$) and there are no heuristic traders ($\lambda = 0$). It becomes equation (3) in section 3 when there are $m$ firms and no heuristic traders, and equation (8) in section 4 when there are heuristic traders but when firms and signals are uncorrelated (i.e., $\Sigma_e$ and $\Sigma_e$ are both diagonal).

Equation (9) defines equilibrium price recursively. To provide a closed-form solution I assume that price is linear in the information available to investors at time $t$. In particular, I assume price can be expressed as follows:

$$P_t = a_0 + A^e_t\left(x_{r+1}\right) + A^y_t + A^y_{t-1}. \quad (10)$$

Given this assumption I substitute this into equation (9) and then equate coefficients across equations (9) and (10) to determine a ‘steady state’ equilibrium price expression. In other words, when the coefficients are equated, if investors anticipate that end of period (time $t + 1$) prices are as in equation (10), their resulting demands and market clearing at time $t$ will result in period $t$ prices that also accord with equation (10). The resulting expression for equilibrium price is given by substituting the following coefficients into equation (10):$^{16}$

$^{15}$ The model can be made slightly more general by, for example, allowing the $\lambda$ ‘s to be different for each firm, and/or allowing $H$ to have non-zero off diagonal elements.

$^{16}$ Details of the derivation are available from the author.
\[ a_0 = -\frac{1}{n} \frac{\rho}{S} \]

\[ A_1 = \frac{1}{r} \left( I + \frac{1}{(1+r)^2} \lambda \rho S C \right) \]

\[ A_2 = \frac{r}{(1+r)^2} \lambda \rho S H \]

\[ A_3 = \frac{1}{1+r} \lambda \rho S H \]

where

\[ S = \text{Var}(P_{t+1} + x_{t+1}) = (1+r)^2 A_1 \text{Var}(x_{t+1})A_1' + (A_1 B + A_2)(C \Sigma_x C' + \Sigma_x)(A_1 B + A_2)' \]

\[ B = \Sigma_x C' (C \Sigma_x C' + \Sigma_x)^{-1}. \]

When there are heuristic traders, as noted in section 4 of the paper, \( S \) appears in the right hand side of \( S = \text{Var}(P_{t+1} + x_{t+1}) \) and so is defined implicitly.\(^{17}\) However, when there are no heuristic traders \( A_1 = \frac{1}{r}, \ A_2 = A_3 = 0 \), and \( S \) does not appear in the right hand side of \( S = \text{Var}(P_{t+1} + x_{t+1}) \), and so is more easily determined.

In each case in the body of the paper (in sections 2, 3 and 4) the expression for equilibrium price is simply a special case of this general solution but also using the following well known expressions based on the multivariate normality assumption:

\[ E_t(x_{t+1}) = E_t(x_{t+1} | y_t, x_t) = x_t + B(y_t - Cx_t) \]

\[ \text{Var}_t(x_{t+1}) = \text{Var}_t(x_{t+1} | y_t, x_t) = (I - BC) \Sigma_x \]

\[ B = \Sigma_x C' (C \Sigma_x C' + \Sigma_x)^{-1}. \]

\(^{17}\) The right hand side of \( S = \text{Var}(P_{t+1} + x_{t+1}) \) is quadratic in the \( m \times m \) \( S \) matrix when there are heuristic traders (i.e., when \( \lambda \) is not zero), which makes it difficult to solve except in simplified settings such as under the assumption in section 4 that each firm is independent.